Sem - 2

CS

Practical no: 1

1) ~~$\lim_{x\to a}$~~ Topic: limits & continuity

1) $\lim_{x\to a}\left[\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$

$\rightarrow$ $= \lim_{x\to a}\left[\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} \times \dfrac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} \times \dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}\right]$

$= \lim_{x\to a} \dfrac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}$

$= \lim_{x\to a} \dfrac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(3a-3x)(\sqrt{a+2x}+\sqrt{3x})}$

$= \dfrac{1}{3}\lim_{x\to a} \dfrac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(a-x)(\sqrt{a+2x}+\sqrt{3x})}$

$= \dfrac{1}{3}\cdot\dfrac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$

$= \dfrac{1}{3}\cdot\dfrac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$

$= \dfrac{1}{3}\cdot\dfrac{2\sqrt{a}+2\sqrt{a}}{\cancel{2}\sqrt{3a}}$

$= \dfrac{1}{3}\cdot\dfrac{\overset{2}{\cancel{4}}\sqrt{a}}{\cancel{2}\sqrt{3a}}$

$= \dfrac{2}{3\sqrt{3}}$

2. $\lim_{y\to 0}\left[\dfrac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

$\rightarrow$ $= \lim_{y\to 0}\left[\dfrac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}} \times \dfrac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$

$= \lim_{y\to 0}\left[\dfrac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}\right]$

$= \lim_{y\to 0}\left[\dfrac{\cancel{y}}{\cancel{y}(\sqrt{a+y})\cdot(\sqrt{a+y}+\sqrt{a})}\right]$

$= \lim_{y\to 0}\left[\dfrac{1}{\sqrt{a+y}\cdot(\sqrt{a+y}+\sqrt{a})}\right]$

$= \dfrac{1}{\sqrt{a+0}\cdot(\sqrt{a+0}+\sqrt{a})}$

$= \dfrac{1}{\sqrt{a}\cdot(\sqrt{a}+\sqrt{a})}$

$= \dfrac{1}{\sqrt{a}(2\sqrt{a})} = \dfrac{1}{2a}$

3. $\lim_{x\to \pi/6} \dfrac{\cos x-\sqrt{3}\sin x}{\pi-6x}$

$\rightarrow$ By substituting $x-\pi/6 = h$

$x = h+\pi/6$

where $h\to 0$

$\lim_{h\to 0} \dfrac{\cos(h+\pi/6)-\sqrt{3}\sin(h+\pi/6)}{\pi-6(h+\pi/6)}$

using

$\cos(A+B) = \cos A\cdot\cos B - \sin A\cdot\sin B$

$\sin(A+B) = \sin A\cdot\cos B + \cos A\cdot\sin B$

$= \lim_{h\to 0} \dfrac{\cos h\cdot\cos\pi/6 - \sin h\cdot\sin\pi/6 - \sqrt{3}\sin h\cos\pi/6 + \cos h\cdot\sin\pi/6}{\pi-\cancel{6}\left(\dfrac{6h+\pi}{\cancel{6}}\right)}$

$= \lim_{h\to 0} \dfrac{\cos h\cdot\frac{\sqrt{3}}{2} - \sinh\cdot\frac{1}{2} - \sqrt{3}(\sin h\frac{\sqrt{3}}{2} + \cos h\cdot\frac{1}{2})}{\cancel{\pi}-6h-\cancel{\pi}}$

$\cos\pi/6 = \sqrt{3}/2$

$\sin\pi/6 = 1/2$

$= \lim_{h\to 0} \dfrac{\cancel{\cos h\cdot\frac{\sqrt{3}}{2}h} - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cancel{\cos\frac{\sqrt{3}}{2}h}}{-6h}$

$= \lim_{h\to 0} \frac{\frac{\cancel{+}\sin 4h}{2}}{\cancel{+}6h}$

$= \lim_{h\to 0} \frac{\sin 4h}{3 \cancel{\times} h}$

$= \frac{1}{3} \lim_{h\to 0} \frac{\sin h}{h} = \frac{1}{3} \times 1 = \frac{1}{3}$

4. $\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right]$

→ By rationalizing Numerator & Denominator both

$= \lim_{x\to\infty} \left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}} \times \frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$

$= \lim_{x\to\infty} \left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$

$= \lim_{x\to\infty} \left[\frac{\cancel{4}8(\sqrt{x^2+3}+\sqrt{x^2+1})}{2(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$

$= 4\cancel{\times 1} = 4$

5. i) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \le \pi/2$
$= \frac{\cos x}{\pi - 2x}$, for $\pi/2 < x < \pi$ } at $x = \pi/2$

→ $f(\pi/2) = \frac{\sin^2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ ∴ $f(\pi/2) = 0$

f at $x = \pi/2$ define

$\lim_{x\to\pi/2^+} f(x) = \lim_{x\to\pi/2^-} \frac{\cos x}{\pi - 2x}$

By substituting method

$x - \pi/2 = h$

$x = \pi/2 + h$

where $h \to 0$

$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - 2(h+\pi/2)}$

$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - \cancel{2}(2h+\pi/\cancel{2})}$

$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{-2h}$ $\quad \cos(A+B) = \cos A\cdot\cos B - \sin A\cdot\sin B$

$\lim_{h\to 0} \frac{\cos h\cdot\cos\pi/2 - \sin h\cdot\sin\pi/2}{-2h}$

$\lim_{h\to 0} \frac{\cos h\cdot 0 - \sin h}{-2h}$

$\lim_{h\to 0} \frac{-\sin h}{\cancel{-}2h}$

$= \frac{1}{2} \lim_{h\to 0} \frac{\sin h}{h} = \frac{1}{2}$

b) $\lim_{x\to\pi/2^-} f(x) = \lim_{x\to\pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$ using $\sin 2x = 2\sin x\cdot\cos x$

$\lim_{x\to\pi/2^-} \frac{2\sin x\cdot\cos x}{\sqrt{2\sin^2 x}}$

$\lim_{x\to\pi/2^-} \frac{2\cos x\cdot\cancel{\sin x}}{\sqrt{2}\cdot\cancel{\sin x}}$

$\lim_{x\to\pi/2^-} \frac{2\cos x}{\sqrt{2}}$

$\frac{2}{\sqrt{2}} \lim_{x\to\pi/2^-} \cos x$

∴ LHL ≠ RHL

∴ f is not continuous at $x = \pi/2$

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5\. ii) $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$= x+3 \quad 3 \le x \le 6$

$= \frac{x^2-9}{x+3} \quad 6 \le x < 9$

at $x = 3$ & $x = 6$.

→ i) $f(3) = \frac{x^2-9}{x-3} = 0$

f at $x = 3$ define

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3$.

$f(3) = x+3 = 3+3 = 6$

f is define at $x = 3$

ii) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$.

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}}$

∴ LHL = RHL

f is continuous at $x = 3$.

for $x = 6$.

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3$.

2\. $\lim_{x\to 6^+} \frac{x^2-9}{x+3} = \lim_{x\to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{x+3}}$

$\lim_{x\to 6^+} (x-3) = 6-3 = 3$

$\lim_{x\to 6^-} x+3 = 3+6 = 9$

∴ LHL ≠ RHL

function is not continuous.

6\. Find value of K, so that the function f(x) is continuous at the indicated point:

i) $f(x) = \frac{1-\cos 4x}{x^2} \quad x < 0$

$= k \quad x = 0$

at $x = 0$.

→ f is continuous at $x = 0$

$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = K$

$2 \lim_{x\to 0} \frac{-2\sin^2 2x}{x^2} = K$

$2 \lim_{x\to 0} \left(\frac{\sin^2 2x}{x}\right)^2 = K$

$2(2)^2 = K$

$K = 8$

2) $f(x) = \cos(\sec^2 x)^{\cot^2 x} \quad x \neq 0$

$= k \quad x = 0$

at $x = 0$.

