The Normal-Form Representation

Problem 3

Hermaphroditic Fish

Members of some species of hermaphroditic fish choose, in each mating encounter, whether to play the role of a male or a female. Each fish has a preferred role, which uses up fewer resources and hence allows more future mating. A fish obtains a payoff of H if it mates in its preferred role and L if it mates in the other role, where $H > L$. (Payoffs are measured in terms of number of offspring, which fish are evolved to maximize.) Consider an encounter between two fish whose preferred roles are the same. Each fish has two possible actions: mate in either role, and insist on its preferred role. If both fish offer to mate in either role, the roles are assigned randomly, and each fish's payoff is $\frac{1}{2}(H + L)$. If each fish insists on its preferred role, the fish do not mate; each goes off in search of another partner and obtains the payoff S. The higher the chance of meeting another partner, the larger is S. Formulate this situation as a strategic game and determine the range of values of S, for any given values of H and L, for which the game differs from the Prisoner's Dilemma only in the names of the actions.

Answer: The Normal-Form Representation

Players

The number of players is $n = 2$. The set of players is—

$$N = \{1, 2\}$$

Strategy Sets

S_i is the set of all pure strategies available to player i. In this game, the fish can choose to (1) mate in either role (pure strategy E) or insist on its preferred role (pure strategy P). Hence—

$$S_i = \{E, P\} \quad \text{for } i \in N$$

Payoff Functions

If both fish choose strategy E, then each fish's expected payoff is $\frac{1}{2}(H + L)$.

$$v_1(E, E) = v_2(E, E) = \frac{1}{2}(H + L)$$

If each fish insists on its preferred role, the fish do not mate; each goes off in search of another partner, and obtains the payoff S.

$$v_1(P, P) = v_2(P, P) = S$$

The higher the chance of meeting another partner, the larger is S.

If one fish chooses strategy P and the other chooses strategy E, then the former gets a payoff of H and the latter gets a payoff of L.

$$v_1(P, E) = v_2(E, P) = H$$

$$v_1(E, P) = v_2(P, E) = L$$

Answer: The Matrix-Form Representation

		Player 2	
		E	P
Player 1	E	$\frac{1}{2}(H+L), \frac{1}{2}(H+L)$	L, H
	P	H, L	S, S

Prisoner's Dilemma

For the game to be a Prisoner's Dilemma, the payoffs must be such that each player will prefer (1) opponent's cooperation (strategy E), and (2) defection for self (strategy P). The preference ranking for the four outcomes must be as follows—

$$(P, E) \succ (E, E) \succ (P, P) \succ (E, P)$$

That is—

$$H > \frac{1}{2}(H+L) > S > L$$