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Practical - 1

Topic :- limits & continuity

1) $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

2) $\lim_{y \to a} \left[\dfrac{\sqrt{a+x} - \sqrt{a}}{y\sqrt{a+y}} \right]$

3) $\lim_{x \to \pi/6} \left[\dfrac{\cos 3x - \sqrt{x^2-3}}{\pi - 6x} \right]$

4) $\lim_{x \to 0} \dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}$

5) Examine the continuity of the following function

i) $f(x) = \begin{cases} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}, & 0 < x \le \pi/2 \\ \dfrac{\cos x}{\pi - 2x}, & \pi/2 < x < \pi \end{cases}$ at $x = \pi/2$

ii) $f(x) = \begin{cases} \dfrac{x^2-9}{x-3}, & 0 < x < 3 \\ x+3 & 3 \le x < 6 \\ \dfrac{x^2-9}{x+3} & 6 \le x \le 9 \end{cases}$ at $x = 3$ & $x = 6$

6) find value of K so that the function $f(x)$ is continous at the indicated point

i) $f(x) = \left.\begin{cases} \dfrac{1-\cos 4x}{x^2}, & x < 0 \\ k, & x = 0 \end{cases}\right\}$ at $x = 0$

ii) $f(x) = \left.\begin{cases} (\sec^2 x)^{\cot^2 x} \\ = k \end{cases}\right\}$ at $x = 0$

iii) $f(x) = \left.\begin{cases} \dfrac{\sqrt{3} - \tan x}{\pi - 3x}, & x \neq \pi/3 \\ = k, & x = \pi/3 \end{cases}\right\}$ at $x = \pi/3$

7) Discuss the continuity of the following function which of these function have removable discontinuity Re-define function so as to remove the discontinuity

i) $f(x) = \left.\begin{cases} \dfrac{1-\cos 3x}{x \tan x}, & x \neq 0 \\ = 9, & x = 0 \end{cases}\right\}$ at $x = 0$

ii) $f(x) = \left.\begin{cases} (e^{3x} - 1) \sin x^\circ, & x \neq 0 \\ = \pi/60, & x = 0 \end{cases}\right\}$ at $x = 0$

8) if $f(x) = \dfrac{(e^{x^2} - \cos x)}{x^2}$

for $x \neq 0$ is continous at $x = 0$ find $f(0)$

9) if $f(x) = \dfrac{\sqrt{2} - \sqrt{(1+\sin x)}}{\cos^2 x}$ for $x \neq \pi/2$

is continous at $x = \pi/2$ find $f\ \pi/2$

Solutions:-

i) $\lim\limits_{x \to a} \dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}$

$= \lim\limits_{x \to a} \dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \dfrac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \times \dfrac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}}$

$= \lim\limits_{x \to a} \dfrac{(a+2x-3x)}{(3a+x-4x)} \dfrac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}}$

$= \lim\limits_{x \to a} \dfrac{(a-x)}{(3a-3x} \dfrac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3}x}$

$= \lim\limits_{x \to a} \dfrac{\cancel{(a-x)}}{3(a-x)} \dfrac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}}$

$= \lim\limits_{x \to a} \dfrac{1}{3} \dfrac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}}$

$$= \frac{\sqrt{3a+9} + 2\sqrt{a}}{3\sqrt{a+2a} + \sqrt{3a}}$$

$$= \frac{\sqrt{4a} + 2\sqrt{a}}{3\sqrt{3a} + \sqrt{3a}}$$

$$= \frac{2\sqrt{a} + 2\sqrt{a}}{3 \times 2\sqrt{3a}} = \frac{2\sqrt{a}}{3\sqrt{3a}} = \frac{2}{3\sqrt{3}}$$

2) $$\lim_{y \to 0} \frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}}$$

$$= \lim_{y \to 0} \frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+x} + \sqrt{a}}{\sqrt{a+x} + \sqrt{a}}$$

$$= \lim_{y \to 0} \frac{(a+y-a)}{(y\sqrt{a+y})(\sqrt{a+y} + \sqrt{a})}$$

$$= \lim_{y \to 0} \frac{y}{y\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$$

$$= \lim_{y \to 0} \frac{1}{\sqrt{a}(\sqrt{a} + \sqrt{a})}$$

$$= \lim_{y \to 0} \frac{1}{\sqrt{a}(2\sqrt{a})}$$

$$= \frac{1}{2a}$$

3) $\lim_{x \to \pi/6} \dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

$x - \pi/6 = h \quad x = h + \pi/6 \quad h \to 0$

$$\lim_{\pi \to 0} \frac{\cos(h+\pi/6) - \sqrt{3}(h+\pi/6)}{\pi - 6(h+\pi/6)}$$

$$\lim_{x \to 0} \frac{(\cos h \cdot \cos\pi/6 - \sin h \cdot \sin\pi/6) - \sqrt{3}(\sin h \cos\pi/6 + \cos h \sin\pi/6)}{-6h}$$

$$\lim_{\pi \to 0} \frac{[(\cos h\, \sqrt{3}/2 - (\sin h\, 1/2)] - \sqrt{3}\,(\sin h\, \sqrt{3}/2 + \cos h\, 1/2)}{-6h}$$

$$\lim_{\pi \to 0} \frac{\left(\cos \frac{\sqrt{3}h}{2} - \sin h/2\right) - \left(\sin \frac{3h}{2} + \cos \frac{\sqrt{3}}{2} h\right)}{-6h}$$

$$= \lim_{h \to 0} \frac{-\frac{\sin 4h}{2}}{-6h}$$

$$= \lim_{h \to 0} \frac{\sin 2h}{6h}$$

$$= \frac{1}{3} \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \frac{1}{3}$$

4) $\lim_{x \to 0} \dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}$

$= \lim_{x \to \infty} \dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \dfrac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}} \times \dfrac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}}$

$= \lim_{x \to \infty} \dfrac{(x^2+5-x^2+3)(\sqrt{x^2+3} + \sqrt{x^2+1})}{(x^2+3-x^2-1)(x^2+5+\sqrt{x^2-3})}$

$= \lim_{x \to \infty} \dfrac{8(\sqrt{x^2+3}) + \sqrt{x^2+1})}{2(\sqrt{x^2+5} + \sqrt{x^2-3})}$

$= \lim_{x \to \infty} \dfrac{4(\sqrt{x^2+3} + \sqrt{x^2+1})}{x^2+5 + \sqrt{x^2-3}}$

$= \lim_{x \to \infty} \dfrac{4\sqrt{x^2(1+3/x^2)} + \sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)} + \sqrt{x^2(1-3/x^2}}$

$= 4 \lim_{x \to \infty} \dfrac{\sqrt{x^2(1+3/x^2)} + \sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)} + \sqrt{x^2(1-3/x^2)}}$

$= 4$

5) $\therefore f(\pi/2) = \dfrac{\sin(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$

1) $f(\pi/2) = 0$

$\therefore f$ at $x = \pi/2$ define

2) $\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2^+} \dfrac{\cos x}{\pi - 2x}$

put $x - \pi/2 = h$

$\therefore x = \pi/2 + h$

as $x \to \pi/2$, $h \to 0^+$

$$\lim_{h\to 0^+} \frac{\cos(\pi/2+h)}{\pi-2(\pi/2+h)} = \lim_{h\to 0^+} \frac{-\sin h}{\pi-\pi-2h}$$

$$\therefore \lim_{h\to 0^+} \frac{-\sin h}{-2h} = \frac{1}{2}$$

$$\lim_{x\to \pi/2^-} f(x) = \lim_{x\to \pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$$

$$= \lim_{x\to \pi/2^-} \frac{2\sin x\cos x}{\sqrt{2\sin^2 x}} = \lim_{x\to \pi/2^-} \frac{2\sin x\cos x}{\sqrt{2}\sin x}$$

$$= \frac{2}{\sqrt{2}} \lim_{x\to \pi/2} \cos x = 0$$

$\therefore$ LHL $\neq$ RHL

$\therefore f$ is not continuous at $= \pi/2$

5) 2) $f(x) = \dfrac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$= x+3 \quad 3 \le x \le 6$

