The Rational Choice Paradigm

Problem 3

Fruit Trees

You have room for up to two fruit-bearing trees in your garden. The fruit trees that can grow in your garden are either apple, orange, or pear. The cost of maintenance is ₹ 1000 for an apple tree, ₹ 700 for an orange tree, and ₹ 1200 for a pear tree. Your food bill will be reduced by ₹ 1300 for each apple tree you plant, by ₹ 1450 for each pear tree you plant, and by ₹ 900 for each orange tree you plant. You care only about your total expenditure in making any planting decisions.

(a) What is the set of possible actions and related outcomes?
(b) What is the payoff of each action/outcome?
(c) Which actions are dominated?
(d) Draw the associated decision tree. What will a rational player choose?
(e) Now imagine that the reduction in your food bill is half for the second tree of the same kind. (You like variety.) That is, the first apple tree still reduces your food bill by ₹ 1300, but if you plant two apple trees your food bill will be reduced by ₹ 1300 + ₹ 650 = ₹ 1950, and similarly for pear and orange trees. What will a rational player choose now?

Introduction

We're given that—

1. We can plant up to two trees.
2. Choices: Apple (A), Orange (O), Pear (P), Nothing (N).
3. Maintenance cost per tree—
    a. Apple: **₹ 1000**
    b. Orange: **₹ 700**
    c. Pear: **₹ 1200**
4. Reduction in food bill per tree—
    a. Apple: **₹ 1300**
    b. Orange: **₹ 900**
    c. Pear: **₹ 1450**

Answer (a). Possible Actions and Related Outcomes

Our garden has two empty spots for planting trees. Each spot can be filled in one of four distinct ways: A, O, P, or N. Each action consists of a pair of choices, one for each spot. Determining the total number of available actions is a problem of selecting two items out of four possibilities with replacement. The total number of actions is equal to (*See Appendix*)—

$$ {}^{4+2-1}C_2 = {}^{5}C_2 = \frac{5!}{2!\,3!} = 10 $$

The action set is—

$$ A = \{(A,A),(A,O),(A,P),(A,N),(O,O),(O,P),(O,N),(P,P),(P,N),(N,N)\} $$

This is a discrete action set with a finite number of elements.

Answer (b). Payoff of each action/outcome

Each action consists of a pair of options. For each action, we can calculate the total cost and total savings, based on the tree-specific information given in the problem statement. The net payoff for each action is then the difference between the total savings and the total cost for that action. These values are summarized below.

Action	Cost Tree 1	Cost Tree 2	Cost Total	Savings Tree 1	Savings Tree 2	Savings Total	Net Payoff
(A,A)	1000	1000	2000	1300	1300	2600	**600**
(A,O)	1000	700	1700	1300	900	2200	500
(A,P)	1000	1200	2200	1300	1450	2750	550
(A,N)	1000	0	1000	1300	0	1300	300
(O,O)	700	700	1400	900	900	1800	400
(O,P)	700	1200	1900	900	1450	2350	450
(O,N)	700	0	700	900	0	900	200
(P,P)	1200	1200	2400	1450	1450	2900	500
(P,N)	1200	0	1200	1450	0	1450	250
(N,N)	0	0	0	0	0	0	0

Answer (c). Dominated actions

We compare actions by comparing their payoffs. The action with the lower payoff is said to be **dominated** by the action with the higher payoff. Based on the payoff values in the table above, all actions except choosing two apple trees are dominated. Conversely, the action of choosing two apple trees is said to be the **dominant** action.

Answer (d). Decision tree and best choice

This problem involves a single decision. The decision tree would have the player node on the left, with ten branches representing each choice. Payoffs would be written to the right of each arrow. The best choice is to plant two apple trees for a payoff of 600.

ANSWER (E). ALTERED PAYOFFS

In the payoff table, for rows containing a pair of the same tree, reduce the savings from the second tree by half.

