Heuristic Optimization

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Introduction to Optimization

Outline

* What is Optimization?
* Problems and Applications
* Optimization
* Personal Examples

What is Optimization?

* Optimization is an important branch of Operations Research (OR), Artificial Intelligence (AI) and Computer Science (CS), which started just before World War II in Britain with the establishment of teams of scientists to study the strategic and tactical problems involved in military operations. The objective was to find the most effective utilization of limited military resources by the use of quantitative techniques.
* Following the war, numerous peacetime applications emerged, leading to the use of optimization and management science in many industries and occupations.

What is Optimization?

- After the WW-II, with the industrial boom, it was aware that many industrial problems are very similar to those encountered in the military applications.

- After 50s of last century, optimization has been widely applied in many areas: transportation, logistics, AI, supply chain, telecommunication, finance, etc.

What is Optimization?

* “application of mathematical techniques, models, and tools to a problem within a system to yield the optimal solution”

* Phases of an Optimization Project
  * develop math model to represent the system
  * solve and derive solution from model
  * put the solution to work

Definitions

- **Model**: A schematic description of a system, theory, or phenomenon that accounts for its known or inferred properties and may be used for further study of its characteristics.

- **Operations Research** (OR) is the study of mathematical models for complex organizational systems.

- **Optimization** is a branch of OR/AI/CS which uses mathematical techniques such as mathematical programming and intelligent heuristics to derive values for system variables that will optimize performance.

Models

* **Linear Programming**
  * Typically, a single objective function, representing either a profit to be maximized or a cost to be minimized, and a set of constraints that circumscribe the decision variables. The objective function and constraints all are linear functions of the decision variables.
  * Software has been developed that is capable of solving problems containing millions of variables and tens of thousands of constraints.
* **Integer Linear Programming**
  * The decision variables are restricted to be integers.

Models

- **Nonlinear Programming**
  - The objective and/or any constraint is nonlinear.
  - In general, much more difficult to solve than linear.
  - Most (if not all) real world applications require a nonlinear model. In order to be make the problems tractable, we often approximate using linear functions.

Models

- **Dynamic Programming**
  - A DP model describes a process in terms of states, decisions, transitions and returns. The process begins in some initial state where a decision is made.
  - The decision causes a transition to a new state. Based on the starting state, ending state and decision a return is realized.
  - The process continues through a sequence of states until finally a final state is reached. The problem is to find the sequence that maximizes the total return.
  - Objectives with very general functional forms may be handled and a global optimal solution is always obtained.
  - The number of states grows exponentially with the number of dimensions of the problem.

Mathematical Programming

- A mathematical model consists of:
  - Decision Variables, Constraints, Objective Function, Parameters and Data
- The general form of a math programming model is:

$$
\begin{aligned}
&min \text{ or } max \quad && f(x_1, \ldots, x_n) \\
&s.t. && g_i(x_1, \ldots, x_n) \left\{ \begin{array}{c} \leq \\ = \\ \geq \end{array} \right\} b_i \\
& && x \in X
\end{aligned}
$$

- Linear program (LP): all functions f and gi are linear and X is continuous.
- Integer program (IP): X is discrete.

Mathematical Programming

* A solution is an assignment of values to variables.

* A feasible solution is an assignment of values to variables such that all the constraints are satisfied.

* The objective function value of a solution is obtained by evaluating the objective function at the given solution.

* An optimal solution (assuming minimization) is one whose objective function value is less than or equal to that of all other feasible solutions.

More General Models

- Given decision variables ***s***, search space **S** (restricted by a set of constraints **C**) and objective function ***f*:*S*->*R***:

  - Given a search space *S* which represents feasible solutions
  - Given an objective function *f*: $S \rightarrow R$
  - Find $s^* \in S$ such as :

  $$f(s^*) \leq f(s) \qquad \forall s \in S$$

  s* : global optimum

- Large scale and complex optimization problems in many areas of science and industry (Military, Telecommunications, Transportation-Logistics, Finance, Design,…).