$= \dfrac{x^2-9}{x+3} \quad 6 \le x < 9$

at $x = 3$, $x = 6$

at $x = 3$ $\quad f(3) = \dfrac{x^2-9}{x-3} = 0$

f at $x = 3$ define

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}$$

$f(3) = x+3 = 3+3 = 6$

f is defined at $x = 3$

2) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x+3) = 6$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{(x-3)}$

$= LHL = RHL$

f is continuous at $x=3$

for $x = 6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9}$

$\lim_{x \to 6^+} = \frac{x^2-9}{x+3}$

$\lim_{x \to 6^+} \frac{(x-3)(x+3)}{(x+3)} = \lim_{x \to 6^+} (x-3) = 6-3 =$

$\lim_{x \to 6^-} x+3 = 3+6 = 9$

$LHL \neq RHL$

function is not continuous

6) $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & x < 0 \\ k & x = 0 \end{cases}$ at $x = 0$

Soln:

f is continuous at $x = 0$

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \frac{\quad}{x^2} = k$

$2 \lim_{x \to 0} \frac{2 \sin^2 x}{x^2} = k \quad \therefore 2 \lim_{x \to 0} \left(\frac{\sin^2 2x}{x}\right)^2 = k$

$2(2)^2 = k \quad k = 8$

(ii) $f(x) = (\sec^2 x)^{\cot^2 x} \quad x \neq 0$, $x = 0$ } at $x = 0$

$= k$

$= 4/3$

(iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x} \quad x \neq \pi/3$, $x = \pi/3$ } at $x = \pi/3$

$= k$

$x - \pi/3 = h \qquad \therefore \lim_{x \to \pi/3} f(x) = f(\pi/3)$

$x = h + \pi/3 \quad h \to 0$

$f(\pi/3 + h) = \sqrt{3} \qquad \lim_{x \to \pi/3} \frac{\sqrt{3} - \tan x}{\pi - 3x} = k$

$\lim_{h \to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

$\lim_{h \to 0} \frac{\sqrt{3}\left(\frac{\tan \pi/3 + \tan h}{1 - \tan \pi/3 \cdot \tan h}\right)}{\pi - \pi - 3h}$

$\lim_{\quad 0} \frac{\sqrt{3}(1 - \tan \pi/3 \cdot \tan h) - \tan(\pi/3 - \tan h)}{(-3h)(1 - \tan \pi/3 \cdot \tan h)}$

$$f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2} \quad x \neq 0$$
$$= \pi/6 \quad x = 0$$
at $x = 0$

at $x = 0$

$$\lim_{x \to 0} (e^{3x}-1) \sin\left(\frac{\pi x}{180}\right) / x^2$$

$$3 \lim_{x \to 0} \frac{e^{3x}-1}{3x} \lim_{x \to 0} \frac{\sin \pi x/180}{x}$$

$$3 \log e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$$

f is continuous at $x = 0$

$$f(x) = \frac{e^{x^2} - \cos x}{x^2} \quad x = 0$$

is continuous at $x = 0$

$$\lim_{x \to 0} f(x) = f(0)$$

$$= \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$= \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$(e^{x^2} - 1) + (1 - \cos x)/x^2$$

$$\frac{e^{x^2}-1}{x^2} + \lim_{x \to 0} 2 \cdot \frac{\sin^2 x/2}{x^2}$$

$$= \log e + 2 \left(\frac{\sin^2 x/2}{x}\right)^2$$

Multiply with 2 on Numerator & denominator

$= 1 + 2 + \frac{1}{4} = 3/2 = f(0)$

$$f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \quad x \neq \pi/2$$

$f(x)$ is continuous at $x = \pi/2$

$$\frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$= \frac{2 - 1 + \sin x}{\cos^2 x\,(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \frac{1 + \sin x}{(1 - \sin x^2)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2} + \sqrt{2})}$$

$$= \frac{1}{2 + 2\sqrt{2}} = \frac{1}{4\sqrt{2}}$$

$$f(\pi/2) = \frac{1}{4\sqrt{2}}$$

6) ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$

$= k$ $\quad x = 0$ $\Big\}$ at $x = 0$

$f(x) = (\sec^2 x)^{\cot^2 x}$

using

$\sec^2 - \tan^2 x = \sec^2 x = 1$

$\therefore \sec^2 x = 1 + \tan^2 x$

$\cot^2 x = 1/\tan^2 x$

$\therefore \lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$

$\therefore \lim_{x \to 0} (1+\tan^2 x)^{1/\tan^2 x}$

we know, that

$\lim_{x \to 0} (1+px)^{1/px} = e$

$\therefore = e$

$k = e$

Practical-2

Topic :- derivative

Q1 show that the following function defined from R to R are differentiable

1) $\cot x$ 2) $\operatorname{cosec} x$ 3) $\sec x$

Q2 if $f(x) = 4x+1 \quad x \le 2$
$= x^2+5 \quad x > 0$ at $x = 2$ then find f is differentiable or not.

Q3 if $f(x) = 4x+7 \quad x < 3$
$= x^2+3x+ \quad x \geq 3$ at $x = 3$ then find f is differentiable or not.

Q4 if $f(x) = 8x-5 \quad x \le 2$
$= 3x^2-4x+7 \quad x < 2$ at $x = 2$ then find f is differentiable or not?

Solution:

Q1 $\cot x$

1) $f(x) = \cot x$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x-a}$$

$$= \lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x-a}$$

$$= \lim_{x \to 0} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$$

put $x - a = h$ $\quad x = a + h \quad$ as $x - a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$$

formula : $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$\tan A - \tan B = \tan(A-B)(1 + \tan A \cdot \tan B)$

$$= \lim_{h \to 0} \frac{\tan(a-a-h) - (1 + \tan a + \tan(a+h))}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{-\tan h}{h} \times \frac{1 + \tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1 + \tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a} = -\frac{1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$Df(a) = -\cos^2 a$

$\therefore$ f is differentiable $\forall a \in R$

2) $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\sin x - 1/\sin a}{x - a}$$

$$= \lim_{x \to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

put $x - a = h$ $\quad x = a+h$ as $x \to a$ $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

formula

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin h/2}{h/2} \times 1/2 \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= \frac{-1/2 \times 2\cos(2a/2)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

3) $\sec x$

$$f(x) = \sec x$$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sec(x) - \sec(a)}{x - a}$$

$$= \lim_{x \to a} \frac{1/\cos x - 1/\cos a}{x - a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$ put $x - a = h$, $x = a + h$

as $x \to a$ $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \cos a \cos(a+h)}$$

formula:

$$-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{\cos a \quad \cos(a+h) \times -h/2}$$

$$= -1/2 \times -2 \quad \frac{\sin a}{\cos a \times \cos a}$$

$= \tan a \sec a$

Q2 Soln:

LHD

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x + 1 - (4 + 2 + 1)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x - 8}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)} = 4$$

$Df(2^-) = 4$

RMD:

$$Df(2^+) = \lim_{x \to 2^+} \frac{x^2 + 5 - 9}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{x^2 - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{x-2}$$

$$= 2 + 2 = 4$$

$Df(2^+) = 4$ RMD = LMD

f is differentiable at $x = 2$

Q3

Soln: RMD:

$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - (3^2 + 3 - 3 + 1)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x - 18}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 6x - 3x - 18}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x(x+6) - 3(x+6)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{(x-3)} = 3 + 6 = 9$$

