Computer Science I

Searching & Sorting

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Outline

Part I: Introduction & Linear Search

- Processing data is a fundamental operation in Computer Science

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- Two fundamental operations in processing data are *searching* and *sorting*

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- Two fundamental operations in processing data are *searching* and *sorting*
- Form the basis or preprocessing step of many algorithms
- Large variety of algorithms have been developed

Searching

- Given a *collection* of *elements* $A = \{a_1, a_2, \ldots, a_n\}$ and a *key* k, find an element that "matches" k

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- Collection: haystack, key: needle

Very general problems statement:

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  - Find extremal elements
- What do you do for unsuccessful searches?

Potential Solution: Linear Search

- Basic idea: iterate through each element

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- Stop at the first match
- If no such element, return a "flag" value

A potential C solution:

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Linear Search

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- Take an array of integers
- An integer key k
- Find the first element equal to k
- Return its index
- Unsuccessful search: -1 as a flag value

Linear Search


```c
/**
 * This function takes an array of integers
 * and searches it for the given key, returning
 * the index at which it finds it, or -1 if no
 * such element exists.
 */
int linearSearch(const int *arr, int n, int key) {

  for(int i=0; i<n; i++) {
    if(arr[i] == key) {
      //you found your needle...
      return i;
    }
  }
  //the needle was not found
  return -1;
}
```

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Solution works

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- Search arrays of `double` or strings or `Student` structures, etc.: copy-pasta
- Different search criteria (search `Student` by NUID or name): yet another implementation
- Ultimate goal: one single "generic" searching (and sorting) solution that will work with arrays of any type of data
- Can we do better?

Part II: Binary Search & Comparison

Binary Search: Basic Idea

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  - If $k < m$: k must lie in the left-half of the array
  - If $m < k$: k must lie in the right-half of the array

Illustration

$$\underbrace{\boxed{a_1}\boxed{\quad\cdots\quad}\boxed{a_{\frac{n}{2}-1}}}_{k<m}\underset{\substack{\uparrow\\ m}}{\boxed{a_{\frac{n}{2}}}}\underbrace{\boxed{a_{\frac{n}{2}+1}}\boxed{\quad\cdots\quad}\boxed{a_n}}_{m<k}$$

Example

Search for $k = 42$:

$$l = 0, \quad r = 10, \quad m = 5$$

index	0	1	2	3	4	5	6	7	8	9	10
contents	-3	2	4	4	9	12	34	42	102	157	180

Example

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Example

$12 < 42 = k$:

$$l = 6, \quad r = 10, \quad m = 5$$

index	0	1	2	3	4	5	6	7	8	9	10
contents	-3	2	4	4	9	12	34	42	102	157	180

Example

$$l = 6, \quad r = 10, \quad m = 8$$

index	0	1	2	3	4	5	6	7	8	9	10
contents	-3	2	4	4	9	12	34	42	102	157	180

Example

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Example

$42 < 102$:

$$l = 6, \quad r = 7, \quad m = 8$$

index	0	1	2	3	4	5	6	7	8	9	10
contents	-3	2	4	4	9	12	34	42	102	157	180

Example

$$l = 6, \quad r = 7, \quad m = 6$$

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Example

$34 < 42 = k$:

$$l = 6, \quad r = 7, \quad m = 6$$

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Example

$$l = 7, \quad r = 7, \quad m = 7$$

index	0	1	2	3	4	5	6	7	8	9	10
contents	-3	2	4	4	9	12	34	42	102	157	180

Example

$a_7 = 42 = k$

$$l = 7, \quad r = 7, \quad m = 7$$

index	0	1	2	3	4	5	6	7	8	9	10
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Recursive Code

```c
int binarySearch(const int *arr, int l, int r, int k) {
  if(l > r) {
    return -1;
  } else {
    int m = (l + r) / 2; //bad in practice

    if(arr[m] == k) {
      return m;
    } else if(k < arr[m]) {
      return binarySearch(arr, l, m-1, k);
    } else if(arr[m] < k) {
      return binarySearch(arr, m+1, r, k);
    }
  }
}
```

