ТЕТРАДЬ

для типового расчета № 2

по лин. алгебре

учени______ класса______

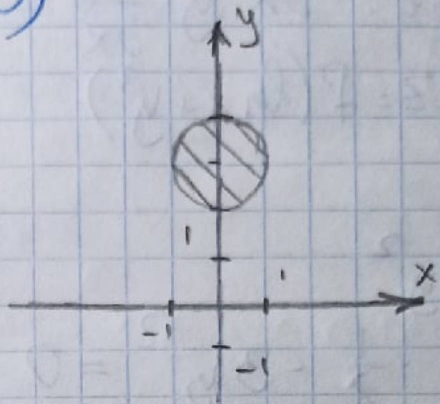
РК6-26Б ______ школы______

15 вариант

Петраков Станислав

№1

$$g(x;y) = \sqrt{1 - x^2 - (y-3)^2}$$



$$1 - x^2 - (y-3)^2 \geq 0$$

$$(y-3)^2 + x^2 \leq 1$$

$$z(x;y) = \frac{x^2 + y^2}{y} = \frac{x^2}{y} + y$$

$$z'_x = \frac{1}{y} \cdot 2x$$

$$z'_y = x^2(-1)\frac{1}{y^2} + 1 = \frac{-x^2 + y^2}{y^2}$$

$$\begin{cases} z'_x = 0 & \Rightarrow x = 0 \\ z'_y = 0 & \Rightarrow z'_y = 0 = \frac{y^2}{y^2} = 1 \end{cases}$$

Точка $(0;1)$ $\notin$ области опр. значения на границах области опред.:

$$\begin{cases} (y-3)^2 + x^2 = 1 & \Rightarrow x^2 = 1 - (y-3)^2 \\ z(x;y) = \frac{x^2}{y} + y \end{cases}$$

$$f(y) = \frac{1 - (y-3)^2}{y} + y = \frac{1 - (y^2 - 6y + 9)}{y} + y =$$

$$= -y + 6 - y + \frac{8}{y} + y = 6 - \frac{8}{y} = \frac{6y - 8}{y}$$

Найдем наиб и наим. знач.: $f'(y) = \left(6 - \frac{8}{y}\right)' = \frac{8}{y^2}$

$f'(y) = 0$; $\frac{8}{y^2} \neq 0 \Rightarrow$ Нет точек экстремума

№ 2

диф. ур-е: $3x^2y\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$

ф-я: $z = f(\ln x + y^3)$

$z'_x = \frac{1}{x}$

$z'_y = 3y^2$

$3xy^2 \cdot \frac{1}{x} - 3y^2 = 0$

$3y^2 - 3y^2 = 0$

$0 = 0$ => ф-я удовл. данному диф. ур-ю

№3

A: $(x+y\ln y)dx+(1+x\ln y)dy=0$

$f_1=x+y\ln y \quad f_2=1+x\ln y$

$\frac{\partial f_1}{\partial y}=\ln y+y\frac{1}{y}=\ln y+1$

$\frac{\partial f_2}{\partial x}=0+\ln y=\ln y$

$\ln y+1\neq\ln y$ => дифференц. форма не явл. полным дифф. ф-ями

Б: $\left(\frac{1}{\sqrt{1-x^2}}+y^2x^{y-1}\right)dx+(x^y+yx^y\ln x+5)dy=0$

$\frac{\partial f_1}{\partial y}=0+2yx^{y-1}+y^2\cdot x^{y-1}\ln x$

$\frac{\partial f_2}{\partial x}=yx^{y-1}+y(yx^{y-1}\ln x+\overbrace{x^y\cdot\frac{1}{x}}^{x^{y-1}})=$

$=yx^{y-1}+y^2x^{y-1}\ln x+x^{y-1}y$

$2yx^{y-1}+y^2x^{y-1}\ln x=yx^{y-1}+y^2x^{y-1}\ln x+x^{y-1}y$

$0=0$ ->

=> диф. форма, явл. полными диф. ф-ями

Найдем ф-ю

$$F = \int \left(\frac{1}{\sqrt{1-x^2}} + y^2 x^{y-1} \right) dx =$$

$$= \int \frac{dx}{\sqrt{1-x^2}} + y^2 \int x^{y-1} dx = \arcsin x + y^2 \frac{x^y}{y} + \varphi(y) =$$

$$= \arcsin x + y x^y + \varphi(y)$$

$$\frac{\partial F}{\partial y} = \left(\arcsin x + y x^y + \varphi(y) \right)' =$$