$Df(3^+) = 9$

LMD $= Df(3^-)$

$$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3} = \lim_{x \to 3^-} \frac{4x-12}{x-3} = \lim_{x \to 3^-} \frac{4(x-3)}{(x-3)}$$

$Df(3^+) = 4$

RHD $\neq$ LHD

f is not differentiable at $x = 3$

04 Sol^n:

$f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

RHD: $$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x-2}$$

$$= 3 \times 2 + 2 = 8$$

$Df(2^+) = 8$

LHD:

$$Df(2^-) = \lim_{x \to 2^-} \frac{8x - 5 - 11}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)} \qquad Df(2^-) = 8$$

$\therefore$ LHD = RHD f is differentiable at $x = 3$

Practical - 3

Topic: Application of derivative

1) find the intervals in which function is increasing or decreasing

i) $f(x) = x^3 - 5x - 11$

ii) $f(x) = x^2 - 4x$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

iv) $f(x) = x^3 - 27x + 5$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

2) find the intervals in which function is concave upwards

i) $y = 3x^2 - 2x^3$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

iii) $y = x^3 - 27x + 5$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

v) $y = 2x^3 + x^2 - 20x + 4$

Solution:

Q1

1) $f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\therefore f$ is increasing iff $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5}/3)(x + \sqrt{5}/3) > 0$

$+ \quad - \quad +$

$-\sqrt{5}/3 \qquad \sqrt{5}/3$

$x \in (-\infty, -\sqrt{5}/3) \cup (\sqrt{5}/3, \infty)$

& f is decreasing iff $f(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore 3(x^2 - 5/3) < 0$

$\therefore (x - \sqrt{5}/3)(x + \sqrt{5}/3) < 0$

$+ \quad - \quad +$

$-\sqrt{5}/3 \qquad \sqrt{5}/3$

$x \in (-\sqrt{5}/3, \sqrt{5}/3)$

2) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\therefore f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 2x - 4 < 0$

$2(x-2) < 0$

$x - 2 < 0$

$x \in (-\infty, 2)$

3) $f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$\therefore 2(3x^2 + x - 10) > 0$

$\therefore 3x^2 + x - 10 > 0$

$\therefore 3x^2 + 6x - 5x - 10 > 0$

$\therefore 3x(x+2) - 5(x+2) > 0$

$\therefore (x+2)(3x-5) > 0$

$x \in (-\infty, -2) \cup (5/3, \infty)$

$+ \quad - \quad +$

$-2 \qquad 5/3$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

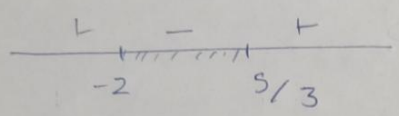
$\therefore 2(3x^2 + x - 10) < 0$

$\therefore 3x^2 + x - 10 < 0$

$\therefore 3x^2 + 6x - 5x - 10 < 0$

$\therefore 3x(x+2) - 5(x+2) < 0$

$\therefore (x+2)(3x-5) < 0$



$x \in (-2, 5/3)$

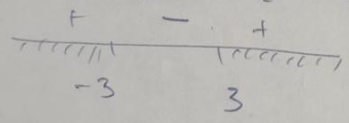
4) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore (x-3)(x+3) > 0$



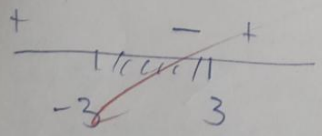
$\therefore x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$(x-3)(x+3) < 0$



$\therefore x \in (-3, 3)$

5) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\therefore$ f is increasing iff $f'(x) > 0$

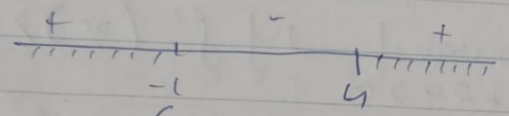
$\therefore 6x^2 - 18x - 24 > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$\therefore x^2 - 4x + x - 4 > 0$

$\therefore x(x-4) + 1(x-4) > 0$

$\therefore (x-4)(x+1) > 0$



$\therefore x \in (-\infty, -1) \cup (4, \infty)$

and f is decreasing iff $f'(x) < 0$

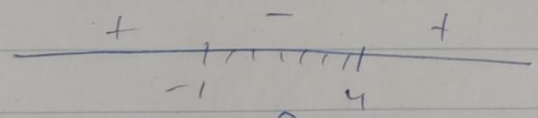
$\therefore 6x^2 - 18x - 24 < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

$\therefore x^2 - 4x + x - 4 < 0$

$\therefore x(x-4) + 1(x-4) < 0$

$\therefore (x-4)(x+1) < 0$



$\therefore x \in (-1, 4)$

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1) $Y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

f is concave upward if $f''(x) > 0$

$\therefore (6 - 12x) > 0$

$\therefore 12(6/12 - x) > 0$

$x - 1/2 > 0$

$x > 1/2$

$\therefore f''(x) > 0$

$\therefore x \in (1/2, \infty)$

2// $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(2) = 12x^2 - 36x + 24$

f is concave upward if $f''(x) > 0$

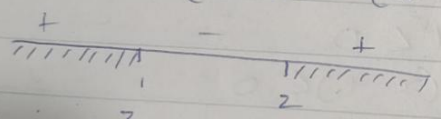
$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore x(x-2) - 1(x-2) > 0$

$\therefore (x-2)(x-1) > 0$



$x \in (-\infty, 1) \cup (2, \infty)$

3// $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x \in (0, \infty)$

4) $y = 64 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 64$

$f'(x) \quad 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0 \qquad x > 3/2 \qquad \therefore x \in (3/2, \infty)$

5) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 12(x + 2/16) > 0$

$\therefore x + 1/6 > 0$

$\therefore x < -1/6$

$\therefore f''(x) \not> 0$

$\therefore$ There exist no interval $(-1/6, \infty)$

11/01/2020

Practical - 4

Topic :- Application of derivative by newton's method

Q1) find maximum & minimum value of the following

i) $f(x) = x^2 + 16/x^2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 + 1$ $[-1/2, 4]$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ $[-2, 3]$

Q2) find the root of the following equation by Newton's (Take 4 iteration only) correct upto 4 decimal

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

a) $f(x) = x^2 + 16/x^2$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$

$\therefore \quad 2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 32/2$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore \quad f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$\therefore \quad f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = -2$

$\therefore$ function reaches minimum value at $x = 2$, & $x = -2$

iii) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

consider, $f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f(\pm 1) = -30 + 60$

$= 30 > 0$ $\therefore f$ has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$\therefore f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$ $\therefore f$ has maximum value at $x = -1$.