Iterative Code


```c
int binarySearch(const int *arr, int n, int k) {
  int l = 0;
  int r = n-1;
  while(l <= r) {
    int m = (l + r) / 2; //bad in practice

    if(arr[m] == k) {
      return m;
    } else if(k < arr[m]) {
      r = m - 1;
    } else if(arr[m] < k) {
      l = m+1;
    }
  }
  return -1;
}
```

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- Which is better? How much better?
- How much "work" does each algorithm perform?
- Suppose we search an array of n elements
- How many *comparisons* does each search perform?

Linear Search Analysis

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Linear Search Analysis

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- Worst Case: you are unlucky and make all n comparisons
- Average case scenario: $\approx \frac{n}{2}$ comparisons
- Called *linear search* because the work is *linearly* proportional to the array size

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$$\frac{n}{2}$$

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- After second:

$$\frac{n}{4}$$

- After third:

$$\frac{n}{8}$$

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Binary Search
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- After third:

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- After k iterations:

$$\frac{n}{2^k}$$

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Binary Search

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- Solve for k:
$$k = \log_2(n)$$

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$$\frac{n}{2^k} = 1$$
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$$k = \log_2(n)$$
- Roughly only $\log_2(n)$ comparisons are made.

Comparison

- Linear: $\approx n$ versus Binary Search: $\log_2(n)$

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Comparison

- Linear: $\approx n$ versus Binary Search: $\log_2{(n)}$
- Linear search is *exponentially worse*
- Binary search is *exponentially faster*

Perspective

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$$\approx 5 \times 10^{11}$$

  comparisons

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- Suppose we have a database of 1 trillion, 10^{12} elements
- Unsorted using linear search:
$$\approx 5 \times 10^{11}$$
comparisons
- Sorted ("indexed") using binary search:
$$\approx \log_2 (10^{12}) \approx 40$$
comparisons

- Suppose we *double* the input size: $n \to 2n$

Another Perspective
Growth Rate

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- Doubling the input size doubles the number of comparisons
- Binary search:
$$\log_2(n) \to \log_2(2n)$$
- $\log_2(2n) = \log_2(n) + 1$
- Doubling the input size only adds one more comparison!

Part III: Selection Sort

Introduction

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- To exploit binary search we need to be able to sort
- Many different sorting algorithms each with different properites
- Bubble Sort, Selection Sort, Insertion Sort, Quick Sort, Merge Sort, Heap Sort, Tim Sort, etc.
- Some efficient, some inefficient
- Start with a simple implementation: Selection Sort

Basic Idea

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- Demonstration

Selection Sort Example

Iteration 1

index	0	1	2	3	4	5	6	7
contents	42	2	62	7	20	102	34	47

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Selection Sort Example

Iteration 1

swap

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Selection Sort Example

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Selection Sort Example

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index	0	1	2	3	4	5	6	7
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Selection Sort Example

Iteration 2

index	0	1	2	3	4	5	6	7
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```c
void selectionSort(int *arr, int n) {

  for(int i=0; i<n-1; i++) {
    int minIndex = i;
    for(int j=i+1; j<n; j++) {
      if(arr[j] < arr[minIndex]) {
        minIndex = j;
      }
    }
    //swap
    int temp = arr[i];
    arr[i] = arr[minIndex];
    arr[minIndex] = temp;
  }
}
```

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Analysis

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- How bad is it?
- How many comparisons does selection sort make on an array of size n?
  - First iteration: $n - 1$ comparisons
  - Second iteration: $n - 2$ comparisons

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  - First iteration: $n-1$ comparisons
  - Second iteration: $n-2$ comparisons
  - i-th iteration: $n-i$ comparisons
  - Last iteration: 1 comparison
  - In total:

$$1+2+3+\cdots+(n-2)+(n-1)=\frac{n(n-1)}{2}=\frac{1}{2}n^2+\frac{1}{2}n$$

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- Not feasible for even "moderately large" inputs

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- Doubling the input *quadruples* the number of operations

Another Perspective

- Double the size of the array: $n \to 2n$
- Number of comparisons grows:

$$n^2 \to (2n)^2 = 4n^2$$

- Doubling the input *quadruples* the number of operations
- Four times slower!

Part IV: Quick Sort

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- Basic Divide & Conquer strategy

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```
quickSort(arr, 0, 3);
```

```
quickSort(arr, 5, 7);
```

