№ 7.7

$$z = \frac{1}{\sqrt{x^2+y^2-R^2}}$$

$x^2+y^2-R^2>0$

$x^2+y^2>R^2$

$y^2>R^2-x^2$

$|y|>\sqrt{R^2-x^2}$

Отв.: $\left[\begin{array}{l} \begin{cases} y \geq 0 \\ y > \sqrt{R^2-x^2} \end{cases} \\ \begin{cases} y<0 \\ y<-\sqrt{R^2-x^2} \end{cases} \end{array}\right.$

№ 7.9

$$z=\sqrt{1-(x^2+y)^2}$$

$1-(x^2+y)^2 \geq 0$

$(x^2+y)^2 \leq 1$

$|x^2+y| \leq 1$

$\left[\begin{array}{l} \begin{cases} x^2+y \geq 0 \\ x^2+y \leq 1 \end{cases} \\ \begin{cases} x^2+y<0 \\ x^2+y \geq -1 \end{cases} \end{array}\right.$ Отв.: $\left[\begin{array}{l} \begin{cases} y \geq -x^2 \\ y \leq 1-x^2 \end{cases} \\ \begin{cases} y<-x^2 \\ y \geq -1-x^2 \end{cases} \end{array}\right.$

№ 7.13

$$z=\sqrt{\log_a(x^2+y^2)}$$

$$\log_a(x^2+y^2)\geq 0$$

$$x^2+y^2\geq 1$$

$$y^2\geq 1-x^2$$

$$1-x^2\geq 0$$

$$x^2\leq 1$$

$$x\in[-1;1]$$

$$\left[\begin{array}{l}\begin{cases}y\geq 0\\ y\geq\sqrt{1-x^2}\end{cases}\\ \begin{cases}y<0\\ y\leq-\sqrt{1-x^2}\end{cases}\end{array}\right.$$

Отв.: $x\in[-1;1]$

$$\left[\begin{array}{l}y\geq\sqrt{1-x^2}\\ y\leq-\sqrt{1-x^2}\end{array}\right.$$

№ 7.33

$$\lim_{\substack{x\to 0\\ y\to 0}}\frac{\sin xy}{xy}=\lim_{a\to 0}\frac{\sin a}{a}=1$$

№ 7.34

$$\lim_{\substack{x\to 0\\ y\to 0}}\frac{\sin(xy)}{y}=\lim_{\substack{x\to 0\\ y\to 0}}\frac{\sin(xy)}{yx}\,x=1\cdot 0=0$$

Ф-ии неск. переменных
частная производная
№7.56

1) $z = xy + \frac{y}{x}$

$\frac{\partial z}{\partial x} = y - \frac{y}{x^2}$; $\frac{\partial z}{\partial y} = x + \frac{1}{x}$; $\frac{\partial z}{\partial x \partial y} = 1 - \frac{1}{x^2}$

2) $\frac{\partial^2 z}{\partial x^2} = \left(y - \frac{y}{x}\right)' = -y(-2)\frac{1}{x^3} = \frac{2y}{x^3}$

$\frac{\partial^2 z}{\partial y^2} = \left(x + \frac{1}{x}\right)' = 0$

№7.58

$z = xe^{-xy}$

1) $\frac{\partial z}{\partial x} = e^{-xy} + e^{-xy}(-y)x = e^{-xy}(1+xy)$

$\frac{\partial z}{\partial y} = xe^{-xy}(-x) = -x^2e^{-xy}$

$\frac{\partial z}{\partial x \partial y} = x(xy-2)e^{-xy}$

2) $\frac{\partial^2 z}{\partial x^2} = \left(e^{-xy}(1-xy)\right)' = e^{-xy}(-y)(1+xy) +$
$+ e^{-xy}(-y) = e^{-xy}(xy^2 - 2y)$

$\frac{\partial^2 z}{\partial y^2} = (-x^2) \cdot e^{-xy}(-x) = -x^3 e^{-xy}$

N 7.59

$z = \frac{\cos y^2}{x}$

1) $\frac{\partial z}{\partial x} = -\frac{\cos y^2}{x^2}$

$\frac{\partial z}{\partial y} = \frac{1}{x}(-s \sin y^2) \cdot 2y = -\frac{2y \sin y^2}{x}$

2) $\frac{\partial^2 z}{\partial x^2} = (-\cos y^2)\left(-\frac{2}{x^3}\right) = \frac{2\cos y^2}{x^3}$

$\frac{\partial^2 z}{\partial y^2} = -\frac{2}{x}(\sin y^2 + y\cos^2 y \cdot 2y)$

№762

$$z = \arcsin\left(\frac{y}{\sqrt{x^2+y^2}}\right)$$

1) $$\frac{\partial z}{\partial x} = \frac{1}{\sqrt{1-\frac{y^2}{x^2+y^2}}} \cdot y\left(-\frac{1}{2}\right) \cdot \frac{1}{\sqrt{(x^2+y^2)^3}} \cdot 2x =$$

$$= \frac{-xy}{\sqrt{(x^2+y^2)^3 - y^2(x^2+y^2)^2}} = -\frac{y \operatorname{Sign} x}{(x^2+y^2)}$$

$$\frac{\partial z}{\partial y} = \frac{1}{\sqrt{1-\frac{y^2}{x^2+y^2}}} \cdot \frac{\sqrt{x^2+y^2} - y \cdot \frac{1}{2} \frac{1}{\sqrt{x^2+y^2}} \cdot 2y}{(x^2+y^2)} =$$

$$= \frac{x}{x^2+y^2}$$

2) $$\frac{\partial^2 z}{\partial x^2} = (-y)(-1) \cdot \frac{1}{(x^2+y^2)^2} = \frac{y \cdot 2|x|}{(x^2+y^2)^2}$$

$$\frac{\partial^2 z}{\partial y^2} = \cancel{(-y)(-1)\frac{1}{(x^2+y^2)^2}} = x \cdot (-1) \frac{1}{(x^2+y^2)^2}$$