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore f$ has maximum value 5 at $x = -1$ &
has the minimum value 1 at $x =$

iv) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$ $\therefore$ f has maximum value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$f'(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has maximum value 1 at $x = 0$ & f has min value -3 at $x = 2$

i) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

consider, $f(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

$f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore$ f has min value at $x = 2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$\therefore f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore$ f has max value at $x = -1$ &

f has min value -19 at $x = 2$

Q2

1) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $\quad x_0 = 0 \rightarrow$ given

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$\therefore x_1 = x_0 - f(x_0) / f'(x_0)$

$x_1 = 0 + 9.5/55$

$x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.895 - 9.4985 + 9.5$

$= -0.0829$

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.00895 - 1.0362 - 55$

$= 55.9467$

$\therefore x_2 = x_1 - f(x_1) / f'(x_1)$

$= 0.1727 - 0.0829 / 55.9467$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$\therefore x_3 = x_2 - f(x_2) / f'(x_2)$

$= 0.1712 + 0.0011 / 55.9393$

$= 0.1712$

The root of the equation is 0.1712

ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation

$\therefore$ By Newton's Method,

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7342)^3 - 4(2.7342) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5046 - 4$

$= 18.5046$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$= 2.7342 - 0.596 / 18.5046$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9851 - 4$

$= 17.9851$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 4$

$= 19.7158 - 10.806 - 4 = -0.0901$

$f'(3) = 3(2.7015)^2 - 4 = 21.8943 \rightarrow 4 = 17.$

$x^4 = 2.7015 + 0.0901 / 17.8943 = 2.7015 + 0.$

$= 2.7065$

3) $f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - (1.8(1)^2 - 10(1) + 17$

$= -1.8 - 10 + 17$

$= 6 - 2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = -2.2$

let $x_0 = 2$ be initial approximation by Newton's Method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x - f(x_2) / f'(x_2)$

$= 2 - 2.2 / 5.2$

$= 2 - 0.4230 = \underline{1.577}$

$f(x_1) = (1.577)^3 - (1.8)(1.577)^2 - 10(1.577) +$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 0.6755$

$= -8.2164$

$\therefore x_2 = x_1 - f(x_1) / f'(x_1)$

$= 1.577 + 0.0822$

$= 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 4$

$= 4.5677 - 4.9553 - 16.592 + 17$

$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 8.2588 - 5.97312 - 10$

$= -7.7143$

$x_3 = x_2 - f(x_2) / f'(x_2)$

$= 1.6592 + 0.0204 / 7.7143$

$= 1.6592 + 0.0026$

$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5842 - 4.9708 - 16.618 + 17$

$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - f(x_3) / f'(x_3)$

$= 1.618 + \frac{0.0004}{7.6977}$

$= 1.6618$

Ans

[illegible]/02/2020

Practical 5

Topic:- integration

a1) Solve the following integration

i) $dx/\sqrt{x^2+2x-3}$

ii) $\int (4e^{3x}+1)\,dx$

iii) $\int (2x^2-3\sin x+5\sqrt{x})\,dx$

iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

v) $\int 6-7\sin(2t+4)\,dt$

vi) $\int \sqrt{x}\,(x^2-1)\,dx$

vii) $\int 1/x^3 \sin(1/x^2)\,dx$

viii) $\int \cos x/3\sqrt{\sin 2x}\,dx$

ix) $\int e^{\cos^2 x}\sin 2x\,dx$

x) $\int (x^2-2x/x^3-3x^2+1)\,dx$

i) $\int \frac{1}{x^2+2x-3} \, dx$

$= \int \frac{1}{\sqrt{x^2+2x-3}} \, dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}$

$a^2+2ab+b^2 = (a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}}$

Substitute

put $x+1=t$

$dx = \frac{1}{t} \times dt$ where $t=1$ $t=x+1$

$\int \frac{1}{\sqrt{t^2-4}} \, dt$

using,

$$= \int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(|x + \sqrt{x^2-a^2}|\right)$$

$$= \ln\left(|t + \sqrt{t^2-4}|\right)$$

$t = x+1$

$$= \ln\left(|x+1 + \sqrt{(x+1)^2-4}|\right)$$

$$= \ln\left(|x+1 + \sqrt{x^2+2x-3}|\right) + c$$

$=$

2) $\int (4e^{3x}+1)\,dx$

$$= \int 4e^{3x}\,dx + \int 1\,dx$$

$$= 4\int e^{3x}\,dx + \int 1\,dx \quad \# \quad \int e^{ax}\,dx = \frac{1}{x} \times e^{ax}$$

$$= \frac{4e^{3x}}{3} + x + c$$

$$= \frac{4e^{3x}}{3} + x + c$$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\,dx \quad \sqrt[n]{a^m} = a^{m/n}$$

$$= \int 2x^2\,dx - \int 3\sin(x)\,dx + \int 5x^{1/2}\,dx$$

$$= 2x^3/3 + 3\cos x + \frac{10 \times \sqrt{x}}{3} + c$$

$$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + c$$

$$\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$$

4/ $$= \int \frac{x^3+3x+4}{x^{1/2}}\,dx$$

split the denominator

$$\int x^3/x^{1/2} + 3x/x^{1/2} + 4/x^{1/2}\,dx$$

$$= \int x^{5/2} + \int 3x^{1/2} + 4/x^{1/2}\,dx$$

$$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$$

5/ $\int t^7 \times \sin(2+4)\,dt$

put $u = 2t^4$

$du = 2 \times 4t^3$

$$= \int t^7 \times \sin(2+4) \times 1/2 \times 4t^3\,du$$

$$= \int t^4 \sin(2t^4) \times 1/2 \times 4\,du$$

$$= \int t^4 \sin(2t^4) \times 1/8\,du =$$

$$= \frac{t^4 \times \sin(2t^4)}{8}\,du$$

substitute t^4 with $u/2$

$$= \int u/2 \times \sin/8\,du$$

$$= \int \frac{u \times \sin(u)}{2}/8\,du$$

$$= \int \frac{u \times \sin(u)}{16}\,du$$

$$= 1/16 \int u \times \sin(u)\,du$$

$\# \int u\,dv = uv - \int v\,du$

where $u = u$

$dv = \sin(u) \times du$

$du = 1du \quad v = -\cos(u)$

$= \frac{1}{16}(u \times (-\cos(u)) - \int -\cos(u)\,du$

$= \frac{1}{16} \times (u \times (-\cos u)) + \int \cos(u)\,du$

$\# \int \cos x\,dx = \sin(x)$

$= \frac{1}{16} \times (u \times (-\cos(u)) + \sin(u))$

Return to substitution $u = 2t^4$

$= \frac{1}{16} \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$

vi) $\int \sqrt{x}\,(x^2 - 1)\,dx$

$= \int \sqrt{x}\,x^2 - \sqrt{x}\,dx$

$= \int x^{1/2} \times x^2 - \sqrt{x}^{1/2}\,dx$

$= \int x^{5/2} - x^{1/2}\,dx$

$= \int x^{5/2}\,dx - \int x^{1/2}\,dx$

$= I_1 \; \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x}\,7}{7} = \frac{2x^3\sqrt{x}}{7}$

$= I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x}\,3}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + C$

(vii) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\, dx$

$= \int \frac{\cos x}{\sin^{2/3} x}\, dx$

put $t = \sin(x)$

$t = \cos x$

$= \int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos(x)\, dt}$

$= \frac{1}{\sin^{3/2}}\, dt$

$= \frac{1}{t^{2/2}}\, dt$

$I = \int \frac{1}{t^{2/3}}\, dt = \frac{-1}{(2/3 - 1)\, t^{2/3 - 1}}$

$= \frac{-1}{-1/3\, t^{2/3 - 1}} = \frac{1}{1/3\, t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$

