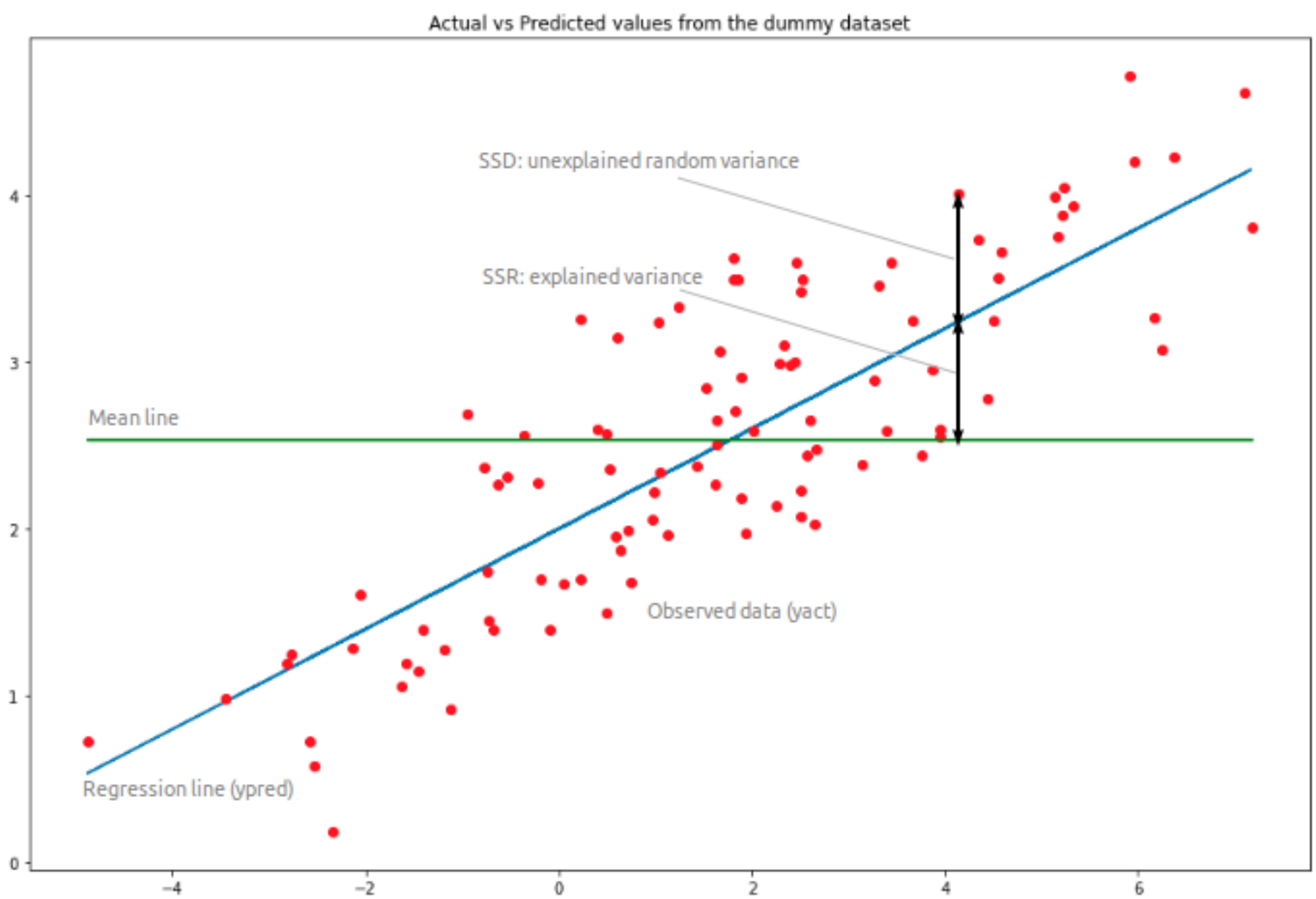
Published in Towards Data Science

Lorraine Li Follow
Oct 24, 2018 · 8 min read · Listen

Introduction to Linear Regression in Python

A quick tutorial on how to implement linear regressions with the Python `statsmodels` & `scikit-learn libraries.`



Example linear regression model using simulated data understood and can be explained using plain English.

Linear regression models have many real-world applications in an array of industries such as economics (e.g. predicting growth), business (e.g. predicting product sales, employee performance), social science (e.g. predicting political leanings from gender or race), healthcare (e.g. predicting blood pressure levels from weight, disease onset from biological factors), and more.

