Filter Design Project using Matlab and Circuit Simulator

This project includes design, implementation, and verification of band pass filters and a band reject filers using active devices. In this project, you need to minimize the number of active devices to reduce monetary cost as an engineer. In the Bode plots, please mark the cutoff frequencies and pass band gain. Please report design procedures, Matlab simulation code/results, circuit schematics, and simulation results, and discussion. The report should be max 10 pages.

1. Specification of Band Pass Filter

a. Pass Band Voltage Gain = 10dB

b. Cutoff Frequencies: Fc_1= 10kHz, Fc_2 = 800kHz

Design a multi-order active band pass filter using a cascade method. You need to choose your own model order for the low-pass filter and the high-pass filter. To verify your design, please code your transfer function and draw the Bode plot (Magnitude plot and phase plot) using Matlab. Then, please perform circuit simulation. If the result from the circuit simulator is different from Matlab simulation, please adjust the value to satisfy the specification.

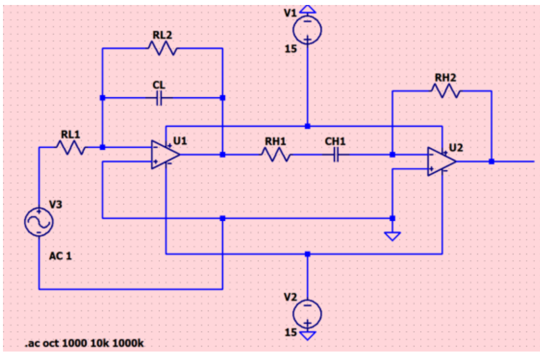
Extra Credit: If you can design a better band pass filter with the same model order, please find the transfer function of it and show the Bode plot (Matlab only).

Solution)

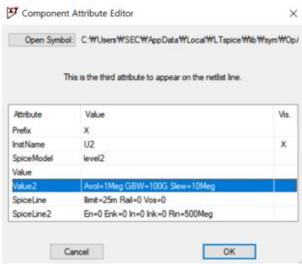
The implementation of this filter is explained through a step-by-step approach.

step 1: Design the overall structure of the circuit.

A second-order active bandpass filter was designed as follows:



Schematic



Basic property of the op-amp (Model:universalopamp2)

step 2: Draw out and solve the mathematical equations to satisfy the conditions stated above.

Recall that the voltage gain of an inverting op-amp is $-\frac{z2}{z1}$, where z1 is the impedance which is on the way to the inverting input of the op-amp and z2 is the impedance which is on the feedback branch of the op-amp.

Since two inverting op-amps are cascaded, the total voltage gain would be the product of the voltage gain of each stage, namely, Gv=$-\frac{ZL2}{ZL1}\times-\frac{ZH2}{ZH1}=\frac{(ZL2)(ZH2)}{(ZL1)(ZH1)}$.

It is established that ZL1=RL1, $ZL2=RL2//\frac{1}{S(CL)}, ZH1=RH1+\frac{1}{S(CH)}, ZH2=RH2$ so

Gv= $\frac{(ZL2)(ZH2)}{(ZL1)(ZH1)}=\frac{(RL2//\frac{1}{S(CL)})(RH2)}{(RL1)(RH1+\frac{1}{S(CH)})}=\frac{(\frac{RL2\frac{1}{S(CL)}}{RL2+\frac{1}{S(CL)}})(RH2)}{(RL1)(RH1+\frac{1}{S(CH)})}=\frac{(RH2)}{(RL1)}\frac{(\frac{RL2\frac{1}{S(CL)}}{RL2+\frac{1}{S(CL)}})}{(RH1+\frac{1}{S(CH)})}=$
$\frac{(RH2)}{(RL1)}\frac{RL2\frac{1}{S(CL)}}{(RH1+\frac{1}{S(CH)})(RL2+\frac{1}{S(CL)})}=\frac{(RH2)(RL2)}{(RL1)}\frac{1}{(RH1+\frac{1}{S(CH)})(RL2+\frac{1}{S(CL)})S(CL)}$

$$=\frac{(RH2)(RL2)}{(RL1)}\frac{1}{\{(RH1)(RL2)+\frac{1}{S}(\frac{RL2}{CH}+\frac{RH1}{CL})+\frac{1}{S^2(CH)(CL)}\}S(CL)}$$

$$=\frac{(RH2)(RL2)(CH)}{(RL1)}\frac{S}{(RH1)(RL2)(CL)(CH)S^2+((CL)RL2+(CH)RH1)S+1}$$

$$=\frac{(RH2)(RL2)(CH)}{(RL1)}\frac{\frac{1}{(RH1)(RL2)(CL)(CH)}S}{S^2+\frac{\{(CL)RL2+(CH)RH1\}}{(RH1)(RL2)(CL)(CH)}S+\frac{1}{(RH1)(RL2)(CL)(CH)}}$$

$$=\frac{(RH2)(RL2)(CH)}{(RL1)\{(CL)RL2+(CH)RH1\}}\frac{\frac{\{(CL)RL2+(CH)RH1\}}{(RH1)(RL2)(CL)(CH)}S}{S^2+\frac{\{(CL)RL2+(CH)RH1\}}{(RH1)(RL2)(CL)(CH)}S+\frac{1}{(RH1)(RL2)(CL)(CH)}}$$