$= 3\sqrt[3]{t}$

Return substitution $t = \sin(x)$

$= 3\sqrt[3]{\sin(x)} + c$

(x) $\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1}\, dx$

put $x^3 - 3x^2 + 1 = dt$

$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 3 \times 2x}\, dt$

$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 6x}\, dt$

$$\int \frac{1}{x^3-3x^2+1} \times \frac{1}{3} dt$$

$$= \int \frac{1}{3(x^3-3x^2+1)} dt = \int \frac{1}{3t} dt$$

$$= \frac{1}{3} \int \frac{1}{t} dt \quad \int \frac{1}{x} dx + \ln|x|$$

$$= \frac{1}{3} \times \ln|t| + C$$

$$= \frac{1}{3} + \ln(|x^3 - 3x^2 + 1|) + C$$

Practical 6

Topic :- applications of integration & numerical integration

1) find the length of following curve

i) $x = \sin t$, $y = 1 - \cos t$ & $t \in [0, 2\pi]$

ii) $y = \sqrt{y - x^2}$ $\quad x \in [-2, 2]$

iii) $y = x^{3/2}$ $\quad [0, \pi]$

iv) $x = 3 \sin t$ $\quad y = 3 \cos t$ & $t \in [0, 2\pi]$

v) $x = \frac{1}{6} y^3 + \frac{1}{2y}$ $\quad y \in [1, 2]$

2) using simpson's rule solve the following

① $\int_0^2 e^{x^2} dx$ with $n = 4$

② $\int_0^4 x^2 dx$ with $n = 4$

③ $\int_0^{\pi/4} \sqrt{\sin x}\, dx$ with $n = 6$

1) $x = t - \sin t \quad y = 1 - \cos t \quad t \in (0, 2\pi)$

$$L = \int_0^y \sqrt{\left(\frac{\delta y}{\delta t}\right)^2 + \left(\frac{\delta x}{\delta t}\right)^2}\, dt$$

$$\frac{\delta y}{\delta t} = \sin t \qquad \frac{dx}{dt} = 1 - \cos t$$

$$L = \int_0^{2\pi} \sqrt{(\sin t)^2 + (1 - \cos t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{\sin^2 t + 1 + \cos^2 t - 2\cos t}$$

$$= \int_0^{2} \sqrt{2 - 2\cos t}$$

2) $\sqrt{4 - x^2}$

$$C = \int \sqrt{1 + (dy/dx)^2}\, dx$$

$$dy/dx = \frac{1(-2x)}{2\sqrt{4 - x}}$$

$$= \frac{-x}{\sqrt{4 - x}}$$

$$L = \int_2^2 \sqrt{1 + (-x/\sqrt{4 - x^2})^2}\, dx$$

$$= \int_2^2 \sqrt{1 + x^2/4 - x^2}$$

$$= 2 \int_{-2}^{2} \sqrt{\frac{1}{\quad}} \, dx$$

$$= 2 \int_{-2}^{2} \frac{1}{\sqrt{(2)^2 - (x)^2}} \, dx$$

$$= 2 \left[\sin^{-1} (x/2) \right]_{-2}^{2}$$

$$= 2 \left[\sin^{-1}(1) - \sin^{-1}(-1) \right]$$

$$= 2 \left[\pi/2 - (-\pi/2) \right]$$

$$= 2\pi$$

(4) $y = x^{3/2}$ $\qquad x = [0, 4]$

$$l = \int_{a}^{b} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$dy/dx = 3/2 \sqrt{x}$$

$$l = \int_{0}^{4} \sqrt{1 + 9/4 x} \, dx$$

$$= 1/2 \int_{0}^{4} \sqrt{4 + 9x} \, dx$$

$$= 1/2 \left[\frac{(4 + 9x)^{3/2}}{3/2} \times 1/9 \right]_{0}^{4}$$

$$= 1/27 \left[40^{3/2} - 8 \right]$$

3 $x = 3\sin t \quad y = 3\cos t \quad t \in [0, 2\pi]$

$$L = \int_0^3 \sqrt{(dy/dt)^2 + (dx/dt)^2}\, dt$$

$dy/dt = -3\sin t \quad dx/dt = 3\cos t$

$$L = \int_0^{2\pi} \sqrt{(3\sin^2 t) + (3\cos^2 t)}\, dt$$

$$L = 3 \int_0^{2\pi} \sqrt{\sin^2 t + \cos^2 t}$$

$$= 3 \int_0^{2\pi} \sqrt{1}\, dt$$

$$= 3[t]_0^{2\pi} = 6\pi$$

4) $x = \frac{1}{6}y^3 + \frac{1}{2y}$

$$L = \int_0^6 \sqrt{1 + (dx/dy)}\, dy$$

$$dx/dy = 3y^2/2 - 1/2y^2$$

$$L = \int_1^2 \sqrt{1 + \left(\tfrac{1}{2}y^2 - \tfrac{1}{2y^2}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{1 + \left(\frac{y^2 - 1}{2y^2}\right)^2}\, dy$$

$$\therefore \int_1^2 \sqrt{\frac{(y^4 - 1)^2 + 4 + y^4 \times 1}{4y^4}}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4 + 1)^2}{(2y^2)^2}}\, dy$$

$= \int_1^2 y^2/2 \, dy + \int_1^2 1/2y^2 \, dy$

$= 1/2 \left[y^3/3 \right]_1^2 + 1/2 \left[1/y \right]_1^2$

$= 1/2 \left[8/3 - 1/3 \right] + 1/2 \left[1/2 - 1 \right]$

$= 7/6 - 1/4$

$= 14 - 3/12$

$= 11/12$

Q2) 1) $\int_0^2 e^{x^2} \delta x$ with $n = 4$

$a = 0, b = 2, n = 4$

$h = \frac{b-a}{n} = 2-0/4 = 1/2$

x	0	0.4	1	1.4	2
y	1	1.2840	2.7182	9.4677	54.5981

by simpson rule:

$\int_0^2 e^{x^2} dx = 0.5/3 \left[(1 + 54.5981) + 4[1.2840 + 9.4677] + 2(2.7182) \right]$

$= 17.3535$

2) $\int_0^4 x^2 dx$ with $n = 4$

$a = 0, b = 4, n = 4$

$h = \frac{b-a}{n} = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16

[illegible] rule

$$\int_0^2 x^2\,dx = \frac{1}{3}\left[0+16) + 4(2+a) + 2(4)\right]$$

$$= \frac{1}{3}(16+40+8)$$

$$= \frac{1}{3}\times 64$$

$$= 64/3$$

2) $\int_0^{\pi/3} \sqrt{\sin x}\,dx$ with $n=6$

$a=0,\ b=\pi/3,\ n=6$

$h = \pi/3 - 0/6 = \pi/18$

$x = 0 \quad \pi/18 \quad 3\pi/18 \quad 3\pi/18 \quad 4\pi/18 \quad 5\pi/18 \quad 6\pi/18$

$y = 0 \quad 0.9167 \quad 0.5848 \quad 0.7071 \quad 0.8017 \quad 0.8752 \quad 0.93$

by simpson rule

$$\int_0^{\pi/18} \sqrt{\sin x} = h/3\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \pi/54\left[(0.09306) + 4(0.4167 + 0.7071 + 0.8752) + 2(0.5848 + 0.5017)\right]$$

$$= \pi/54 \times 11.6096$$

$$= 0.6806$$

$$\int_0^{\pi/3} \sqrt{\sin x} = 0.6806$$

Practical 7

Differential equation

1) Solve the following differential equation

1) $x \frac{dy}{dx} + y = e^x$

2) $e^x \, dy/dx + 2e^x y = 1$

3) $x \, dy/dx = \cos x / x - 2y$

4) $x \, dy/dx + 3y = \sin x / x^2$

5) $e^{2x} \frac{dy}{dx} + 2e^{2x} y = 2x$

6) $\sec^2 x \tan y + \sec^4 y \tan x \cdot dy = 0$

11) $dy/dx = \sin^2 (x - y + 1)$

12) $dy/dx = 2y + 5y - 1 / 6x + 9y + 6$

$x\, dy/dx + y = e^x$

$\therefore dy/dx + 1/x\, y = e^x/x$

Comparing with $dy/dx + P(x)y = Q(x)$

$P(x) = 1/x \quad , \quad Q(x) = e^x/x$

$I.F = e^{\int 1/x\, dx}$

$= e^{\log x}$

$I.F = x$

$\therefore y(I.F) = \int Q(x)(I.F)dx$

$y(x) = \int e^x/x \cdot x\, dx$

$y\, x = \int e^x dx$

$xy = e^x + c$

11 $e^x\, dy/dx + 2e^x y = 1$

$e^x(dy/dx + 2y) = 1$

$dy/dx + 2y = 1/e^x$

Comparing with $dy/dx + P(x)y = Q(x)$

v) $e^{2x}\, dy/dx + 2e^{2x} y = 2x$

$e^{2x}(dy/dx + 2y) = 2x$

$dy/dx + 2y = 2x/e^{2x}$

comparing with $dy/dx + P(x)\, y = Q(x)$

$\therefore P(x) = 2 \quad Q(x) = 2x/e^{2x}$

$\therefore I.F = e^{\int 2\, dx}$

$= e^{2x}$

$\therefore y(I.F) = \int Q(x)\,(I.F)\, dx$

$y(e^{2x}) = \int \frac{2x}{e^{2x}} \cdot (e^{2x})\, dx$

$y e^{2x} = 2\frac{x^2}{x} + c$

$y e^{2x} = x^2 + c$

vi) $\sec^2 x + \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$\sec^2 x \tan y\, dx = -\sec^2 y \tan x\, dy$

