ТЕТРАДЬ

для типового расчета №2

по ИиТУ

учени_______ класса_______

РК6-26Б школы_______

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16 вариант

$$y'' = \frac{1}{y^3}$$

$$y' = p(y)$$

$$y'' = p' \cdot p$$

$$p' \cdot p = \frac{1}{y^3}$$

$$\frac{dp}{dy} \cdot p = \frac{1}{y^3}$$

$$p\,dp = \frac{1}{y^3}\,dy$$

$$\int p\,dp = \int \frac{1}{y^3}\,dy$$

$$\frac{p^2}{2} = \frac{1}{y^2} \cdot \frac{1}{-2} + C_1$$

$$p^2 = -\frac{1}{y^2} + C_1$$

$$p = \sqrt{-\frac{1}{y^2} + C_1}$$

$$\frac{dy}{dx} = \sqrt{-\frac{1}{y^2} + C_1}$$

$$\frac{dy}{\sqrt{-\frac{1}{y^2} + C_1}} = dx$$

$$\int \frac{dy}{\sqrt{-\frac{1}{y^2} + C_1}} = \int dx$$

№1

$$\int \frac{1}{\sqrt{-\frac{1}{y^2} + C_1}}\,dy = \int \frac{1}{\sqrt{\frac{-1 + C_1 y^2}{y^2}}}\,dy =$$

$$= \int \frac{y}{\sqrt{C_1 y^2 - 1}}\,dy = \frac{1}{2}\int \frac{1}{\sqrt{C_1 y^2 - 1}}\,dy^2 =$$

$$= \frac{1}{2C_1}\int \frac{1}{\sqrt{C_1 y^2 - 1}}\,d(C_1 y^2 - 1) = \frac{1}{2C_1} \cdot \sqrt{C_1 y^2 - 1} \cdot 2 =$$

$$= \frac{1}{C_1}\sqrt{C_1 y^2 - 1}.$$

$$\frac{1}{C_1}\sqrt{C_1 y^2 - 1} = x + C_2$$

$$x = \frac{1}{C_1}\sqrt{C_1 y^2 - 1} + C_2$$

№2

$xy'' - (x+1)y' + e^x = 0 \quad y(1) = e; y'(1) = 2e$

$y' = p \quad y'' = p'$

$xp' - (x+1)p + e^x = 0$

$p = u \cdot v$

$p' = u'v + v'u$

$xu'v + v'ux - (x+1)uv + e^x = 0$

$u(v'x - (x+1)v) + xu'v + e^x = 0$

$v'x - (x+1)v$

$\frac{dv}{dx}x = (x+1)v$

$\frac{dv}{v} = \frac{x+1}{x}dx \to \left(1 + \frac{1}{x}\right)dx$

$\ln v = x + \ln x$

$v = e^{x + \ln x}$

$v = e^x x$

$u' x v = -e^x$

$u' \cdot x \cdot e^x x = -e^x$

$u' = -\frac{1}{x^2}$

$\frac{du}{dx} = -\frac{1}{x^2}$

$du = -\frac{1}{x^2}dx$

$u = -\frac{1}{x} \cdot \frac{1}{-1} + C_2$

$u = -\frac{1}{x} + C_2$

$p = e^x x\left(\frac{1}{x} + C_2\right)$

$p = e^x + e^x x C_2 = y'$

$y'(1) = 2e$

$e + eC_2 = 2e$

$\underline{C_2 = 1}$

$\frac{dy}{dx} = e^x + e^x x$

$y = e^x + e^x(x+1) + C_3$

$y(1) = e$

$e = e + e \cdot 0 + C_3$

$\underline{C_3 = 0}$

Ответ: $y = e^x \cdot x$

№3

$y'' - 2y' - 8y = 16e^{-2x} + 17\sin x$

$y'' - 2y' - 8y = 0$

$D = 4 + 8\cdot 4 = 36$

$x_1 = \frac{2-6}{2} = -2$

$x_2 = 4$

Общ. решение: $y = C_1 e^{-2x} + C_2 e^{4x}$

Частн. решения:

$y'' - 2y' - 8y = 16e^{-2x}$

$y_1 = Ae^{-2x}x$

$y'' - 2y' - 8y = 17\sin x$

$y_2 = B\cos x + C\sin x$

$y = y_1 + y_2 = Ae^{-2x}x + B\cos x + C\sin x$

$y' = Ae^{-2x} - 2Axe^{-2x} - B\sin x + C\cos x$

$y'' = 4Ae^{-2x}x - 4Ae^{-2x} - B\cos x - C\sin x$

$(4Ae^{-2x}x - 4Ae^{-2x} - B\cos x - C\sin x) - 2(Ae^{-2x} - 2Axe^{-2x} -$
$- B\sin x + C\cos x) - 8(Ae^{-2x}x + B\cos x + C\sin x) =$
$= 16e^{-2x} + 17\sin x$

4Ae

+2B

=

–

=

$$\cancel{4Ae^{-2x}x} - 4Ae^{-2x} - B\cos x - C\sin x + \cancel{4Ae^{-2x}x} - 2Ae^{-2x} +$$
$$+ 2B\sin x - 2C\cos x - \cancel{8Ae^{-2x}x} - 8B\cos x - 8C\sin x =$$
$$= 16e^{-2x} + 17\sin x$$

$$-6Ae^{-2x} + \cos x(-B - 2C - 8B) + \sin x(-C + 2B - 8C) =$$
$$= 16e^{-2x} + 17\sin x$$

$$-6A = 16 \Rightarrow A = -\frac{8}{3}$$

$$\begin{cases} -9B - 2C = 0 \\ 2B - 9C = 17 \end{cases} \qquad 2C = -9B \quad C = -\frac{9}{2}B$$

$$2B + 9\frac{9}{2}B = 17$$
$$4B + 81B = 34$$
$$85B = 34$$
$$B = \frac{34}{85} \Rightarrow C = -\frac{9}{2}\cdot\frac{34}{85} = -\frac{162}{85}$$

$$y = C_1 e^{-2x} + C_2 e^{4x} - \frac{8}{3}e^{-2x}x + \frac{34}{85}\cos x - \frac{162}{85}\sin x$$

№4

$4y'' + 4y' + y = 2x^2 - 4 \qquad y(0) = 4;\ y'(0) = 0$

$D = 16 - 4\cdot 4 = 0$

$\lambda = -\frac{1}{2}$

Общ. решение: $y = C_1 e^{-\frac{x}{2}} + C_2 x e^{-\frac{x}{2}}$

$y(0) = 4$

$4 = C_1$

$y' = C_1 e^{-\frac{x}{2}} \cdot \left(-\frac{1}{2}\right) + C_2 x e^{-\frac{x}{2}} \left(-\frac{1}{2}\right) + C_2 e^{-\frac{x}{2}}$

$y'(0) = 0$

$0 = 4\left(-\frac{1}{2}\right) + C_2 \Rightarrow C_2 = 2$

$y = 4e^{-\frac{x}{2}} + 2x e^{-\frac{x}{2}}$

Частные решения:

$y = Ax^2 + Bx + C$

$y' = 2Ax + B$

$y'' = 2A$

$4\cdot 2A + 4(2Ax + B) + Ax^2 + Bx + C = 2x^2 - 4$

$Ax^2 + x(8A + B) + 8A + 4B + C = 2x^2 - 4$

$A = 2$

$8A + B = 0 \Rightarrow B = -16$

$16 - 4\cdot 16 + C = -4$

$C = 44$

Ответ: $4e^{-\frac{x}{2}} + 2x e^{\frac{x}{2}} + 2x^2 - 16x + 44$

№5

$x^2 y'' - 4xy' + 6y = x^4 \sin x \qquad y_1(x) = x^2$

Общ. решение:

$x = e^t \qquad y' = e^{-t}\left(\frac{dy}{dt}\right) \quad y'' = e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$

$e^{2t} \cdot e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) - 4e^t \cdot e^{-t}\frac{dy}{dt} + 6y = 0$