From this, β=$\frac{\{(CL)RL2+(CH)RH1\}}{(RH1)(RL2)(CL)(CH)}$, ω0=$\frac{1}{\sqrt{(RH1)(RL2)(CL)(CH)}}$, $K=\frac{(RH2)(RL2)(CH)}{(RL1)\{(CL)RL2+(CH)RH1\}}$ can be derived, where Gv=K$\frac{\beta S}{S^2+\beta S+\omega 0^2}$. Indeed, this model is of order 2.

By the condition that the pass band gain is $10dB=\sqrt{10}$, K= $\sqrt{10}$ is obtained, that is, $\frac{(RH2)(RL2)(CH)}{(RL1)\{(CL)RL2+(CH)RH1\}}=\sqrt{10}$

Also, recall that the two cut-off frequencies of a band pass filter are:

ωc1,c2=$\pm\frac{\beta}{2}+\sqrt{(\frac{\beta}{2})^2+\omega 0^2}$

By this, ωc2-ωc1=β and ωc2+ωc1=2$\sqrt{(\frac{\beta}{2})^2+\omega 0^2}=\sqrt{\beta^2+4\omega 0^2}$ can be derived, that is,
1580kπ=$\frac{\{(CL)RL2+(CH)RH1\}}{(RH1)(RL2)(CL)(CH)}$, 1620kπ=$\sqrt{\beta^2+4\omega 0^2}$=$\sqrt{(1580k\pi)^2+4\omega 0^2}$

Solving these 3 equations yields 3 relations, which are $(RH1)(RL2)(CL)(CH)$=3.16629× 10^{-12}, $(CL)RL2+(CH)RH1$=1.57166× 10^{-5}, and $\frac{(RH2)(RL2)(CH)}{(RL1)}$=4.97003× 10^{-5}.

step 3: Draw the Bode plot for Gv (both the magnitude plot and the phase plot)

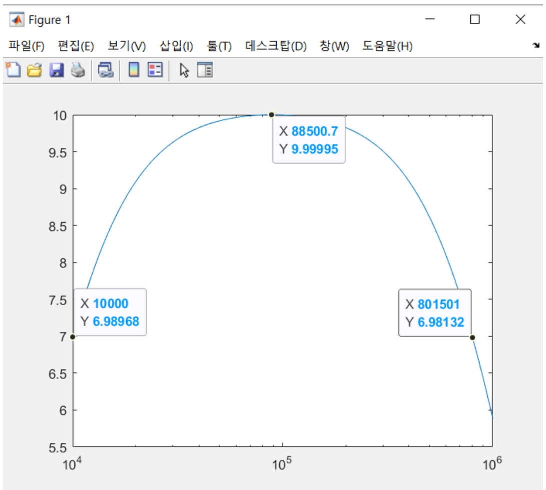
Note that the magnitude of Gv is

$$|Gv|=K\left|\frac{\beta S}{S^2+\beta S+\omega 0^2}\right| = \frac{K\beta\omega}{\sqrt{(\omega 0^2-\omega^2)^2+(\beta\omega)^2}} = \frac{15696.6k\omega}{\sqrt{(3.15827\times 10^{11}-\omega^2)^2+(1580k\pi\omega)^2}}$$

This expression of $|Gv|$ was examined on Matlab and the Bode plot was obtained as follows:

```
f = logspace(4,6,1000);
w=2*pi*f;
numerator=15696600.*w;
denominator=sqrt((3.15827*10^11-w.^2).^2+(1580000*pi.*w).^2);
Gv=numerator./denominator;
GdB=20*log10(Gv);
semilogx(f,GdB)
```

MATLAB code



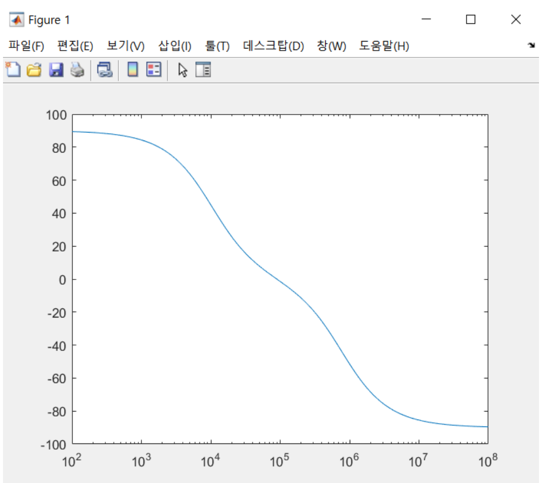
Bode plot for the magnitude of the voltage gain

Indeed, the passband gain is nearly 10 in dB, the frequencies at which the gain is -3dB from the passband gain is nearly 10kHz and 800kHz, which are referred to as the cut-off frequencies.