$\frac{\sec^2 x}{\tan x} dx = \frac{-\sec^2 y}{\tan y} dy$

$\therefore \int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy$

$\therefore P(x) = 2 \qquad Q(x) = 1/e^x$

$I.F = e^{\int 2\,dx}$

$= e^{2x}$

$y(I.F) = \int Q(x)\,(I.F)\,dx$

$y e^{2x} = \int 1/e^x \; e^2 x\,dx$

$y e^{2x} = \int e^{-x} \cdot e^{ex}\,dx$

$y e^{2x} = e^x + C$

iii) $dy/dx = \frac{\cos x}{x} - 2y$

$x\,dy/dx + 2y = \cos x/x$

$dy/dx + (2/x)y = \cos x/x^2$

comparing with $dy/dz + P(x)y = Q(x)$

$\therefore P(x) = 2/x \quad ; \quad Q(x) = \frac{\cos x}{x}$

$I.F = e^{\int 2/x\,dx}$

$= e^{2\log x}$

$= e^{\log x^2}$

$= I.F = x^2$

$\therefore y(I.F) = \int Q(x)\,(I.F)\,dx$

$y(x^2) = \int \frac{\cos x}{x^2}(x^2)\,dx$

$x^2 y = \sin x + C$

$x\,dy/dx + 3y = \sin x / x^2$

$dy/dx + (3/x)\,y = \sin x / x^2$

comparing with $dy/dz + P(x)\,y = Q(z)$

$P(x) = 3x \qquad Q(x) = \dfrac{\sin x}{x^3}$

$$I.F = e^{\int 3/x\,dx}$$
$$= e^{3\log x}$$
$$= e^{\log x^3}$$
$$= x^3$$

$$\therefore y(I.F) = \int Q(x)(I.F)\,dx$$
$$y(x^3) = \int \frac{\sin x}{x^3}(x^3)\,dx$$
$$x^3 y = -\cos x + c$$

viii) $dy/dx = \dfrac{2+3y-1}{6x+9y+6}$

$dy/dx = \dfrac{2x+3y-1}{3(2x+3y+2)}$

Put $\dfrac{2x+3y-4}{2+3dy/dx + \dfrac{dv}{dx}}$

$\dfrac{dy}{dx} = 1/3\left(dv/dx - 2\right)$

$1/3\left(dv/dx - 2\right) = \dfrac{v-1}{3(v+2)}$

$\dfrac{dv}{dx} = \dfrac{v-1}{v+2} + 2$

$\dfrac{dv}{dx} = \dfrac{v-1+2v+4}{v+2}$

$\dfrac{dv}{dx} = \dfrac{3v+3}{v+2}$

$\dfrac{v+2dv}{3(v+1)} = dx$

$\log|\tan x| = -\log|\tan y| + c$

$\log|\tan x| + \log|\tan y| = c$

$\log|\tan x \cdot \tan y| = c \Rightarrow \tan x \cdot \tan y = e^c$

ii) $dy/dx = \sin^2(x-y+1)$

Put $x-y+1 = u$

$1 - dy/dx = du/dx$

$dy/dx = 1 - du/dx$

$\therefore 1 - du/dx = \sin^2 u$

$1 - \sin^2 u = du/dx$

$dx = du/(1-\sin^2 u)$

$\int dx = \int \sec^2 u \, du$

$x = \tan u + c$

but $u = x+y-2$

$x = \tan(x+y-1) + c$

$\frac{1}{3}\int\left(\frac{u+1+1}{u+1}\right)du + \int dx$

$\frac{1}{3}\int 1 + \frac{1}{u+1}\,du = \int dx$

$\frac{1}{3}(u + \log(u+1)) = x + c$

But $u = 2x + 3y$

$\therefore\ 2x+3y+\log|2x+3y+1| = 3x+c$

or

$3y-x-\log|2x+3y+1| + c$

11/01/2020

Practical - 8

Using Euler's Method for the following

1) $dy/dx = y + e^{x} - 2$, $y(0) = 2$, $h = 0.5$ find $y(2)$

$f(x) = y f e^{x} - 2$ $\quad x_0 = 0$

$y(0) = 2$ $\quad h = 0.5$

$y(2) = ?$

n	x_n	y_0	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$y(2) = 9.8215$

Q2 $dy/dx = 3x^2+1$ $y(1)=2$ find $y(2)$

$h = 0.5$ & $h = 0.25$

$y_0 = 2$ $x_0 = 1$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$y(2) = 7.7875$

n	x_n	y_n	$f(x_n, y_n)$	x_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.6426	299.960
4	2	299996		

$y(2) = 299.960$

Q3) $dy/dx = 1+y^2$ $y(0)=0$, $h=0.2$ find $y(1)$

$x_0 = 0$, $y_0 = 0$

$h = 0.2$

x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0.2
0.2	0.2	1.04	0.408
0.4	0.408	1.1664	0.6412
0.6	0.6412	1.411	0.9234
0.8	0.9234	1.8526	1.2939
0.1	1.2939		

$y(1) = 1.2939$

04 $dy/dx = \sqrt{x/y}$ $y(0)=1$, $h=0.2$ find $y(1)$

$x_0 = 0$ $y(0)=1$ $h=0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.0059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3517	0.7694	1.5051
5	1	1.5051		

$y(1) = 1.5051$

Q5) $dy/dx = \sqrt{xy} + 2$

$y(1) = 1$ find $h = 0.2$

$y_0 = 1$ $x_0 = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(2) = 3.6$

17/01/2020

Practical-9

Aim:- limits & partial order derivatives

i] $\lim_{(x,y) \to (-4,7)} \frac{x^3 - 3y + y^2 - 1}{xy + 5}$

$\lim_{(x,y) \to (-4,7)} \frac{x^3 - 3y + y^2 - 1}{xy + 5}$

At $(-4,-1)$ Denominator $\neq 0$

$\therefore$ By applying limit

$= \frac{(-4)^3 - 3(-4) + (-4)^2 - 1}{-4(7) + 5}$

$= -61/9$

ii] $\lim_{(x,y) \to (-2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

$\rightarrow \lim_{(x,y) \to (-2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