→ using $\tan^2 x - \sec^2 x \neq 1$

∴ $\sec^2 x = 1 + \tan^2 x$

& $\cot^2 x = 1/\tan^2 x$

∴ $\lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$

$= \lim_{x\to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$

we know that

$$\lim_{x\to 0} (1+px)^{1/px} = e$$

$\therefore K = e$

3) $f(x) = \left.\begin{array}{ll} \dfrac{\sqrt{3}-\tan x}{\pi - 3x} & x \neq \pi/3 \\ = K & x = \pi/3 \end{array}\right\}$ at $x = \pi/3$

→ put $x - \pi/3 = h$

$x = \pi/3 + h$

where $h \to 0$

$$f(\pi/3+h) = \frac{\sqrt{3} - \tan(\pi/3+h)}{\pi - 3(\pi/3+h)}$$

$$\lim_{h\to 0} \frac{\sqrt{3} - \tan(\pi/3+h)}{\pi - 3(\pi/3+h)}$$

using $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$$\lim_{h\to 0} \frac{\sqrt{3} \cdot \dfrac{\tan\pi/3 + \tan h}{1-\tan\pi/3\cdot\tan h}}{\pi - \pi - 3h}$$

$$\lim_{h\to 0} \frac{\dfrac{\sqrt{3}(1-\tan\pi/3\cdot\tan h) - (\tan\pi/3 + \tan h)}{1-\tan\pi/3\cdot\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3} - \sqrt{3}\times\sqrt{3}\cdot\tan h) - (\sqrt{3}+\tan h)}{1-\tan\pi/3\cdot\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3} - \sqrt{3}\cdot\tan h) - (\sqrt{3}+\tan h)}{1-\sqrt{3}\cdot\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{\cancel{\sqrt{3}} - 3\tan h - \cancel{\sqrt{3}} - \tan h}{1-\sqrt{3}\cdot\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{-4\tan h}{-3h(1-\sqrt{3}\tan h)}$$

$$\lim_{h\to 0} \frac{4\tan h}{3h(1-\sqrt{3}\tan h)}$$

$$\frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \cdot \lim_{h\to 0} \frac{1}{(1-\sqrt{3}\tan h)} \qquad \lim_{h\to 0} \frac{\tan h}{h} = 1$$

$$= \frac{4}{3} \frac{1}{(1-\sqrt{3}(0))} = \frac{4}{3}\left(\frac{1}{1}\right) = \frac{4}{3}$$

7. Discuss the continuity of the following functions which of these functions have a removable discontinuity? Redefine the function so as to remove the discontinuity.

1) $f(x) = \left.\begin{array}{ll} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ = 9 & x = 0 \end{array}\right\}$ at $x = 0$

→ $f(x) = \dfrac{1-\cos 3x}{x \tan x}$

$$\lim_{x\to 0} \frac{2\sin^2 3/2\,x}{x \tan x}$$

$$\lim_{x\to 0} \frac{\dfrac{2\sin^2 3x/2}{x^2} \times x^2}{\dfrac{x\cdot\tan x}{x^2} \times x^2}$$

$$= 2 \lim_{x\to 0} \left(\frac{3}{2}\right)^2 = 2\times\frac{9}{4} = \frac{9}{2}$$

$$\lim_{x\to 0} f(x) = \frac{9}{2} \qquad 9 = f(0)$$

$\therefore f$ is not continuous at $x=0$.

Redefine function

$$f(x) = \frac{1-\cos 3x}{x \cdot \tan x} \quad x \neq 0$$

$$= \frac{9}{2} \quad x = 0$$

Now, $\lim_{x\to 0} f(x) = f(0)$

f has removable discontinuity at $x=0$.

2) $$f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2} \quad x \neq 0 \quad \Big\}\text{ at } x=0$$

$$= \frac{\pi}{60} \quad x = 0$$

$$\rightarrow f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2} \quad x\neq 0 \quad \Big\}\text{ at } x=0.$$

$$= \pi/6 \quad x=0$$

$$\lim_{x\to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$\lim_{x\to 0} \frac{e^{3x}-1}{x} \cdot \lim_{x\to 0} \frac{\sin(\pi x/180)}{x}$$

$$\lim_{x\to 0} 3 \cdot \frac{e^{3x}-1}{3x} \cdot \lim_{x\to 0} \frac{\sin(\pi x/180)}{x}$$

$$3 \lim_{x\to 0} \frac{e^{3x}-1}{3x} \cdot \lim_{x\to 0} \frac{\sin(\pi x/180)}{x}$$

$$3 \log e \cdot \frac{\pi}{180} = \frac{\pi}{60} = f(0)$$

f is continuous at $x=0$.

8. If $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is continuous at $x=0$. Find $f(0)$.

$\rightarrow \therefore$ Given f is continuous at $x=0$

$$\lim_{x\to 0} f(x) = f(0)$$

$$= \lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$= \lim_{x\to 0} \frac{e^{x} - \cos x - 1 + 1}{x^2} = f(0)$$

$$= \lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$= \lim_{x\to 0} \frac{(e^{x^2}-1) + (1-\cos x)}{x^2}$$

$$= \lim_{x\to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \frac{1-\cos x}{x^2}$$

$$\log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$$

$$\log e + 2\lim_{x\to 0} \left(\frac{\sin x/2}{x}\right)^2$$

Multiply with 2 on Numerator & Denominator

$$= 1 + \cancel{2} \times \frac{1}{\cancel{4}\,2} = \frac{3}{2} = f(0)$$

9. If $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \pi/2$ is continuous at $x = \pi/2$. Find $f(\pi/2)$.

$\rightarrow f(0)$ is continuous at $x = \pi/2$

$$\lim_{x\to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$\lim_{x \to \pi/2} \frac{2-1+\sin x}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{1+\sin x}{1-\sin^2 x(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{\cancel{1+\sin x}}{(1-\sin x)\cancel{(1+\sin x)}(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \quad = \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$= \frac{1}{2(2\sqrt{2})} = \frac{1}{4\sqrt{2}}$$

$$\therefore f(\pi/2) = \frac{1}{4\sqrt{2}}$$

Practical no: 2

Topic: Derivative

Q.1 Show that the following function defined from R to R are differentiable

i) $\cot x$

$\rightarrow f(x) = \cot x$

$$Df(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x-a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x-a}$$

$$= \lim_{x \to a} \frac{\tan a - \tan x}{(x-a)\tan a \cdot \tan x}$$

put $x-a=h$

$x = a+h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$$

formula: $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \cdot \tan B}$

$\tan A - \tan B = \tan(A-B)\ (1+\tan A \cdot \tan B)$

$$= \lim_{h \to 0} \frac{\tan(a-a-h) - (1+\tan a + \tan(a+h))}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h\to 0} \frac{-\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\cdot \tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= -\frac{\sec^2 a}{\tan^2 a} = \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$\therefore Df(a) = -\cos^2 a$

$\therefore f$ is differentiable $\forall a \in R$.

ii) $\operatorname{cosec} x$

$\rightarrow f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x-a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \cdot \sin x}$$

put $x-a=h$

$x = a+h$

as $x\to a$, $h\to 0$

$$Df(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

formula:

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\cdot \sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\cdot \sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h\to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \cdot \sin(a+h)}$$

$$= -\frac{1}{\cancel{2}} \times \cancel{2} \frac{\cos(2a+h/2)}{\sin(a+h)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \cdot \operatorname{cosec} a.$$

iii) $\sec x$

$\rightarrow f(x) = \sec x$

$$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x-a}$$

$$= \lim_{x\to a} \frac{\cos a - \cos x}{\cos a \cdot \cos x (x-a)}$$

put $x-a=h$

$x = a+h$

as $x\to a$, $h\to 0$

$$Df(h) = \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cdot \cos(a+h)}$$

formula: $-2\sin\left(\frac{C+D}{2}\right)\cdot \sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\cdot \sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cdot \cos(a+h)}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\cdot \sin(h/2)}{\cos a \cdot \cos(a+h) \times -h/2} \times -\frac{1}{2}$$

$$= -\frac{1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cdot \cos(a+0)}$$

$$= +\frac{1}{\cancel{2}} \times +\cancel{2} \frac{\sin a}{\cos a \cdot \cos a}$$

$= \tan a \cdot \cancel{\tan} \sec a.$

Q.2. If $f(x) = 4x+1 \quad , x \le 2$
$\quad = x^2+5 \quad x > 0$ at $x = 2$ then find function is differentiable or not.

