The Expected Utility Theory

Problem 2

Marketing Strategies

A company is launching a new product and must choose one of three possible marketing strategies: A, B, or C. Each strategy has a different cost and a different impact on product sales. Strategy A costs ₹5,000, and the sales revenue (in ₹) follows a uniform distribution over the interval $[₹10,000, ₹20,000]$. Strategy B has a cost of ₹10,000, and the sales revenue follows a triangular distribution over the interval $[₹10,000, 30,000]$, peaking at $₹20,000$. Strategy C has a cost of $₹15,000$, and the sales revenue follows a normal distribution with a mean of $₹25,000$ and a standard deviation of $₹5000$. Which marketing strategy should the company choose?

Introduction

To determine the best marketing strategy, the company should compare expected profits for each strategy. The expected profit is calculated as—

$$Expected\ Profit = Expected\ Sales\ Revenue - Cost$$

The sales revenue (x) is a random variable that follows a uniform distribution for strategy A, a triangular distribution for strategy B, and a normal distribution for strategy C.

Strategy A: Uniform Distribution

- Cost: 5
- Sales revenue distribution: Uniform in $[10, 20]$

Probability density function

For a uniform distribution, the probability density function is given by—

$$f(x) = \begin{cases} \dfrac{1}{b-a} & if\ x \in [a,\ b] \\ 0 & otherwise \end{cases}$$

The sales revenue (x) is a random variable that follows a uniform distribution over the interval $[10, 20]$.

$$a = 10$$

$$b = 20$$

For $x \in [10, 20]$, the probability density function is—

$$f(x) = \frac{1}{b-a} = \frac{1}{20-10} = \frac{1}{10}$$

Expected value of Sales Revenue

$$E[x] = \int_{10}^{20} x f(x) dx$$

$$\therefore E[x] = \int_{10}^{20} x \frac{1}{10} dx$$

$$\therefore E[x] = \frac{1}{10}\left[\frac{x^2}{2}\right]_{10}^{20}$$

$$\therefore E[x] = 15$$

Expected payoff (profit)

$$v(A) = 15 - 5 = 10$$

Strategy B: Triangular Distribution

- Cost: 10
- Sales revenue distribution: Triangular in $[10, 30]$, peak at 20

Probability density function

For a triangular distribution, the probability density function is given by—

$$f(x) = \begin{cases} \dfrac{2(x-a)}{(b-a)(p-a)} & a \le x \le p \\ \dfrac{2(b-x)}{(b-a)(b-p)} & p < x \le b \\ 0 & otherwise \end{cases}$$

The sales revenue (x) is a random variable that follows a triangular distribution over the interval $[10, 30]$ with a peak at $x = 20$.

$$a = 10$$

$$b = 30$$

$$p = 20$$

The probability density function is—

$$f(x) = \begin{cases} \dfrac{2(x-10)}{(30-10)(20-10)} & 10 \le x \le 20 \\ \dfrac{2(30-x)}{(30-10)(30-20)} & 20 < x \le 30 \\ 0 & otherwise \end{cases}$$

$$\therefore f(x) = \begin{cases} \dfrac{x-10}{100} & 10 \le x \le 20 \\ \dfrac{30-x}{100} & 20 < x \le 30 \\ 0 & otherwise \end{cases}$$

Expected value of Sales Revenue

$$E[x] = \int_{10}^{30} x f(x) dx$$

$$E[x] = \int_{10}^{20} x f(x) dx + \int_{20}^{30} x f(x) dx$$

$$E[x] = \int_{10}^{20} x \left[\frac{x-10}{100}\right] dx + \int_{20}^{30} x \left[\frac{30-x}{100}\right] dx$$

$$\therefore E[x] = \frac{1}{100}\int_{10}^{20} (x^2 - 10x) dx + \frac{1}{100}\int_{20}^{30} (30x - x^2) dx$$

$$\therefore E[x] = \frac{1}{100}\left[\frac{x^3}{3} - 10\frac{x^2}{2}\right]_{10}^{20} + \frac{1}{100}\left[30\frac{x^2}{2} - \frac{x^3}{3}\right]_{20}^{30}$$

$$\therefore E[x] = \frac{1}{100}\left\{\left[\frac{20^3}{3} - 10\frac{20^2}{2}\right] - \left[\frac{10^3}{3} - 10\frac{10^2}{2}\right]\right\} + \frac{1}{100}\left\{\left[30\frac{30^2}{2} - \frac{30^3}{3}\right] - \left[30\frac{20^2}{2} - \frac{20^3}{3}\right]\right\}$$

$$\therefore E[x] = 20$$

Expected payoff (profit)

$$v(B) = 20 - 10 = 10$$

Strategy C: Normal Distribution

- Cost: 15
- Sales revenue distribution: Normal with mean at 25 and a standard deviation of 5

Probability density function

For a normal distribution, the probability density function is given by—

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The sales revenue (x) is a random variable that follows a normal distribution with a mean of 25 and a standard deviation of 5.

$$\mu = 25$$

$$\sigma = 5$$

Expected value of Sales Revenue

$$E[x] = \int_{-\infty}^{\infty} xf(x)dx$$

$$E[x] = \int_{-\infty}^{\infty} x\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx$$

$$E[x] = \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{\infty} xe^{-\frac{(x-\mu)^2}{2\sigma^2}}dx$$

Substitute $z = \frac{x-\mu}{\sigma}$. Then, $x = \mu + \sigma z$, and $dx = \sigma dz$. $z = -\infty$ when $x = -\infty$ and $z = \infty$ when $x = \infty$. Hence,

$$E[x] = \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{\infty} (\mu + \sigma z)e^{-\frac{z^2}{2}}(\sigma dz)$$

$$E[x] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} (\mu + \sigma z)e^{-\frac{z^2}{2}}dz$$

$$E[x] = \frac{\mu}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{z^2}{2}}dz + \frac{\sigma}{\sqrt{2\pi}}\int_{-\infty}^{\infty} ze^{-\frac{z^2}{2}}dz$$

The first integral is the Gaussian integral—

$$\int_{-\infty}^{\infty} \mu e^{-\frac{z^2}{2}}dz = \sqrt{2\pi}$$

The integrand of the second integral is an odd function, hence the second integral is zero.

$$\therefore E[x] = \frac{\mu}{\sqrt{2\pi}}\left(\sqrt{2\pi}\right) + \frac{\sigma}{\sqrt{2\pi}}(0)$$

$$\therefore E[x] = \mu = 25$$

Expected payoff (profit)

$$v(B) = 25 - 15 = 10$$

Conclusion

All strategies are equally viable based on expected profit.