At $(-2,0)$ denominator $\neq 0$

$\therefore$ By applying limit

$= \frac{(0+1)(4+0-8)}{2}$

$= -4/2 = -2$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}$$

At $(1,1,1)$ denominator $\neq 0$

$$= \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}$$

$$= \lim_{(x,y,z)\to(1,1,1)} x+yz / x^2$$

= on applying limit

$$= \frac{1+1(1)}{1^2} = 1$$

ii] $f(x,y) = xye^{x^2}+y^2$

$$\therefore f(x) = \frac{\delta f}{\delta x}(x,y)$$

$$= \frac{\delta(xye^{x^2}+y^2)}{\delta x}$$

$$= ye^{x^2+y^2}(2x)$$

$$= f_x = 2xye^{x^2+y^2}$$

$$f_y = \delta f(x,y)/\delta y$$

$$= \frac{\delta(xye^{x^2}+y^2)}{\delta x}$$

$$= xe^{x^2+y^2}(2y)$$

$$\therefore f_y = 2yxe^{2x^2+y}$$

iv] $f(x,y) = e^x\cos y$

$$\Rightarrow f_x = \frac{\delta x}{\delta x} f(x,y)$$

$$= \frac{\delta(e^x\cos y)}{\delta x}$$

$$f_x = e^x\cos y$$

$$f_y = \frac{\delta f(x,y)}{\delta y}$$

$$= \delta(e^x\cos y)/\delta y$$

$$f_y = -e^x\sin^2 y$$

iii) $f(x,y) = x^3y^2-3x^2y+y^3+1$

$$f(x) = \delta f(x,y)/\delta x$$

$$= \frac{\delta f(x^3y^3-3x^2y+y^3}{\delta x}$$

$$= 3x^2y^2-6xy$$

$$t_y = \frac{\delta f(x,y)}{\delta y}$$

$$= \frac{\delta f\,(x^3y^2 - 3x^2y + y^3 + 1)}{\delta y}$$

$$= 2x^3y - 3x^2 + 3y^2$$

Q3 i) $t(x,y) = 2x/1+y^2$

$$\Rightarrow f_x = \frac{\delta f\,(x,y)}{\delta x}$$

$$= \frac{\delta f(2x)(1+y^2)}{\delta x}$$

$$= \frac{\dfrac{1+y^2\delta(2x)}{\delta x} - 2x\,\dfrac{\delta(1+y^2)}{\delta x}}{(1+y^2)^2}$$

$$= 2+2y^2/(1+y^2)^2$$

$$= 2/1+y^2 //$$

At $f(0,0)$

$= 2/1+0 = 2$

$$\delta y = \frac{\delta + (2x)(1+y^2)}{\delta y}$$

$$= 1+y^2 \quad \frac{\frac{\delta(2x)}{\delta x} - 2x \, \delta/\delta x \, (1+y^2)}{(1+y^2)^2}$$

$$\therefore \quad \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

$$= \quad -4xy/(1+y^2)^2$$

At $(0,0)$

$$= \frac{-4(0)(0)}{(1+0)^2} = 0//$$

4 i] $t(x,y) = y^2xy/x^2$

$$tx = \frac{x^2 \frac{\delta + (y^2 - xy)}{\delta x} - (y^2 - xy) \, \delta/\delta x \, (x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$\Rightarrow \quad \frac{-x^2y - 2x(y^2 - xy)}{x^4}$$

$f_y = 2y - x/x^2$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2y - 2x(y^2 - xy)}{x^4}\right)$$

$$= \frac{x^4\left(\partial/\partial x\,(-x^2y - 2xy^2 + 2x^2y)\right) - (-x^2y - 2xy + 2x^2y)}{(x^4)^2}$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2x^2y)}{x^6} \quad \text{---(i)}$$

$$f_{yy} = \partial/\partial y\,(2y - x/x^2)$$

$$= \frac{2-0}{x^2} = 2/x^2 \quad \text{---(ii)}$$

$$f_{yx} = \partial/\partial x\,(2y - x/x^2)$$

$$= \frac{x^2\,\partial/\partial x\,(xy - x) - (2y - x)\,\partial/\partial x\,(x^2)}{(x^2)^2}$$

$$= \frac{-x^2 - 4xy - 2x^2}{x^4} \quad \text{---(iii)}$$

from (iii) & (ii)

$f_{xy} = f_{yx}$

$f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$\Rightarrow f_x = \frac{\partial}{\partial x}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 3x^2 + 6xy^2 - 2x/x+1$

$f_y = \frac{\partial}{\partial y}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 6x^2y$

$$f_{xx} = 6x + 6y^2 - \frac{\left(x^2+1\frac{\partial(2x)}{\partial x} - 2x\left(\frac{\partial}{\partial x}(x^2+1)\right)\right)}{(x^2+1)^2}$$

$$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right) \quad \ldots (i)$$

$f_{yy} = \frac{\partial}{\partial y}(6x^2y)$

$= 6x^2 \ldots (ii)$

$f_{xy} = \frac{\partial}{\partial y}(8x^2 + 6xy^2 - 2x/x^2+1)$

$= 0 + 12xy + 0$

$= 12xy \ldots (iii)$

$f_{yx} = \frac{\partial}{\partial x}(6x^2y)$

$= 12xy \ldots (iv)$

from iii & iv

$f_{xy} = f_{yx}$

③ $f(x,y) = \sin(x,y) + e^{x+y}$

$\rightarrow f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{x+y}$

$f_x = \frac{\partial}{\partial x}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)(y) + e^{1+x}$

$= -y^2\sin(xy) + e^{x+y} \quad \ldots (i)$

$f_{yy} = \frac{\partial}{\partial y}(x\cos(xy) + e^{x+y})$

$= -x \cdot \sin(xy) + e^{x+y} \ldots (ii)$

$f_{xy} = \frac{\partial}{\partial y}(y\cos(xy) + e^{x+y})$

$= -y^2\sin(xy) + \cos(xy) + e^{x+y} \ldots (iii)$

$f_{yx} = \frac{\partial}{\partial x}(x\cos(xy) + e^{x+y})$

$= -x^2\sin(xy) + \cos(xy) + e^{x^2+y} \ldots (iv)$

from (iii) & (iv)

$f_{xy} \neq f_{yx}$

1) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\Rightarrow f(1,1) = \sqrt{1^2+1^2} = \sqrt{2}$

$\Rightarrow f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$

$= x/\sqrt{x^2+y^2}$

$f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= y/\sqrt{x^2+y^2}$

f_x at $(1,1) = 1/\sqrt{2}$

f_y at $(1,1)$ ~~at~~ $= 1/\sqrt{2}$

$l(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - 2/\sqrt{2}$

$= x + y/\sqrt{2}$

2) $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

$\Rightarrow f(\pi/2, 6) = 1 - \pi/2, 0 - 1 - \pi/2$

$f_x = 0 - 1 + y\cos x$

f_x at $(\pi/2, 0)$

$f_y = 0 - 0 + \sin x$

f_y at $(\pi/2, 0) = \sin \pi/2$

$L(x,y) = f(a,b) + f_x(a,b)(x-a)(x-a) + f_y(a,b)(y-b)$

$= 1 - \pi/2 + (x-1)(x - \pi/2) + 1(y-b)$

$= 1 - \pi/2 - x + \pi/2 + y$

$= 1 - x + y$

3) $f(x,y) = \log x + \log y$ at $(1,1)$

$\Rightarrow f(1,1) = \log(1) + \log(1) = 0$

$f_x = 1/x$

f_x at $(1,1) = 1/$

$f_y = 1/y$

f_y at $(1,1) = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= \underline{\underline{x + y - 2}}$

Practical-10

Find the directional derivative of the following function at given points & in the direction of given vector

i) $f(x,y) = x + 2y - 3$ $\quad a = (1,-1)$ $\quad u = 3i - j$

Here, $u = 3i - j$ is not a unit vector

$$|\bar{u}| = \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$f(a) = f(1,-1) = 1 + 2(-1) - 3 = 1 - 2 - 3 = -4$$

$$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$= f\left(1 + \frac{3}{\sqrt{10}}, -1 - \frac{h}{\sqrt{10}}\right)$$

$$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$$

$$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$$

$$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$$

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{4 + h/\sqrt{10} + 4}{h}$$

$$D_u f(a) = 1/\sqrt{10}$$

ii) $f(x) = y^2 - 4x + 1$ $\quad a(3,4)$ $\quad u = i + 5j$

Hence $u = i + 5j$ is not a unit vector

$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

Unit vector along u is $\frac{u}{|u|} = 1/\sqrt{26}\,(1, 5)$

$= \left(1/\sqrt{26},\ 5/\sqrt{26}\right)$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$f(a+hu) = f(3,4) + h\left(1/\sqrt{26},\ 5/\sqrt{26}\right)$

