UM-SJTU Joint Institute
PHYSICS LABORATORY
(VP241)

LABORATORY REPORT

EXERCISE 2

The Hall Probe: Characteristics and Applications

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1 Introduction

This lab is designed to study and verify the principle and effects of Hall effect. Also we will learn the applications of Hall effect using a Hall probe. In detail, it is verifying that the Hall voltage is proportional to the magnetic field. What's more, we will calculate the magnetic field at the center of a solenoid, to find the sensitivity of an integrated Hall probe. The magnetic field distribution along the axis of the solenoid will be measured and compared with the corresponding theoretical curve.

2 Theoretical Background

2.1 Hall Effect

When we put a conducting sheet in a magnetic field, that the direction of the magnetic field **B** is perpendicular to the sheet, and let a electric current **I** passes through the sheet, we can observe a electric potential difference between the sides a and b of the sheet. The directon of electric field is both perpendicular to the current and the directio of the magnetic field. This is called the Hall effect, and the electric potential difference is Hall voltage U_H.

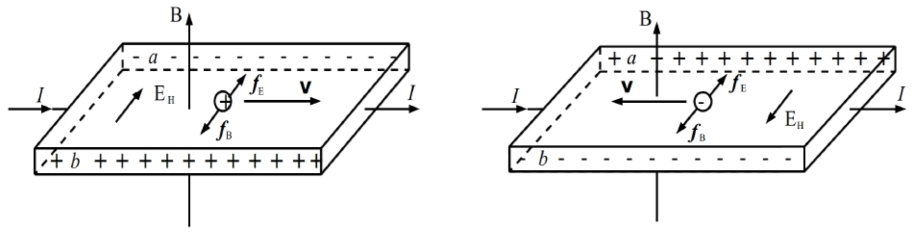

Figure 1: The principle of Hall effect

The Hall effect is caused by Lorenz force. When the charges move in the magnetic field, the Lorenz force F_B leads to the deflection of the moving charges, so that they accumulate on one side of the sheet. The accumulated elctron results in a transverse electric field E_H (The Hall field). Both electric force F_E and the Lorenz force F_B acts on the charges, and finally the two opposite forces will come to a balance when U_H is stable. When the external magnetic field is not too strong, the Hall voltage is proportional to both the current and the magnitude of the magnetic field, and inversely proportional to the thickness of the sheet d:

$$U_H = R_H \frac{IB}{d} = KIB \tag{1}$$

Where R_H is Hall coefficient and $K = R_H/d = K_H/I$, where K_H is the sensitivity of the Hall element.

2.2 Integrated Hall Probe

When the sensitivity K_H and current I of a Hall probe are fixed, we can find the magnitude of the magnetic field by measuring the Hall voltage. We need to amplify the Hall voltage before measuring since it is usually very small.

The following picture is a SS495A integrated Hall probe, which consists of a Hall sensor, an amplifier and a voltage compensator. The output voltage U can be read ignoring the residual voltage. The working voltage US = 5 V, and the output voltage U0 is approximately 2.5 V when the magnetic field is zero. The relation between the output voltage U and the magnitude of the magnetic field is:

$$B = \frac{U - U_0}{K_H} \tag{2}$$

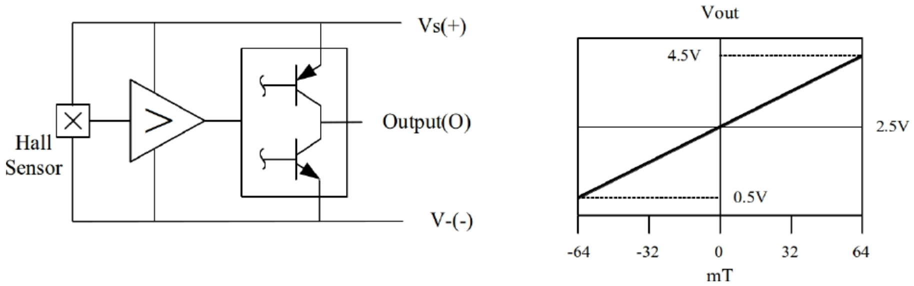

Figure 2: The integrated Hall probe SS495A (left). The relation between the output voltage U and the magnitude of the magnetic field B (right).