Understanding how to implement linear regression models can unearth stories in data to solve important problems. We'll use Python as it is a robust tool to handle, process, and model data. It has an array of packages for linear regression modelling.

The basic idea is that if we can fit a linear regression model to observed data, we can then use the model to predict any future values. For example, let's assume that we have found from historical data that the price (***P***) of a house is linearly dependent upon its size (***S***) — in fact, we found that a house's price is exactly 90 times its size. The equation will look like this:

$P = 90*S$

With this model, we can then predict the cost of any house. If we have a house that is 1,500 square feet, we can calculate its price to be:

$P = 90*1500 = \$135,000$

In this blog post, we cover:

1. The basic concepts and mathematics behind the model
2. How to implement linear regression from scratch using simulated data
3. How to implement linear regression using `statsmodels`
4. How to implement linear regression using `scikit-learn`

. . .

Basic concepts and mathematics

There are two kinds of variables in a linear regression model:

- The **input** or **predictor variable** is the variable(s) that help predict the value of the output variable. It is commonly referred to as ***X***.
- The **output variable** is the variable that we want to predict. It is commonly referred to as ***Y***.

To estimate *Y* using linear regression, we assume the equation:

$$Y_e = \alpha + \beta X$$

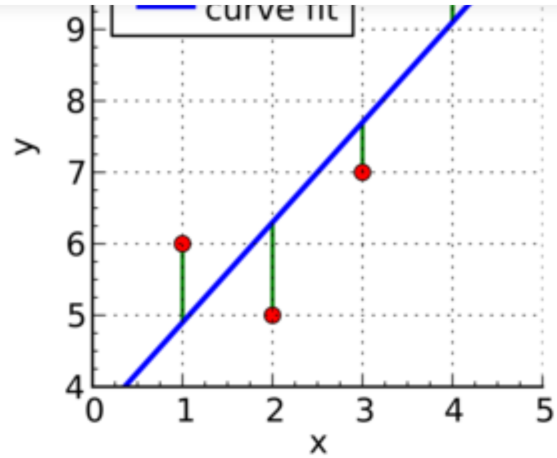
where Y_e is the estimated or predicted value of *Y* based on our linear equation.

Our goal is to find statistically significant values of the **parameters** $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ that minimise the difference between *Y* and Y_e.

If we are able to determine the optimum values of these two parameters, then we will have the **line of best fit** that we can use to predict the values of *Y*, given the value of *X*.

So, how do we estimate α and β? We can use a method called **ordinary least squares**.

Ordinary Least Squares



Green lines show the difference between actual values Y and estimate values Y_e

The objective of the least squares method is to find values of α and β that minimise the sum of the squared difference between Y and Y_e. We will not go through the derivation here, but using calculus we can show that the values of the unknown parameters are as follows:

$$\beta = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{n}(X_i - \bar{X})^2}$$

$$\alpha = \bar{Y} - \beta * \bar{X}$$

where X is the mean of X values and $\bar{Y}$ is the mean of Y values.

If you are familiar with statistics, you may recognise β as simply *Cov(X, Y) / Var(X)*.

Linear Regression From Scratch

In this post, we'll use two Python modules:

- `statsmodels` — a module that provides classes and functions for the estimation of many different statistical models, as well as for conducting statistical tests, and

Before we dive in, it is useful to understand how to implement the model from scratch. Knowing how the packages work behind the scenes is important so you are not just blindly implementing the models.

To get started, let's simulate some data and look at how the predicted values (Y_e) differ from the actual value (Y):

```
import pandas as pd
import numpy as np
from matplotlib import pyplot as plt

# Generate 'random' data
np.random.seed(0)
X = 2.5 * np.random.randn(100) + 1.5   # Array of 100 values with mean = 1.5, stddev = 2.5
res = 0.5 * np.random.randn(100)       # Generate 100 residual terms
y = 2 + 0.3 * X + res                  # Actual values of Y

# Create pandas dataframe to store our X and y values
df = pd.DataFrame(
    {'X': X,
     'y': y}
)

# Show the first five rows of our dataframe
df.head()
```

lr1.py hosted with ❤ by GitHub view raw

If the above code is run (e.g. in a Jupyter notebook), this would output something like:

	X	y
0	5.910131	4.714615
1	2.500393	2.076238
2	3.946845	2.548811

the values for `alpha` and `beta`.

```
# Calculate the mean of X and y
xmean = np.mean(X)
ymean = np.mean(y)

# Calculate the terms needed for the numator and denominator of beta
df['xycov'] = (df['X'] - xmean) * (df['y'] - ymean)
df['xvar'] = (df['X'] - xmean)**2

# Calculate beta and alpha
beta = df['xycov'].sum() / df['xvar'].sum()
alpha = ymean - (beta * xmean)
print(f'alpha = {alpha}')
print(f'beta = {beta}')
```



```
Out:
alpha = 2.0031670124623426
beta = 0.32293968670927636
```

Great, we now have an estimate for `alpha` and `beta`! Our model can be written as $Y_e = 2.003 + 0.323\ X$, and we can make predictions:

```
ypred = alpha + beta * X
```

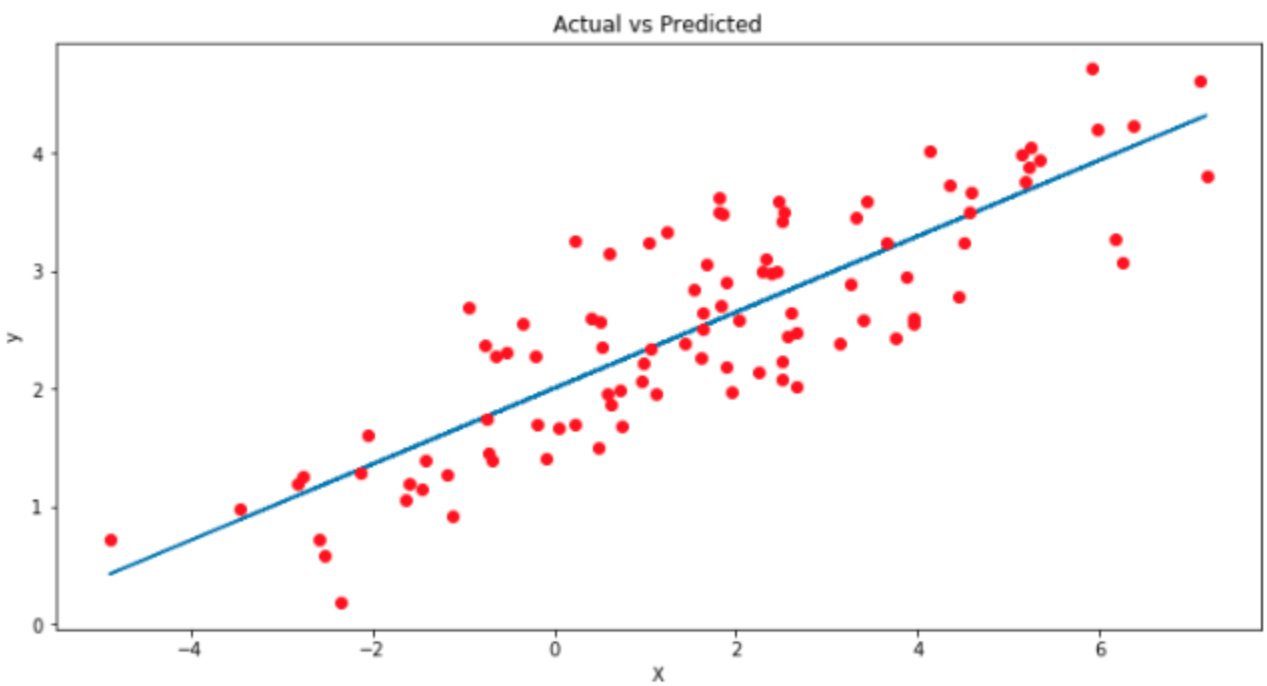


```
Out:
array([3.91178282, 2.81064315, 3.27775989, 4.29675991, 3.99534802,
       1.69857201, 3.25462968, 2.36537842, 2.40424288, 2.81907292,
       ...
       2.16207195, 3.47451661, 2.65572718, 3.2760653 , 2.77528867,
```

```python
# Plot regression against actual data
plt.figure(figsize=(12, 6))
plt.plot(X, ypred)     # regression line
plt.plot(X, y, 'ro')   # scatter plot showing actual data
plt.title('Actual vs Predicted')
plt.xlabel('X')
plt.ylabel('y')

plt.show()
```

lr4.py hosted with ❤ by GitHub view raw



The blue line is our line of best fit, $Y_e = 2.003 + 0.323\,X$. We can see from this graph that there is a positive linear relationship between X and y. Using our model, we can predict y from any values of X!

