Computer Science I

Recursion

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Outline

1. Introduction
2. Designing Recursive Functions
3. Avoiding Recursion

Part I: Introduction

Challenge

Challenge: write code to count down from 10 to 1 *without* using a loop.

Challenge


```
1  void countDown(int n) {
2
3    if(n < 0) {
4      printf("Error: cannot count down from negatives!");
5    } else if(n == 0) {
6      printf("Blast Off\n");
7    } else {
8      printf("%d\n", n);
9      countDown(n-1);
10   }
11   return;
12 }
```

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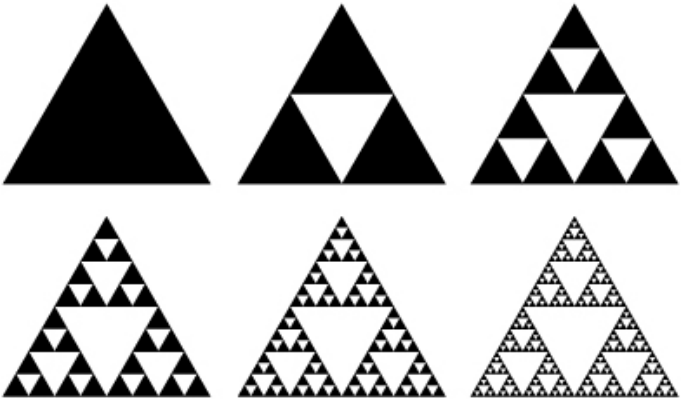
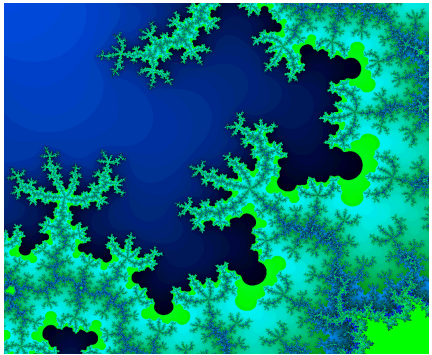
$$1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ 55 \tag{1}$$

Fractals



Serpinski Triangles

Fractals



Mandelbrot Set

Recursive Functions

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- Generally recursive calls pass in different parameter values
- More generally: divide-and-conquer style problem solving

Part II: Writing Recursive Functions

Recursion

In general, every recursive function must have:

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1. A *base case* – a condition after which the recursion stops
2. Each recursive call must make progress toward the base case
3. Corner cases may need to be handled separately

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- A recursive solution requires you to think about a general "case"
- Similar to loops: at the i-th iteration what do you do?
- Given the input, how do you *divide-and-conquer* it?

Examples

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- Write a recursive function to find the largest element in an array of integers (simulate a traditional loop)
- Write a recursive, *divide-and-conquer*-style function to sum the elements in an array of integers

Fibonacci Solution


```
1  int fibonacci(int n) {
2
3    if(n < 1) {
4      return -1;
5    } else if(n == 1 || n == 2) {
6      return 1;
7    } else {
8      return fibonacci(n-1) + fibonacci(n-2);
9    }
10
11 }
```

Largest Element Solution


```
1  int largestElement(const int *arr, int largest, int index) {
2    if(index < 0) {
3      return largest;
4    } else {
5      if(arr[index] > largest) {
6        return largestElement(arr, arr[index], index-1);
7      } else {
8        return largestElement(arr, largest, index-1);
9      }
10   }
11 }
```

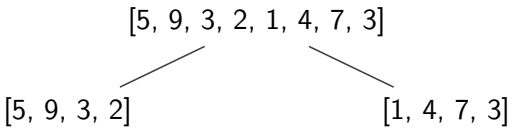
Summation Solution


```
1  int recursiveSum(const int *arr, int l, int r) {
2    if(l > r) {
3      return 0;
4    } else if(l == r) {
5      return arr[l];
6    } else {
7      int m = (l + r) / 2;
8      return recursiveSum(arr, l, m) + recursiveSum(arr, m+1, r);
9    }
10 }
```

Divide & Conquer

[5, 9, 3, 2, 1, 4, 7, 3]

Divide & Conquer



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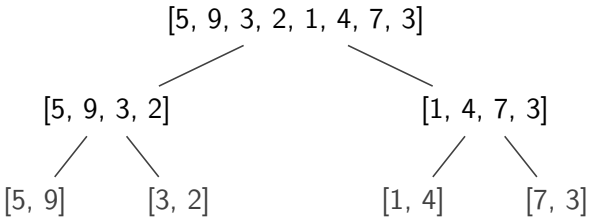

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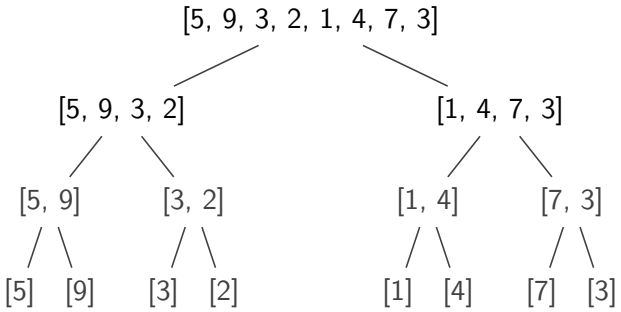

