

The Expected Utility Theory

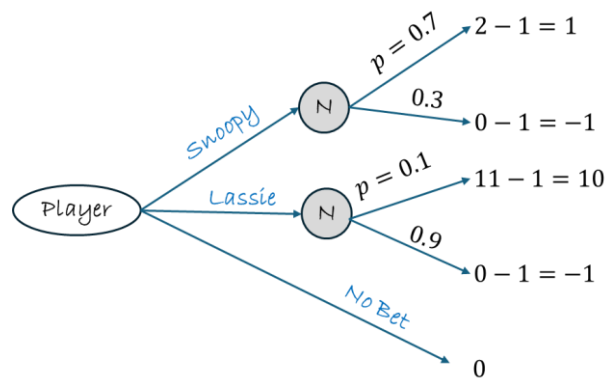
Problem 3

AT THE DOG RACES

You're in Las Vegas, and you must decide what to do at the dog-racing betting window. You may choose not to participate, or you may bet on one of two dogs as follows. Betting on Snoopy costs Rs. 1, and you will be paid Rs. 2 if he wins. Betting on Lassie costs Rs. 1, and you will be paid Rs. 11 if she wins. You believe that Snoopy has probability 0.7 of winning and that Lassie has probability 0.1 of winning (there are other dogs on which you are not considering betting). Your goal is to maximize the expected monetary return of your action.

- Draw the decision tree for this problem.
- What is your best course of action, and what is your expected value?
- Someone offers you gambler's 'anti-insurance,' which you may accept or decline. If you accept it, you get paid Rs. 2 up front and you agree to pay back 50% of any winnings you receive. Draw the new decision tree and find the optimal action.

ANSWER (A). DECISION TREE



ANSWER (B). BEST COURSE OF ACTION & EXPECTED VALUE

Expected value for betting on Snoopy:

$$v(\text{Snoopy}) = 0.7 \times 1 + 0.3 \times -1 = 0.4$$

Expected value for betting on Lassie:

$$v(\text{Lassie}) = 0.1 \times 10 + 0.9 \times -1 = 0.1$$

Expected value for not betting

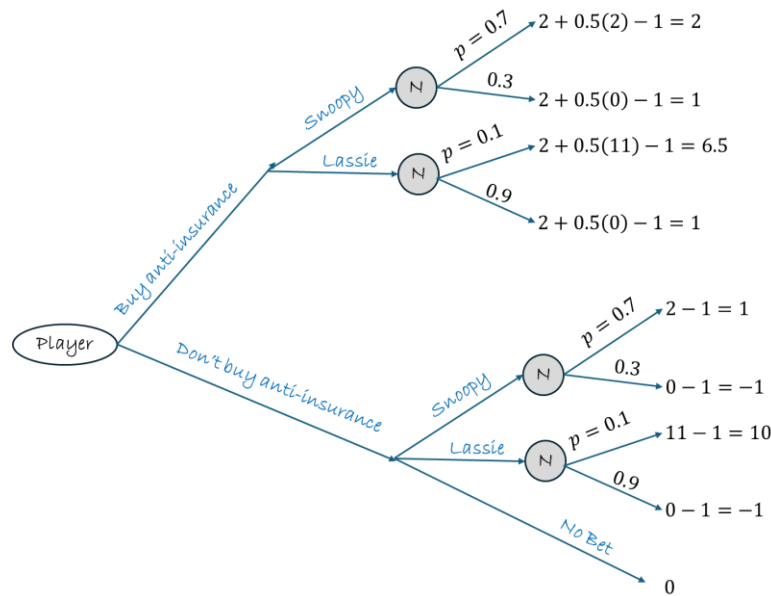
$$v(\text{No Bet}) = 0$$

Hence, the best course of action is to bet on Snoopy.

ANSWER (C). ANTI-INSURANCE

Assume that if we purchase anti-insurance, then we must place a bet.

To write the payoff for each outcome, add 2 to your payoff (from the anti-insurance), subtract 1 (for the betting cost), and add half of the winnings (2 for Snoopy and 11 for Lassie). The new decision tree is as follows—



The five actions and their respective expected payoffs are as follows—

- (1) Buy anti-insurance. Bet on Snoopy.

$$E[BS] = 0.7 \times 2 + 0.3 \times 1 = 1.7$$

- (2) Buy anti-insurance. Bet on Lassie.

$$E[BL] = 0.1 \times 6.5 + 0.9 \times 1 = 1.55$$

- (3) Don't buy anti-insurance. Bet on Snoopy.

$$E[DS] = 0.7 \times 1 + 0.3 \times -1 = 0.4$$

- (4) Don't buy anti-insurance. Bet on Lassie.

$$E[DL] = 0.1 \times 10 + 0.9 \times -1 = 0.1$$

- (5) Don't bet.

$$E[NB] = 0$$

Thus, the best action is to buy anti-insurance and bet on Snoopy to get an expected payoff of 1.7.