$\rightarrow$ LHD:

$$D^+(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-(4 \times 2+1)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2} = \lim_{x \to 2} \frac{4(x-2)}{x-2} = 4$$

RHD:

$$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$$

$$= 2+2 = 4$$

RHD = LHD.
f is differentiable at $x = 2$.

Q.3. If $f(x) = 4x+7 \quad , x < 3$
$\quad = x^2+3x+1 \quad x \ge 3$ at $x = 3$ then find f is differentiable ~~at~~ or not?

$\rightarrow$ RHD:

$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x(x+6) - 3(x+6)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{x-3}} = 3+6 = 9$$

LHD = $Df(3^-)$

$$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4\cancel{(x-3)}}{\cancel{x-3}}$$

$$= 4$$

RHD $\ne$ LHD.
f is not differentiable at $x = 3$.

Q.4. If $f(x) = 8x-5 \qquad x \le 2$
$\quad = 3x^2-4x+7 \qquad x > 2$ at $x = 2$ then find f is differentiable or not.

$\rightarrow$ $f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

RHD:

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2-4x+7-11}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2-4x-\cancel{18}\,4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2-6x+2x-4}{x-2}$$

$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x-2}$

$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{x-2}}$

$= 3 \times 2 + 2 = 8$

LHD:

$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x-2}$

$= \lim_{x \to 2^-} \frac{8x - 16}{x-2}$

$= \lim_{x \to 2^-} \frac{8\cancel{(x-2)}}{\cancel{x-2}}$

$= 8$

LHD = RHD

f is differentiable at $x = 3$.

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Practical no: 3

Topic: Application of derivatives.

Q.1. Find the intervals in which function is increasing or decreasing.

a) $f(x) = x^3 - 5x - 11$

b) $f(x) = x^2 - 4x$

c) $f(x) = 2x^3 + x^2 - 20x + 4$

d) $f(x) = x^3 - 27x + 5$

e) $f(x) = 69 - 24x - 9x^2 + 2x^3$

Q.2. Find the intervals in which the function is concave upwards:

a) $y = 3x^2 - 2x^3$

b) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

c) $y = x^3 - 27x + 5$

d) $y = 69 - 24x - 9x^2 + 2x^3$

e) $y = 2x^3 + x^2 - 20x + 4$

Solution

Q.1. a) $f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\therefore$ f is increasing iff $f'(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x - \sqrt{5/3}) > 0$

$+ \quad - \quad +$

$-\sqrt{5/3} \quad \sqrt{5/3}$

$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$3(x^2 - 5/3) < 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$

$+ \quad - \quad +$

$-\sqrt{5/3} \quad \sqrt{5/3}$

$x \in (-\sqrt{5/3}, \sqrt{5/3})$

b) $f(x) = x^2 - 4x$

$\therefore f'(x) = 2x - 4$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$\therefore 2(x-2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

and ~~f is~~ it is decreasing iff $f'(x) < 0$

$\therefore 2x - 4 < 0$

$2(x-2) < 0$

$x - 2 < 0$

$x \in (-\infty, 2)$

c) $f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$\therefore 2(3x^2 + x - 10) > 0$

$\therefore 3x^2 + x - 10 > 0$

$\therefore 3x^2 + 6x - 5x - 10 > 0$

$\therefore 3x(x+2) - 5(x+2) > 0$

$(x+2)(3x-5) > 0$

+ − + ; −2, 5/3

$x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$\therefore 2(3x^2 + x - 10) < 0$

$\therefore 3x^2 + 6x - 5x - 10 < 0$

$\therefore 3x(x+2) - 5(x+2) < 0$

$\therefore (3x-5)(x+2) < 0$

+ − + ; −2, 5/3

$x \in (-2, 5/3)$

d) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\because f$ is increasing iff $f'(x) > 0$

$3(x^2 - 9) > 0$

$(x-3)(x+3) > 0$

$x = 3, -3$

+ − + ; −3, 3

$x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore (x^2 - 9) < 0$

$\therefore (x+3)(x-3) < 0$

+ − + ; −3, 3

$\therefore x \in (-3, 3)$

e) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$\therefore x^2 - 3x - 4 > 0$

$\therefore x^2 - 4x + x - 4 > 0$

$\therefore x(x-4) + 1(x-4) > 0$

$\therefore (x+1)(x-4)$

$x = 4, -1$

+ − + ; −1, 4

$x \in (-\infty, -1) \cup (4, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 - 18x - 24 < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

$\therefore x^2 - 4x + x - 4 < 0$

$\therefore x(x-4) + 1(x-4) < 0$

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$\therefore (x-4)(x+1)$

$x = 4, -1$

$x \in (-1, 4)$

Q.2. a) $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

f is concave upwards if $f''(x) > 0$

$\therefore (6 - 12x) > 0$

$\therefore 12(6/12 - x) > 0$

$\therefore x - 1/2 > 0$

$\therefore x > 1/2$

$\therefore f''(x) > 0$

$x \in (1/2; \infty)$

b) $y = x^4 - 6x^3 + 12x^2 + 5x - 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward if $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 3x + 2 > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore x(x-2) - 1(x-2) > 0$

$(x-2)(x-1) > 0$

$x = 1, 2$

$x \in (-\infty, 1) \cup (2, \infty)$

c) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x \in (0, \infty)$

d) $y = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0$ $\quad x > 3/2$

$x \in (3/2, \infty)$

e) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 12(x + 2/12) > 0$

$\therefore x + 1/6 > 0$

$\therefore x \not> -1/6$

$\therefore f''(x) \not> 0$ $\quad x \in (-1/6, \infty)$

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2/1/2/19

~~There exist no interval.~~

Practical no: 4

Topic: Application of derivative & Newtons method

Q.1. Find minimum and maximum value of following function:

a) $f(x) = x^2 + \frac{16}{x^2}$

b) $f(x) = 3 - 5x^3 + 3x^5$

c) $f(x) = x^3 - 3x^2 + 1$ in $[\frac{-1}{2}, 4]$

d) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

Q.2. Find the root of following equation by 'Newtons Method (Take 4 iteration) correct upto 4 decimal

a) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

b) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

c) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

Solution

Q.1. a) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$\therefore 2x = 32/x^3$

$\therefore x^4 = 32/2$

$\therefore x^4 = 16$

$\therefore x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/2^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = 2$

$\therefore f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$\therefore f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = -2$.

$\therefore$ Function reaches minimum value at $x = 2$, and $x = -2$

b) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

Consider, $f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$\therefore x^2 = 1$

$\therefore x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$ $\therefore f$ has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$\therefore f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore f$ has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore$ f has maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$

c) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

Consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$\therefore x = 0$ or $x = 2$

$\therefore f''(x) = 6x - 6$

$\therefore f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore$ f has maximum value at $x = 0$

$\therefore f(0) = 0^3 - 3(0)^2 + 1 = 1$

$\therefore f''(2) = 6(2) - 6$

$= 12 - 6 = 6 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore f(2) = 2^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore$ f has maximum value 1 at $x = 0$ and f has minimum value -3 at $x = 2$.

d) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

Consider, $f''(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 2) = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore x^2 + x - 2x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$\therefore (x-2)(x+1) = 0$

$\therefore x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$\therefore f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$\therefore f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore$ f has maximum value 8 at $x = -1$ and f has minimum value -19 at $x = 2$

Q.2. a) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $\quad x_0 = 0 \rightarrow$ given

$f'(x) = 3x^2 - 6x - 55$

By Newtons method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$\therefore x_1 = x_0 - f(x_0)/f'(x_0)$