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- Often referred to as *quasilinear*

Part V: Sorting in Practice

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- One sorting function to sort them all

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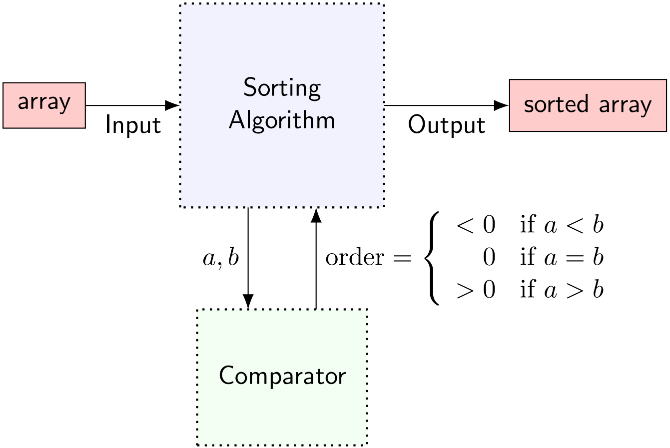
Comparator Illustration



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- Recall: `malloc()`

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- Cast the `void *` to a particular data type
- Use the data's *state* to determine the proper order
- Return an integer value that expresses the proper order

Best Practice

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- Use descriptive function names
- Be explicit in your comparisons
- Avoid "tricks"
- Reuse comparator functionality when possible

Examples

- Write a comparator to order integers in non-decreasing order
- Write a comparator to order integers in non-increasing order
- Write a comparator to order `Student` structures by NUID
- Write a comparator to order `Student` structures by GPA
- Write a comparator to order `Student` structures by last name/first name

Part VI: Function Pointers

Function Pointers

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- Easy to pass variables by value or by reference
- How do we pass a function?
- We need *function pointers*

- Recall: a *pointer* refers to a memory location

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- Demonstration

Function Pointers: Demo


```c
//create a pointer called ptrToFunc that can point to a
//function that returns an integer and takes three arguments:
//(int, double, char)

int (*ptrToFunc)(int, double, char) = NULL;

//declare a pointer that can point to math's sqrt function
double (*ptrToSqrt)(double) = NULL;

//let's make ptrToSqrt point to the sqrt function
ptrToSqrt = sqrt;

//you can call a function via its pointer:
double x = ptrToSqrt(2.0);

//careful: you can reassign standard library functions:
sqrt = sin;
double y = sqrt(3.14159); //0
//don't do this

//a function that takes another function:
void runAFunction(double x, double (*func)(double)) {
  //run func on x:
  double y = func(x);
}
```

Part VII: Searching & Sorting in C

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- Function pointers allow us to do this
- The array to be searched/sorted is also generic, `void *`
- Demonstration: generic linear search

Generic Linear Search


```c
/**
 * This function takes an array of integers
 * and searches it for the given key, returning
 * the index at which it finds it, or -1 if no
 * such element exists.
 */
int linearSearch(const void *key, const void *arr, int n, int size, int (*compar) (const

  for(int i=0; i<n; i++) {
    if(compar(key, (arr + i * size) ) == 0) {
      return i;
    }
  }
  return -1;
}
```

Sorting in C

- The standard C library provides a generic sorting function

```c
void qsort(void *base,
           size_t nel,
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