

# Mathematical Foundations of Data Sciences



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## Chapter 18

# Optimal Transport

TODO.

Refs [\[37, 33\]](#)



# Bibliography

- [1] P. Alliez and C. Gotsman. Recent advances in compression of 3d meshes. In N. A. Dodgson, M. S. Floater, and M. A. Sabin, editors, *Advances in multiresolution for geometric modelling*, pages 3–26. Springer Verlag, 2005.
- [2] P. Alliez, G. Ucelli, C. Gotsman, and M. Attene. Recent advances in remeshing of surfaces. In *AIM@SHAPE repport*. 2005.
- [3] Amir Beck. *Introduction to Nonlinear Optimization: Theory, Algorithms, and Applications with MATLAB*. SIAM, 2014.
- [4] Stephen Boyd, Neal Parikh, Eric Chu, Borja Peleato, and Jonathan Eckstein. Distributed optimization and statistical learning via the alternating direction method of multipliers. *Foundations and Trends® in Machine Learning*, 3(1):1–122, 2011.
- [5] Stephen Boyd and Lieven Vandenbergh. *Convex optimization*. Cambridge university press, 2004.
- [6] E. Candès and D. Donoho. New tight frames of curvelets and optimal representations of objects with piecewise  $C^2$  singularities. *Commun. on Pure and Appl. Math.*, 57(2):219–266, 2004.
- [7] E. J. Candès. The restricted isometry property and its implications for compressed sensing. *Compte Rendus de l’Académie des Sciences, Serie I*(346):589–592, 2006.
- [8] E. J. Candès, L. Demanet, D. L. Donoho, and L. Ying. Fast discrete curvelet transforms. *SIAM Multiscale Modeling and Simulation*, 5:861–899, 2005.
- [9] A. Chambolle. An algorithm for total variation minimization and applications. *J. Math. Imaging Vis.*, 20:89–97, 2004.
- [10] Antonin Chambolle, Vicent Caselles, Daniel Cremers, Matteo Novaga, and Thomas Pock. An introduction to total variation for image analysis. *Theoretical foundations and numerical methods for sparse recovery*, 9(263-340):227, 2010.
- [11] Antonin Chambolle and Thomas Pock. An introduction to continuous optimization for imaging. *Acta Numerica*, 25:161–319, 2016.
- [12] S.S. Chen, D.L. Donoho, and M.A. Saunders. Atomic decomposition by basis pursuit. *SIAM Journal on Scientific Computing*, 20(1):33–61, 1999.
- [13] F. R. K. Chung. Spectral graph theory. *Regional Conference Series in Mathematics, American Mathematical Society*, 92:1–212, 1997.
- [14] Philippe G Ciarlet. Introduction à l’analyse numérique matricielle et à l’optimisation. 1982.
- [15] P. L. Combettes and V. R. Wajs. Signal recovery by proximal forward-backward splitting. *SIAM Multiscale Modeling and Simulation*, 4(4), 2005.