For example, if we had a value $X = 10$, we can predict that:
$Y_e = 2.003 + 0.323\,(10) = 5.233$.

To demonstrate this method, we will be using a very popular `advertising` dataset about various costs incurred on advertising by different mediums and the sales for a particular product. You can download this dataset here.

We will only be looking at the `TV` variable in this example — we will explore whether TV advertising spending can predict the number of sales for the product. Let's start by importing this csv file as a `pandas` dataframe using `read_csv()`:

```python
# Import and display first five rows of advertising dataset
advert = pd.read_csv('advertising.csv')
advert.head()
```

lr5.py hosted with ❤ by GitHub view raw

	TV	Radio	Newspaper	Sales
0	230.1	37.8	69.2	22.1
1	44.5	39.3	45.1	10.4
2	17.2	45.9	69.3	9.3
3	151.5	41.3	58.5	18.5
4	180.8	10.8	58.4	12.9

First, we use `statsmodels`' `ols` function to initialise our simple linear regression model. This takes the formula `y ~ X`, where `X` is the predictor variable (`TV` advertising costs) and `y` is the output variable (`Sales`). Then, we fit the model by calling the OLS object's `fit()` method.

```
# initialise and fit linear regression model using `statsmodels`
model = smf.ols('Sales ~ TV', data=advert)
model = model.fit()
```

lr6.py hosted with ❤ by GitHub view raw

We no longer have to calculate `alpha` and `beta` ourselves as this method does it automatically for us! Calling `model.params` will show us the model's parameters:

```
Out:
Intercept    7.032594
TV           0.047537
dtype: float64
```

In the notation that we have been using, α is the intercept and β is the slope i.e. α = 7.032 and β = 0.047.

Thus, the equation for the model will be: ***Sales = 7.032 + 0.047*TV***

In plain English, this means that, on average, if we spent $100 on TV advertising, we should expect to sell 11.73 units.

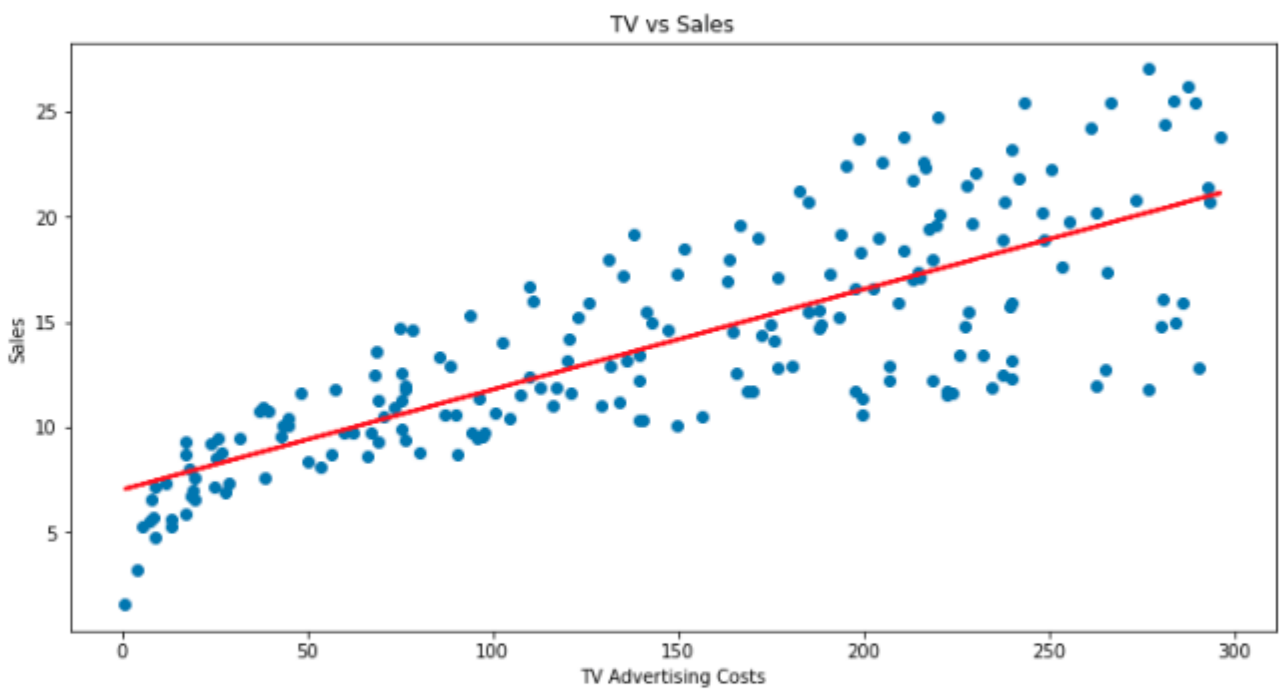
Now that we've fit a simple regression model, we can try to predict the values of sales based on the equation we just derived using the `.predict` method.