$y'' - 5y' + 6y = 0$

$D = 25 - 4 \cdot 6 = 1$

$t_1 = \frac{5-1}{2} = 2 \qquad t_2 = 3$

Общ. реш. одн. ур.: $y = C_1 e^{2x} + C_2 e^{3x}$

Ответ: $y = C_1 e^{2x} + C_2 e^{3x} + x^2$

№6

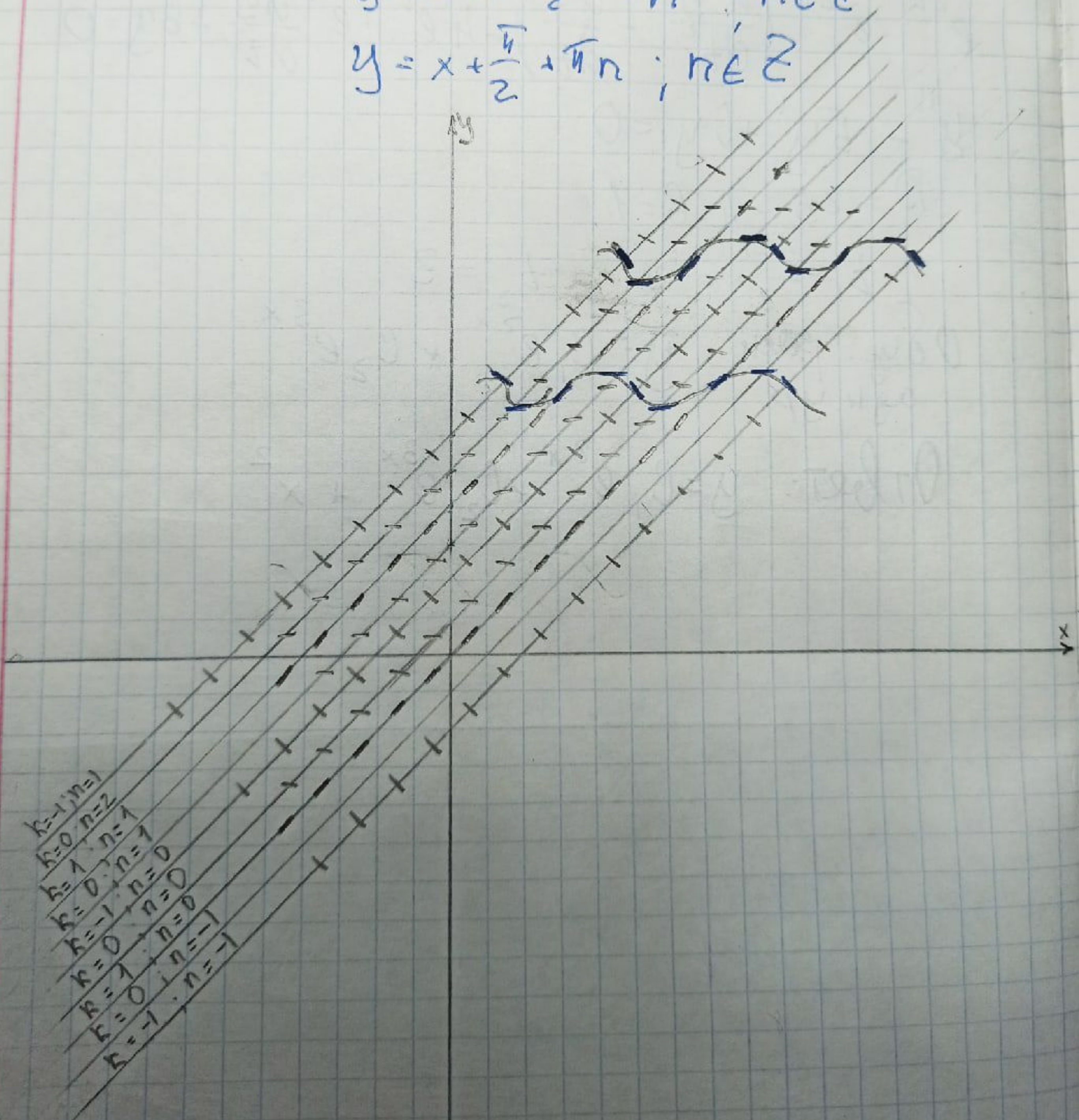
$y' = \cos(y - x)$

$\cos(y - x) = k \quad ; \; k \in [-1; 1]$

При $k = 0$: $\cos(y - x) = 0$

$y - x = \frac{\pi}{2} + \pi n \; ; \; n \in \mathbb{Z}$

$y = x + \frac{\pi}{2} + \pi n \; ; \; n \in \mathbb{Z}$



N5

$x^2y'' - 4xy' + 6y = x^4 \sin x \qquad y_1(x) = x^2$

$x^2y'' - 4xy' + 6y = 0$

$y = z y_1$

$y' = z' y_1 + y_1' z$

$y'' = z'' y_1 + z' y_1' + y_1'' z + y_1' z' = z'' y_1 + 2z' y_1' + y_1'' z$

$x^2(z'' x + 2z') - 4x(z' x + z) + 6zx = 0$

$z'' x^3 + 2x^2 z' - 4x^2 z' - 4xz + 6zx = 0$

$z'' x^3 - 2x^2 z' + 2zx = 0$

$z'' x^2 - 2x z' + 2z = 0$

$z = x^\lambda \; ; \; z' = \lambda x^{\lambda-1} \; ; \; z'' = \lambda(\lambda-1) x^{\lambda-2}$

$x^2\left(\frac{d^2(x^\lambda)}{dx^2}\right) - 2x \frac{d(x^\lambda)}{dx} + 2x^\lambda = 0$

$x^2 \cdot \lambda(\lambda-1) x^{\lambda-2} - 2x \lambda x^{\lambda-1} + 2x^\lambda = 0$

$x^\lambda(\lambda(\lambda-1) - 2\lambda + 2) = 0$