Outline

* What is Optimization?
* **Problems and Applications**
* Optimization and Algorithms

International Optimization Competition

* **学术界和工业界的融合：**
  * ROADEF/EURO每1-2年举办一次国际竞赛
  * 学术界：简单问题复杂求解；工业界：复杂问题简单求解.
  * 竞赛题目由大公司提供，获奖者将得到公司的合同
  * 几乎所有入围算法都是启发式算法
* **检验算法优劣和学者水平最客观公正的平台：**
  * 学术界往往过于强调创新，有时“为了发表论文而创新，为了创新而创新”
  * 欧美优化学界走在应用的最前沿

不同竞赛的区别

* 奥运会: Olympic Game: 挑战人类体力极限的比赛
* ACM Programming Contest: 专家考验学生的比赛
* Mathematical Modeling Contest:专家考验学生的比赛
* Timetabling Scheduling Competition: 专家考验专家的比赛
* ROADEF/EURO Challenge: 资本家考验专家的比赛

ROADEF/EURO竞赛主题

2018 Saint-Gobain: Cutting Optimization（玻璃切割优化）

2016 Air Liquid: Inventory Routing Problem （天燃气配送）

2014 SNCF: Trains Don’t Vanish（火车站综合调度）

2012 Google: Machine Reassignment（云计算负载均衡）

2010 EDF: Production Management （核电站生产调度）

2009 AMADEUS: Airline Disruption Management （航空中断管理）

2007 France Telecom: Personnel Scheduling （人员排班调度）

2005 Renault: Car Sequencing problem （车间调度）

2003 ONERA: Missions of Earth Observation Satellites（卫星拍照片）

2001 CELAR: Frequency Assignment （军事频率分配）

1999 Bouygues: Inventory Management Problem （库存管理）

Outline

* What is Optimization?
* Problems and Applications
* **Optimization and Algorithms**

Computational Complexity: Algorithm

* **Polynomial time algorithm:** shortest path, max flow, sorting.

* **Exponential time algorithm:** Linear Programming, Integer Linear Programming, Dynamic Programming Branch and Bound.

Complexity	Size=10	20	30	40	50
$O(x)$	0.00001 s	0.00002 s	0.00003 s	0.00004 s	0.00005 s
$O(x^2)$	0.0001 s	0.0004 s	0.0009 s	0.0016 s	0.0025 s
$O(x^5)$	0.1 s	0.32 s	24.3 s	1.7 mn	5.2 mn
$O(2^x)$	0.001 s	1.0 s	17.9 mn	12.7 days	35.7 years
$O(3^x)$	0.059 s	58.0 mn	6.5 years	3855 centuries	2×10^8 centuries

Computational Complexity：Problem

* **Problem complexity** = the time complexity of the best algorithm to solve the problem exactly

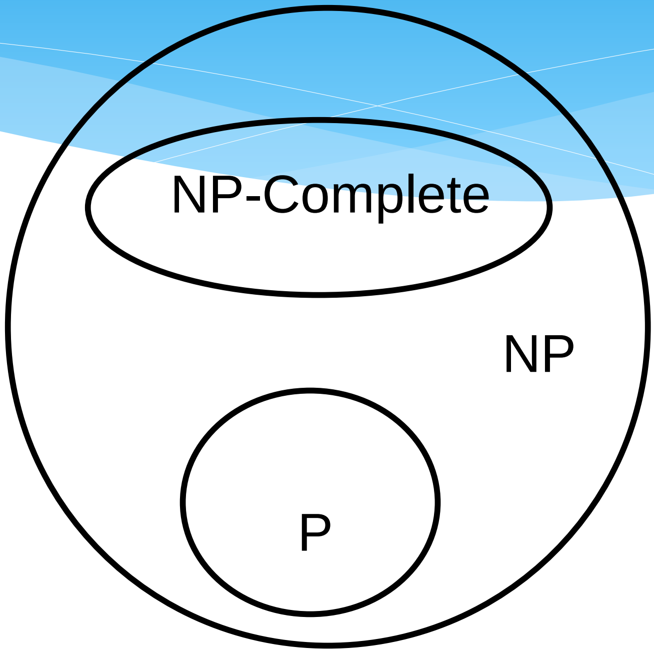
* **P（easy）** = polynomial time complexity algorithm
* **NP（difficult）** = no polynomial time complexity algorithm

* Most of the optimization problems in real applications are NP hard.

NP-Hardness

- Classification of Problems:

  P and NP



- Up to now, there does not exist polynomial exact algorithm for NP-Complete and NP-Hard problems.

Optimization Algorithms