The MATLAB code and plot for the phase response would be as follows:

```
f = logspace(2,8,1000);
w=2*pi*f;
phase=90-atan2d(1580000*pi.*w,3.15827*10^11-w.^2);
semilogx(f,phase)
```

MATLAB code



Bode plot for the phase of the voltage gain

step 4) Determine values of the parameters and implement the circuit on LTspice

So far, 3 relations have been obtained for 6 parameters, which are RL1, RL2, CL, RH1, RH2, CH. Therefore, 3 values of these parameters can be set arbitrarily, and then the other 3 can be obtained from the 3 relations.

Again, recall $(RH1)(RL2)(CL)(CH)$=3.16629× 10^{-12},
$(CL)RL2 + (CH)RH1$=1.57166× 10^{-5}, and $\frac{(RH2)(RL2)(CH)}{(RL1)}$=4.97003× 10^{-5}.

Since CH and RL2 exist in all of the 3 relations, they can be set arbitrarily, for example as CH=1nF, RL2=1kΩ. Once these values are plugged into the relations, RH1(CL)=3.16629 × 10^{-6} and $10^{12}(CL) + RH1 = 15716.6$ are obtained. Solving these 2 equations yields either (RH1,CL)=(204.112Ω,15.5125nF) or (RH1,CL)=(15.5125kΩ,0.204112nF). The second pair is selected for the solution.

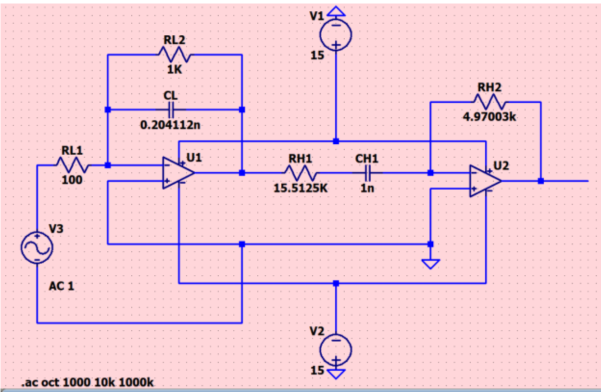
At this stage, it remains to determine RH1 and RL1, for which the relationship $\frac{(RH2)}{(RL1)}$=49.7003 is provided. Assuming RL1=100Ω, the relationship yields RH2=4.97003kΩ.

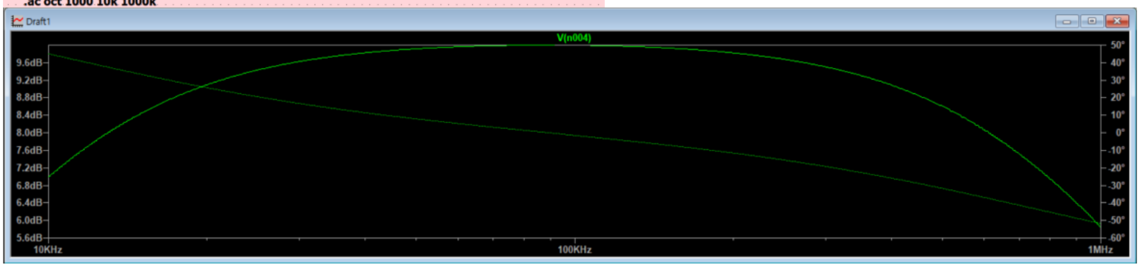
Now all the values of the 6 parameters are obtained as follows:

[Table 1] Parameter values of the bandpass filter

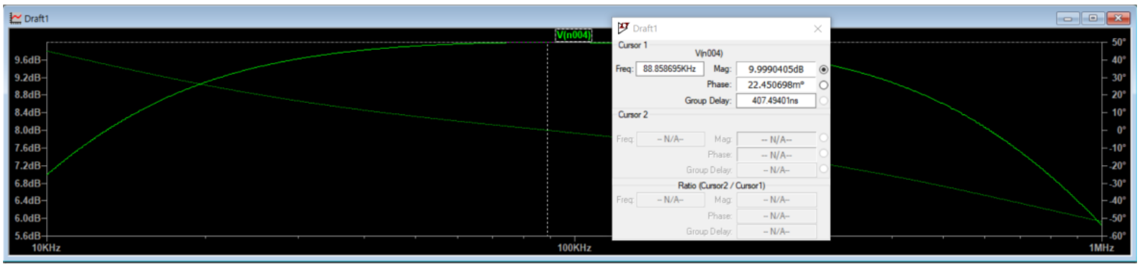
RL1	RL2	CL	RH1	RH2	CH
100Ω	1kΩ	0.204112nF	15.5125kΩ	4.97003kΩ	1nF

These values were plugged in on LTspice and the simulation was conducted. The resulting circuit and the plots are as follows:

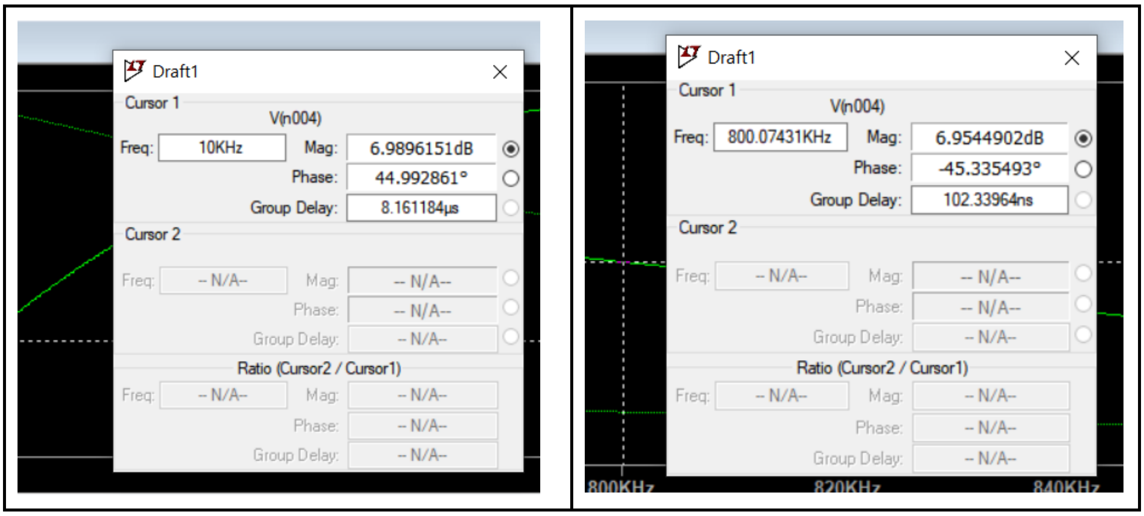




The solid line indicates the magnitude response, and the dotted line indicates the phase response.



It could be verified that the passband gain is 9.9990405dB, which is almost identical to 10dB.



It could be also verified that the two cut-off frequencies are 10kHz, and 800kHz, as intended.

Conclusion:

A bandpass filter was successfully implemented that meets all the specified conditions. Numerous combinations of resistor and capacitor values could have been utilized, provided they adhere to the three established relationships. In terms of cost efficiency, this design appears to be more economical compared to other second-order bandpass filter designs, due to the minimal number of resistors and capacitors employed in the circuit.

2. Specification of Band Reject Filter

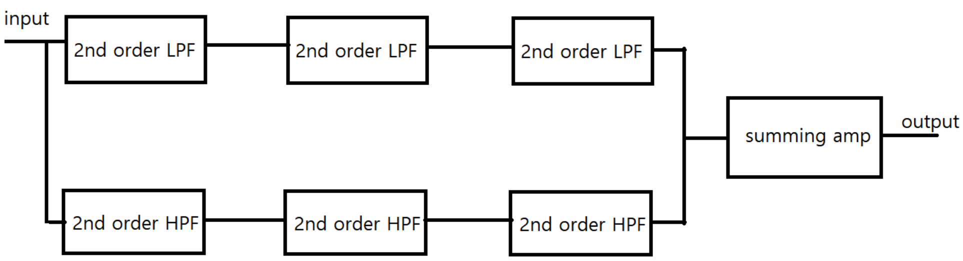
a. Pass Band Gain = 10dB
b. Cutoff Frequencies: $Fc1$ = 10kHz, $Fc2$ = 500kHz
c. Minimum attenuation at 100kHz = - 15dB

Design a multi-order active band-reject filter using a cascade method. You need to determine the model order of your transfer function. To verify your design, please code your transfer function and draw the Bode plot (Magnitude plot and phase plot) using Matlab first. Then, please perform circuit simulation. Please mark the max gain and the gain at 100 KHz. In addition, please mark the cutoff frequencies. If the result from the circuit simulator is different from Matlab simulation, please adjust the value to satisfy the specification.

