The Expected Utility Theory

Problem 5

Recreation Choices

A player has three possible activities from which to choose: going to a football game, going to a boxing match, or going for a hike. The payoff from each of these alternatives will depend on the weather. The following table gives the agent's payoff in each of the two relevant weather events:

Alternative	Payoff if rain	Payoff if shine
Football game	1	2
Boxing match	3	0
Hike	0	1

Let p denote the probability of rain.

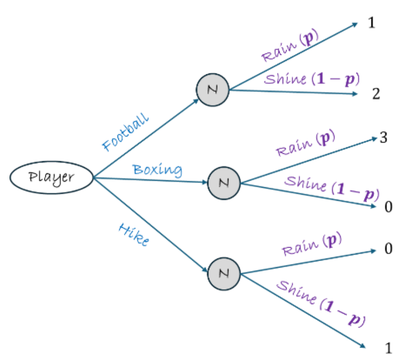
(a) Is there an alternative that a rational player will never take regardless of p?

(b) What is the optimal decision, or best response, as a function of p?

Introduction

This is a single-person decision problem. The decision-maker has three actions to choose from: (1) football, (2) boxing, (3) hike. Each action leads to a Nature node with two outcomes: (1) Rain, (2) Sunshine.

This is what the decision tree looks like—



Decisions outside the control of our decision-maker (Rain or Shine) are assigned to the hypothetical player, Nature.

Answer (a). Alternative Never Taken

Consider the two actions that Nature can pick from—

- Case 1. It rains: Comparing payoffs, the preference relation is $Boxing \succ Football \succ Hike$.
- Case 2. It shines: The preference relation is $Football \succ Hike \succ Boxing$.

Looking at the preference relations for the two cases, it's clear that the hike is always worse than going to the football game. The Hike alternative is said to be dominated by the Football alternative. He should never go on a hike.

Answer (B). Best Response

Since hiking is dominated by football, we can disregard the expected payoff for hiking. The expected payoffs for the remaining two actions are as follows—

$$v(F) = (p \times 1) + \big((1-p) \times 2\big) = p + 2 - 2p = 2 - p$$

$$v(B) = (p \times 3) + \big((1-p) \times 0\big) = 3p$$

Football is the better option if—

$$v(F) > v(B)$$

$$2 - p > 3p$$

$$2 > 4p$$

$$4p < 2$$

$$p < \frac{1}{2}$$

Thus, football is better if $p < \frac{1}{2}$ and boxing is better if $p > \frac{1}{2}$. The decision-maker is indifferent if $p = \frac{1}{2}$.

Key Takeaways

- Drawing decision trees for uncertain outcomes.
- Calculating expected payoffs for uncertain outcomes.
- The idea of dominated actions.
- The idea of the best response.