$\therefore x_1 = 0 + 9.5/55$

$\therefore x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$
$= 0.0051 - 0.0895 - 9.4985 + 9.5$
$= -0.0829$
$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$
$= 0.0895 - 1.0362 - 55$
$= -55.9467$
$\therefore x_2 = x_1 - f(x_1) / f'(x_1)$
$= 0.1727 - 0.0829 / 55.9467$
$= 0.1712$
$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$
$= 0.0050 - 0.0879 - 9.416 + 9.5$
$= 0.0011$
$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$
$= 0.0879 - 1.0272 - 55$
$= -55.9393$
$\therefore x_3 = x_2 - f(x_2) / f'(x_2)$
$= 0.1712 - 0.0011 / 55.9393$
$= 0.1712$
$\therefore$ The root of the equation is 0.1712

b) $f(x) = x^3 - 4x - 9 \quad [2,3]$
$f'(x) = 3x^2 - 4$
$f(2) = (2)^3 - 4(2) - 9$
$= 8 - 8 - 9$
$= -9$
$f(3) = 3^3 - 4(3) - 9$
$= 27 - 12 - 9$
$= 6$
Let $x_0 = 3$ be the initial approximation
$\therefore$ By Newton's method,
$x_{n+1} = x_n - f(x_n) / f'(x_n)$
$x_1 = x_0 - f(x_0) / f'(x_0)$

$= 3 - 6/23$
$= 2.7392$
$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$
$= 20.5528 - 10.9568 - 9$
$= 0.596$
$f'(x_1) = 3(2.7392)^2 - 4$
$= 22.5096 - 4$
$= 18.5096$
$x_2 = x_1 - f(x_1) / f'(x_1)$
$= 2.7392 - 0.596 / 18.5096$
$= \cancel{2.7092}\ 2.7071$
$f(x_2) = (2.7071)^3 - 4(2.7071)$
$= 19.8386 - 10.8284$
$= 0.0102$
$f'(x_2) = 3(2.7071)^2 - 4$
$= 21.9851 - 4$
$= 17.9851$
$x_3 = x_2 - f(x_2) / f'(x_2)$
$= 2.7071 - 0.0102 / 17.9851$
$= 2.7071 - 0.0056$
$= 2.7015$
$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= 19.7158 - 10.806 - 9 = -0.0901$
$f'(x_3) = 3(2.7015)^2 - 4 = 21.8943 - 4 = 17.8943$
$x_4 = x_3 - f(x_3) / f'(x_3)$
$= 2.7015 + 0.0901 / 17.8943$
$= 2.7015 + 0.0050$
$= 2.7065$

c) $f(x) = x^3 - 1.8x^2 - 10x + 17$ [1, 2]

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = 1^3 - 1.8(1)^2 - 10(1) + 17$
$= -1.8 - 10 + 17$
$= 6.2$

$f(2) = 2^3 - 1.8(2)^2 - 10(2) + 17$
$= 8 - 7.2 - 20 + 17 = -2.2$

Let $x_0 = 2$ be initial approximation.

By newton's method,

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$
$= 2 - \cancel{0.6320}\ 2.2/5.2$
$= 2 - 0.4230 = 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$
$= 3.9219 - 4.4764 - 15.77 + 17$
$= 0.6755$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$
$= 7.4608 - 5.6772 - 10$
$= -8.2164$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$
$= 1.577 + 0.6755 / 8.2164$
$= 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$
$= 4.5677 - 4.9553 - 16592 + 17$
$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$
$= 8.2588 - 5.97312 - 10$
$= -7.7143$

$x_3 = x_2 - f(x_2) / f'(x_2)$
$= 1.6592 + 0.0204 / 7.7143$
$= 1.6592 + 0.0026$
$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$
$= 4.5892 - 4.9708 - 16.618 + 17$
$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$
$= 8.2847 - 5.9824 - 10$
$= -7.6977$

$x_4 = x_3 - f(x_3) / f'(x_3)$
$= 1.8618 + 0.0004 / 7.6977$
$= 1.6618$

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Practical no: 5

Topic: Integration

1) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

2) $\int (4e^{3x}+1)\,dx$

3) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

4) $\int \frac{x^3+3x-4}{\sqrt{x}}\,dx$

5) $\int t^7 \cdot \sin(2t^4)\,dt$

6) $\int \sqrt{x}\cdot(x^2-1)\,dx$

7) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

8) $\int \frac{\cos x}{\sqrt[3]{\sin x^2}}\,dx$

9) $\int e^{\cos^2 x} \cdot \sin 2x\,dx$

10) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right) dx$

Solution

1) $\int \frac{1}{\sqrt{x^2-2x-3}}\,dx$

$= \int \frac{1}{\sqrt{x^2-2x+1-4}}\,dx$

$(a+b)^2 = a^2+2ab+b^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}}\,dx$

substitute put $x+1=t$

$dx = \frac{1}{t} \times dt$ where $t=1$ $t=x+1$

$\int \frac{1}{\sqrt{t^2-4}}\,dt$

using,

$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln(|x+\sqrt{x^2-a^2}|)$

$= \ln(|t+\sqrt{t^2-4}|)$

$t = x+1$

$= \ln(|x+1+\sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)+c.$

2) $\int (4e^{3x}+1)\,dx$

$= \int 4e^{3x}\,dx + \int 1\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx$ # $\int e^{ax}\,dx = \frac{1}{a} \times e^{ax}$

$= 4\frac{e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + c.$

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3) $\int 2x^2 - 3\sin x + 5\sqrt{x}\, dx$

$= \int 2x^2 - 3\sin x + 5x^{1/2}\, dx$

$= \int 2x^2\, dx - \int 3\sin x\, dx + \int 5(x)^{1/2}\, dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + c$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + c.$

4) $\int \frac{x^3+3x+4}{\sqrt{x}}\, dx$

$= \int \frac{x^3+3x+4}{x^{1/2}}\, dx$

spilt the denominator

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\, dx$

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}}\, dx$

$= \int x^{5/2}\, dx + 3\int x^{1/2}\, dx + 4\int \frac{1}{x^{1/2}}\, dx$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

5) $\int t^7 \cdot \sin(2t^4)\, dt$

put $u = 2t^4$

$du = 2\times 4t^3$

$= \int t^7 \cdot \sin(2t^4) \times \frac{1}{2\times 4t^3}\, du$

$= \int t^4 \cdot \sin(2t^4) \times \frac{1}{8}\, du = \frac{t^4 \times \sin(2t^4)}{8}\, du$

substitute t^4 with $u/2$.

$= \int \frac{u/2 * \sin(u)}{8}\, du$

$= \int \frac{u \times \sin(u)}{2} / 8\, du$

$= \int \frac{u \times \sin(u)}{16}\, du$

$= \frac{1}{16} \int u \times \sin(u)\, du$

$\int u\, dv = uv - \int v\, du$

where $u = v$

$dv = \sin(u) * du$

$du = 1\, du \quad v = -\cos(u)$

$= \frac{1}{16} \times (u \times (-\cos u)) - \int -\cos(u)\, du$

$= \frac{1}{16} \times (u \times (-\cos u)) + \int \cos u\, du$

$\int \cos x\, dx = \sin x$

$= \frac{1}{16} \times (u \times (-\cos u)) + \sin(u))$

Return the substition $u = 2t^4$

$= \frac{1}{16} \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + c$

6) $\int \sqrt{x} \cdot (x^2 - 1)\, dx$

$= \int \sqrt{x} \cdot x^2 - \sqrt{x}\, dx$

$= \int x^{1/2} \cdot x^2 - x^{1/2}\, dx$

$= \int x^{5/2}\, dx - \int x^{1/2}\, dx$

$= \frac{x^{5/2+1}}{5/2+1} + \frac{x^{1/2+1}}{1/2+1} + c$

$$= \frac{x^{7/2}}{7/2} + \frac{x^{3/2}}{3/2} + C$$

$$= \frac{2x^3 \cdot \sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + C$$

8) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}} dx$

$$= \int \frac{\cos x}{\sin x^{2/3}} dx$$

put $t = \sin x$

$t = \cos x$

$$= \int \frac{\cos x}{\sin(x)^{3/2}} \times \frac{1}{\cos x} dt$$

$$= \frac{1}{\sin x^{3/2}} dt$$

$$= \frac{1}{t^{2/3}} dt$$

$$I = \int \frac{1}{t^{2/3}} dt$$

$$= \frac{-1}{(2/3-1)\, t^{2/3-1}}$$

$$= \frac{-1}{-1/3\, t^{2/3-1}} = \frac{1}{1/3\, t^{-1/3}}$$

$$= \frac{t^{1/3}}{\cancel{1/3}} = 3t^{1/3}$$

$$= 3\sqrt[3]{t}$$

Return substitution $t = \sin x$

$$= 3\sqrt[3]{\sin x} + C.$$

1a) $\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} dx$

put $x^3 - 3x^2 + 1 = dt$

$$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 3 \times 2x} dt$$

$$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3(x^2 - 2x)} dt$$

$$= \int \frac{1}{x^3 - 3x^2 + 1} \times \frac{1}{3} dt$$

$$= \int \frac{1}{3(x^3 - 3x^2 + 1)} dt$$

$$= \int \frac{1}{3t} dt$$

$$= \frac{1}{3} \int \frac{1}{t} dt \qquad \int \frac{1}{x} dx = \ln|x|$$

$$= \frac{1}{3} \times \ln|t| + C$$

$$= \frac{1}{3} \times \ln(|x^3 - 3x^2 + 1|) + C$$

7) $I = \int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

Let $\frac{1}{x^2} = t$

$x^{-2} = t$

$-\frac{2}{x^3} dx = dt$

$\therefore I = -\frac{1}{2} \int -\frac{2}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$= -\frac{1}{2}\int \sin t$

$= -\frac{1}{2}(-\cos t) + c$

$= \frac{1}{2}\cos t + c$

Resubstitution $t = 1/x^2$

$\therefore I = \frac{1}{2}\cos\left(\frac{1}{x^2}\right) + c$

9) $I = \int e^{\cos^2 x} \sin 2x \, dx$

Let $\cos^2 x = t$

$-2\cos x \cdot \sin x \, dx = dt$

$-\sin 2x \, dx = dt$

$I = \int -\sin 2x \, e^{\cos^2 x} dx$

$= -\int e^t dt$

$= \cancel{e^t + c}$

Resubstituting $t = \cos^2 x$

$I = -e^{\cos^2 x} + c$

04/01/2020

Practical no: 6

Topic: Application of integration & Numerical integration.

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Q.1. Find the length of following curve:

① $x = t\sin t$, $y = 1 - \cos t$ $t \in [0, 2\pi]$

② $y = \sqrt{4 - x^2}$ $x \in [-2, 2]$

③ $y = x^{3/2}$ in $[0, 4]$

④ $x = 3\sin t$, $y = 3\cos t$ $t \in [0, 2\pi]$

⑤ $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in [1, 2]$

Q.2. Using Simpson's Rule to solve the following:

① $\int_0^2 e^{x^2} dx$ with $n = 4$

② $\int_0^4 x^2 dx$ with $n = 4$

③ $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n = 6$.

Solution

Q.1.4) $x = 3\sin t$, $y = 3\cos t$ $t \in [0, 2\pi]$

$\rightarrow \frac{dx}{dt} = 3\cos t$ $\frac{dy}{dt} = 3\sin t$

$I = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}$

$= 3\int_0^{2\pi} \sqrt{\sin^2 t + \cos^2 t}$

$= 3\int_0^{2\pi} dx$

$= 3[x]_0^{2\pi}$

$= 3(2\pi - 0)$

$= 6\pi$ units

(2) $\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}} \cdot (-2x)$

$= \frac{-\cancel{2}x}{\cancel{2}\sqrt{4-x^2}}$

$L = \int_{-2}^{2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_{-2}^{2} \sqrt{1+\left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$

$= \int_{-2}^{2} \sqrt{\frac{4-x^2+x^2}{4-x^2}}\, dx$

$= \int_{-2}^{2} \sqrt{\frac{4}{4-x^2}}\, dx$

$= \int_{-2}^{2} \frac{2}{\sqrt{4-x^2}}\, dx$

$= \int_{-2}^{2} \frac{1}{\sqrt{2^2-x^2}}\, dx$

$= 2[\sin^{-1}(x/2)]_{-2}^{2}$

$= 2[\sin^{-1}(1) - \sin^{-1}(-1)]$

$= 2[\pi/2 - (-\pi/2)]$

$= 2\left[\frac{\pi}{2} + \frac{\pi}{2}\right]$

$= 2[\pi]$

$= 2\pi$

3) $\frac{dy}{dx} = \frac{3}{2} x^{1/2}$

$L = \int_0^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_0^4 \sqrt{1+\left(\frac{3\sqrt{x}}{2}\right)^2}\, dx$

$= \int_0^4 \sqrt{1+\frac{9x}{4}}\, dx$

$= \int_0^4 \sqrt{\frac{4+9x}{4}}\, dx$

$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\, dx$

$= \frac{1}{2}\left[\frac{(4+9x)^{1/2+1}}{1/2+1}\right]_0^4$

$= \frac{1}{2}\left[\frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9}\right]_0^4$

$= \frac{1}{27} \cdot [(4+9x)^{3/2}]_0^4$

$= \frac{1}{27}\left[(4+0)^{3/2} - (40)^{3/2}\right]$

$= \frac{1}{27}\left[4^{3/2} - (40)^{3/2}\right]$

$= \frac{1}{27} \cdot \left[\cancel{4^{3/2}} - 8\right]$

(5) $\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4-1}{2y^2}$

$$L = \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$= \int_1^2 \sqrt{1+\frac{(y^4-1)^2}{4y^4}}\,dy$$

$$= \int_1^2 \sqrt{\frac{(y^4-1)+4\times y^4\times 1}{4y^4}}\,dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\,dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\,dy$$

$$= \frac{1}{2}\int_1^2 y^2\,dy + \frac{1}{2}\int_1^2 y^{-2}\,dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-1}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} + \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{17}{6}\right]$$

$$= \frac{17}{12}$$

Q.2 ① $\int_0^2 e^{x^2}\,dx$ with $n=4$

$h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$

x	0	0.5	1	1.5	2
y	1	1.284	2.7183	9.9877	54.5982
	y_0	y_1	y_2	y_3	y_4

$$\int_0^2 e^{x^2}\,dx = \frac{0.5}{3}\left[(y_0+y_4)+4(y_1+y_3)+2(y_2)\right]$$

$$= \frac{0.5}{3}\left[(1+54.5982)+4(1.284+9.487)+2.23885\right]$$

$$= \frac{0.5}{3}\left[55.5982+43.0866+5.4366\right]$$

$$= 17.3535$$

② $\int_0^4 x^2\,dx$ with $n=4$

$h = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16
	y_0	y_1	y_2	y_3	y_4

$$\int_0^4 x^2\,dx = \frac{h}{3}\left[(y_0+y_4)+4(y_1+y_3)+2y_2\right]$$

$$= \frac{1}{3}\left[0+16+4(1+9)+2\times 4\right]$$

$$= \frac{1}{3}\left[16+4(10)+8\right]$$

$$= \frac{64}{3}$$

$$= 21.3333$$

③ $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n = 6$

$h = \frac{\pi/3 - 0}{6} = \pi/18$

x	0	$\frac{\pi}{18}$	$\frac{2\pi}{18}$	$\frac{3\pi}{18}$	$\frac{4\pi}{18}$	$\frac{5\pi}{18}$	$\frac{6\pi}{18}$
	0	0.4167	0.5848	0.7071	0.8017	0.9752	0.9306
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$\int_0^{\pi/3} \sqrt{\sin x}\, dx = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{\pi/18}{3}\left[0.4167+0.9306+4(0.4167+0.7071+0.9752) + 2(0.5848+0.8017)\right]$$

$$= \frac{\pi}{54}\left[1.3473+4(1.999)+2(1.3865)\right]$$

$$= \frac{\pi}{54}\left[1.3473+7.996+2.773\right]$$

$$= \frac{\pi}{54} \times 12.1163$$

$$= 0.7049$$

Practical no: 7

Topic: Differential Equation

1) $x\frac{dy}{dx} + y = e^x$

2) $e^x \frac{dy}{dx} + 2e^x y = 1$

3) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

4) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

5) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

6) $\sec^2 x \cdot \tan y\, dx + \sec^2 y \tan x\, dy = 0$

7) $\frac{dy}{dx} = \sin^2(x-y+1)$

8) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

Solution

① $x\frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$P(x) = \frac{1}{x}$, $Q(x) = \frac{e^x}{x}$

