Model Identification and Data Analysis (Academic year 2014-2015)

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Laboratory lecture

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Outline

- Time series and systems
  - Definition and generation of time series and dynamic systems
  - Analysis of systems
  - Analysis of time series (sampled covariance and spectral analysis)
- Optimal predictors
- Identification
  - Parameter identification
  - Structure selection
  - Model validation
- Exercizes

Time series and systems

– White noise

```
>> N=100;     % number of samples
>> lambda=1;  % standard deviation
>> mu=1;      % expected value
```

- I method

```
>> u=mu+lambda*randn(N,1);
```

- II method

```
>> u=wgn(M,N,P); % zero-mean signal
M: number of signals
P: signal power (dBW) (10log10(lambda^2))
```

- Sum of a WGN and a generic signal

```
>> u=awgn(X,SNR,SIGPOWER);
X:        original signal
SRN:      signal-to-noise ratio (dB)
SIGPOWER: power of signal X (dBW) (10log10(γ(0)))
          - if SIGPOWER='measured', AWGN estimates the power
               of signal X.
```

Time series and systems

- Definition of dynamic linear discrete-time systems: external representation (transfer function – *LTI format*)

```
>> system = tf(num,den,Ts)
```

  - `num`: numerator
  - `den`: denominator
  - `Ts`: sampling time (-1 default)

```
>> system=tf([1 1],[1 -0.8],1)
Transfer function:
                     z + 1
                    -------
                    z - 0.8
```

Time series and systems

- Step response of a LTI system

```
>> T=[0:Ts:Tfinal];
>> [y,T]=step(system,Tfinal); % step response
>> step(system,Tfinal)        % plot
```

- Response of a LTI system to a general input signal

```
>> T=[0:Ts:Tfinal];
>> u1=sin(0.1*T);              % e.g., sinusoidal input
>> u2=1+randn(T,1);            % e.g., white gaussian noise
>> [y,T]=lsim(system,u1+u2,T);
```

Time series and systems

The SYSTEM IDENTIFICATION TOOLBOX uses different formats for models (detto IDPOLY *format*) and signals (detto IDDATA *format*)

Definition of IDDATA *objects*:

```
>> yu_id=iddata(y,u,Ts);
u: input data
y: output data
Ts: sampling time
```

Time series and systems

– Definition of *IDPOLY objects*:

```
>> model_idpoly = idpoly(A,B,C,D,F,NoiseVariance,Ts)
A,B,C,D,F are vectors (i.e., containing coefficients of the
  polynomials in incresing order of the exponent of z^-1)
Model_idpoly:
A(q) y(t) = [B(q)/F(q)] u(t-nk) + [C(q)/D(q)] e(t)
(If ARMAX: F=[], D=[])
The delay nk is defined in polynomial nk.

>> conversion from LTI format:
Model_idpoly = idpoly(model_LTI)
model_LTI is a LTI model (defined, e.g., using tf, zpk, ss)
```

– Response of an IDPOLY *system* to a generic input signal (vector u)

```
>> y=sim(model_idpoly,u);
```

Time series and systems

- Analysis of systems (both for LTI and IDPOLY *formats*):
  - Zeri e poli:

```
>> [Z,P,K]=zpkdata(model)
Z =   [-1] % positions of the zeros
P = [0.8000] % positions of the poles
K = 1 transfer constant
```