$$= x^y + y x^y \ln x + \varphi'(y) -$$

$$x^y + y x^y \ln x + \varphi'(y) = x^y + y x^y \ln x + 5$$

$$\varphi'(y) = 5$$

$$\varphi(y) = \int 5 dy = 5y$$

Ответ: $\arcsin x + y x^y + 5y$

N 4

$u = \arccos(xyz) + z(x^2+y^2)^{-1} \cdot \ln(yz)$

$A(0;-1;-1)\ B(2;0;1)$

$\overline{AB}(2;1;2)$

$\overline{AB_0} = \frac{\overline{AB}}{|\overline{AB}|} = \frac{2i+j+2k}{\sqrt{4+1+4}} = \frac{2}{3}i + \frac{1}{3}j + \frac{2}{3}k$

$\cos\alpha = \frac{2}{3}\ \cos\beta = \frac{1}{3}\ \cos\gamma = \frac{2}{3}$

$u'_x = \frac{-1}{\sqrt{1-(xyz)^2}} \cdot yz + \ln(yz) \cdot 2 \cdot (-1) \frac{2x}{(x^2+y^2)^2} =$

$= \frac{-yz}{\sqrt{1-(xyz)^2}} - \frac{4x\ln(yz)}{(x^2+y^2)^2}$

$u'_x(A) = \frac{-1}{\sqrt{1-0}} = -1 \quad u'_x(A) = -1$

$u'_y = \frac{-1}{\sqrt{1-(xyz)^2}} xz + z\left(\frac{1}{yz} \cdot z \cdot \frac{1}{x^2-y^2} + \ln(yz)(-1) \cdot \right.$

$\left. \frac{1}{(x^2+y^2)^2} \cdot 2y\right) = \frac{-xz}{\sqrt{1-(xyz)^2}} + z\left(\frac{1}{y(x^2-y^2)} - \frac{\ln(yz) \cdot 2y}{(x^2+y^2)^2}\right)$

$u'_y(A) = 2\left(\frac{1}{(-1)\cdot(0-1)} - \frac{\ln(1) \cdot 2(-1)}{1}\right) = 2$

$u'_y(A) = 2$

$$u'_z = \frac{-1}{\sqrt{1-(xyz)^2}} \cdot xy + \frac{1}{(x^2+y^2)} \cdot 2 \cdot \frac{1}{y^2} \cdot y =$$

$$= \frac{-xy}{\sqrt{1-(xyz)^2}} + \frac{2}{z(x^2+y^2)}$$

$$u'_z(A) = \frac{2}{-1(0+1)} = -2 \qquad u'_z(A) = -2$$

$$\frac{\partial u}{\partial l_{AB}} = -1\frac{2}{3} + 2\frac{1}{3} - 2\frac{2}{3} = -\frac{4}{3}$$

$$\text{grad } u(A) = (-1)\bar{i} + 2\bar{j} - 2\bar{k}$$

$$\frac{\partial u}{\partial l_{grad}} = |\text{grad } u(A)| = \sqrt{1+4+4} = 3$$

Ответ: $-\frac{4}{3}$; 3

№6(a)

$f(x,y) = xy^2 - 9x^3 + 18x^2 - y^2 - 9x$

$f'_x = y^2 - 9\cdot 3x^2 + 18\cdot 2x - 9 = y^2 - 27x^2 + 36x - 9$

$f'_y = x\cdot 2y - 2y$

$\begin{cases} x\cdot 2y - 2y = 0 \\ y^2 - 27x^2 + 36x - 9 = 0 \end{cases}$ $\to$ $2y(x-1) = 0$

$\Downarrow \left[\begin{array}{l} y = 0 \\ x = 1 \end{array}\right.$

$y = 0:$

$-27x^2 + 36x - 9 = 0$

$-3x^2 + 4x - 1 = 0$

$D = 4$

$x_1 = 1;\ x_2 = \frac{1}{3}$

$x = 1:$

$y^2 - 27 + 36 - 9 = 0$

$y^2 = 0$

$y = 0$

Точки: $(1;0);\ (\frac{1}{3};0)$

$\frac{\partial^2 f}{\partial x^2} = -27\cdot 2\cdot x + 36 = -54 + 36;\ \frac{\partial^2 f}{\partial y^2} = 2x - 2$