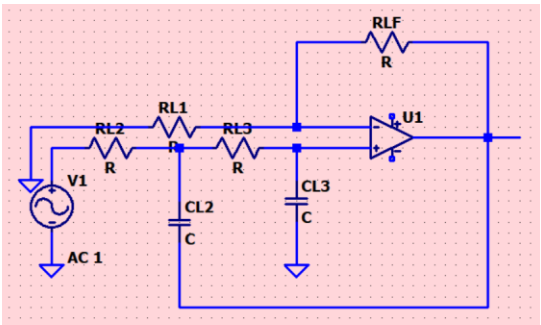
Solution)
The design steps are as follows:

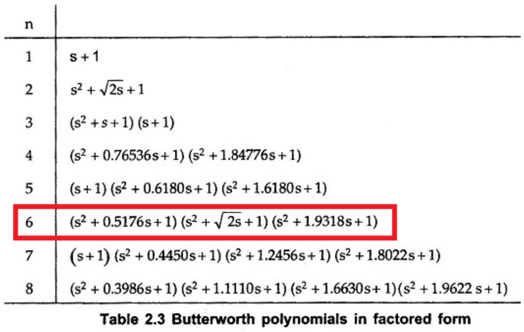
Step 1: Design the overall structure

To create a sixth-order band-reject filter, three identical second-order LPF's and three identical HPF's were cascaded. The outputs of these filters were then connected to a summing amplifier.



(1) LPF compartment





n	
1	$s+1$
2	$s^2+\sqrt{2}s+1$
3	$(s^2+s+1)(s+1)$
4	$(s^2+0.76536s+1)(s^2+1.84776s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$
6	$(s^2+0.5176s+1)(s^2+\sqrt{2}s+1)(s^2+1.9318s+1)$
7	$(s+1)(s^2+0.4450s+1)(s^2+1.2456s+1)(s^2+1.8022s+1)$
8	$(s^2+0.3986s+1)(s^2+1.1110s+1)(s^2+1.6630s+1)(s^2+1.9622s+1)$

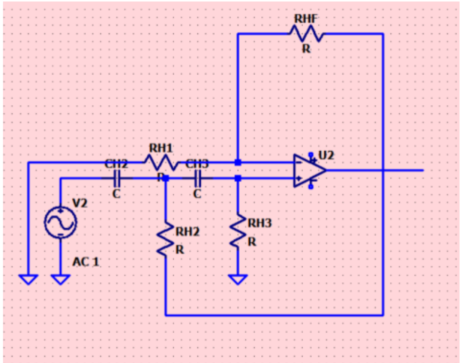
Table 2.3 Butterworth polynomials in factored form

This design employs a second-order Butterworth low-pass filter (LPF) with a transfer function corresponding to ωc=1, as detailed in the Butterworth transfer function table referenced from an online source (the website link is provided at the end of this document). To achieve a sixth-order low-pass filter, three of these second-order Butterworth LPFs will be cascaded.

The passband gain of this LPF is said to be $A = 1 + \frac{RLF}{RL1}$
Provided that the passband gain throughout the whole stage of the filter is $\sqrt{10}$, it becomes obvious that the passband gain for each stage is $(\sqrt{10})^{1/3}$, so $A = 1 + \frac{RLF}{RL1} = (\sqrt{10})^{1/3}$ is obtained, which results in $\frac{RLF}{RL1}$=0.4678.

By the condition that the cut-off frequency is 10kHz, the transfer function of this filter would be Glv= $\frac{(\sqrt{10})^{1/3}}{(\frac{s}{20k\pi})^2+\sqrt{2}(\frac{s}{20k\pi})+1}\frac{(\sqrt{10})^{1/3}}{(\frac{s}{20k\pi})^2+0.5176(\frac{s}{20k\pi})+1}\frac{(\sqrt{10})^{1/3}}{(\frac{s}{20k\pi})^2+1.9318(\frac{s}{20k\pi})+1}$ (which can be obtained by taking s=$\frac{s}{20k\pi}$ on the sixth order Butterworth polynomial)

(2) HPF compartment



This is Butterworth second order HPF which will be cascaded. (Again, the source link is provided at the end of this document).

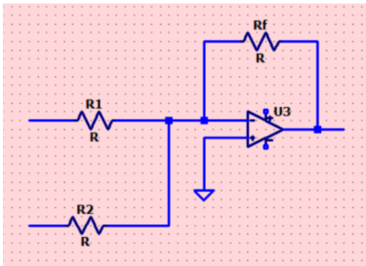
The passband gain is $|Gv|=1+\frac{RHF}{RH1}$.