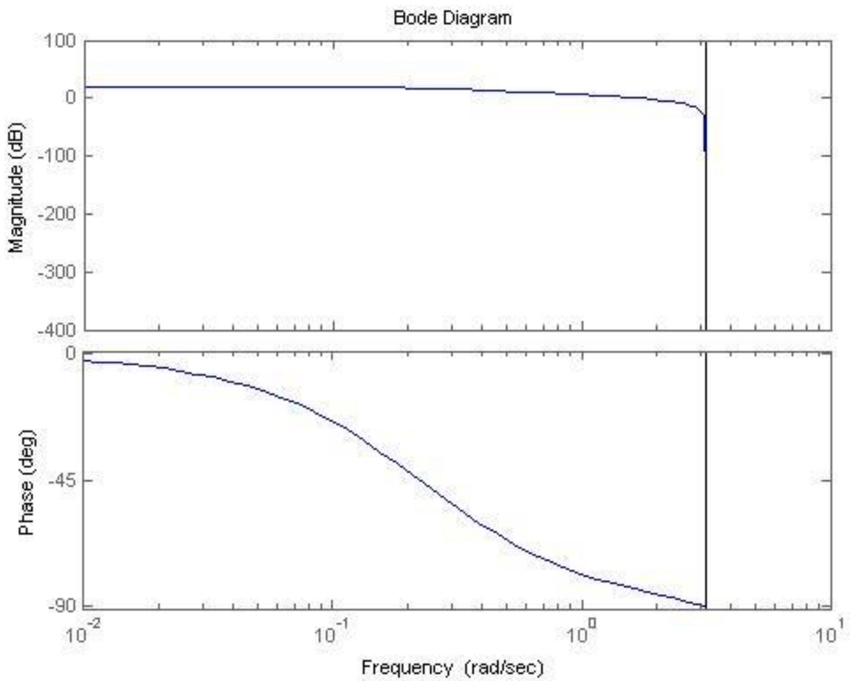
  - Bode diagrams:

```
>> [A,phi] = bode(model,W)
W: values of the angular frequency 'w' where to compute 'A(w)'
   and 'phi(w)'
A:   amplitude (A(w))
Phi: phase (phi(w))
```

Time series and systems

– Bode diagrams:

```
>> [A,phi] = bode(model,[0.1 1])

A(:,:,1) =9.1179
A(:,:,2) =1.9931

phi(:,:,1) =-24.2456
phi(:,:,2) =-78.5036

>> bode(model)
>> % draws the Bode plots
      (amplitude -dB- and phase)
```



Time series and systems

- Analysis of time series
  - `m=mean(X)                  % Average or mean value.`
  - `Y=detrend(X,'constant') % Remove constant trend from data sets.`
  - `Y=detrend(X,'linear')    % Remove linear trend from data sets.`
  - Covariance function $\Upsilon(\tau)$ ˘

```
>> gamma=covf(y,10) % tau=0,1,…,10-1
>> plot([0:9],gamma)
```

Warning: `covf` computed the correlation function!
TO OBTAIN THE COVARIANCE FUNCTION THE SIGNAL MUST HAVE ZERO MEAN

  - Sectrum $\Gamma(\omega)$ –periodogram method

```
>> periodogram(y);          % plots the periodogram
>> [Pxx,w]=periodogram(y);
>> plot(w/pi,10*log10(Pxx))
```

Warning: `no regularization technique is applied: the estimator is not consistent!`

Predictors

- Given the system

$$y(t) = \frac{B(z)}{A(z)}u(t-k) + \frac{C(z)}{A(z)}e(t), e \sim WN(0, \sigma^2)$$