Action	Cost Tree 1	Cost Tree 2	Cost Total	Savings Tree 1	Savings Tree 2	Savings Total	Net Payoff
(A, A)	1000	1000	2000	1300	650	1950	-50
(A, O)	1000	700	1700	1300	900	2200	500
(A, P)	1000	1200	2200	1300	1450	2750	**550**
(A, N)	1000	0	1000	1300	0	1300	300
(O, O)	700	700	1400	900	450	1350	-50
(O, P)	700	1200	1900	900	1450	2350	450
(O, N)	700	0	700	900	0	900	200
(P, P)	1200	1200	2400	1450	725	2175	-225
(P, N)	1200	0	1200	1450	0	1450	250
(N, N)	0	0	0	0	0	0	0

Thus, an apple tree is still the best choice for the first tree, but now the second tree should be a pear tree.

KEY TAKEAWAYS

- **Dominated Actions**. An action is dominated if it results in a lower payoff compared to another action. A rational decision-maker never selects a dominated action.
- **Altered Payoffs**. Revising payoffs due to diminishing returns.

APPENDIX: COUNTING ACTIONS

REFERENCE: APPLIED COMBINATORICS BY ALAN TUCKER (CHAPTER 5)

1.1 EXAMPLE 1: BINARY SEQUENCES

How many different 8-digit binary sequences are there with six 1s and two 0s?

Imagine a sack containing 8 balls, each labeled with a unique integer from 1 to 8. Each ball represents a position in the binary sequence. Select any 6 balls from the sack and assign the digit 1 to the corresponding positions in the sequence. Assign the digit 0 to the remaining 2 positions. This produces one 8-digit binary sequence with six 1s and two 0s.

Different selections of 6 balls correspond to different binary sequences. Since the order in which the balls are selected doesn't matter – for example, selecting balls $\{1, 2, 3, 4, 5\ 6\}$ gives the same binary sequence as selecting balls $\{3, 1, 2, 4, 5, 6\}$ – we're counting combinations, not permutations.

Therefore, the number of different 8-digit binary sequences with six 1s and two 0s is ${}^{8}C_{6} = 28$.

1.2 Example 2: Ordering Hot Dogs

How many different ways are there to select six hot dogs from three varieties: regular, chili, and super?

Each selection of six hot dogs can be described by specifying how many are regular, how many are chili, and how many are super. For example, an order of one regular, four chili, and one super can be recorded on an order form as—

Regular	Chili	Super
x	$xxxx$	x

Each x represents a hot dog. Since the chef already knows that the order of varieties on the form is regular, chili, super, the request can simply be written as $x|xxxx|x$ without column headings.

In general, given that there are three varieties, any selection of r hot dogs will consist of some sequence of r xs and two |s. Conversely, any sequence of r xs and two |s represents a unique selection: the xs before the first | count the number of regular dogs; the xs between the two |s count chilis; and the final xs count supers. So, there is a one-to-one correspondence between orders and such sequences.

The problem, therefore, reduces to the following counting question: How many different ways are there of arranging six xs and two |s in a 8-slot sequence? As in Example 1, the answer is ${}^{8}C_{6} = 28$.

1.3 Example 3: Fruit Trees

You've exactly two planting slots in your garden. Each slot may be occupied by one fruit tree – Apple, Orange, or Pear – or it may be left empty. The order of the slots doesn't matter. How many distinct planting decisions are possible?

A planting decision is determined by how many slots are occupied by each of the four possibilities: Apple, Orange, Pear, or Nothing. Such a decision can be represented in a table with four columns—

Apple	Orange	Pear	Nothing
x	x		

Each x represents a tree and there must be exactly two xs in each row. For example, the above table represents the decision to plant one apple tree and one orange tree. Since the order of categories is fixed, the same planting decision can be written more compactly as $x|x||$. Another example of a planting decision is $|||xx$ which represents the decision to leave both planting slots empty.

In general, a planting decision – selection of r trees from n types – will consist of r xs and $(n-1)$ |s. Conversely, any sequence of r xs and $(n-1)$ |s represents a unique planting decision. So, there is a one-to-one correspondence between planting decisions and such sequences.

The problem, therefore, reduces to the following counting question: How many different ways are there of arranging r xs [and $(n-1)$ |s] in a sequence of length $(n-1)+r$? The answer is ${}^{(n-1+r)}C_{r}$.

For the fruit tree problem, $n = 4$ and $r = 2$. Hence the number of planting decisions are ${}^{(4-1+2)}C_{2} = {}^{5}C_{2} = 10$.