We can also visualise our regression model by plotting `sales_pred` against the TV advertising costs to find the line of best fit:

```python
# Plot regression against actual data
plt.figure(figsize=(12, 6))
plt.plot(advert['TV'], advert['Sales'], 'o')           # scatter plot showing actual data
plt.plot(advert['TV'], sales_pred, 'r', linewidth=2)   # regression line
plt.xlabel('TV Advertising Costs')
plt.ylabel('Sales')
plt.title('TV vs Sales')

plt.show()
```

lr7.py hosted with ❤ by GitHub view raw



We can see that there is a positive linear relationship between TV advertising costs and Sales — in other words, spending more on TV advertising predicts a higher number of sales!

With this model, we can predict sales from any amount spent on TV advertising. For example, if we increase TV advertising costs to $400, we can predict that sales will increase to 26 units:

lr12.py hosted with ❤ by GitHub view raw

```
Out:
0    26.04725
dtype: float64
```

Linear Regression with `scikit-learn`

We've learnt to implement linear regression models using `statsmodels` …now let's learn to do it using `scikit-learn`!

For this model, we will continue to use the `advertising` dataset but this time we will use two predictor variables to create a **multiple linear regression model**. This is simply a linear regression model with more than one predictor, and is modelled by:

$Y_e = \alpha + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p$, where p is the number of predictors.

In our example, we will be predicting `Sales` using the variables `TV` and `Radio` i.e. our model can be written as:

$Sales = \alpha + \beta_1 * TV + \beta_2 * Radio$.

First, we initialise our linear regression model, then fit the model to our predictors and output variables:

```
# Build linear regression model using TV and Radio as predictors
# Split data into predictors X and output Y
predictors = ['TV', 'Radio']
X = advert[predictors]
y = advert['Sales']

# Initialise and fit model
lm = LinearRegression()
model = lm.fit(X, y)
```

lr8.py hosted with ❤ by GitHub view raw

Again, there is no need to calculate the values for `alpha` and `betas` ourselves – we just have to call `.intercept_` for `alpha`, and `.coef_` for an array with our coefficients `beta1` and `beta2`:

```
print(f'alpha = {model.intercept_}')
print(f'betas = {model.coef_}')
```

lr9.py hosted with ❤ by GitHub view raw

```
Out:
alpha = 2.921099912405138
betas = [0.04575482 0.18799423]
```

Therefore, our model can be written as:

*Sales = 2.921 + 0.046*TV + 0.1880*Radio.*

We can predict values by simply using `.predict()`:

```
model.predict(X)
```

lr10.py hosted with ❤ by GitHub view raw

```
12.512084, 11.718212, 12.105515,  3.709379, 12.551696,
...
12.454977,  8.405926,  4.478859, 18.448760, 16.4631902,
 5.364512,  8.152375, 12.768048, 23.792922, 15.15754285])
```


Now that we've fit a multiple linea... to our data, we can predict sales from any combination of TV and Radio advertising costs! For example, if we wanted to know how many sales we would make if we invested $300 in TV advertising and $200 in Radio advertising…all we have to do is plug in the values!

```
new_X = [[300, 200]]
print(model.predict(new_X))
```

lr11.py hosted with ❤ by GitHub view raw

. . .

I hope you enjoyed this brief tutorial about the basics of linear regression!

We covered how to implement linear regression from scratch and by using `statsmodels` and `scikit-learn` in Python. In practice, you will have to know how to validate your model and measure efficacy, how to select significant variables for your model, how to handle categorical variables, and when and how to perform non-linear transformations.