ТЕТРАДЬ

для Типовой расчёт по ИиДУ

16 Вариант

учени______ класса______

РК6-26Б школы______

Петрасова

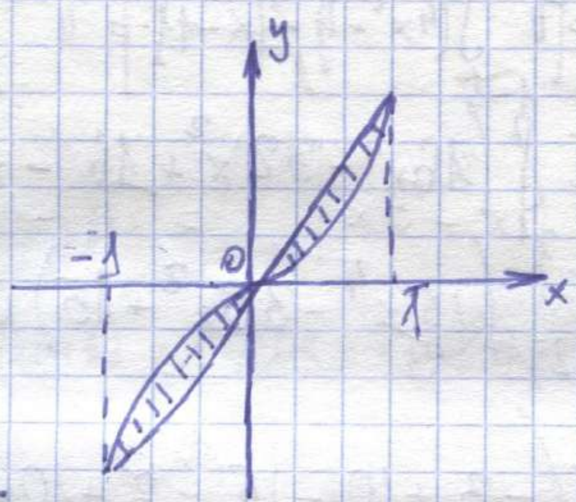
Станислава

№1

$y = \arcsin x$

$y = \frac{\pi}{2} x$

$S - ?$



$S = \left| 2\int_0^1 \frac{\pi}{2}x - \arcsin x \, dx \right| =$

$= \left| 2\left(\frac{\pi}{4}x^2 - x\arcsin x \Big|_0^1 + \int_0^1 x \frac{1}{\sqrt{1-x^2}} dx \right) \right| =$

$= \left| 2\left(-\frac{\pi}{4} + \arcsin 1 - \sqrt{1-x^2} \Big|_0^1 \right) \right| =$

$\int \frac{x}{\sqrt{1-x^2}} dx = \left| \begin{matrix} t = 1-x^2 \\ dt = -2x\,dx \end{matrix} \right| = -\frac{1}{2}\int \frac{1}{\sqrt{t}} dt = -\sqrt{t} = -\sqrt{1-x^2}$

$= \left| 2\left(-\frac{\pi}{4} + \frac{\pi}{2} - 1 \right) \right| = \left| \frac{\pi}{2} - 2 \right| = 2 - \frac{\pi}{2}$

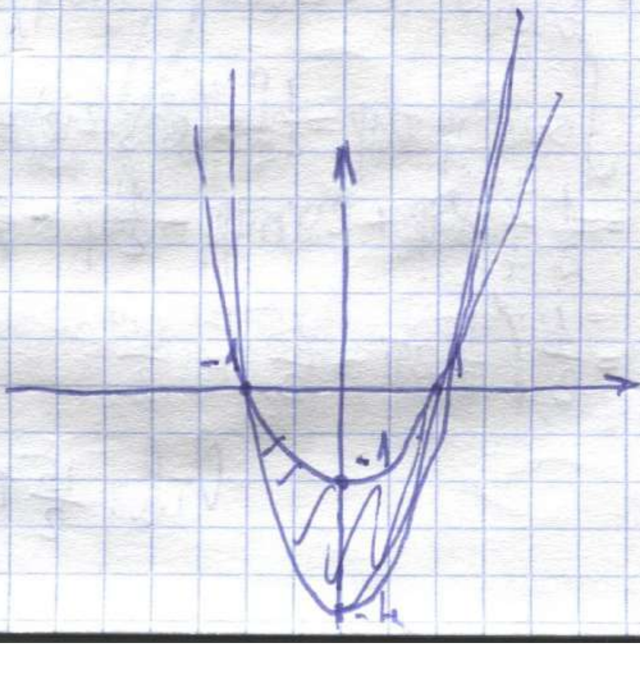
Ответ: $2 - \frac{\pi}{2}$

№2

$y = 4x^2 - 4$

$y = x^2 - 1$

$V - ?$



$$V = \left|\pi \int_{-1}^{1} (4x^2-4)^2 - (x^2-1)^2 dx\right| =$$

$$= \left|\pi \int_{-1}^{1} (16x^4 - 32x^2 + 16 - x^4 + 2x^2 - 1)\, dx\right| =$$

$$= \left|\pi \int_{-1}^{1} (15x^4 - 30x^2 + 15)\, dx = \pi \left(3x^5 - 10x^3 + 15x \Big|_{-1}^{1}\right)\right| =$$