The optimal k-steps predictor is given by

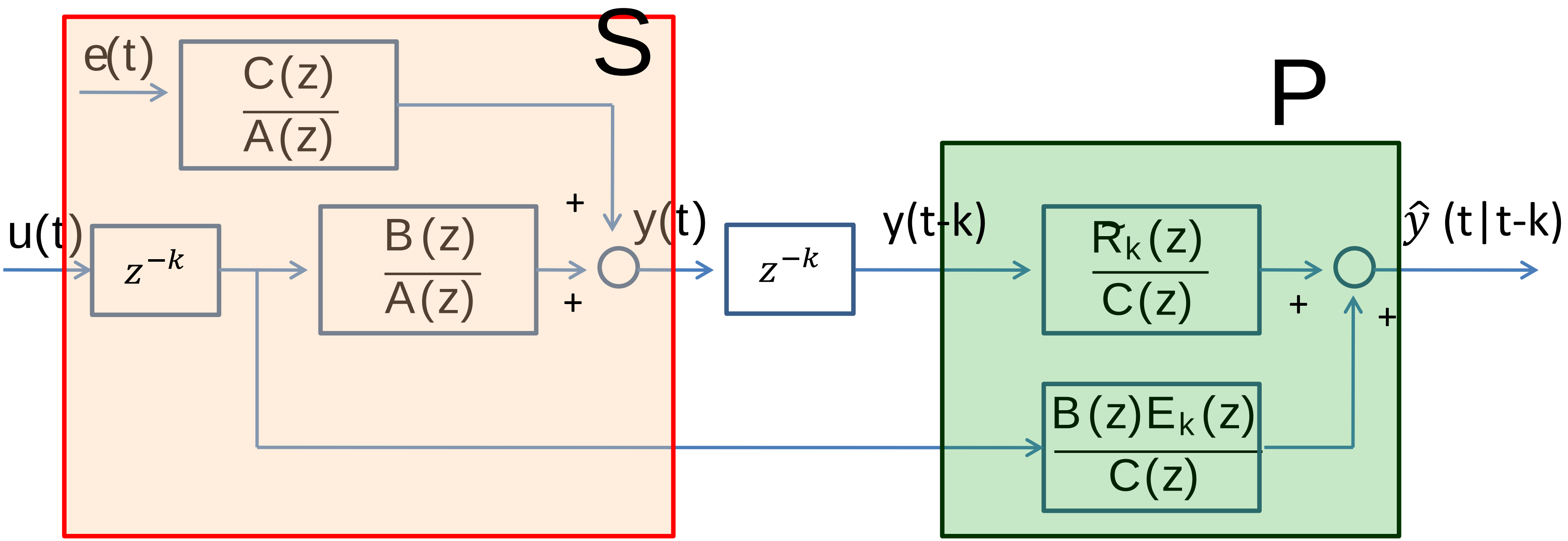
$$\hat{y}(t|t-k) = \frac{B(z)\boxed{E_k(z)}}{C(z)}u(t-k) + \frac{\boxed{R_k(z)}}{C(z)}y(t)$$

$$= \frac{B(z)\boxed{E_k(z)}}{C(z)}u(t-k) + \frac{\boxed{\tilde{R}_k(z)}}{C(z)}y(t-k)$$

where $E_k(z)$ e $R_k(z)$ are the result and the remainder, respectively, of the long division, and

$$R_k(z) = \tilde{R}_k(z)z^{-k}$$

Predictors



S
P
e(t)
$\frac{C(z)}{A(z)}$
u(t)
z^{-k}
$\frac{B(z)}{A(z)}$
+
+
y(t)
z^{-k}
y(t-k)
$\frac{\tilde{R}_k(z)}{C(z)}$
$\frac{B(z)E_k(z)}{C(z)}$
+
+
$\hat{y}$ (t|t-k)

Predictors

```
- MATLAB function
>> [Yp,X0e,PREDICTOR] = predict(model,yu_id,k)    % optimal k-steps
                                                      predictor.
   inputs     model:              IDPOLY format
              yu_id:   IDDATA object
              k:                  # of prediction steps
   outputs    Yp:                 IDDATA format
              XOe:                Initial states
              PREDICTOR:          IDPOLY format

PREDICTOR{1}: Discrete-time IDPOLY model with two inputs:
         - u(t)
         - y(t)

Outputs:

A(q)  = …      -> C(q)

B1(q)  = …     -> R(q)

B2(q)  = …     -> E(q)B(q)q^-k
```

$$\hat{y}(t|t-k) = \frac{B(z)E_k(z)}{C(z)}u(t-k) + \frac{R_k(z)}{C(z)}y(t)$$

Predictors

- ESEMPIO:

```
>> modello=idpoly([1 0.3],[0 0 1 0.2],[1 0.5],[],[],1,1)
Discrete-time IDPOLY model: A(q)y(t) = B(q)u(t) + C(q)e(t)

A(q) = 1 + 0.3 q^-1
B(q) = 1 + 0.2 q^-1
C(q) = 1 + 0.5 q^-1
k     = 2



>> [yd,xoe,modpred]=predict(modello,dati,2);
>> modpred{1}
Discrete-time IDPOLY model: A(q)y(t) = B(q)u(t) + e(t)
A(q) = 1 + 0.5 q^-1

B1(q) = -0.06 q^-2

B2(q) = q^-2 ( 1 + 0.4 q^-1 + 0.1 q^-2 )
```

