The Rational Choice Paradigm

Problem 2

City Parks

A city's mayor has to decide how much money to spend on parks and recreation. City codes restrict this spending to no more than 5% of the budget, and the yearly budget of the city is ₹ 2 Crore. The mayor wants to please his constituents, who have diminishing returns from parks. The money-equivalent benefit from spending ₹ c on parks is $v(c) = \sqrt{400c} - \frac{1}{80}c$.

(a) What is the action set for the city's mayor?
(b) How much should the mayor spend?
(c) The movie *An Inconvenient Truth* has shifted public opinion, and now people are more willing to pay for parks. The new preferences of the people are given by $v(c) = \sqrt{1600c} - \frac{1}{80}c$. What now is the action set for the mayor, and how much spending should he choose to cater to his constituents?

Answer (a). Action Set

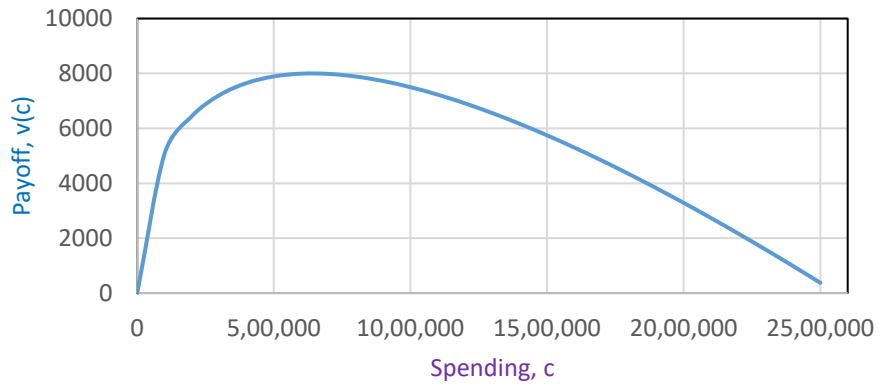
The city's total yearly budget is ₹ 2 Crore. The spending limit for parks is capped at 5% of the total budget, amounting to ₹ 10 Lakhs. The action set is continuous and defined as—

$$c \in [0, 1000000]$$

Answer (b). Optimal Spending

The payoff function represents the satisfaction levels of the people as a function of the money spent on parks. It exhibits diminishing returns, and is given by—

$$v(c) = \sqrt{400c} - \frac{1}{80}c$$



We assume that the mayor is rational and will choose a value of c that maximizes the satisfaction levels of people – and, by extension, his own satisfaction. To determine the value of c that gives the highest payoff, we solve the following maximization problem—

$$\max_{c\in[0,1000000]} \sqrt{400c} - \frac{1}{80}c$$

The first-order condition for this problem is—

$$\frac{dv}{dc} = 0$$

$$\therefore \frac{d}{dc}\left(\sqrt{400c} - \frac{1}{80}c\right) = 0$$

$$\therefore \frac{10}{\sqrt{c}} - \frac{1}{80} = 0$$

$$\therefore c = 640000$$

To check the second-order condition, let's determine $\frac{d^2v}{dc^2}$.

$$\frac{d^2v}{dc^2} = \frac{d}{dc}\left(\frac{dv}{dc}\right) = \frac{d}{dc}\left(\frac{10}{\sqrt{c}} - \frac{1}{80}\right) = -5c^{-\frac{3}{2}} - 0 < 0$$

Hence, the mayor should spend ₹ 6.4 Lakhs on parks.

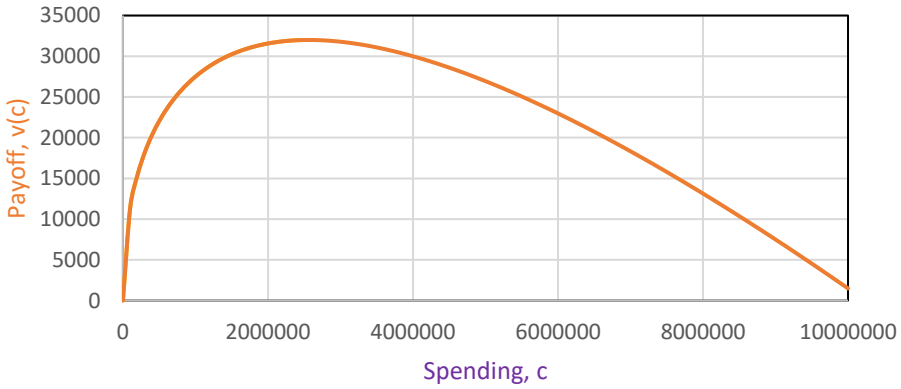
ANSWER (C). NEW PREFERENCES

Since the restriction on spending hasn't changed, the action set is the same as before—

$$c \in [0, 1000000]$$

Thanks to *An Inconvenient Truth*, the new payoff function is—

$$v(c) = \sqrt{1600c} - \frac{1}{80}c$$



The maximization problem is

$$\max_{c\in[0,1000000]} \sqrt{1600c} - \frac{1}{80}c$$

To find the value of c that results in a maximum for $v(c)$, the first-order condition is—

$$\frac{dv}{dc} = 0$$

$$\therefore \frac{d}{dc}\left(\sqrt{1600c} - \frac{1}{80}c\right) = 0$$

$$\therefore \frac{20}{\sqrt{c}} - \frac{1}{80} = 0$$

$$\therefore c = 2560000$$

Since this exceeds the permissible budget of 10 Lakhs, the optimal solution is to spend ₹ 10 Lakhs on parks.

KEY TAKEAWAYS

- **Continuous Action Spaces**. Decision-makers often face an infinite range of possible actions within a specified interval.
- **Decision-Making within Constraints**. Practical constraints – financial, regulatory, or otherwise – can limit the achievable payoff. In such cases, decision-makers can either optimize within these limits or work to remove the constraints.
- **Maximizing Payoffs**. Optimization involves applying first- and second-order conditions, while accounting for potential 'corner cases' when maximizing an 'ever-growing' function.
- **Dynamic Decision Problems**. Problem parameters can evolve over time (for example, shifts in public preferences due to awareness campaigns like *An Inconvenient Truth*), requiring decisions to adapt to new conditions.
- **Diminishing Returns**. Satisfaction from spending on parks decreases with each additional unit of expenditure, highlighting the trade-off between higher spending and smaller incremental benefits.
- **Unit Consistency**. Ensure that the units of c in the payoff function align with the specified scale.

EXTRAS

1. Department of Parks and Recreation (DPR): These are departments of government bodies that oversee and maintain public parks, nature preserves, open space areas, greenways, trails, and recreational public works for sports, recreation, arts, etc.
2. The movie *An Inconvenient Truth* (2006) highlights the urgent need to address climate change and the role of human actions in accelerating global warming.