$\frac{\partial^2 f}{\partial x \partial y} = 2y$

$\underline{(1;0)}$: $\begin{pmatrix} -18 & 0 \\ 0 & 0 \end{pmatrix} = 0$ – точка экстремума

$(\frac{1}{3};0)$: $\begin{pmatrix} 18 & 0 \\ 0 & -\frac{4}{3} \end{pmatrix} = 18\left(-\frac{4}{3}\right) < 0$ точка не явл. экстремумом

Ответ: $(1;0)$

№6(б)

$x^3-18x^2-3y^2-6z^2+12xy+6xz-6yz-$
$-18x+6y+24z+11=f(x;y;z)$

$f'_x=3x^2-36x+12y+6z-18$

$f'_y=-6y+12x-6z+6$

$f'_z=-12z+6x-6y+24$

$$\begin{cases}3x^2-36x+12y+6z-18=0 & \Big| :3\\ -6y+12x-6z+6=0 & \Big| :6\\ -12z+6x-6y+24=0 & \Big| :6\end{cases}$$

$$\begin{cases}x^2-12x+4y+2z-6=0\\ -y+2x-z+1=0\\ -2z+x-y+4=0\end{cases}$$

$-y=2z-x-4$

$2z-x-4+2x-z+1=0$

$z=3-x$

$y=-2z+x+4$

$y=-2(3-x)+x+4$

$y=-6+2x+x+4$

$y=3x-2$

$x^2 - 12x - 4(3x-2) + 2(3-x) - 6 = 0$

$x^2 - 12x + 12x - 8 + 6 - 2x - 6 = 0$

$x^2 - 2x - 8 = 0$

$D = 36$

$x_1 = -2 \quad x_2 = 4$

$(-2; -8; 10)$ и $(4; 10; -1)$

$\frac{\partial^2 f}{\partial x^2} = 6x - 36 \qquad \frac{\partial^2 f}{\partial y \partial x} = 12$

$\frac{\partial^2 f}{\partial y^2} = -6 \qquad \frac{\partial^2 f}{\partial z \partial x} = 6$

$\frac{\partial^2 f}{\partial z^2} = -12 \qquad \frac{\partial^2 f}{\partial y \partial z} = -6$

$(-2; -8; 10)$

$\delta_1 = -48 < 0$

$\delta_2 = \begin{pmatrix} -48 & 12 \\ 12 & -6 \end{pmatrix} = 48 \cdot 6 - 12 \cdot 12 = 144 > 0$

$\delta_3 = \begin{pmatrix} -48 & 12 & 6 \\ 12 & -6 & -6 \\ 6 & -6 & -12 \end{pmatrix} = \begin{pmatrix} -24 & 0 & -6 \\ 12 & -6 & -6 \\ -6 & 0 & -6 \end{pmatrix} = -6(24 \cdot 6 - 36) < 0 \Rightarrow$

$\Rightarrow$ точка экстремума (максимума

$(4; 10; 1)$

$\delta_1 = -12 < 0$

$\delta_2 = \begin{pmatrix} -12 & 12 \\ 12 & -6 \end{pmatrix} = -72 < 0$

$\delta_3 = \begin{pmatrix} -12 & 12 & 6 \\ 12 & -6 & -6 \\ 6 & -6 & -12 \end{pmatrix} = \begin{pmatrix} -12 & 0 & 6 \\ 12 & 6 & -6 \\ 6 & 0 & -12 \end{pmatrix} = 6(144 - 36) > 0$

точка не экстремума

№5

$x^2 - xy - 8x - z + 5 = 0$ $S(1;2;1)$

$\frac{\partial F}{\partial x} = 2x - y - 8$ $\frac{\partial F}{\partial z} = -1$

$\frac{\partial F}{\partial y} = -x$

$\frac{2x - y - 8}{1} = \frac{-x}{2} = -1$ $\boxed{x = 2}$

$2x - y - 8 = -1$

$4 - y - 8 = -1$

$\boxed{y = -3}$

$4 + 6 - 16 - z + 5 = 0$

$-z - 1 = 0 \Rightarrow \boxed{z = -1}$

Точка $(2;-3;-1)$

Касат: $(x-2)\cdot(-1) + (y+3)(-2) + (z+1)(-1) = 0$

$-x + 2 - 2y + 6 - z - 1 = 0$

$-x - 2y - z - 5 = 0$

Нормаль: $\frac{x-2}{-1} = \frac{y+3}{-2} = \frac{z+1}{-1}$