Predictors

– Prediction error

```
>> Epsilon = pe(model_idpoly,yu_iddata) % k-steps prediction error
```

– ARDERSON TEST FOR THE PREDICTION ERROR (99% confidence reguion)

```
>> e=resid(model,data)
```

<u>inputs</u> model: IDPOLY *format*
data: IDDATA *format* (input-output data)
<u>outputs</u> e: IDDATA ([errore di predizione,input])

If the outputs are not specified, a plot is shown:

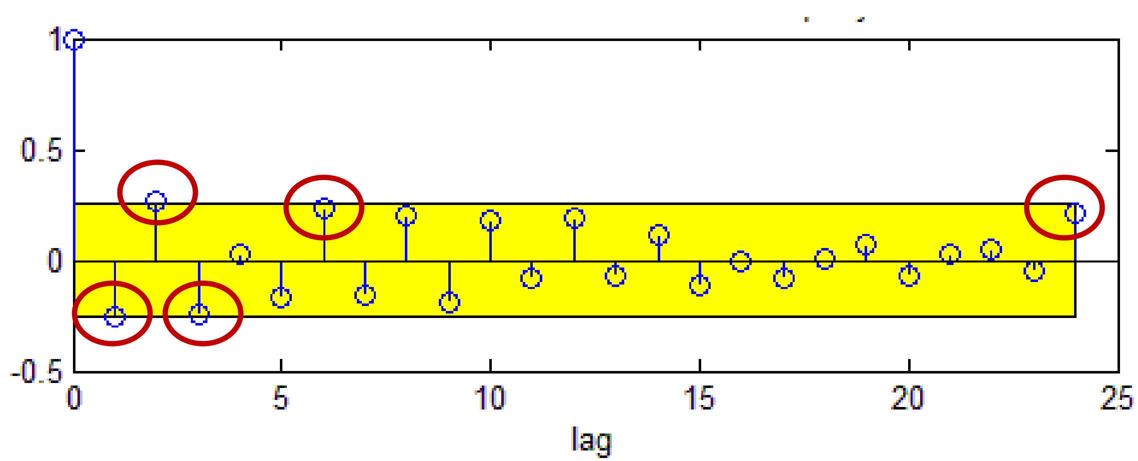
- Empirical correlation function (empirical - sampled) of the prediction error
- 99% confidence region

  -> if more than 99% of the samples of the normalized correlation function lie in the confidence region: positive results of the whiteness Test!