2.3 Magnetic Field Distribution Inside a Solenoid

The magnetic field distribution on the axis of a single layer solenoid can be calculated from the following formula:

$$B(x) = \mu_0 \frac{N}{L} I_M \left\{ \frac{L + 2x}{2[D^2 + (L + 2x)^2]^{\frac{1}{2}}} + \frac{L - 2x}{2[D^2 + (L - 2x)^2]^{\frac{1}{2}}} \right\} = C(x) I_M \tag{3}$$

where N is the number of turns of the solenoid, L is its length, I_M is the current through the solenoid wire, and D is the solenoid's diameter. The magnetic permeability of vacuum is $\mu_0 = 4\pi \times 10^{-7} H/m$.

3 Experimental setup and Measurement procedure

3.1 Apparatus

The experimental setup shown below consists of an integrated Hall probe SS495A with $K_H = 31.25 \pm 1.25 V/T$ or $K_H = 3.125 \pm 0.125 mV/G$, a solenoid, a power supply, a voltmeter, a DC voltage divider, and a set of connecting wires.

Figure 3: Measurement setup.

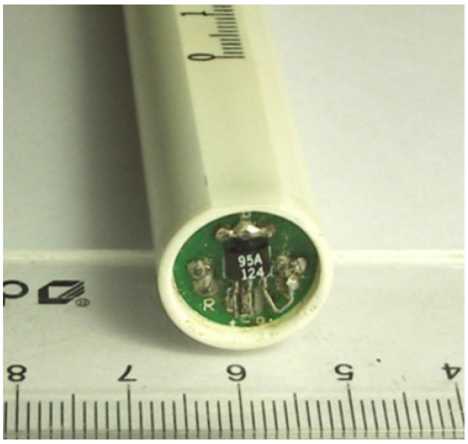

Figure 4: Integrated Hall probe SS495A.

3.2 Measurement Procedure

3.2.1 Relation Between Sensitivity K_H and Working Voltage U_S

First place the integrated Hall probe at the center of the solenoid(15 cm). Set the working voltage at 5V and measure the output voltage U_0(when $I_M = 0$) and U (when $I_M = 250$ mA). Calculate the sensitivity of the probe K_H

Then Measure K_H for different U_S from 2.8V to 10V. Calculate K_H/U_S and plot the curve K_H/U_S vs U_S.

3.2.2 Relation Between Output Voltage U and Magnetic Field B

Set $B = 0$ and $U_S = 5$V, connect the 2.4 2.6 V output terminal of the DC voltage divider and the negative port of the voltmeter, and adjust it until $U_0 = 0$. Then place the integrated Hall probe at the center of the solenoid(15 cm). Measure the output voltage U for I_M ranging from 0 to 500mA with intervals of 50 mA. Plot the curve U vs. B and find the sensitivity K_H.

3.2.3 Magnetic Field Distribution Inside the Solenoid

Set $I_M = 250$ mA, record the output voltage U with the position of the solenod x varying.

4 Results

The data sheet is attached to the end of this report.
The uncertainty calculation is included in the next part.