$I.f = e^{\int p\, dx}$

$= e^{\int 1/x\, dx}$

$= e^{\ln|x|}$

I.f $= x$

$y(\text{I.f}) = \int Q(x) \times \text{I.f}\ dx + c$

$y.x = \int \frac{e^x}{\cancel{x}} \cdot \cancel{x}\ dx + c$

$xy = e^x + c$

② $e^x \frac{dy}{dx} + 2e^x y = 1$

$\frac{dy}{dx} + \frac{2e^x y}{e^x} = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2$, $Q(x) = e^{-x}$

I.f $= e^{\int 2\, dx}$

$= e^{2x}$

$y.\text{If} = \int Q(x) \cdot (\text{I.f})\ dx + c$

$y \cdot e^{2x} = \int e^{-x} \cdot e^{2x}\ dx + c$

$= \int e^x\ dx + c$

$y \cdot e^{2x} = e^x + c$

③ $x\frac{dy}{dx} + \frac{\cos x}{x} = 2y$

$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2/x$ $\quad Q(x) = \cos x / x^2$

I.f $= e^{\int P dx}$.

$= e^{\int 2/x\, dx}$

$= e^{\ln|x^2|}$

$= x^2$

$y(\text{I.f}) = \int Q(x) \cdot (\text{I.f})\ dx + c$

$= \int \frac{\cos x}{\cancel{x^2}} \times \cancel{x^2}\ dx + c$

$= \int \cos x\ dx + c$

$x^2 y = \sin x + c$

④ $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$

$P(x) = 3/x$ $\quad Q(x) = \sin x / x^3$

I.f $= e^{\int P dx}$

$= e^{\int 3/x\, dx}$

$= e^{\ln|x^3|}$

$= x^3$

$y \cdot \text{If} = \int Q(x) \cdot (\text{If})\ dx + c$

$= \int \frac{\sin x}{\cancel{x^3}} \cdot \cancel{x^3}\ dx + c$

$x^3 y = -\cos x + c$

(5) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \qquad Q(x) = 2x/e^{2x}$

$I.f = e^{\int P dx}$

$= e^{\int 2 dx}$

$= e^{2x}$.

$y(I.f) = \int Q(x)\,(I.f)\,dx + c$

$y \cdot e^{2x} = \int \frac{2x}{\cancel{e^{2x}}} \cdot \cancel{e^{2x}}\,dx + c$

$= \int 2x\,dx + C$

$e^{2x} \cdot y = x^2 + c$

(6) $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$\sec^2 x \cdot \tan y\,dx = -\sec^2 y \tan x\,dy$

$\frac{\sec^2 x}{\tan x}\,dx = -\frac{\sec^2 y}{\tan y}\,dy$

On Integrating we get

$\int \frac{\sec^2 x}{\tan x}\,dx = -\int \frac{\sec^2 y}{\tan y}\,dy$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\log|\tan x \cdot \tan y| = c$

$\therefore \tan x \cdot \tan y = e^c$

(7) $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$

Differentiating on both sides

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$\frac{dv}{dx} = 1 - \sin^2 v \qquad \frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v\,dv = \int dx$

$\tan v = x + c$

$\tan(x+y-1) = x + c$.

(8) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

put $2x+3y = v$

$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\frac{(v-1)}{(v+2)}$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$

$= \frac{3v+3}{v+2}$

$\frac{dv}{dx} = \frac{3(v+1)}{v+2}$

$\frac{v+2}{v+1}\,dv = 3\,dx$

On Integrating we get,

$$\int \frac{v+2}{v+1}\,dv = \int 3\,dx$$

$$\int \frac{v+1}{v+1}\,dx + \int \frac{1}{v+1}\,dv = 3x$$

$$v + \log|v+1| = 3x + c$$

$$2x + \cancel{\log}\, 3y + \log|2x+3y+1| = 3x + c$$

$$3y = x - \log|2x+3y+1| + c$$

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1/01/2024

Practical no: 8

Topic: Euler's method

① $\frac{dy}{dx} = y + e^{x} - 2$ $y(0) = 2$, $h = 0.5$ find $y(2)$

② $\frac{dy}{dx} = 1 + y^2$ $y(0) = 0$ $h = 0.2$ find $y(1)$.

③ $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$ $h = 0.2$ find $y(1)$

④ $\frac{dy}{dx} = 3x^2 + 1$ $y(1) = 2$ find $y(2)$
For $h = 0.5$ & $h = 0.25$

⑤ $\frac{dy}{dx} = \sqrt{xy} + 2$, $y(1) = 1$ find $y(1.2)$ with $h = 0.2$

Solution.

① $y(0) = 2$ ~~x(0)~~ $x_0 = 0$ $h = 0.5$
find $y(2)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$y(2) = 9.8215$

(2) $\frac{dy}{dx} = 1 + y^2$ $\quad y(0) = 0 \quad h = 0.2$

find $y(1)$

$y_0 = 0 \quad x_0 = 0 \quad h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	↓ 1.8526	1.2939
5	1	1.2939		

$y(1) = 1.2939$

(3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $\quad y(0) = 1 \quad h = 0.2 \quad$ find $y(1)$

$x_0 = 0, \quad y_0 = 1 \quad h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7694	1.5051
5	1	1.5051		

$y(1) = 1.5051$

(4) $\frac{dy}{dx} = 3x^2 + 1 \quad y(1) = 2 \quad$ find $y(2)$

$h = 0.5$ & $h = 0.25$.

$y_0 = 2 \quad x_0 = 1$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$y(2) = 7.875$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6509	19.3360
3	1.75	19.3360	1122.6426	299.9966
4	2	299.996		

$y(2) = 299.996$

(5) $\frac{dy}{dx} = \sqrt{xy} + 2 \quad y(1) = 1 \quad$ find $y(1.2)$

$h = 0.2$

11/01/2020

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	~~1.86~~ 3.6		

$y(1.2) = \cancel{1.86}\ 3.6$

Practical no: 9

Topic: Limits & partial order derivative.

Q.1 Evaluate the following limits.

1) $\lim\limits_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

2) $\lim\limits_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

3) $\lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

Q.2 Find f_x, f_y for each of the following f.

1) $f(x,y) = xy\,e^{x^2+y^2}$
2) $f(x,y) = e^x \cos y$
3) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

Q.3 Using definition find values of f_x, f_y at $(0,0)$ for

$$f(x,y) = \frac{2x}{1+y^2}$$

Q.4 Find all second order partial derivatives of f. Also verify whether $f_{xy} = f_{yx}$.

1) $f(x,y) = \dfrac{y^2-xy}{x^2}$
2) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$
3) $f(x,y) = \sin(xy) + e^{x+y}$

Q.5 Find the linearization of $f(x,y)$ at given point.

1) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$
2) $f(x,y) = 1-x+y\sin x$ at $(\pi/2, 0)$
3) $f(x,y) = \log x + \log y$ at $(1,1)$

Solution

Q.1.1) $\lim\limits_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

At $(-4,-1)$ denominator $\neq 0$

$\therefore$ By applying limit

$$= \frac{(-4)^3 - 3(-4) + (-4)^2 - 1}{-4(-1)+5}$$

$$= \frac{+61}{9}$$

2) $\lim\limits_{(x,y)\to(-2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

At $(-2,0)$ denominator $\neq 0$

$\therefore$ By applying limit

$$= \frac{(0+1)(4+0+8)}{-2}$$

$$= \frac{+12}{-2} = -6$$

3) $\lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

At $(1,1,1)$ denominator $\neq 0$.