$= f\left(3 + h/\sqrt{26},\ 4 + 5h/\sqrt{26}\right)$

$f_{x,y}(a+hu) = \left(4 + 5h/\sqrt{26}\right)^2 - 4\left(3 + h/\sqrt{26}\right) + 1$

$= 16 + 25h^2/26 + 40h/\sqrt{26} - 12 - 4h/\sqrt{26} + 1$

$= 25h^2/26 + 40h/\sqrt{26} - 4h/\sqrt{26} + 5$

$\frac{25h^2}{26} + 40h/\sqrt{26} - 4h/\sqrt{26} + 5$

$25h^2/26 + \frac{40h - 4h}{\sqrt{26}} + 5$

$25h^2/26 + 36h/\sqrt{26} + 5$

$Df(a) = \lim_{h \to 0} \; 25h^2/26 + 36h/\sqrt{26} \Big/ h$

$$\frac{\cancel{h}\left(25h/26 + 36/\sqrt{26}\right)}{\cancel{h}}$$

$\therefore Df(a) = 25h/26 + 36/\sqrt{26}$

) $2x+3y$ $\quad a = (1,2),\ u = (3i + 4j)$

Here $u = 3i + 4j$ is not a unit vector

$|\bar{u}| = \sqrt{3^2 + 4^2} = \sqrt{26} = 5$

Unit vector along u is $\frac{u}{|u|} = 1/5\,(3,4)$

$= 3/5,\ 4/5)$

$f(a) = f(1,2) = 2(1) + 3(2) = 8$

$f(a+hu) = f(1,2) + h(3/5,\ 4/5)$

$= f(1 + 3h/5,\ 2 + 4/5)$

$f(a+hu) = 2(1 + 3h/5) + 3(2 + 4h/5)$

$= 2 + 6h/5 + 6 + 12h/5$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{18h}{5} + 8 - 8}{h}$$

$$= 18/5$$

22 find gradient vector for the following fun

i) $f(x,y) = x^y + y^x$ # $a = (1,1)$

$$f(x) = y \cdot x^{y-1} + y^x \log y$$

$$f_y = x^y \log x = x y^{x-1}$$

$$\nabla f(x,y) = (f_x, f_y)$$

$$= (y x^{y-1} + y^x \log y, x^y \log x + x y^{x-1})$$

$$f(1,1) = (1+0, 1+0)$$

$$= 1, 1$$

ii) $f(x,y) = (\tan^{-1} x) y^2$ $a = (1,-1)$

$$f_x = y^2/(1+x^2)$$

$$f_y = 2y \tan^{-1} x$$

$$\nabla f(x,y) = (f_x, f_y)$$

$$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x\right)$$

$f(1,-1) = (1/2, \tan^{-1}(1)(-2))$

$= \left(\frac{1}{2}, \frac{\pi}{4}(-2)\right)$

$= \left(\frac{1}{2}, -\frac{\pi}{2}\right)$

$f(x,y,z) = xyz - e^{x+y+z}$, $a = (+1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z},$

$xy - e^{x+y+z}$

$f(1,-1,0) = (-1)(0) - e^{(1+(-1)),0)}, (1)(0) - e^{1+(-1)+0}),$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

23) Find the eq. of tangent and normal to each of the following curves at given pts.

$x^2 - xy + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y\, 2x + e^{xy}\, y$

$f_y = x^2(-\sin y) = e^{xy} \cdot x$

$(x_0, y_0) = (1, 0) = \therefore x_0 = 1,\ y_0 = 0$

eq^n of tangent

$$f_x(x - x_0) + f_y(y - y_0) = 0$$

$f_x(x_0, y_0) = \cos 0 \quad 2(1) + e^0\, 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2 \quad (-\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1) + 1(y - 0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ It is the required eq^n of tangent

eq^n of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$(1)(1) + 2(y) + d = 0$

$1 + 2y + d = 0$ at $(1, 0)$

$= 1 + 2(0) + d = 0$

$d + 1 = 0$

$\therefore d = -1$

$x^2+y^2-2x+3y+2=0$ at $(2,-2)$

$f_x = 2x+0-2+0+0$

$= 2x-2$

$f_y = 0+2y-0+3+0$

$= 2y+3$

$(x_0, y_0) = (2,-2)$ $\therefore x_0 = 2$, $y_0 = -2$

$f_x(x_0, y_0) = 2(2)-2 = 2$

$f_y(x_0, y_0) = 2(-2)+3 = -1$

eq^n of tangent

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2x-2-y-2 = 0$

$2x-y-4=0$ → It is a eq. eq^n of tangent

eq^n of Normal

$= ax+by+c=0$

$bx+ay+d=0$

$= -1(x)+2(y)+d=0$

$-x+2y+d=0$ at $(2,-2)$

$-2+2(-2)+d=0$

$-2-4+d=0$

$-6+d=0$

$d=6$

21 Find the eq^n of tangent & normal line to each of the following surface:-

i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 + 0$

$= 2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2, 1, 0) \therefore x_0 = 2, y_0 = 1, z_0 = 0$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

eq^n of tangent

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$= 4(x - 2) + 3(y - 1) + 0(z - 0) = 0$

$= 4x - 8 + 3y - 3$

$4x + 3y - 11 = 0 \rightarrow$ req. eqn of tangent

Eq^n of normal at $(4, 3, -11)$

$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{0}$

$3xyz - x - y + z = 4$ at $(1, -1, 2)$

$3xyz - x - y + z + 4 = 0$ at $(1, -1, 2)$

$f_x = 3yz - 1 - 0 + 0 + 0$
$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$
$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$
$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2) \therefore x_0 = 1, y_0 = -1, z_0 = 2$

$f_x(x_0, y_0, z_0) = -3(-1)(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eq^n of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$

req eq^n of tangent.

Eq^n of normal at $(-7, 5, -2)$

$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$

$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$

Q5 Find the local maxima & minima for following function.

i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$
$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$
$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$
$3(2x - y + 2) = 0$
$2x - y + 2 = 0$
$2x - y = -2$ — ①

$f_y = 0$

$2y - 3x - 4 = 0$
$2y - 3x = 4$ → ②

Multiply eqn 1 with 2

$4x - 2y = -4$
$2y - 3x = 4$
$x = 0$

Substitute value of x in eqn ①

$2(0) - y = -2$
$-y = -2$
$y = 2$

critical point are $(0,2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore f$ has maximum at $(0, 2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0, 2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= -4$

) $f(x, y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0 \rightarrow (1)$

$f_y = 0$

$3x^2 - 2y = 0 \rightarrow (2)$

Multiply equ (1) with (3)

(2) with (4)

$12x^2 + 9y = 0$

$-12x^2 - 8y = 0$

$11y = 0$

$y = 0$

Substitute value of y in eq^n ①

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

Critical pts is $(0,0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0,0)$

$= 24(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - (s)^2$

$= 0 - 0 = 0$

$r = 0$ & $rt - s^2 = 0$

(nothing to say)

$f(x,y)$ at $(0,0)$

$2(0)^4 + 3(0^2/0)$

$= 0 + 0 + 0$

$= 0$

iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 7$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \therefore 2x + 2 = 0$

$x = -2/2 \therefore x = -1$

$f_y = 0 \qquad -2y + 8 = 0$

$y = -8/-2 = 4$

$\therefore y = 4$

~~or~~ Critical point is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x,y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) = 70$

$1 + 16 - 2 + 32 - 70$

$17 + 30 - 70$

$37 - 70$

$= 33$

01/02/2022