4.1 Relation between Sensitivity K_H and Working Voltage U_S

The measurement data for U is in the following table:

U_S[V]	$U_0(I_M = 0)$[V]	$U(I_M = 250\text{mA})$[V]
5.00	2.517	2.636

Table 1: Data for U_0 and U with $U_S = 5$V

The uncertainties of these three data are:

u_{U_S}[V]	u_{U_0}[V]	u_U[V]
0.03	0.007	0.007

Table 2: Uncertainties for U_0 and U with $U_S = 5$V

K_H can be calculated by the following formula:

$$K_H = \frac{U - U_0}{B} = \frac{2.636 - 2.517}{2.5 \times 1.4366 \times 10^{-3}} = 33.13 \pm 2.76V/T, \quad u_{K_H,r} = 8.31\%$$

We measured K_H for different U_S and calculated K_H/U_S based on the following formula:

$$\frac{K_H}{U_S} = \frac{U - U_0}{B \times U_S}$$

The calculation results and scatter plot are listed below:

	U_S	u_{U_s}	K_H/U_S	u_{K_H/U_S}
1	2.79	0.01	7.83	0.99
2	3.21	0.02	6.65	0.86
3	3.59	0.02	6.67	0.77
4	4.00	0.02	6.67	0.69
5	4.41	0.02	6.69	0.63
6	4.80	0.02	6.50	0.58
7	5.22	0.03	6.56	0.53
8	5.58	0.03	6.39	0.49
9	6.07	0.03	6.24	0.45
10	6.36	0.03	6.13	0.43
11	6.79	0.03	5.99	0.41
12	7.17	0.04	5.90	0.38
13	7.64	0.04	5.76	0.36
14	7.99	0.04	5.65	0.35
15	8.41	0.04	5.36	0.33
16	8.81	0.04	5.34	0.31
17	9.21	0.05	5.11	0.30
18	10.05	0.05	4.88	0.27

Table 3: Results for K_H/U_S

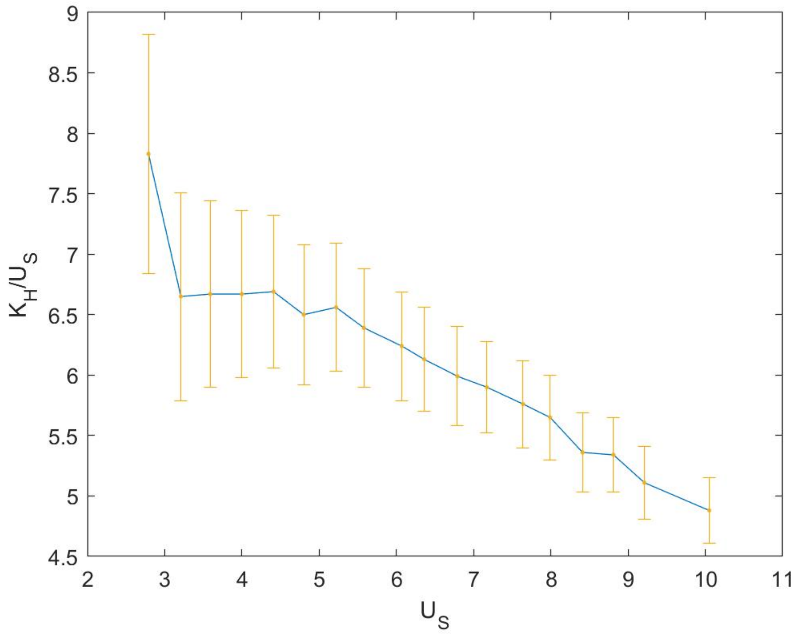

Figure 5: Scatter Plot for K_H/U_S vs U_S

4.2 Relation Between Output Voltage U and Magnetic Field B

The original data and its uncertainties are listed below:

	I_M[mA]	U_A[A]	U[mV]	u_U[V]
1	0	0.000	0.00	0.000600
2	50	0.001	24.40	0.000612
3	100	0.002	49.22	0.000625
4	150	0.003	71.08	0.000636
5	200	0.004	94.42	0.000647
6	250	0.005	117.00	0.000659
7	300	0.006	140.86	0.000670
8	350	0.007	162.36	0.000681
9	400	0.008	184.90	0.000692
10	450	0.009	208.32	0.000704
11	500	0.010	232.4	0.000716