By applying limit

$$= \frac{1^2 - 1^2(1)^2}{1^3 - 1^2(1)(1)} = 1$$

Q.2.1) $f(x,y) = xy\,e^{x^2+y^2}$

$\therefore f_x = \frac{\partial f}{\partial x}(x,y)$

$= \frac{\partial}{\partial x}(xy e^{x^2+y^2})$

$= y e^{x^2+y^2}(2x)$

$= 2xy\,e^{x^2+y^2}$

$f_y = \frac{\partial f(x,y)}{\partial y}$

$= \frac{\partial (xye^{x^2+y^2})}{\partial y}$

$= xe^{x^2+y^2}.(2y)$

$= 2yxe^{x^2+y^2}$

② $f(x,y) = e^x \cos y$.

$\rightarrow f_x = \frac{\partial f}{\partial x}(x,y)$

$= \frac{\partial (e^x \cos y)}{\partial x}$

$= e^x \cos y$

$f_y = \frac{\partial f(x,y)}{\partial y}$

$= \frac{\partial (e^x \cos y)}{\partial y}$

$= -e^x \sin y$

③ $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$\rightarrow f_x = \frac{\partial f}{\partial x}(x,y)$

$= \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$

$= 3x^2y^2 - 6xy + y^3$

$f_y = \frac{\partial f}{\partial y}(x,y)$

$= \frac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1)$

$= 2yx^3 - 3x^2 + 3y^2$

Q.3. $f(x,y) = \frac{2x}{1+y^2}$

$\rightarrow f_x = \frac{\partial f}{\partial x}(x,y)$

$= \frac{\partial}{\partial x}\left(\frac{2x}{1+xy^2}\right)$

$= \frac{1+y^2 \frac{\partial}{\partial x} 2x - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$

$= ($

$= \frac{(1+y^2)2 - 2x(1+y^2)}{(1+y^2)^2}$

$= \frac{2+2y^2 - \cancel{2x} + 2xy^2}{(1+y^2)^2}$

$= \frac{2+2y^2}{(1+y^2)^2}$

$= \frac{2}{1+y^2}$

At $(0,0)$

$f_x = \frac{2}{1+0} = 2$

$f_y = \frac{\partial t}{\partial y} (2x/1+y^2)$

$= \frac{(1+y^2)\frac{\partial}{\partial y}(2x) - 2x\frac{\partial}{\partial y}(1+y^2)}{}$

$= \frac{(1+y^2)(0) - 2x(2y)}{(1+y^2)^2}$

$= \frac{-4xy}{(1+y^2)^2}$

At (0,0)

$f_y = \frac{-4(0)(0)}{(1+0^2)^2} = 0$

Q4 ① $f(x,y) = \frac{y^2 - xy}{x^2}$

$f_x = \frac{x^2\frac{\partial}{\partial x}(y^2-xy) - (y^2-xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$

$= \frac{x^2(-y) - (y^2-xy)(2x)}{x^4}$

$= \frac{-x^2y - 2x(y^2-xy)}{x^4}$

$f_y = \frac{2y-x}{x^2}$

$f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2y - 2x(y^2-xy)}{x^4}\right)$

$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2x^2y)}{x^6}$ — ①

$f_{yy} = \frac{\partial}{\partial y}\left(2y - \frac{x}{x^2}\right)$

$= \frac{2-0}{x^2} = \frac{2}{x^2}$ — ②

$f_{xy} = \frac{\partial}{\partial y}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$

$= \frac{x^2 - 4xy + 2x^2}{x^4}$ — ③

$f_{yx} = \frac{\partial}{\partial x}\left(2y - \frac{x}{x^2}\right)$

$= \frac{x^2\frac{\partial}{\partial x}(x(y-x)) - (xy-x)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$

$= \frac{-x^2 - 4xy - 2x^2}{x^4}$ — ④

from ③ & ④

$f_{xy} = f_{yx}$

③ $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$\rightarrow f_x = \frac{\partial}{\partial x}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 3x^2 + 6xy^2 - \frac{2x}{x+1}$

$f_y = \frac{\partial}{\partial y}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 6x^2y$

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$$f_{xx} = 6x + 6y^2 - \left(x^2 \frac{\partial}{\partial x}(2x - 2x \frac{\partial}{\partial x}(x^2+1)\right)$$

$$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right) \text{ — (1)}$$

$$f_{yy} = \frac{\partial}{\partial y}(6x^2 y)$$

$$= 6x^2 \text{ — (2)}$$

$$f_{xy} = \frac{\partial}{\partial y}(3x^2 + 6xy^2 - 2x/x^2+1)$$

$$= 0 + 12xy - 0$$

$$= 12xy \text{ — (3)}$$

$$f_{yx} = \frac{\partial}{\partial x}(6x^2 y)$$

$$= 12xy \text{ — (4)}$$

from (3) & (4)

$f_{yx} = f_{xy}$

(3) $f(x,y) = \sin(xy) + e^{x+y}$

$$f_x = y\cos(xy) + e^{x+y}(1)$$

$$= y\cos(xy) + e^{x+y}$$

$$f_y = x\cos(xy) + e^{x+y}(1)$$

$$= x\cos(xy) + e^{x+y}$$

$$f_x = \frac{\partial}{\partial x}(y\cos(xy) + e^{x+y})$$

$$= -y\sin(xy)(y) + e^{y+x}(1)$$

$$= -y^2\sin(xy) + e^{x+y} \text{ — (1)}$$

$$f_{yy} = \frac{\partial}{\partial y}(x\cos(xy) + e^{x+y})$$

$$= -x\sin(xy)\cdot(x) + e^{x+y}(1)$$

$$= -x^2\sin(xy) + e^{x+y}$$

$$f_{xy} = \frac{\partial}{\partial y}(y\cos(xy) + e^{x+y})$$

$$= (-y^2\sin(xy) + \cos(xy) + e^{x+y} \text{ — (3)}$$

$$f_{yx} = \frac{\partial}{\partial x}(x\cos(xy) + e^{x+y})$$

$$= -x^2\sin(xy) + \cos(xy) + e^{x+y} \text{ — (4)}$$

from (3) & (4)

$f_{xy} \neq f_{yx}$

Q.5.) $f(x,y) = \sqrt{x^2+y^2}$

$f(1,1) = \sqrt{1^2+1^2} = \sqrt{2}$

$$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$$

$$= \frac{x}{\sqrt{x^2+y^2}}$$

$$f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$$

$$= \frac{y}{\sqrt{x^2+y^2}}$$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$

f_y at $(1,1) = \frac{1}{\sqrt{2}}$

$f(x,y) = f(a,b) + f_x(a,b)(x-a) +$
$f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$

(2) $f(x,y) = 1 - x + y\sin x$

$f(\frac{\pi}{2}, 0) = 1 - \frac{\pi}{2} + 0$

$= 1 - \frac{\pi}{2}$

$f_x = -1 + y\cos x$

f_x at $(\pi/2, 0) = -1$

$f_y = 0 - 0 + \sin x$

f_y at $(\pi/2, 0) = \cancel{0}\ 1$

$f(x,y) = f(a,b) + f_x(a,b)(x-a) +$
$f_y(a,b)(y-b)$

$= 1 - \frac{\pi}{2} + (-1)(x - \pi/2) + 1(y-0)$

$= 1 - \pi/2 - x + \pi/2 + y$

$= 1 - x + y$

(3) $f(x,y) = \log x + \log y$

$\rightarrow f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x}$ $\qquad f_y = \frac{1}{y}$

f_x at $(1,1) = 1$ $\qquad f_y$ at $(1,1) = 1$

$f(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

Practical no: 10

Topic: Directional derivative, Gradient vector & maxima, minima Tangent & normal vectors.

Q.1. Find the directional derivative of the following function at given points & in the direction of given vector.