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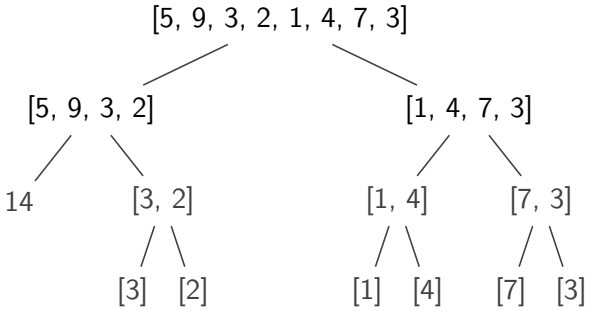

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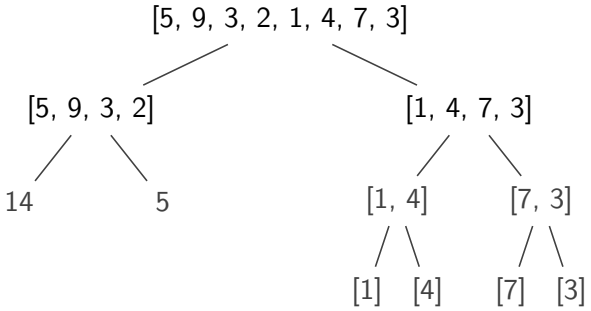

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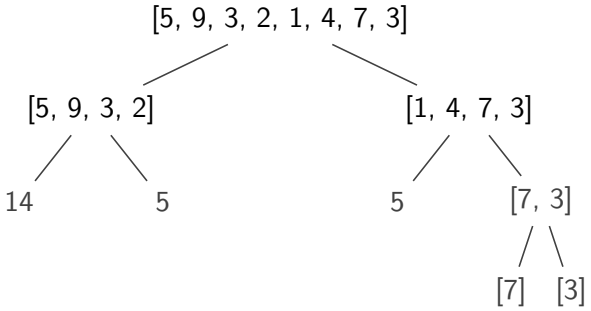

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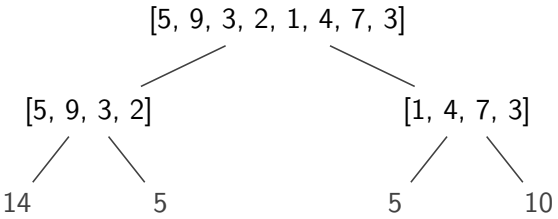

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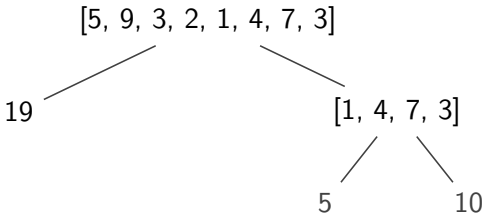

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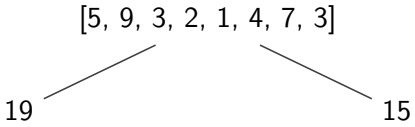

Divide & Conquer

Part III: Avoiding Recursion

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- Functional programming languages (Lisp, Haskell) use recursion as a fundamental control flow mechanism

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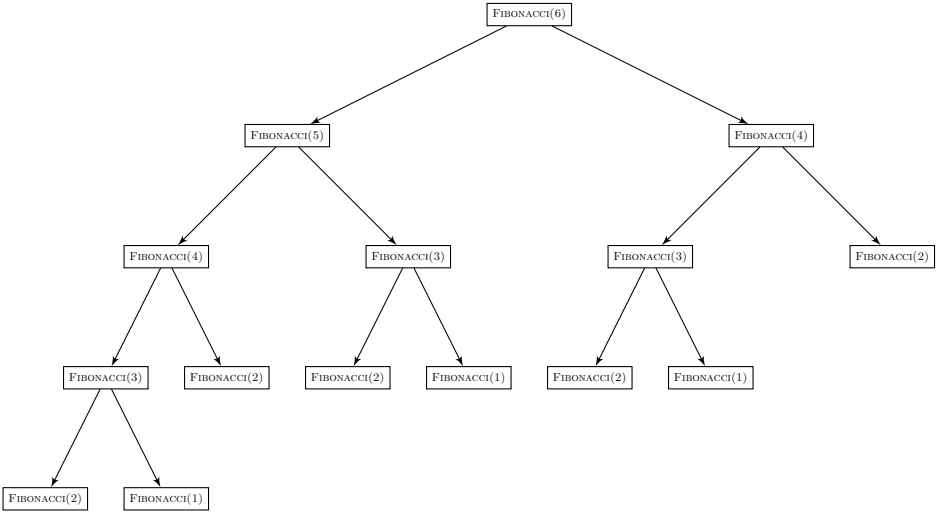

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