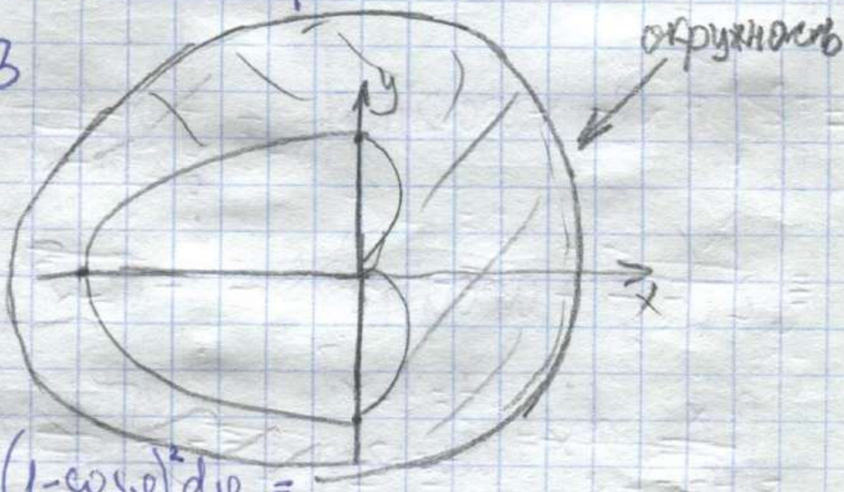
$$= \left|\pi(-3 + 10 - 15 - 3 + 10 - 15)\right| = |-16\pi| = 16\pi$$

№3

$\rho = 3$ внутри

$\rho = 1 - \cos\varphi$ вне

S - ?



$$S = \frac{1}{2}\int_0^{2\pi} 3^2 d\varphi - \frac{1}{2}\int_0^{2\pi} (1-\cos\varphi)^2 d\varphi =$$

$$= \frac{1}{2}\int_0^{2\pi} (\overbrace{9 - 1}^{8} + 2\cos\varphi - \cos^2\varphi)\, d\varphi =$$

$$\int \cos^2\varphi\, d\varphi = \frac{1}{2}\int (\cos(2\varphi) + 1)\, d\varphi = \frac{1}{2}\varphi + \frac{1}{4}\sin(2\varphi)$$

$$= \frac{1}{2}\left(8\varphi + 2\sin\varphi - \frac{1}{2}\varphi - \frac{1}{4}\sin 2\varphi \Big|_0^{2\pi}\right) =$$

$$= \frac{1}{2}\left(\frac{15}{2} 2\pi = \frac{15}{2}\pi\right.$$

Отв.: $\frac{15}{2}\pi$

№4

$$\begin{cases} x = t - \sin t \\ y = 1 + \cos t \end{cases} \qquad 0 \le t \le \pi$$

l - ?

$$l = \int_0^{\pi} \sqrt{(x')^2 + (y')^2}\, dt$$

$$(x')^2 + (y')^2 = (1-\cos t)^2 + (\sin t)^2 = 1 - 2\cos t + \cancel{\cos^2 t} + 1 - \cancel{\cos^2 t} =$$

$$= 2 - 2\cos t = 2(1-\cos t)$$

$$\int \sqrt{1-\cos t}\, dt = \sqrt{2} \int \sqrt{\sin^2 \frac{t}{2}}\, dt = $$ т.к. $t \in [0;\pi]$, где $\sin t \ge 0$ модуль не нужен

$$= \sqrt{2} \int \sin \frac{t}{2}\, dt = -2\sqrt{2} \cos \frac{t}{2}$$

$$l = -4\sqrt{2} \cos \frac{t}{2} \Big|_0^{\pi} = -4\sqrt{2} \cdot \cancelto{0}{\cos \frac{\pi}{2}} + 4\sqrt{2} \cos 0 = 4\sqrt{2}$$

Ответ: $4\sqrt{2}$

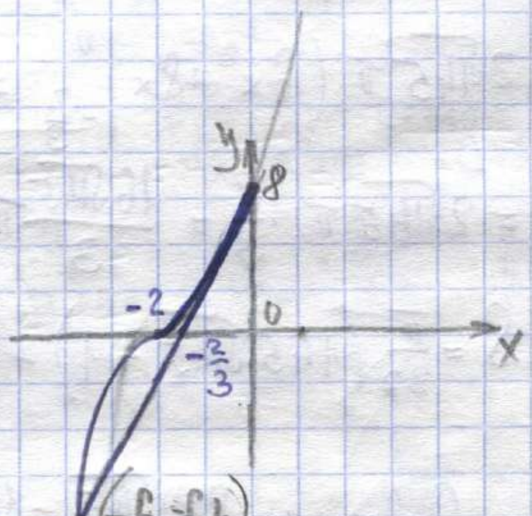
№5

$y = (x+2)^3$ $\quad x \le 0$, $y \ge 0$

кас. в т. пересеч. с осью OY

вращ. вокруг OX

$S_{пов}$ - ?