Table 4: Measurement data for U vs I_M

Then we can calculate the magnitude of magnetic field for each electric current:

$$B = \frac{U - U_0}{K_H} = \frac{U}{K_H}$$

The calculation results and liner fit plot are listed below:

	B[T]	$u_B[T]$	U[mV]	u_U[V]
1	0	0.000000	0.00	0.000600
2	0.0007183	0.000015	24.40	0.000612
3	0.0014366	0.000030	49.22	0.000625
4	0.0021549	0.000045	71.08	0.000636
5	0.0028732	0.000059	94.42	0.000647
6	0.0035915	0.000074	117.00	0.000659
7	0.0043098	0.000089	140.86	0.000670
8	0.0050281	0.000104	162.36	0.000681
9	0.0057464	0.000119	184.90	0.000692
10	0.0064647	0.000134	208.32	0.000704
11	0.007183	0.000144	232.4	0.000716

Table 5: Results for U vs. B

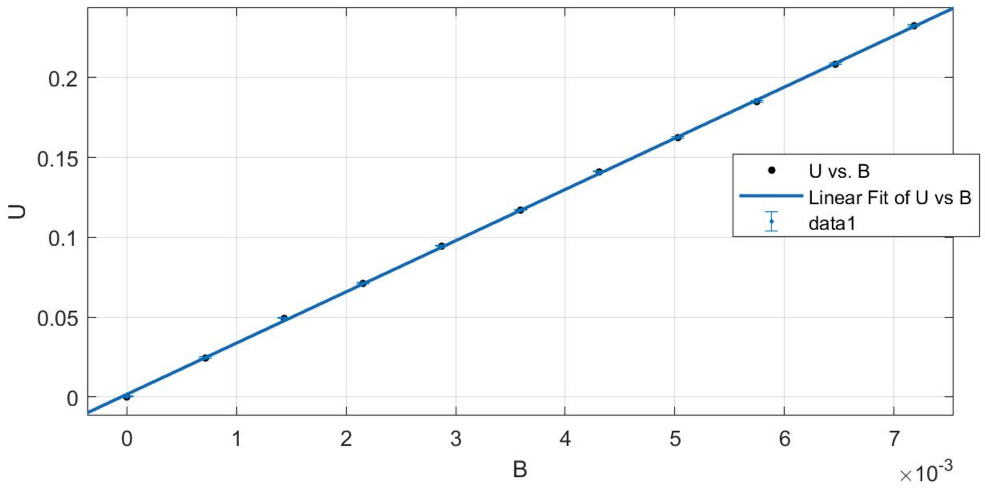

Figure 6: Linear Fit of U-B

Since $K_H = U/B$. We can read the sensitivity from the slope of the linear fit:

$$K_H = 32.07 \pm 0.29V/T$$

The deviation of experimental sensitivity form the theoretical value is:

$$\begin{aligned}\Delta K &= \frac{K_{H,experimental} - K_{H_theory}}{K_{H_theory}} \times 100\% \\ &= \frac{32.07 - 33.13}{33.13} \times 100\% \\ &= -3.20\%\end{aligned}$$

So we can see the experimental data is very close to the theoretical value, and the error is relativly small.

4.3 Magnetic Field Distribution Inside the Solenoid

In this section, we studied the magnetic field distribution inside the solenoid by measuring the corresponding magnetic field. We can calculate the magnitude of magnetic field in relation to the distance. The results are listed in the tables below:

	x[cm]±0.05[cm]	U[mV]	u_U[V]		x[cm]±0.05[cm]	U[mV]	u_U[V]
1	0.00	13.20	0.000607	27	16.00	118.21	0.000659
2	0.50	19.15	0.000610	28	17.00	118.07	0.000659
3	1.00	27.79	0.000614	29	18.00	117.85	0.000659
4	1.50	39.30	0.000620	30	19.00	117.38	0.000659
5	2.00	55.13	0.000628	31	19.50	117.19	0.000659
6	2.50	73.88	0.000637	32	20.00	117.08	0.000659
7	3.00	86.60	0.000643	33	20.50	116.98	0.000658
8	3.50	96.53	0.000648	34	21.00	116.78	0.000658
9	4.00	102.50	0.000651	35	21.50	116.50	0.000658
10	4.50	107.11	0.000654	36	22.00	116.27	0.000658
11	5.00	110.03	0.000655	37	22.50	115.90	0.000658
12	5.50	112.20	0.000656	38	23.00	115.36	0.000658
13	6.00	113.71	0.000657	39	23.50	114.51	0.000657
14	6.50	114.68	0.000657	40	24.00	113.29	0.000657
15	7.00	115.42	0.000658	41	24.50	112.15	0.000656
16	7.50	116.06	0.000658	42	25.00	110.20	0.000655
17	8.00	116.53	0.000658	43	25.50	107.61	0.000654
18	8.50	117.09	0.000659	44	26.00	103.71	0.000652
19	9.00	117.31	0.000659	45	26.50	97.55	0.000649
20	9.50	117.44	0.000659	46	27.00	89.20	0.000645
21	10.00	117.54	0.000659	47	27.50	75.34	0.000638
22	11.00	117.61	0.000659	48	28.00	62.15	0.000631
23	12.00	117.70	0.000659	49	28.50	43.60	0.000622
24	13.00	117.90	0.000659	50	29.00	30.38	0.000615
25	14.00	118.11	0.000659	51	29.50	20.55	0.000610
26	15.00	118.20	0.000659	52	30.00	14.94	0.000607

Table 6: Measurement data for the distribution of magnetic field

	x[cm]±0.05[cm]	B(x)[T]	u_B[T]		x[cm]±0.05[cm]	B(x)[T]	u_B[T]
1	0.00	0.000398430	0.00004	27	16.00	0.003568065	0.00030
2	0.50	0.000578026	0.00005	28	17.00	0.003563839	0.00030
3	1.00	0.000838817	0.00007	29	18.00	0.003557199	0.00030
4	1.50	0.001186236	0.00010	30	19.00	0.003543012	0.00030
5	2.00	0.001664051	0.00014	31	19.50	0.003537277	0.00030
6	2.50	0.002230003	0.00019	32	20.00	0.003533957	0.00030
7	3.00	0.002613945	0.00022	33	20.50	0.003530939	0.00029
8	3.50	0.002913673	0.00024	34	21.00	0.003524902	0.00029
9	4.00	0.003093873	0.00026	35	21.50	0.003516450	0.00029
10	4.50	0.003233021	0.00027	36	22.00	0.003509508	0.00029
11	5.00	0.003321159	0.00028	37	22.50	0.003498340	0.00029
12	5.50	0.003386659	0.00028	38	23.00	0.003482040	0.00029
13	6.00	0.003432237	0.00029	39	23.50	0.003456384	0.00029
14	6.50	0.003461515	0.00029	40	24.00	0.003419559	0.00029
15	7.00	0.003483851	0.00029	41	24.50	0.003385149	0.00028
16	7.50	0.003503169	0.00029	42	25.00	0.003326290	0.00028
17	8.00	0.003517356	0.00029	43	25.50	0.003248113	0.00027
18	8.50	0.003534259	0.00030	44	26.00	0.003130395	0.00026
19	9.00	0.003540899	0.00030	45	26.50	0.002944461	0.00025
20	9.50	0.003544823	0.00030	46	27.00	0.002692424	0.00023
21	10.00	0.003547842	0.00030	47	27.50	0.002274072	0.00019
22	11.00	0.003549955	0.00030	48	28.00	0.001875943	0.00016
23	12.00	0.003552671	0.00030	49	28.50	0.001316028	0.00011
24	13.00	0.003558708	0.00030	50	29.00	0.000916994	0.00008
25	14.00	0.003565047	0.00030	51	29.50	0.000620284	0.00005
26	15.00	0.003567763	0.00030	52	30.00	0.000450951	0.00004