Likewise, from the conditions, $\frac{RHF}{RH1}$=0.4678 can be obtained.

By the condition regarding the cut-off frequency, the transfer function of this filter would be

Ghv=$\frac{(\sqrt{10})^{1/3}(\frac{s}{1M\pi})^2}{(\frac{s}{1M\pi})^2+\sqrt{2}(\frac{s}{1M\pi})+1}\frac{(\sqrt{10})^{1/3}(\frac{s}{1M\pi})^2}{(\frac{s}{1M\pi})^2+0.5176(\frac{s}{1M\pi})+1}\frac{(\sqrt{10})^{1/3}(\frac{s}{1M\pi})^2}{(\frac{s}{1M\pi})^2+1.9318(\frac{s}{1M\pi})+1}$ (which can be obtained by taking s=$\frac{s}{1M\pi}$ on the sixth order Butterworth polynomial)

(3) Summing amplifier

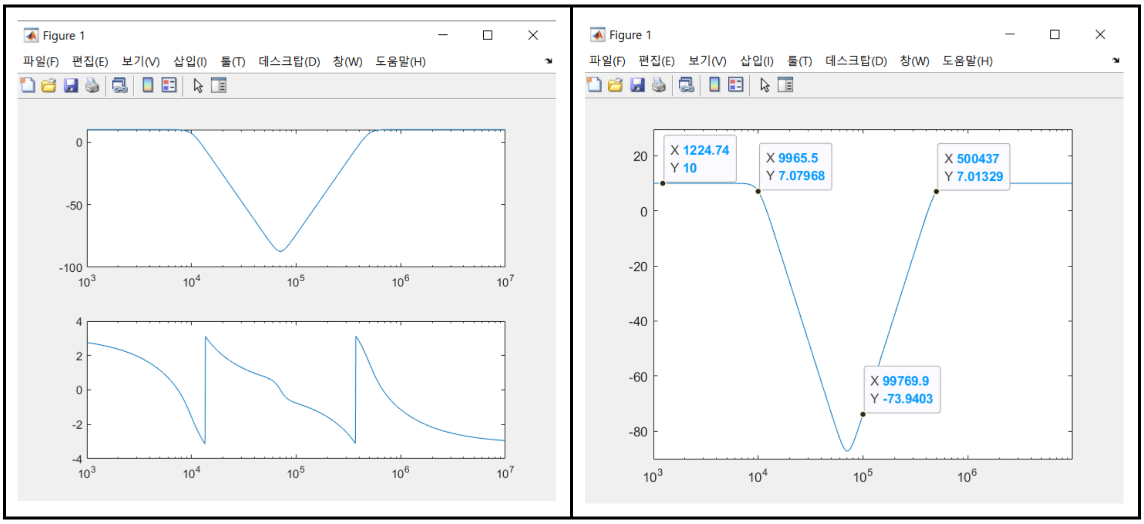


This is a summing amplifier, of which the transfer function is given as $Vo=-(V1\frac{Rf}{R1}+V2\frac{Rf}{R2})$. Since the output signals of LPF&HPF will be just added, Rf=R1=R2 would be appropriate. The resistor value was arbitrarily set to 10kΩ.

step 2: Draw the Bode plot for Gv (both the magnitude plot and the phase plot)

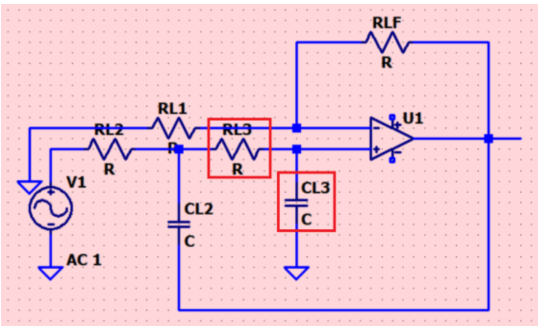
The MATLAB code&plot for the magnitude response and the phase response of the transfer function are as follows:

```
f=logspace(3,7,1000);
wcl=2*pi.*f/(20000*pi);
wch=2*pi.*f/(1000000*pi);
Glv=(10^(1/6)./(1-wcl.^2+sqrt(2).*wcl*1i)).*(10^(1/6)./(1-wcl.^2+0.5176.*wcl*1i))...
    .*(10^(1/6)./(1-wcl.^2+1.9318.*wcl*1i));
Ghv=(-10^(1/6)*wch.^2./(1-wch.^2+sqrt(2).*wch*1i)).*(-10^(1/6)*wch.^2./(1-wch.^2+0.5176.*wch*1i))...
    .*(-10^(1/6)*wch.^2./(1-wch.^2+1.9318.*wch*1i));
Gv=-(Glv+Ghv);
Gvmag=abs(Gv);
Gvphase=angle(Gv);
GdB=20*log10(Gvmag);
subplot(2,1,1)
semilogx(f,GdB)
subplot(2,1,2)
semilogx(f,Gvphase)
```