$y = ax + b \quad 8 = 0x + b \Rightarrow b = 8$

$$y_к = f'(x_0)x \underbrace{- x_0 f'(x_0) + f(x_0)}_{8}$$

$$y' = 3(x+2)^2 \cdot 1 = 12$$

$$y_к = 12x + 8$$

$$S = 2\pi \int_a^b f(x) \sqrt{1 + (f'(x))^2}\, dx$$

$$S_{пов} = S_{(x+2)^3} + S_{2\; 12x+8}$$

1) $S_1 = 2\pi \int\limits_{-2}^{0} (x+2)^3 \sqrt{1+\left(3(x+2)^2\right)^2}\, dx =$

$= 2\pi \int\limits_{-2}^{0} (x+2)^3 \sqrt{1+9(x+2)^4}\, d(x+2) = \left| t = x+2 \right| =$

$= 2\pi \int\limits_{0}^{2} t^3 \sqrt{1+9t^4}\, dt = \left| \begin{array}{l} u = 1+9t^4 \\ du = 9\cdot 4t^3 dt \\ t^3 = \dfrac{du}{36dt} \end{array} \right| = \dfrac{\pi}{18} \int\limits_{1}^{145} \sqrt{u}\, du =$

$= \dfrac{\pi}{18} \left(\dfrac{u^{3/2}}{3/2} \Big|_{1}^{145} \right) = \dfrac{\pi}{27} \left(\sqrt{145^3} - 1 \right) = \dfrac{\pi}{27} \left(145\sqrt{145} - 1 \right)$

2) $S_2 = 2\pi \int\limits_{-2/3}^{0} (12x+8) \sqrt{1+12^2}\, dx = 2\sqrt{145} \int\limits_{-2/3}^{0} (12x+8) =$

$= 2\sqrt{145}\,\pi \left(6x^2 + 8x \Big|_{-\frac{2}{3}}^{0} = 2\sqrt{145}\,\pi \cdot \left(6\cdot 0 + 8\cdot 0 - 6\cdot\dfrac{4}{9} + 8\cdot\dfrac{2}{3} \right) = \right.$

$= 2\sqrt{145}\,\pi \cdot \dfrac{8}{3} = \dfrac{16\sqrt{145}}{3}\pi$

$S = \dfrac{\pi}{27}\left(145\sqrt{145} - 1\right) + \dfrac{16\sqrt{145}}{3}\pi = \pi\, \dfrac{289\sqrt{145} - 1}{27}$

Ответ: $\pi\, \dfrac{289\sqrt{145} - 1}{27}$

№6(а)

$$\int_1^{+\infty} \ln\frac{x^2+5}{x^2+4}\,dx = \lim_{a\to+\infty}\int_1^a \ln\frac{x^2+5}{x^2+4}\,dx \;=$$

$$\int \ln\frac{x^2+5}{x^2+4}\,dx = x\ln\frac{x^2+5}{x^2+4} + \int x\,\frac{\cancel{x^2+4}}{x^2+5}\cdot\frac{2x}{(x^2+4)^{\cancel{2}}}\,dx =$$

$$= x\ln\frac{x^2+5}{x^2+4} + 2\int\frac{x^2}{(x^2+5)(x^2+4)} = x\ln\frac{x^2+5}{x^2+4} + 2\int\left(\frac{5}{x^2+5} - \frac{4}{x^2+4}\right)dx =$$

$$= x\ln\frac{x^2+5}{x^2+4} + 2\left(\frac{1}{\sqrt5}\operatorname{arctg}\frac{x}{\sqrt5} - \frac{1}{2}\operatorname{arctg}\frac{x}{2}\right)$$

$$= \lim_{a\to+\infty}\left(x\ln\frac{x^2+5}{x^2+4} + \frac{2}{\sqrt5}\operatorname{arctg}\frac{x}{\sqrt5} - \operatorname{arctg}\frac{x}{2}\,\Big|_1^a\right) =$$