Predictors

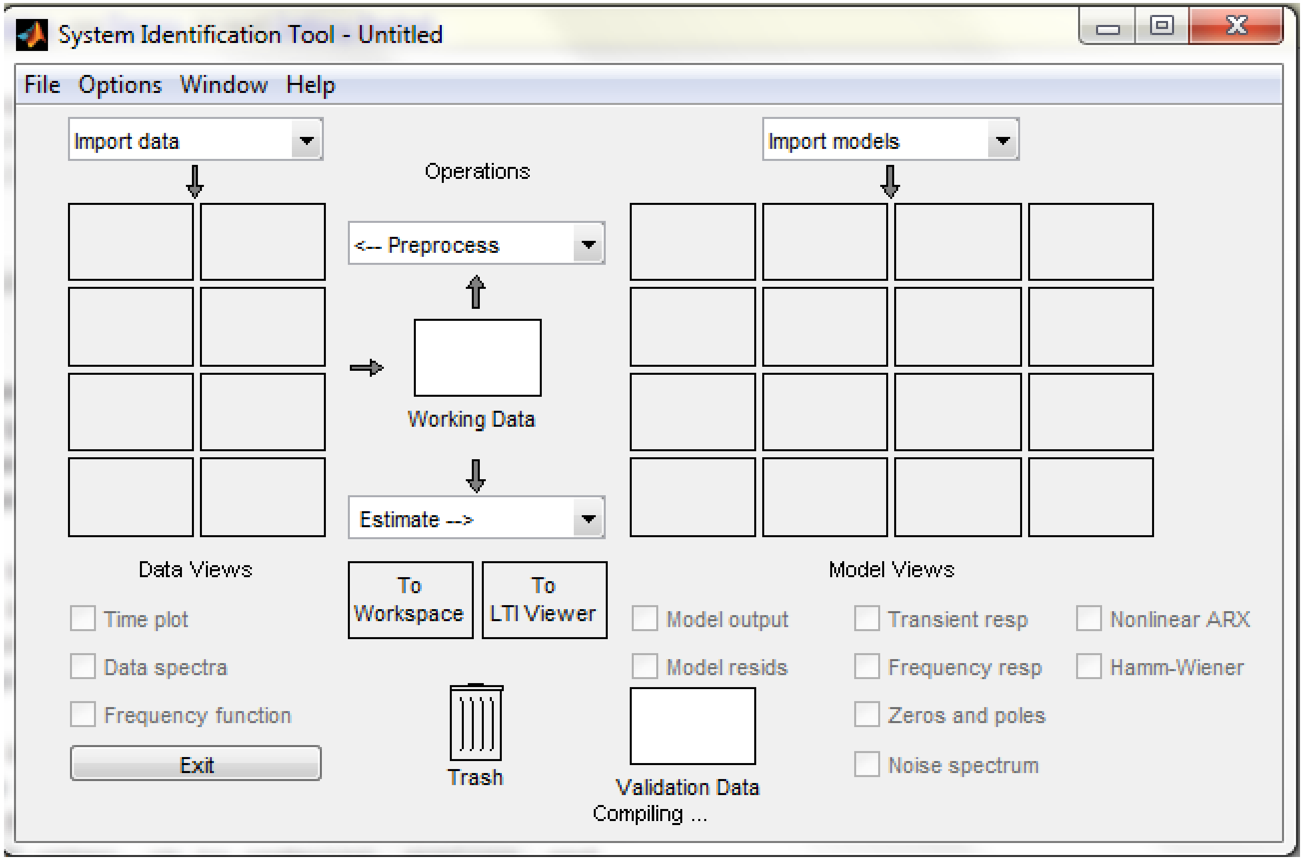
- Example:

```
>> model_idpoly=idpoly([1 0.5],[0 1 0.3],[1 0.6],[],[],1,1)

Discrete-time IDPOLY model: A(q)y(t) = B(q)u(t) + C(q)e(t)
A(q) = 1 + 0.5 q^-1
B(q) = q^-1 + 0.3 q^-2
C(q) = 1 + 0.6 q^-1
This model was not estimated from data.
Sampling interval: 1

>> data=randn(100,2);
>> resid(model_idpoly,
```