Table 7: Data for distribution of magnetic field

Since $I_M = 250mA$, we can derive that $B_{theoretical} = 2.5B_{0.1}(x)$.
The calculation results are shown below:

x[m]	B_{theo}[T]	x[m]	B_{theo}[T]
±0.000	0.0035915	±0.080	0.0035143
±0.010	0.0035908	±0.090	0.0034640
±0.020	0.0035890	±0.100	0.0033695
±0.030	0.0035858	±0.110	0.0031713
±0.040	0.0035808	±0.120	0.0029908
±0.050	0.0035730	±0.130	0.0027158
±0.060	0.0035613	±0.140	0.0023153
±0.070	0.0035433	±0.150	0.0018083

Table 8: Theoretical value of the magnetic field inside the solenoid($I_M = 250mA$)

Finally we can plot the scatter plot of B vs. x:

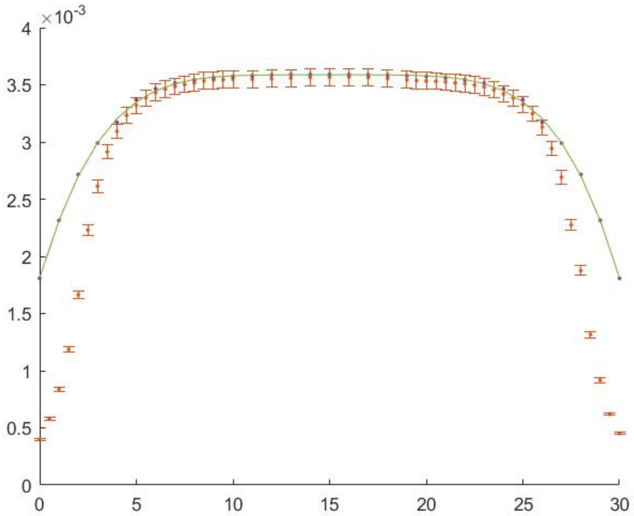

Figure 7: The magnetic field in side the solenoid(I_M = 250mA).

5 Uncertainty analysis

5.1 Relation between Sensitivity K_H and Working Voltage U_S

The uncertainties of $U_0(I_M = 0[V])$ and $U_0(I_M = 250mA)[V]$ can be calculated as:

$$u_{U_S} = 5.00 \times 0.5\% = 0.03V$$
$$u_{U_0} = 2.517 \times 0.05\% + 6 \times 10^{-3} = 0.007V$$
$$u_U = 2.636 \times 0.05\% + 6 \times 10^{-3} = 0.007V$$

Then the uncertainty for K_H can be calculated by the equation $K_H = \frac{U-U_0}{B}$ as:

$$\begin{aligned} u_{K_H} &= \sqrt{\left(\frac{\partial K_H}{\partial U}\right)^2 u_U^2 + \left(\frac{\partial K_H}{\partial U_0}\right)^2 u_{U_0}^2} \\ &= \left(\frac{1}{B}\right) \times \sqrt{u_U^2 + u_{U_0}^2} \\ &= 2.76\,\mathrm{V/T} \\ u_{K_H,r} &= 8.31\% \end{aligned}$$

So that $K_H = 33.13 \pm 2.76V/T, \quad u_{K_H,r} = 8.3\%$

The uncertainty of K_H/U_S can be calculated by the formula $K_H/U_S = (U - U_0)/(B \times U_S)$:

$$\begin{aligned} u_{K_H/U_S} &= \sqrt{\left(\frac{\partial K_H/U_S}{\partial K_H}\right)^2 u_{K_H}^2 + \left(\frac{\partial K_H/U_S}{\partial U_S}\right)^2 u_{U_S}^2} \\ &= \sqrt{\left(\frac{1}{U_S}\right)^2 u_{K_H}^2 + \left(\frac{1}{U_S^2}\right)^2 u_{U_S}^2} \end{aligned}$$

