GD f^o quadratique non fortement cvx

$$f(x) = \frac{1}{2}\langle Cx, x\rangle - \langle x, b\rangle$$

$x^* = C^{-1}b$ (check cas non inversible)

$$x_{k+1} = x_k - \tau(Cx_k - b).$$

$$x_k - x^* = (Id - \tau C)^k (x_0 - x^*).$$

$$\frac{1}{2}\langle C(x_k - x^*), x_k - x^*\rangle = \frac{1}{2}\langle Cx_k, x_k\rangle - \underbrace{\langle Cx_k, x^*\rangle}_{=\langle x_k, b\rangle} + \underbrace{\frac{1}{2}\langle Cx^*, x^*\rangle}_{\frac{1}{2}\langle b, x^*\rangle = -f(x^*)}$$

$$= f(x_k) - f(x^*)$$

$$\Rightarrow f(x_k) - f(x^*) = \frac{1}{2}\langle \underbrace{(Id - \tau C)^k C (Id - C)^k}_{M_k} (x_0 - x^*), x_0 - x^*\rangle$$

$$\cancel{\leq \frac{1}{2}\mathrm{tr}\left((Id - \tau C)^{2k} C\right) \ldots}$$

$$\leq \frac{1}{2}\sigma_{max}(M_k) \cdot \|x_0 - x^*\|^2$$

$$\boxed{\text{car } (1-t)^{2k} t \leq \frac{1}{4k} \text{ pour } t \in [0,1]} \Rightarrow \underbrace{\sigma (1 - \tau\sigma)^{2k}}_{\shortparallel} \leq \frac{1}{\tau 4k}$$

<u>Démo</u> : $(1-t)^{2k} t \leq (e^{-t})^{2n} t = \frac{1}{2n}(2nt)e^{-2nt} \leq \frac{1}{2n}\sup_{u \geq 0} ue^{-u} = \frac{1}{2en} \leq \frac{1}{4n}$