Identification

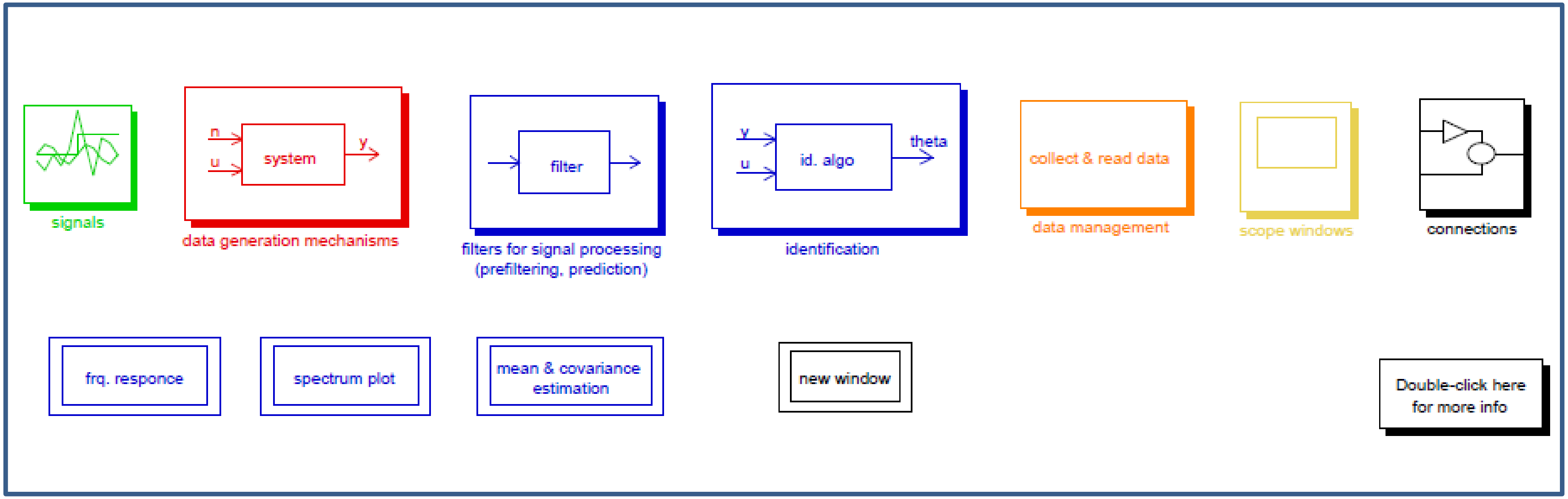
```
>> ident % start the MATLAB identification toolbox GUI
```



Identification

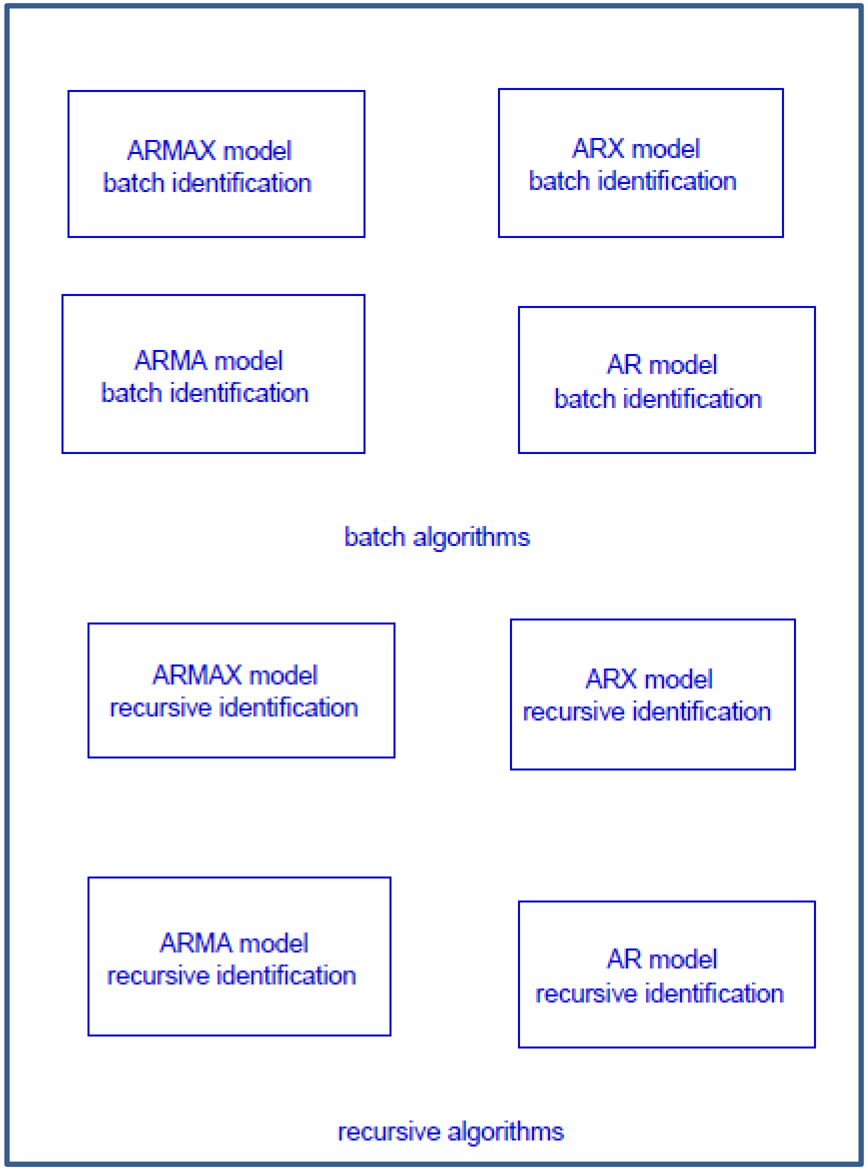
- IMAD_LAB simulink toolbox. To open the GUI the toolbox type:

```
>> imad_lab
```