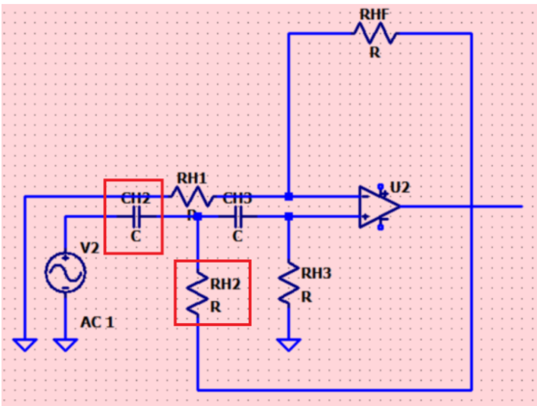


It could be verified that the passband gain is almost 10dB and the two cut-off frequencies are 10kHz and 500kHz. Also, the gain at 100kHz is -73.9dB, which satisfies minimum attenuation required on this filter.

step 3) Determine values of the parameters and implement the circuit on LTspice



LPF



HPF

Regarding the LPF, the relation $\frac{RLF}{RL1}$=0.4678 was utilized. RL1 was arbitrarily set to RL1=1kΩ, which resulted in RLF=0.4678kΩ. The values of the other parameters were determined by repetitive trial and error, and it was found that RL3 along with CL3(which are marked with red squares) have a significant effect on the value of the cut-off frequency. The values of those two parameters were adjusted until the cut-off frequency reached 10kHz, with other parameters being fixed. For all three stages, corresponding parameter values have been set the same for the sake of simplicity.

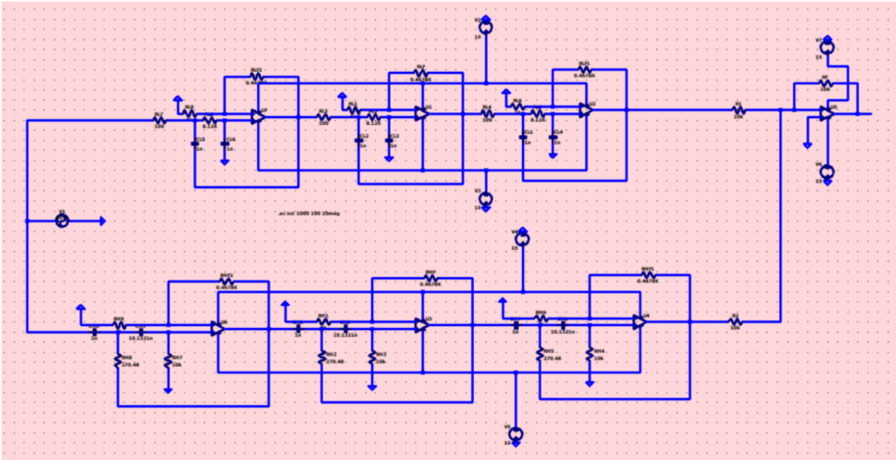
Regarding the HPF, the relation $\frac{RHF}{RH1}$=0.4678 was utilized. Again, RH1 was arbitrarily set to

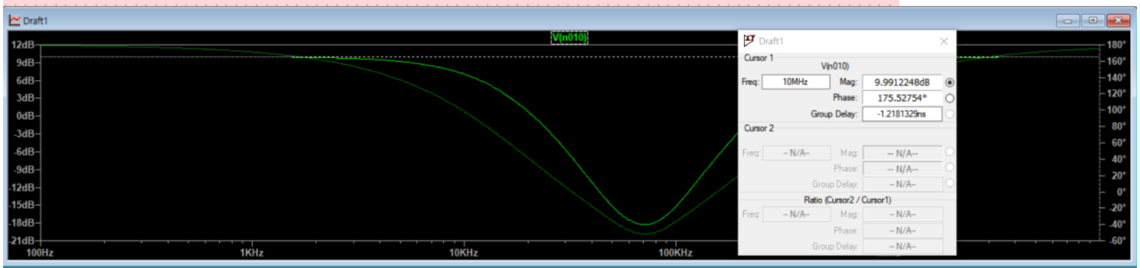
RH1=1kΩ, which resulted in RHF=0.4678kΩ. In terms of the other parameters, it was found that RH2 along with CH2(also marked with red squares) have a significant effect on the value of the cut-off frequency. The values of those two parameters were adjusted until the cut-off frequency reached 500kHz, with other parameters being fixed. For all three stages, corresponding parameter values are set the same as well.