1) $f(x,y) = x + 2y - 3$ $a = (1,-1)$ $u = 3i - j$

2) $f(x,y) = y^2 - 4x + 1$ $a = (3,4)$ $u = i + 5j$

3) $f(x,y) = 2x + 3y$ $a = (1,2)$ $u = 3i + 4j$

Q.2. Find gradient vector for following function at given points.

① $f(x,y) = x^y + y^x$, $a = (1,1)$

② $f(x,y) = (\tan^{-1} x) \cdot y^2$ $a = (1,-1)$

③ $f(x,y,z) = xyz - e^{x+y+z}$, $a = (1,-1,0)$

Q.3. Find the equation of tangent & normal to each of the following curves at given points.

① $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

② $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

Q.4. Find the equation of tangent & normal line to each of the following surfaces.

① $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

② $3xyz - x - y + z = -4$ at $(1,-1,2)$

Q.5. Find the local maxima & minima for the following functions.

① $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

② $f(x,y) = 2x^4 + 3x^2 y - y^2$

③ $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

Solution

Q.1.1) Here $u = 3i - j$ is not a unit vector

$|\bar{u}| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = 1 + 2(-1) + 3 = -1 - 2 - 3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{-4 + \cancel{h}/\sqrt{10} + 4}{\cancel{h}}$$

$$= \frac{1}{\sqrt{10}}$$

② Here $u = i + 5j$ is not a unit vector

$|\bar{u}| = \sqrt{1^2 + 5^2} = \sqrt{26}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$f(a) = f(3,4) = 4^2 - 4(3) + 1 = 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$f(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h\to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{\cancel{h}}$$

$$= \frac{\cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{\cancel{h}}$$

$$= \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

③ Here $u = 3i + 4j$ is not a unit vector

$|\bar{u}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3, 4)$

$$= \left(\frac{3}{5}, \frac{4}{5}\right)$$

$f(a) = f(1, 2) = 2(1) + 3(2) = 8$

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$$= f\left(1 + \frac{3h}{5}, 2 + \frac{4}{5}\right)$$

$$= 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$$

$$= 2 + \frac{6h}{5} + 6 + \frac{12h}{15}$$

$$= \frac{18h}{5} + 8$$

$$D_u f(a) = \lim_{h\to 0} \frac{\frac{18\cancel{h}}{5} + 8 - 8}{\cancel{h}}$$

$$= \frac{18}{5}$$

Q.2.

① $f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = y \cdot x^y \log x + xy^{x-1}$

$\nabla f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^2 \log y, x^y \log x + xy^{x-1})$

$f(1,1) = (1+0, 1+0) = (1,1)$

(2) $f_x = \frac{1}{1+x^2} \cdot y^2$ $\quad f_y = 2y \cdot \tan^{-1} x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y\tan^{-1} x\right)$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \frac{\pi}{\cancel{4}\,2}(-\cancel{2})\right)$

$= \left(\frac{1}{2}, -\frac{\pi}{2}\right)$

(3) $f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1,-1,0) = ((-1)(0) - e^{(1-1+0)}, (1)(0)\, e^{(1+-1-0)}, (1)(-1) - e^{1+(-1)+0})$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

Q.3.

(1) $f_x = \cos y \cdot 2x + e^{xy} \cdot y$

$f_y = x^2(-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1, 0)$ $\quad \therefore x_0 = 1,\ y_0 = 0.$

eqn of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \cdot 2(1) + e^0 \cdot 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2 \times (-\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0 \rightarrow$ is eqn of tangent.

eqn of normal

$= ax + by + c = 0$

$= bx + ay + d = 0.$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$ at $(1,0)$

$= 1 + 2(0) + d = 0$

$d + 1 = 0$

$\therefore d = -1$

(2) $f_x = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2, -2)$ $\quad x_0 = 2,\ y_0 = 2$

$f_x(x_0, y_0) = 2(2) - 2 = 2$

$f_y(x_0, y_0) = 2(-2) + 3 = -1$

eqn of tangent

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2(x-2) + 1(y+2) = 0$

$2x-2-y-2=0$

$2x-y-4=0 \longrightarrow$ is eqn of tangent.

eqn of normal

$ax+by+c=0$

$bx+ay+d=0$

$=-1(x)+2(y)+d=0$

$-x+2y+d=0$ at $(2,-2)$

$-2+2(-2)+d=0$

$-2-4+d=0$

$-6+d=0$

$d=6$

Q.4.

(1) $f_x = 2x-0+0+z = 2x+z$

$f_y = 0-2z+3+0 = 2z+3$

$f_z = 0-2y+0+x$

$= -2y+x$

$(x_0,y_0,z_0) = (2,1,0)$ $x_0=2, y_0=1, z_0=0$

$f_x(x_0,y_0,z_0) = 2(2)+0 = 4$

$f_y(x_0,y_0,z_0) = 2(0)+3 = 3$

$f_z(x_0,y_0,z_0) = -2(1)+2 = 0$

eqn of tangent

$f_x(x-x_0) + f_y(y-y_0) + f_z(z-z_0) = 0$

$4(x-2)+3(y-1)+0(z-0)=0$

$= 4x-8+3y-3$

$= 4x+3y-11 \longrightarrow$ is eqn of tangent.

Eqn of normal at $(4,3,-11)$

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

(2) $f_x = 3yz-1-0+0+0 = 3yz-1$

$f_y = 3xz-0-1+0+0 = 3xz-1$

$f_z = 3xy-0-0+1+0 = 3xy+1$

$(x_0,y_0,z_0) = (1,-1,2)$ $x_0=1, y_0=-1, z_0=2$

$f_x(x_0,y_0,z_0) = 3(-1)(2)-1 = -7$

$f_y(x_0,y_0,z_0) = 3(1)(2)-1 = 5$

$f_z(x_0,y_0,z_0) = 3(1)(-1)+1 = -2$

eqn of tangent

$-7(x-1)+5(y+1)-2(z-2)=0$

$-7x+7+5y+5-2z+4=0$

$-7x+5y-2z+16=0 \longrightarrow$ is eqn of tangent.

Eqn of normal at $(-7,5,-2)$

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$\frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Q.5.

(1) $f_x = 6x+0-3y+6-0 = 6x-3y+6$

$f_y = 0+2y-3x+0-4 = 2y-3x-4$

$f_x = 0$

$6x-3y+6=0$

$3(2x-y+2)=0$

$2x-y+2=0$

$2x-y=-2$ —— (1)

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$f_y = 0$

$2y - 3x - 4 = 0$

$\therefore 2y - 3x = 4$ —— ②

multiply eqn ① & ②

$4x - 2y = -4$

$\underline{2y - 3x = 4}$

$x = 0$

substitute value of x in eqn ①

$2(0) - y = -2$

$y = 2$

Critical points are $(0, 2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = 3$

Here $r > 0$ $\quad rt - s^2 = 6(2) - (3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore f$ had maximum at $(0, 2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0, 2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= -4.$

② $f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y.$

$f_x = 0.$

$8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0$ —— ①

$f_y = 0$

$3x^2 - 2y = 0$ —— ②

multiply eqn ① with 3

② with 4

$12x^2 + 9y = 0$

$\underline{-12x^2 - 8y = 0}$

$11y = 0$

$y = 0.$

substitute value of y in eqn ①

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0.$

Critical point is $(0, 0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0, 0)$

$= 24(0) + 6(0) = 0$

$r = 0.$

$rt - s^2 = 0(-2) - (0)^2$

$= 0 - 0 = 0.$

$r = 0$ & $rt - s^2 = 0$

(nothing to say).

$f(x, y)$ at $(0, 0)$

$2(0)^4 + 3(0)^2(0) - 0$

$= 0 + 0 - 0$

$= 0.$

③ ~~f(x)~~. $f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0$

$2x + 2 = 0$

$x = -1.$

$f_y = 0$

$-2y + 8 = 0$

$y = 4$

$\therefore$ critical points is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - 0^2$

$= -4 < 0$

$f(x, y)$ at $(-1, 4)$

$(-1)^2 - 4^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 47 - 70$

$= -33$

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