# The Normal-Form Representation

Problem 1

## Public Goods Distribution

Three players live in a town, and each can choose to contribute to fund a streetlamp. The value of having the streetlamp is 3 for each player, and the value of not having it is $0$. The mayor asks each player to contribute either $1$ or nothing. If at least two players contribute, then the lamp will be erected. If one player or no players contribute then the lamp will not be erected, in which case any person who contributed will not get his money back. Write down the normal form of this game.

## Introduction

This is a classic public goods game that illustrates the tension between collective well-being and individual incentives, as well as the free-rider problem in which individuals benefit from a public good without contributing, relying on others to bear the cost. The streetlamp, if erected, provides benefits that are—

- Non-excludable: Even those who don't contribute still enjoy the light.
- Non-rivalrous: One person's use of the light doesn't reduce its availability to others.

The contributions are non-refundable.

## Answer: The Normal-Form Representation

### Players

The number of players is $n = 3$. The set of players is—

$$N = \{1, 2, 3\}$$

### Strategy Sets

$S_i$ is the set of all pure strategies available to player $i$. In this game, this is the amount each player can contribute. This is either $0$ or $1$. Hence—

$$S_i = \{0, 1\} \quad \text{for } i \in N$$

### Payoffs

For each strategy profile – every possible combination of strategies chosen by the three players – each player receives a corresponding payoff. There are $2^3 = 8$ possible strategy profiles, each represented as an ordered tuple with three components. Payoffs are determined based on whether the lamp is erected or not, and on whether the player contributed or not.

| Strategy Profile | Payoff for player 1 | Payoff for player 2 | Payoff for player 3 |
|---|---|---|---|
| $(\mathbf{0},\mathbf{0},\mathbf{0})$ | $0$ | $0$ | $0$ |
| $(\mathbf{0},\mathbf{0},\mathbf{1})$ | $0$ | $0$ | $-1$ |
| $(\mathbf{0},\mathbf{1},\mathbf{0})$ | $0$ | $-1$ | $0$ |
| $(\mathbf{0},\mathbf{1},\mathbf{1})$ | $3$ | $2$ | $2$ |
| $(\mathbf{1},\mathbf{0},\mathbf{0})$ | $-1$ | $0$ | $0$ |
| $(\mathbf{1},\mathbf{0},\mathbf{1})$ | $2$ | $3$ | $2$ |
| $(\mathbf{1},\mathbf{1},\mathbf{0})$ | $2$ | $2$ | $3$ |
| $(\mathbf{1},\mathbf{1},\mathbf{1})$ | $2$ | $2$ | $2$ |

## Player $\boldsymbol{i}$'s Payoff Matrix

Although not explicitly required, let's construct a payoff matrix for player $i$ by simplifying the problem into an equivalent two-player game. To do this, we merge player $i$'s two opponents into a single 'mega-opponent.' This will allow us to analyse player $i$'s decisions as if he were playing against a single entity, making it easier to represent the game in matrix form.

In this setup, the mega-opponent's strategy is actually a strategy profile consisting of two elements – one strategy for each of the original opponents. These strategy profiles will form the four columns of the payoff matrix.

| | | Opponents of player $i$ | | | |
|---|---|---|---|---|---|
| | | $(0,0)$ | $(0,1)$ | $(1,0)$ | $(1,1)$ |
| Player $i$ | $0$ | $0$ | $0$ | $0$ | $3$ |
| | $1$ | $-1$ | $2$ | $2$ | $2$ |