$x^\lambda(\lambda^2 - 3\lambda + 2) = 0$

$\lambda_1 = 1 \quad ; \; \lambda_2 = 2$

$y = C_1 x + C_2 x^2$ - общ. реш. однор. ур-я

$$\begin{pmatrix} x & x^2 \\ 1 & 2x \end{pmatrix} \begin{pmatrix} C_1' \\ C_2' \end{pmatrix} = \begin{pmatrix} 0 \\ x^4 \sin x \end{pmatrix}$$

$$\begin{cases} C_1' x + C_2' x^2 = 0 \\ C_1' + C_2' 2x = x^4 \sin x \end{cases}$$

$C_1' = x^4 \sin x - C_2' 2x$

$x^5 \sin x - C_2' \cdot 2x^2 + C_2' x^2 = 0$

$x^5 \sin x = C_2' \cdot x^2$

$C_2' = x^3 \sin x$

$C_2 = \int x^3 \sin x \, dx = -\int x^3 d(\cos x) =$

$= -x^3 \cos x + \underbrace{3\int \cos x \cdot x^2 dx}_{3\int x^2 d\sin x} = -x^3 \cos x + 3x^2 \sin x -$

$- 6\int \sin x \cdot x \, dx = -x^3 \cos x + 3x^2 \sin x + 6x \cos x -$

$- 6\int \cos x \, dx = -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6\sin x =$

$= \cos x (6x - x^3) + \sin x (3x^2 - 6) + \tilde{C}_2$

$C_1' = x^4 \sin x - 2x x^3 \sin x = -x^4 \sin x$

$C_1 = -\int x^4 \sin x \, dx = \ldots = (x^4 - 12x^2 + 24)\cos x -$

$- 4x(x^2 - 6)\sin x + \tilde{C}_1$

$y = ((x^4 - 12x^2 + 24)\cos x - 4x(x^2 - 6)\sin x + \tilde{C}_1)x$

$+ (\cos x (6x - x^3) + \sin x (3x^2 - 6) + \tilde{C}_2)x^2$