Identification

IMAD_LAB simulink toolbox (*Identification* subblock)



Identification

Other related Matlab functions ($\in$ Identification toolbox)

- Parametric model estimation

```
– ar        - AR-models of signals using various approaches.
– armax     - Prediction error estimate of an ARMAX model.
– arx       - LS-estimate of ARX-models.
– bj        - Prediction error estimate of a Box-Jenkins model.
– ivar      - IV-estimates for the AR-part of a scalar time
                    series (with instrumental variable method).
– iv4       - Approximately optimal IV-estimates for ARX-models.
– n4sid     - State-space model estimation using a sub-space
                    method.
– nlarx     - Prediction error estimate of a nonlinear ARX model.
– nlhw      - Prediction error estimate of a Hammerstein-Wiener
                    model.
– oe        - Prediction error estimate of an output-error model.
– pem       - Prediction error estimate of a general model.
```

Identification

- Model structure selection

  - `aic            - Compute Akaike's information criterion.`
  - `fpe            - Compute final prediction error criterion.`
  - `arxstruc       - Loss functions for families of ARX-models.`
  - `selstruc       - Select model structures according to`
    `                         various criteria.`
  - `idss/setstruc  - Set the structure matrices for idss`
    `                         objects.`
  - `struc          - Generate typical structure matrices for`
    `                         ARXSTRUC.`

- For more details please see:

```
>> help ident
```

Exercizes

Linear systems

1. Consider the discrete-time linear system, characterized by the following transfer function

$$W(z) = \frac{1+z^{-1}}{1-0.8z^{-1}}$$

1.1 Plot the step response of the system. What are its features, in the light of the properties (stability, gain, etc.) of the systems?

1.2 Plot the response of the system to two different sinusoidal inputs, i.e., $u(t)$=sin(0.1t) and $u(t)$=sin(t). Analyze the properties of the two output signals in the light of the frequency response theorem.

Exercizes

White noise

1. Define a 1000-samples realization of a Gaussian white noise with zero mean and unity variance $\lambda^2 = 1$. Verify that 68% of samples has absolute value smaller than λ and that 95% has absolute value smaller than 2λ.

2. Repeat the previous exercize in case $\lambda^2 = 4$.

3. Define a 1000-samples realization of a Gaussian white noise with mean $\mu = 10$ and variance $\lambda^2 = 4$. Compare the obtained realization with that obtained at Exercize 2. Estimate the covariance function from the available data and verify that it corresponds with that expected in case of white noise.

Exercizes

Stationary stochastic processes

1. Consider the stochastic process *y*(*t*), generated from the dynamic system

$$W(z) = \frac{1+z^{-1}}{1-0.8z^{-1}}$$

with a white noise *e*(*t*) as input.

1.1 Assume that *e*(*t*) is a white Gaussian noise with mean μ and variance λ^2. Setting $\mu = 0$ and $\lambda^2 = 1$, generate different realizations of both *e*(*t*) and *y*(*t*). Estimate, from the available data, mean and variance of the processes *e*(*t*) and *y*(*t*).

1.2 Show the results to the questions below in case $\mu = 10$ and $\lambda^2 = 1$. What is the relationship between the mean of *y*(*t*) and the gain of the dynamic system *W*(*z*)?