When $U_S = 2.79V$, u_{K_H/U_S} can be calculated as:

$$\begin{aligned} u_{K_H/U_S} &= \sqrt{(1/2.79)^2 \times (2.76)^2 + (1/2.79^2)^2 \times 0.01^2} \\ &= 0.99T^{-1} \end{aligned}$$

5.2 Relation Between Output Voltage U and Magnetic Field B

The uncertainty for I_M can be calclated as:

$$u_{I_M} = I_M \times 2\%$$

When $I_M = 500mA$:

$$u_{I_M} = 500mA \times 2\% = 0.01A$$

The uncertainty for U can be calculated as:

$$u_U = U \times 0.05\% + 6 \times 10^{-4}$$

When $U = 232.4mV$:

$$u_U = 232.4 \times 0.05\% + 6 \times 10^{-4} = 0.000117V$$

The uncertainty for B_{I_M} can be calclated as:

$$u_B = \frac{B_{0.1A}}{0.1} \times u_{I_M}$$

When $I_M = 500mA$:

$$u_B = \frac{1.4366}{0.1} \times 0.01 = 0.000144$$

5.3 Magnetic Field Distribution Inside the Solenoid

We have already found the uncertainty of U:

$$u_U = U \times 0.05\% + 6 \times 10^{-4}$$

The uncertanty of $B(x)$ can be calculated as:

$$\begin{aligned} u_B &= \sqrt{\left(\frac{\partial B}{\partial U} u_U\right)^2 + \left(\frac{\partial B}{\partial K_H} u_{K_H}\right)^2} \\ &= \sqrt{\left(\frac{u_U}{K_H}\right)^2 + \left(-\frac{U}{K_H^2} u_{K_H}\right)^2} \end{aligned}$$

When $x = 0.00cm$:

$$u_B = \sqrt{(0.000607/33.12)^2 + (0.01320 \times 2.76/33.12^2)^2} = 0.00004$$

6 Conclusion and discussion

In this lab we studied the principle of Hall effect and its applications using a Hall probe. It is used to verify that the Hall voltage is proportional to the magnetic field. The sensitivity of an integrated Hall probe is studied by calculating the magnetic field at the center of the solenoid, and the magnetic field distribution along the axis of the solenoid is measuerd and compared with the theoretical value.

The experimental errors mainly come from the experimental apparatuses. Since the numbers on the equipments are changing constantly, it's very hard to find a stable and reliable data to record. On most cases I have to estimate the value so that the result is very inaccurate. If only the equipments are more accurate or if it can evaluate the mean value for me, I can make the result more accurate.

Also since the magnetic field generated in the Hall Probe is quite small, the magnetic field of the Earth can not be ignored.

In the experiment **Relation Between Sensitivity K_H and Working Voltage U_S**, We find the K_H with respect to U_S and plot the scatter graph. We then find that when $U_S = 5V$, K_H is most stable, so we choose 5V as the working voltage in the following experiments.

In the experiment **Relation Between Output Voltage U and Magnetic Field B**, we find the relation between Hall voltage U_H and the magnetic field B. According to the linear fit, we can see that U_H and B has a linear relationship, which fits the theory. The experimental value K_H can be read from the slope of liner fit as 32.07±0.29 V/T. The deviation of experimental value from theoretical value is -3.20%, which is relativly small and proves the theory.

In the experiment **Magnetic Field Distribution Inside the Solenoid**, we measured the magnetic field at different places along the solenoid. We can find that the magnetic field is strongest in the middle of the solenoid, and it decreses quickly when it goes far from the middle. Then we plot the experimental and theoretical on the same graph, and we can find them fit well.

7 References

1. Qin Tian, Cao Jianjun, Yi Hankun, Wu Ziyou, Zhang Yifei, Yao Yuan, Mateusz Krzyzosiak