$$= \lim_{a\to+\infty}\left(a\ln\frac{a^2+5}{a^2+4} + \frac{2}{\sqrt5}\operatorname{arctg}\frac{a}{\sqrt5} - \operatorname{arctg}\frac{a}{2}\right) - \underbrace{\left(\ln\frac{6}{5} + \frac{2}{\sqrt5}\operatorname{arctg}\frac{1}{\sqrt5} - \operatorname{arctg}\frac{1}{2}\right)}_{k} = \lim_{a\to+\infty} a\ln\frac{a^2+5}{a^2+4} + \frac{2}{\sqrt5}\lim_{a\to\infty}\operatorname{arctg}\frac{a}{\sqrt5} -$$

$$- \lim_{a\to+\infty}\operatorname{arctg}\frac{a}{2} - k = \lim_{a\to+\infty} a\ln\left(\frac{a^2}{a^2+4} + \frac{5}{a^2+4}\right) + \frac{2\cdot\pi}{\sqrt5\cdot 2} - \frac{\pi}{2} - k =$$

$$= \lim_{a\to+\infty} a\ln\left(\frac{1}{1+\frac{4}{a^2}} + \underbrace{\frac{5}{a^2+4}}_{\to 0}\right) + \frac{\pi}{\sqrt5} - \frac{\pi}{2} - k =$$

$$= \frac{\pi}{\sqrt5} - \frac{\pi}{2} - \ln\frac{6}{5} - \frac{2}{\sqrt5}\operatorname{arctg}\frac{1}{\sqrt5} + \operatorname{arctg}\frac{1}{2} \Rightarrow \text{интеграл сходящийся}$$

№6(б)

$$\int_1^3 \frac{\ln^2 x\,dx}{(3-x)^3} = \lim_{a\to 0+} \int_1^{3-a} \frac{\ln^2 x\,dx}{(3-x)^3} \circleddash$$

$$\int \frac{\ln^2 x\,dx}{(3-x)^3} = \frac{\ln^2 x}{(3-x)^3}\,x - \int x\,d\frac{\ln^2 x}{(3-x)^3}$$

$$\lim_{a\to 0+}\left(\frac{\ln^2 x}{(3-x)^3}\,x\Big|_1^{3-a}\right) = \lim_{a\to 0+}\left(\frac{\ln^2(3-a)\cdot(3-a)}{a^3} - \frac{\ln^2 1}{8}\right) =$$

$$= \lim_{a\to 0+}\left(\frac{\ln^2 3\cdot 3}{a^3} - \frac{\ln^2 1}{8}\right) = +\infty$$

$$\circleddash \lim_{a\to 0+}\left(\frac{\ln^2 x}{(3-x)^3}\,x\Big|_1^{3-a}\right) - \underbrace{\lim_{a\to 0+}\int_1^{3-a} x\,d\frac{\ln^2 x}{(3-x)^3}}_{m} =$$

$$= +\infty - m$$

$$m = \lim_{a\to 0+}\int_1^{3-a} x\,d\frac{\ln^2 x}{(3-x)^3}$$

чему равен предел

$m \to +\infty$: $+\infty + (+\infty) = +\infty$

$\to$ число : $+\infty + m = +\infty$

$\to -\infty$: $+\infty + (-\infty)$ нет смысла

$\to$ не существует

$\Rightarrow$ предел не имеет конечный предел $\Rightarrow$

$\Rightarrow$ интеграл расходится.

№6(в)

$$\int_0^{+\infty} \frac{e-e^{\cos x}}{x^{\frac{5}{2}}}\,dx = \int_0^1 \frac{e-e^{\cos x}}{x^{\frac{5}{2}}} + \int_1^{+\infty} \frac{e-e^{\cos x}}{x^{\frac{5}{2}}} =$$

$$= \lim_{a\to 0+} \int_{0+a}^1 \frac{e}{x^{\frac{5}{2}}} - \underbrace{\left(\lim_{a\to 0+} \int_{a+0}^1 \frac{e^{\cos x}}{x^{\frac{5}{2}}} - \lim_{a\to +\infty} \int_1^a \frac{e-e^{\cos x}}{x^{\frac{5}{2}}} \right)}_{\text{Пусть это выраж. равно } m} =$$

$$= \lim_{a\to 0+} \left(-\frac{2e}{3x^{\frac{3}{2}}} \Big|_a^1 \right) - m = \lim_{a\to 0+} \left(-\frac{2}{3}e + \frac{2e}{3a} \right) - m =$$

$$= +\infty - m$$

Далее аналогично пункту б№6 $\Rightarrow$

$\Rightarrow$ интеграл расходится