Exercizes

2. Consider the stochastic process $\bar{y}(t)$, generated from the dynamic system

$$\bar{W}(z) = \frac{1-z^{-1}}{1+0.8z^{-1}}$$

with a white noise $e(t)$ as input.

2.1 Assume that $e(t)$ is a white Gaussian noise with mean μ and variance λ^2. Setting $\mu = 0$ and $\lambda^2 = 1$, generate different realizations of both $e(t)$ and $\bar{y}(t)$. Estimate, from the available data, mean and variance of the processes $e(t)$ and $\bar{y}(t)$.

2.2 Show the results to the questions below in case $\mu = 10$ and $\lambda^2 = 1$.

2.3 Plot the spectra of the processes $y(t)$ and $\bar{y}(t)$, i.e., $\Phi_y(\omega) = |W(e^{j\omega})|^2$ and $\Phi_{\bar{y}}(\omega) = |\bar{W}(e^{j\omega})|^2$. Is it possible to establish a relationship between the realizations of $y(t)$ and $\bar{y}(t)$, are their respective spectra $\Phi_y(\omega)$ and $\Phi_{\bar{y}}(\omega)$?

Exercizes

Predictors

1. Consider the stochastic process *y*(*t*), generated from the dynamic system

$$W(z) = \frac{1+0.25z^{-1}}{1-0.9z^{-1}}$$

with input *e*(*t*), i.e., white Gaussian noise with mean $\mu = 0$ and variance $\lambda^2 = 1$.

1.1 Define the optimal 1-step predictor of *y*(*t*).

1.2 Define a realization of *y*(*t*) and the relative 1-step prediction $\hat{y}(t|t-1)$. Compare the signals *y*(*t*) and $\hat{y}(t|t-1)$, by drawing them in the same plot.

1.3 Define the signal $\varepsilon(t) = y(t) - \hat{y}(t|t-1)$ and plot it. Estimate mean and variance of $\varepsilon(t)$. Compute analytically the theoretical values of the latter quantities and compare them with those obtaned from data.

Exercizes

1.4 Define now the trivial – non optimal – predictor $\hat{y}_T(t|t-1) = E[y(t)] = 0$. Estimate mean and variance of the estimation error in this case.

1.5 Compare the performances of the predictors defined above.

1.6 Define the optimal 2-step predictor $\hat{y}(t|t-2)$ of $y(t)$.

1.7 Define a realization of $y(t)$ and the relative 1-step and 2-step predictions $\hat{y}(t|t-1)$ and $\hat{y}(t|t-2)$, respectively. Compare the signals $y(t)$ and $\hat{y}(t|t-1)$, and $\hat{y}(t|t-2)$, by drawing them in the same plot.

1.8 Define the signal $\varepsilon_2(t) = y(t) - \hat{y}(t|t-2)$ and plot it. Estimate mean and variance of $\varepsilon_2(t)$. Compute analytically the theoretical values of the latter quantities and compare them with those obtaned from data.

Exercizes

Identification of time series

Consider the MA(1) stochastic process *y*(*t*), generated from the dynamic system

$$y(t) = e(t) + 0.5e(t-1)$$

with input *e*(*t*), i.e., white Gaussian noise with mean $\mu = 0$ and variance $\lambda^2 = 1$.

Define a realization of *y*(*t*) consisting of *N*=1000 samples.

1. Assume that the data generation system is unknown and the model class, selected for identification purposes, consists of a AR(1) process, i.e.,

$$y(t) = \vartheta y(t-1) + \xi(t)$$

where $\xi(t)$ is a zero mean white noise process.

1.1 Compute, using the recursive least square method, the optimal estimate $\hat{\vartheta}_N$ of ϑ.

Exercizes

1.2 Define the evolution of the prediction error $\varepsilon(t) = y(t) - \hat{y}(t|t-1)$. Estimate its mean and variance.

1.3. Assuming that the data generation system is known, compute analytically the value of ϑ that minimizes the cost function $\bar{J}(\vartheta) = E[\varepsilon(t)^2]$, and the corresponding value of $\bar{J}(\vartheta)$.