

Optimization Technique and Applications

Fleet Routing and Assignment:

Determine which aircraft to fly on each route, and the sequence of segments flown by each aircraft.

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■ ABSTRACT

The fleet planning decision-making process is one of the most problematic issues for airline industry. An over-large fleet size would cause an airline unnecessary expense since the increasing capital assets account for a large proportion of the airline operational costs. Moreover, considering the profit margin of the airline industry around the world continuously pressured by a long-term exposure to a high-cost and low-fare environment, the irrational fleet composition would necessarily deteriorate the airline's operation. Therefore, airlines may have to develop a more practical fleet planning approach to meet passenger demand with lower costs and more controllable risks at a strategic level.

Here in this paper we considered 3 planes moving between 6 airports so that maximum number of passengers can be flown between these airports.

The main contribution by this paper is as follows:

- Cost minimization of the company by considering the total distance travelled by all the planes and the number of airports used by them.
- The segment flown by each aircraft.

In the next section, the problem is presented in detail. The formulation of the problem is solved on MATLAB software.

■ PROBLEM

Airline fleet planning process is one of the most problematic issues for airline industry. Considering the profit margin of the airline industry and the high competition in the industry around the world, if the aircraft planning is not optimized properly it can be very detrimental to the company. Therefore, airlines must develop a more practical fleet planning approach to meet market demand with lower costs and more controllable risks at a strategic level. From the plethora of problems faced by the airline companies, some are:

- **Network Design problem**, which consists in mainly deciding which airports should be served by the airline.
- **Fleet Design problem** which consists in deciding the size and the composition of the fleet of the airline.
- **Flight Scheduling problem** which consists in finding when and how often should each leg flight be operated in a planning period.

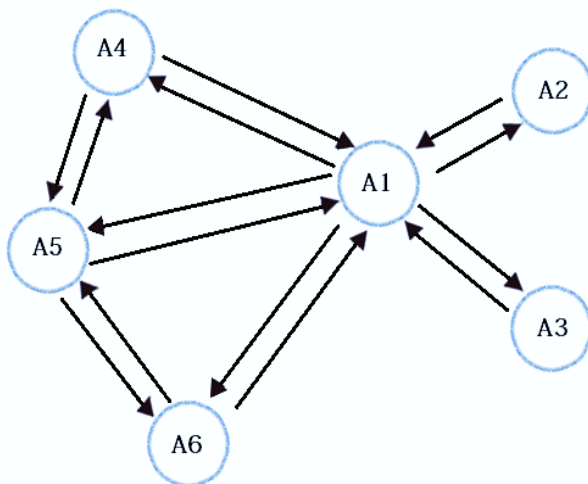
- **Fleet Assignment problem** which consists in assigning a well-suited aircraft type to each flight leg.
- **Aircraft Routing problem** which consists in designing the sequence of flight legs that each aircraft will have to operate on.
- **Crew Management problem** which consists in designing a work schedule for each crew member to be able to operate each flight leg.
- **Cost Design problem** which consists of deciding the minimum cost that the company must bear for total distance covered by an aircraft.

We present a model for the formulation and solution of **Cost Design** and **Flight Scheduling** problems that arise in air transportation. Through this report we will analyse and optimize the problem using LPP model and with the help of MATLAB software.

■ FORMULATION

For the formulation of **Cost Design and Flight Scheduling** problem, the assumptions and variables that we have taken will be explained in this section.

Let us assume an airline would like to use mathematical programming to schedule its flights to minimize cost (Cost Design). The following map shows the city pairs that act as bases:



Here, we are tackling the problem keeping 3 planes in mind, namely Plane1, Plane2 and Plane3. In the above diagram all the encircled parts depict the various airline bases where the planes will pass through. From here on we will consider **Cost Design** problem as **Problem Part – I** and **Flight Scheduling** problem as **Problem Part – II**.

❖ NOMENCLATURE

x = Total distance covered by Plane1 in km (where $x=1$ means 100 kms).

y = Total distance covered by Plane2 in km (where $y=1$ means 100 kms).

z = Total distance covered by Plane3 in km (where $z=1$ means 100 kms).

a = Number of airports used by Plane1.

b = Number of airports used by Plane2.

c = Number of airports used by Plane3.