№ 764

$$u = \left(\frac{y}{x}\right)^z = \frac{y^z}{x^z}$$

1) $$\frac{\partial u}{\partial x} = y^z(-z)\frac{1}{x^{z+1}}$$

$$\frac{\partial u}{\partial y} = \frac{1}{x^z} \cdot z \cdot y^{z-1}$$

$$\frac{\partial u}{\partial z} = \left(\frac{y}{x}\right)^z \ln\frac{x}{y}$$

2) $$\frac{\partial^2 u}{\partial x^2} = -y^z z(-z-1)\frac{1}{x^{z+2}} = \frac{y^z z(z+1)}{x^{z+2}}$$

$$\frac{\partial u}{\partial y^2} = \frac{1}{x^z} \cdot z(z-1)y^{z-2}$$

$$\frac{\partial^2 u}{\partial z^2} = \ln^2\frac{x}{y}\left(\frac{y}{x}\right)^z$$

$$\frac{\partial^2 u}{\partial z \partial x} = y^z\left(\ln\frac{x}{y}\frac{1}{x^z}\right)' = y^z \cdot \frac{1 - z\ln\frac{x}{y}}{x^{z+1}}$$

$$\frac{\partial^2 u}{\partial z \partial y} = \frac{1}{x^z}\left(y^z \ln\frac{x}{y}\right)' = \frac{y^{z-1}}{x^z}\left(z\ln\frac{x}{y} - 1\right)$$

№ 7.88

$z=\ln(x^2+y^2)$ x от 2 до 2,1 $\Delta x=0,1$
y от 1 до 0,9 $\Delta y=0,1$

$z(x+\Delta x, y+\Delta y)=\lg((x+\Delta x)^2+(y+\Delta y)^2)=$
$=\lg(x^2+y^2+\Delta x^2+\Delta y^2+2(x\Delta x+y\Delta y))$.

$\Delta z(x,y)=\cancel{\ } + \lg(x^2+y^2)=0,0187$

№ 7.90

$z=tg\frac{y^2}{x}$

$\frac{\partial z}{\partial x}=\frac{1}{\cos^2\frac{y^2}{x}}\cdot y^2(-1)\frac{1}{x^2}=-\frac{y^2}{x^2}\cdot\frac{1}{\cos^2\left(\frac{y^2}{x}\right)}$

$\frac{\partial z}{\partial y}=\frac{1}{\cos^2\left(\frac{y^2}{x}\right)}\frac{1}{x}\cdot 2y=\frac{2y}{x}\frac{1}{\cos^2\left(\frac{y^2}{x}\right)}$

$dz=\frac{-y^2}{x^2\cos^2\left(\frac{y^2}{x}\right)}dx+\frac{2y}{x\cos^2\left(\frac{y^2}{x}\right)}dy$

№ 7.92

$u=(xy)^z$

$\frac{\partial u}{\partial x}=y^z\cdot z\cdot x^{z-1}$; $\frac{\partial u}{\partial y}=x^z\cdot z\, y^{z-1}$

$\frac{\partial u}{\partial z}=(xy)^z\ln(xy)$

$du=y^z z x^{z-1}dx+x^z z y^{z-1}dy+(xy)^z\ln(xy)$

№ 7.102

$z = \frac{y}{x} - \frac{x}{y}$

$\frac{\partial z}{\partial x} = y(-1)\frac{1}{x^2} - \frac{1}{y} = \frac{-y}{x^2} - \frac{1}{y}$

$\frac{\partial z}{\partial y} = \frac{1}{x} + \frac{x}{y^2}$

1) $dz = \frac{-y^2 - x^2}{x^2 y} dx + \frac{y^2 + x^2}{x y^2} dy$

$\frac{\partial^2 z}{\partial x^2} = \frac{2y}{x^3}$; $\frac{\partial^2 z}{dy^2} = \frac{-2x}{y^3}$

2) $d^2 z = \frac{2y}{x^3} dx^2 - \frac{2x}{y^3} dy^2$

№ 7.107

$z = \operatorname{arcctg} \frac{x}{x+y}$

$\frac{\partial z}{\partial x} = \frac{-1}{1 + \frac{x^2}{(x+y)^2}} \cdot \frac{x+y-x}{x+y} = \frac{-y(x+y)}{(x+y)^2 + x^2}$

$\frac{dz}{dy} = \frac{-1}{1 + \frac{x^2}{(x+y)^2}} \cdot x(-1)\frac{1}{(x+y)^2} = \frac{-x}{(x+y)^2 + x^2}$

$dz = \frac{-y(x+y)}{(x+y)^2 + x^2} dx - \frac{x}{(x+y)^2 + x^2} dy$