

# The Normal-Form Representation

## Problem 2

### BIDDING FOR A PROJECT

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Two companies, *Alpha Builders* and *Beta Constructions*, are competing for a government construction project. The contract will be awarded to the company that submits the lower bid, and the winning company's profit depends on its bid amount and the actual project cost. Each company must submit a sealed bid without knowing the other company's choice.

The companies have three available bidding strategies: ₹50 lakhs (high bid), ₹40 lakhs (medium bid), or ₹30 lakhs (low bid). The actual project cost is ₹25 lakhs, and the company that wins the contract earns a profit equal to its bid minus ₹25 lakhs. The losing company earns nothing. If both companies submit the same bid, the contract is awarded randomly with equal probability, and the expected profit for each is the average of the winning profit.

Write the normal-form representation of the game and construct the payoff matrix.

### INTRODUCTION

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Lower bids increase the chance of winning the contract but reduce profit margins.

### ANSWER: THE NORMAL-FORM REPRESENTATION

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#### PLAYERS

The number of players is  $n = 2$ . The set of players is—

$$N = \{1, 2\}$$

#### STRATEGY SETS

$S_i$  is the set of all pure strategies available to player  $i$ . In this game, each company has three options: bid high (strategy  $H$ ), bid medium (strategy  $M$ ), and bid low (strategy  $L$ ). Hence—

$$S_i = \{H, M, L\} \quad \text{for } i \in N$$

#### PAYOFF FUNCTIONS: UNEQUAL BIDS

If the two companies bid differently, then the company with the lower bid wins.

$$v_1(M, H) = v_2(H, M) = 40 - 25 = 15$$

$$v_1(L, M) = v_2(M, L) = 30 - 25 = 5$$

$$v_1(L, H) = v_2(H, L) = 30 - 25 = 5$$

...and the company with the higher bid gets nothing.

$$v_1(H, M) = v_2(M, H) = 0$$

$$v_1(M, L) = v_2(L, M) = 0$$

$$v_1(H, L) = v_2(L, H) = 0$$

### PAYOFF FUNCTIONS: EQUAL BIDS

When both companies bid the same amount, then the probability that a company will win the contract is 0.5 and the probability that it will lose the contract is  $1 - 0.5 = 0.5$ . If the company wins the contract, it earns a profit based on its bid. If the company loses the contract, it earns nothing. The expected payoff for each company is calculated as the average of the earnings for the two outcomes (winning or losing the contract) weighted by the probabilities of the respective outcomes.

$$v_1(H, H) = v_2(H, H) = \frac{1}{2}(50 - 25) + \frac{1}{2}(0) = 12.5$$

$$v_1(M, M) = v_2(M, M) = \frac{1}{2}(40 - 25) + \frac{1}{2}(0) = 7.5$$

$$v_1(L, L) = v_2(L, L) = \frac{1}{2}(30 - 25) + \frac{1}{2}(0) = 2.5$$

### ANSWER: THE MATRIX-FORM REPRESENTATION

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		Player 2		
		L	M	H
Player 1	L	2.5, 2.5	5, 0	5, 0
	M	0, 5	7.5, 7.5	15, 0
	H	0, 5	0, 15	12.5, 12.5