❖ CONSTRAINTS

For Problem Part – I

- *Constraint 1:* The sum of the total distance travelled by all three planes should be greater than 7200 km provided plane 1 and plane 3 individually cover double the distance travelled by plane 2.

$$200x + 100y + 200z > 7200$$

- *Constraint 2:* On analysing every plane on their present conditions it is found that the price spent on each plane's fuel consumption per km is \$102, \$204 and \$102 for Plane1, Plane2 and Plane3 respectively and the total expenditure allowed on fuel for all planes combined must be less than or equal to \$6120.

$$102x + 204y + 102z \leq 6120$$

- *Constraint 3:* Maintenance cost per km spent on Plane1 is \$200, on Plane2 is \$100 and on Plane3 is \$100. Airlines can spend at most \$8000 on maintenance which should be shared among those three planes.

$$200x + 100y + 100z \leq 8000$$

- *Non-negativity Conditions:* Each Plane should not fly more than 2000 km.

$$20 \geq x \geq 0$$

$$20 \geq y \geq 0$$

$$20 \geq z \geq 0$$

For Problem Part – II

- *Path Function:* Path function helps to minimize the cost so that we have total no. of airports travelled by each plane. It states that four times the sum of the no. of airports used by planes is equal to sum of the total distances covered by each plane. Hence, Mathematically

$$4(a + b + c) = x + y + z$$

❖ COST FUNCTION

For Problem part - I:

Our objective is to minimize total operating cost. The operating cost per km for Plane1, Plane2 and Plane3 are \$200, \$150 and \$400 per km, respectively.

Hence, $C_1 = 200x + 150y + 400z$

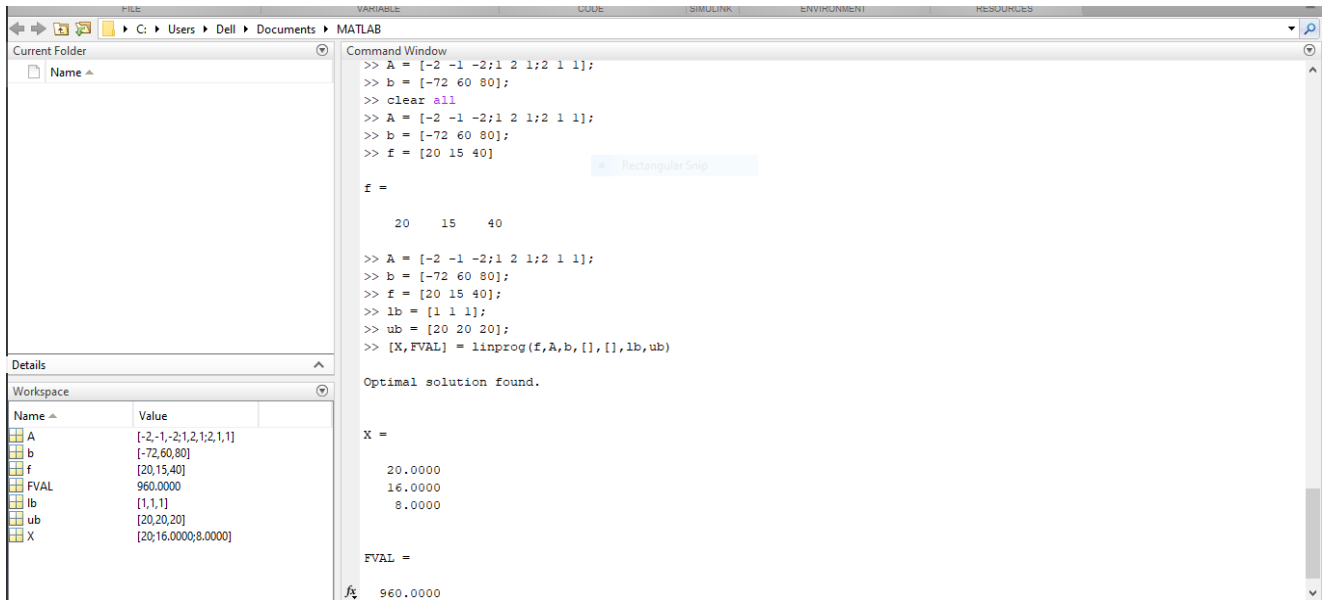
For Problem part - II:

Our objective is to minimize the total number of airports used while maximising the total number of distances covered.

Hence, $C_2 = (a-1)^2 + (b-1)^2 + c^2$

■ SOLUTION

For Problem part - I:



The screenshot shows the MATLAB Command Window and Workspace for Problem part - I. The Command Window displays the following code and output:

```
>> A = [-2 -1 -2; 1 2 1; 2 1 1];
>> b = [-72 60 80];
>> clear all
>> A = [-2 -1 -2; 1 2 1; 2 1 1];
>> b = [-72 60 80];
>> f = [20 15 40]

f =

    20    15    40

>> A = [-2 -1 -2; 1 2 1; 2 1 1];
>> b = [-72 60 80];
>> f = [20 15 40];
>> lb = [1 1 1];
>> ub = [20 20 20];
>> [X, FVAL] = linprog(f,A,b,[],[],lb,ub)

Optimal solution found.

X =

    20.0000
    16.0000
     8.0000

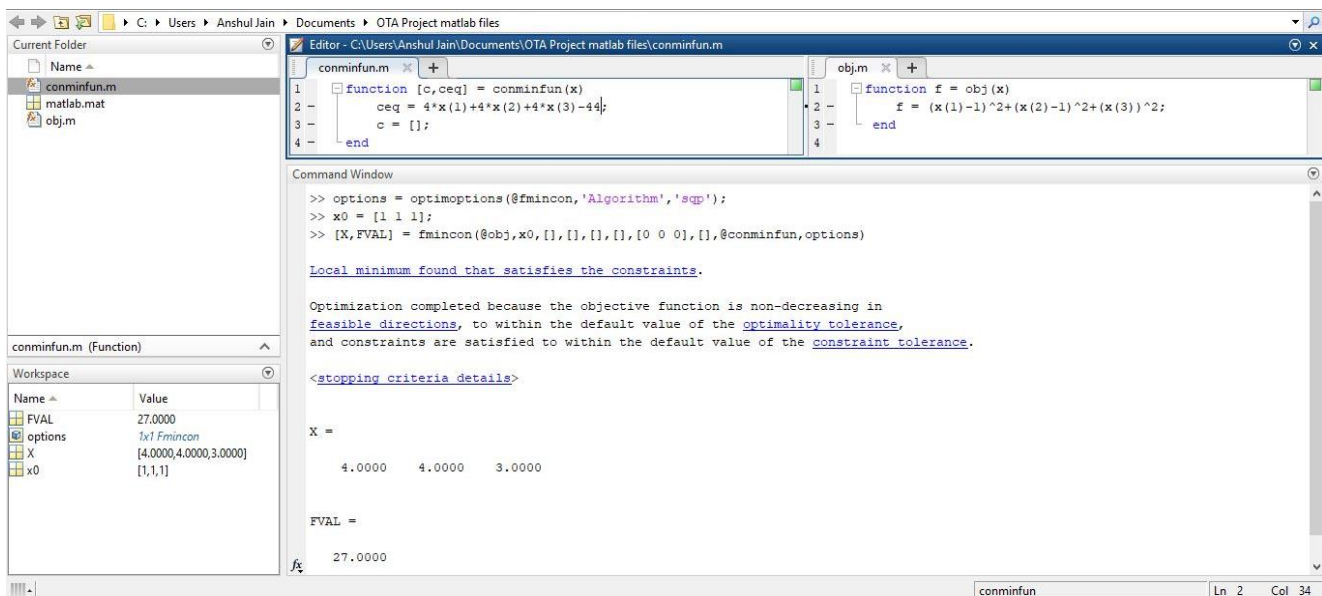
FVAL =

    960.0000
```

The Workspace shows the following variables:

Name	Value
A	$\begin{bmatrix} -2 & -1 & -2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$
b	$\begin{bmatrix} -72 \\ 60 \\ 80 \end{bmatrix}$
f	$\begin{bmatrix} 20 & 15 & 40 \end{bmatrix}$
FVAL	960.0000
lb	$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$
ub	$\begin{bmatrix} 20 & 20 & 20 \end{bmatrix}$
X	$\begin{bmatrix} 20 & 16 & 8 \end{bmatrix}$

For Problem part - II:



The screenshot shows the MATLAB Editor and Command Window for Problem part - II. The Editor displays the following code in `conminfun.m`:

```
function [c,ceq] = conminfun(x)
ceq = 4*x(1)+4*x(2)+4*x(3)-44;
c = [];
end
```

The Command Window displays the following code and output:

```
>> options = optimoptions(@fmincon,'Algorithm','sqp');
>> x0 = [1 1 1];
>> [X,FVAL] = fmincon(@obj,x0,[],[],[],[0 0 0],[],@conminfun,options)

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the optimality tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.

<stopping criteria details>

X =

    4.0000    4.0000    3.0000

FVAL =

    27.0000
```

The Workspace shows the following variables:

Name	Value
FVAL	27.0000
options	1x1 fmincon
X	$\begin{bmatrix} 4.0000 & 4.0000 & 3.0000 \end{bmatrix}$
x0	$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

■ Conclusion

The solution of the given problem comes out to be:

$$x = 20$$

$$y = 16$$

$$z = 8$$

Minimum Cost: \$960,000

Values of:

$$a = 4$$

$$b = 4$$

$$c = 3$$

So, Plane1 can take 4 paths to travel a total distance of 20,000km.

Plane2 can take 4 paths to travel a total distance of 16,000km.

Plane3 can take 3 paths to travel a total distance of 8,000km.

So, Plane1 can follow sequence: A1 - A6 - A5 - A1 - A4 - A5 - A1 - A3

Plane2 can follow sequence: A1 - A5 - A4 - A1 - A4 - A5 - A1

Plane3 can follow sequence: A1 - A4 - A1 - A2 - A1

■ Contribution

- Ansh Mittal -> *Formulation, MATLAB and Constraints.*
- Anshul Jain -> *Formulation, Airline Design and Constraints.*
- Ayush Gupta -> *Problem setting, Conclusion and MATLAB.*
- Daksh Balyan -> *Problem Setting and Abstract.*
- Harshit Garg -> *MATLAB and Abstract.*