[Table 2] Parameter values of the band reject filter

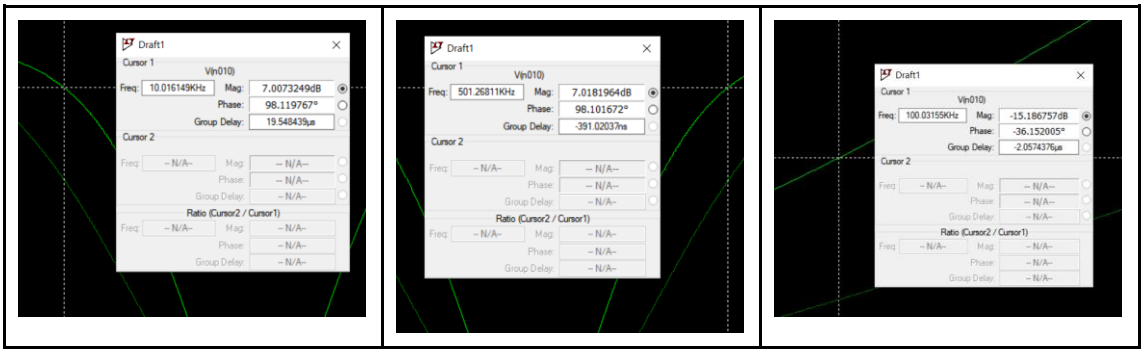
	R1	R2	R3	Rf	C2	C3
LPF	1kΩ	100Ω	8.12kΩ	0.4678kΩ	1nF	1nF
HPF	1kΩ	270.48Ω	10kΩ	0.4678kΩ	1nF	10.1321nF

The final schematic and simulation of the circuit is as follows:





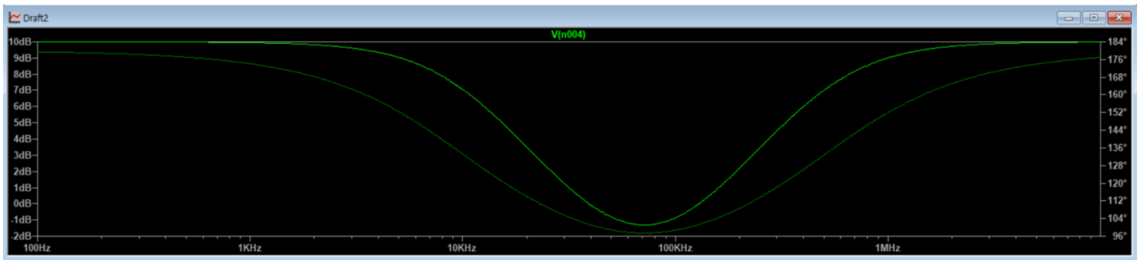
As the reader can see in the graph, the passband gain is almost 10dB



It could be verified that the two cut-off frequencies are nearly 10kHz and 500kHz.
Lastly, the gain at frequency 100kHz is -15.19dB, so that the attenuation from the edge of the passband is -{7dB–(-15.19dB)}=-22.19dB, which satisfies the minimum attenuation condition.

Conclusion:

The circuit that was used in this problem was not as nearly simple as the one that was used in the previous problem. Also, there must have been a loading effect since multiple filters were cascaded in the design. For such reasons, it was not possible to conduct a complete numerical analysis on the circuit. However, with the aid of Butterworth filters and their transfer function table, a sixth order band reject filter was successfully implemented which consists of three second order LPF's, HPF's, and a summing amplifier. The filter is found to satisfy all the conditions specified in the problem.
In terms of the cost efficiency, it is fair to say that higher order filters in general are more expensive, since they call for more resistors and capacitors. It was found that with lower order filters, however, the attenuation would not be as great as -15dB. The picture below is the magnitude response obtained when a second order filter was hired for this problem.



Whatever values were applied for the circuit components, the attenuation was only about -8dB, which obviously falls short of the required attenuation, -15dB. This observation is consistent with the principle that as the order of a filter increases, the slope of the magnitude response becomes steeper. For that reason, the lower order filters would not effectively block the signals on the unwanted range of frequency and therefore hiring a higher(sixth) order filter was inevitable in this problem.

Reference:

Second Order Low Pass Butterworth Filter | Transfer Function (eeeguide.com) : 2nd order Butterworth LPF
Butterworth Polynomials - EEEGUIDE.COM : Transfer function Table
Second Order High Pass Butterworth Filter - EEEGUIDE.COM : 2nd order Butterworth HPF
How to Derive the Transfer Function of the Inverting Summing Amplifier – Mastering Electronics Design: Summing amplifier