- [16] P. Schroeder et al. D. Zorin. Subdivision surfaces in character animation. In *Course notes at SIGGRAPH 2000*, July 2000.
- [17] I. Daubechies, M. Defrise, and C. De Mol. An iterative thresholding algorithm for linear inverse problems with a sparsity constraint. *Commun. on Pure and Appl. Math.*, 57:1413–1541, 2004.
- [18] I. Daubechies and W. Sweldens. Factoring wavelet transforms into lifting steps. *J. Fourier Anal. Appl.*, 4(3):245–267, 1998.
- [19] D. Donoho and I. Johnstone. Ideal spatial adaptation via wavelet shrinkage. *Biometrika*, 81:425–455, Dec 1994.
- [20] Heinz Werner Engl, Martin Hanke, and Andreas Neubauer. *Regularization of inverse problems*, volume 375. Springer Science & Business Media, 1996.
- [21] M. Figueiredo and R. Nowak. An EM Algorithm for Wavelet-Based Image Restoration. *IEEE Trans. Image Proc.*, 12(8):906–916, 2003.
- [22] M. S. Floater and K. Hormann. Surface parameterization: a tutorial and survey. In N. A. Dodgson, M. S. Floater, and M. A. Sabin, editors, *Advances in multiresolution for geometric modelling*, pages 157–186. Springer Verlag, 2005.
- [23] Simon Foucart and Holger Rauhut. *A mathematical introduction to compressive sensing*, volume 1. Birkhäuser Basel, 2013.
- [24] I. Guskov, W. Sweldens, and P. Schröder. Multiresolution signal processing for meshes. In Alyn Rockwood, editor, *Proceedings of the Conference on Computer Graphics (Siggraph99)*, pages 325–334. ACM Press, August8–13 1999.
- [25] A. Khodakovsky, P. Schröder, and W. Sweldens. Progressive geometry compression. In *Proceedings of the Computer Graphics Conference 2000 (SIGGRAPH-00)*, pages 271–278, New York, July 23–28 2000. ACM Press.
- [26] L. Kobbelt.  $\sqrt{3}$  subdivision. In Sheila Hoffmeyer, editor, *Proc. of SIGGRAPH’00*, pages 103–112, New York, July 23–28 2000. ACM Press.
- [27] M. Lounsbery, T. D. DeRose, and J. Warren. Multiresolution analysis for surfaces of arbitrary topological type. *ACM Trans. Graph.*, 16(1):34–73, 1997.
- [28] S. Mallat. *A Wavelet Tour of Signal Processing, 3rd edition*. Academic Press, San Diego, 2009.
- [29] Stephane Mallat. *A wavelet tour of signal processing: the sparse way*. Academic press, 2008.
- [30] D. Mumford and J. Shah. Optimal approximation by piecewise smooth functions and associated variational problems. *Commun. on Pure and Appl. Math.*, 42:577–685, 1989.
- [31] Neal Parikh, Stephen Boyd, et al. Proximal algorithms. *Foundations and Trends® in Optimization*, 1(3):127–239, 2014.
- [32] Gabriel Peyré. *L’algèbre discrète de la transformée de Fourier*. Ellipses, 2004.
- [33] Gabriel Peyré and Marco Cuturi. Computational optimal transport. 2017.
- [34] J. Portilla, V. Strela, M.J. Wainwright, and Simoncelli E.P. Image denoising using scale mixtures of Gaussians in the wavelet domain. *IEEE Trans. Image Proc.*, 12(11):1338–1351, November 2003.
- [35] E. Praun and H. Hoppe. Spherical parametrization and remeshing. *ACM Transactions on Graphics*, 22(3):340–349, July 2003.

- [36] L. I. Rudin, S. Osher, and E. Fatemi. Nonlinear total variation based noise removal algorithms. *Phys. D*, 60(1-4):259–268, 1992.
- [37] Filippo Santambrogio. Optimal transport for applied mathematicians. *Birkhäuser, NY*, 2015.
- [38] Otmar Scherzer, Markus Grasmair, Harald Grossauer, Markus Haltmeier, Frank Lenzen, and L Sirovich. *Variational methods in imaging*. Springer, 2009.
- [39] P. Schröder and W. Sweldens. Spherical Wavelets: Efficiently Representing Functions on the Sphere. In *Proc. of SIGGRAPH 95*, pages 161–172, 1995.
- [40] P. Schröder and W. Sweldens. Spherical wavelets: Texture processing. In P. Hanrahan and W. Purgathofer, editors, *Rendering Techniques '95*. Springer Verlag, Wien, New York, August 1995.
- [41] C. E. Shannon. A mathematical theory of communication. *The Bell System Technical Journal*, 27(3):379–423, 1948.
- [42] A. Sheffer, E. Praun, and K. Rose. Mesh parameterization methods and their applications. *Found. Trends. Comput. Graph. Vis.*, 2(2):105–171, 2006.
- [43] Jean-Luc Starck, Fionn Murtagh, and Jalal Fadili. *Sparse image and signal processing: Wavelets and related geometric multiscale analysis*. Cambridge university press, 2015.
- [44] W. Sweldens. The lifting scheme: A custom-design construction of biorthogonal wavelets. *Applied and Computation Harmonic Analysis*, 3(2):186–200, 1996.
- [45] W. Sweldens. The lifting scheme: A construction of second generation wavelets. *SIAM J. Math. Anal.*, 29(2):511–546, 1997.