The Expected Utility Theory

Problem 1

GETTING AN MBA

Imagine that you have been working after college and now face the decision of whether or not to get an MBA at a prestigious institution. The cost of getting the MBA is 10. Your future value is your stream of income, which depends on the strength of the labor market for the next decade. If the labor market is strong then your income value from having an MBA is 32, while your income value from your current status is 12. If the labor market is average then your income value from having an MBA is 16, while your income value from your current status is 8. If the labor market is weak then your income value from having an MBA is 12, while your income value from your current status is 4.

Assume that the probability of a strong labor market is p, that of an average labor market is 0.5, and that of a weak labor market is $0.5 - p$.

(a) For which values of p will you decide not to get an MBA?
(b) If $p = 0.4$, what is the highest price the university can charge for you to be willing to go ahead and get an MBA?

INTRODUCTION

This is a single-person decision problem. The action set is—

$$A = \{m, d\}$$

in which m represents the action of getting an MBA and d represents the action of not getting an MBA. In either case, there are three possibilities: (1) Strong market (s, probability p), (2) Average market (a, probability 0.5), (3) Weak market (w, probability $(0.5 - p)$).

The payoffs of the respective outcomes of the two actions are summarized below—

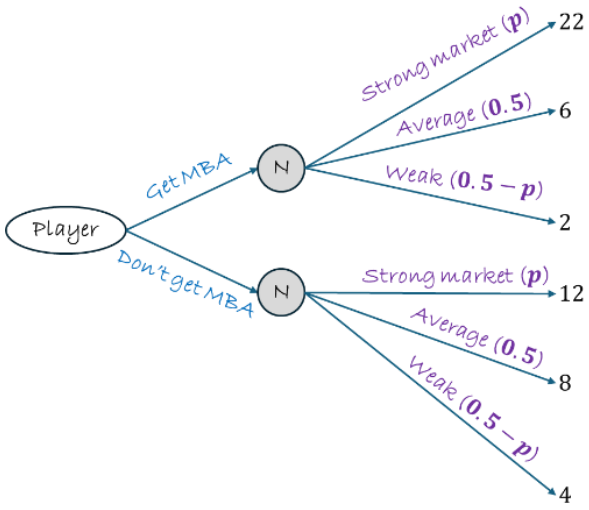
Payoffs for the outcomes of action m:

- Outcome 1: Strong market. $u(m, s) = 32 - 10 = 22$
- Outcome 2: Average market. $u(m, a) = 16 - 10 = 6$
- Outcome 3: Weak market. $u(m, w) = 12 - 10 = 2$

Payoffs for the outcomes of action d:

- Outcome 1: Strong market. $u(d, s) = 12 - 0 = 12$
- Outcome 2: Average market. $u(d, a) = 8 - 0 = 8$
- Outcome 3: Weak market. $u(d, w) = 4 - 0 = 4$

The decision tree is shown below—



ANSWER (A). VALUE OF p FOR NOT GETTING AN MBA

The expected payoff for not getting an MBA is

$$v(d) = (p \times 12) + (0.5 \times 8) + (0.5 - p) \times 4 = 8p + 6$$

The expected payoff for getting an MBA is

$$v(m) = (p \times 22) + (0.5 \times 6) + (0.5 - p) \times 2 = 20p + 4$$

For deciding not to get an MBA,

$$v(d) > v(m)$$

$$8p + 6 > 20p + 4$$

$$\therefore 8p - 20p > 4 - 6$$

$$\therefore -12p > -2$$

$$\therefore 12p < 2$$

$$\therefore p < \frac{1}{6}$$

You should decide not to get an MBA for values of p less than $\frac{1}{6}$.

ANSWER (B). HIGHEST COST FOR GETTING AN MBA FOR $p = 0.4$

Let the cost of getting the MBA be c (instead of $c = 10$ as given for Part (A)). The expected payoff for getting the MBA is given by—

$$v(m) = \big(p \times (32 - c)\big) + \big(0.5 \times (16 - c)\big) + (0.5 - p) \times (12 - c)$$

Plug in $p = 0.4$.

$$v(m) = \big(0.4 \times (32 - c)\big) + \big(0.5 \times (16 - c)\big) + (0.5 - 0.4) \times (12 - c)$$

$$\therefore v(m) = (12.8 - 0.4c) + (8 - 0.5c) + (1.2 - 0.1c)$$

$$\therefore v(m) = 22 - c$$

The expected payoff for not getting the MBA is given by

$$v(d) = 8p + 6 = 8(0.4) + 6 = 9.2$$

For it to be worthwhile to get the MBA

$$v(m) \geq v(d)$$

$$\therefore 22 - c \geq 9.2$$

$$\therefore -c \geq 9.2 - 22$$

$$\therefore c \leq 22 - 9.2$$

$$\therefore c \leq 12.8$$

The university can charge 12.8 and you would still be willing to get an MBA.