The Normal-Form Representation

Problem 4

Price Competition

Imagine a market where the price is given by $p(q) = 100 - q$. There are two firms, 1 and 2, and each firm i has to simultaneously choose its price p_i. If $p_i < p_j$, then firm i gets all of the market while no one demands the good of firm j. If the prices are the same then both firms split the market demand equally. Imagine that there are no costs to produce any quantity of the good. (These are two large dairy farms, and the product is manure.) Write down the normal form of this game.

Introduction

This scenario aligns with the Bertrand duopoly model, where firms compete by setting prices rather than quantities. Unlike the variation of Cournot Duopoly discussed in class, where firms choose how much to produce, here each firm independently sets its price, p_i. That is, the strategy for each firm is the price that the firm sets for the product. Since prices can take any value between 0 and ∞, the game has an infinite strategy space and cannot be represented in a standard matrix form.

Once prices are set, the quantity each firm produces and sells is determined by the price (market demand) given by $q_i = 100 - p_i$. A lower price leads to a higher demand, and vice versa. If both firms set the same price, they split the market equally. However, if one firm undercuts the other (sets a lower price), it captures the entire market while the higher-priced firm produces nothing and sells nothing.

Answer: The Normal-Form Representation

Players

The number of players is $n = 2$. The set of players is—

$$N = \{1, 2\}$$

Strategy Sets

S_i is the set of all pure strategies available to player i. In this game, the strategy sets refer to the prices set by the two firms; firms choose prices $p_i \in S_i$. Each firm is free to set their price from 0 to ∞. Thus, each player has infinitely many pure strategies (actions) to choose from.

$$S_1 = \{0, \infty\}$$

$$S_2 = \{0, \infty\}$$

Payoffs

For every combination of strategies (prices) that the two players decide to play (a strategy profile), each player has a payoff. In this game, payoffs represent the total revenue obtained by the firm from selling its product; this will be the price p_i multiplied by the quantity sold q_i (minus the production cost, which is zero). This payoff will depend on the price (strategy) set by the firm as well as that set by the other firm.

To calculate payoffs, we need to know what the quantities will be for each firm given prices (p_i, p_j). Given the assumption on ties, the quantities are given by,

$$q_i(p_i, p_j) = \begin{cases} 100 - p_i & if\ p_i < p_j \\ 0 & if\ p_i > p_j \\ \dfrac{100 - p_i}{2} & if\ p_i = p_j \end{cases}$$

which in turn means that the payoff function is given by quantity times price (there are no costs)—

$$v_i(p_i, p_j) = \begin{cases} (100 - p_i)p_i & if\ p_i < p_j \\ 0 & if\ p_i > p_j \\ \left(\dfrac{100 - p_i}{2}\right)p_i & if\ p_i = p_j \end{cases}$$