子矩阵修改问题

问题描述

给定一个 $n \times m$ 大小的矩阵 A。

给定 q 组操作,每次操作为给定 5 个正整数 x_1,y_1,x_2,y_2,d , A_{x_1,y_1} 是子矩阵左上角端点, A_{x_2,y_2} 是子矩阵右下角端点,你需要给其中每个元素都增加 d。

输出操作结束后的矩阵 A。

输入格式

```
第一行输入 3 个正整数 n,m,q。 (1 \le n,m \le 10^3,1 \le q \le 10^5)接下来 n 行每行输入 m 个整数,表示 A_{i,j}。 (-10^3 \le A_{i,j} \le 10^3,1 \le i \le n,1 \le j \le m)接下来 q 行,每行输入 5 个正整数 x_1,y_1,x_2,y_2,d。 (1 \le x_1 \le x_2 \le n,1 \le y_1 \le y_2 \le m,-10^3 \le d \le 10^3)
```

输出格式

輸出 n 行 m 个整数,表示操作结束后的矩阵 A。

暴力

暴力实现该问题的时间复杂度为 $O(q \times n \times m)$ 。

二维差分

明确一点:

前缀和数组求差分后是原数组,原数组求差分后是差分数组。

差分数组求前缀和是原数组,原数组求前缀和后是前缀和数组。

C++

```
1 #include <iostream>
   using namespace std;
 2
   const int N = 1005;
 4
 5
    int a[N][N], b[N][N];
 7
    int main() {
 8
        ios::sync_with_stdio(false);
9
        cin.tie(0);
10
        int n, m, q;
11
12
        cin >> n >> m >> q;
13
        // 读取初始矩阵
14
        for (int i = 1; i <= n; ++i) {
15
16
            for (int j = 1; j <= m; ++j) {
17
                cin >> a[i][j];
            }
18
19
        }
```

```
20
21
        // 计算初始差分数组
22
        for (int i = 1; i <= n; ++i) {
23
            for (int j = 1; j <= m; ++j) {
24
                b[i][j] = a[i][j] - a[i - 1][j] - a[i][j - 1] + a[i - 1][j - 1]
    1];
25
            }
26
        }
27
28
        // 处理 q 次增量修改
29
        while (q--) {
30
            int x1, y1, x2, y2, c;
31
            cin >> x1 >> y1 >> x2 >> y2 >> c;
32
            b[x1][y1] += c;
33
            b[x2 + 1][y1] -= c;
34
            b[x1][y2 + 1] -= c;
35
            b[x2 + 1][y2 + 1] += c;
36
        }
37
38
        // 还原前缀和得到最终矩阵
        for (int i = 1; i <= n; ++i) {
39
            for (int j = 1; j <= m; ++j) {
40
41
                a[i][j] = b[i][j] + a[i - 1][j] + a[i][j - 1] - a[i - 1][j - 1]
    1];
42
            }
43
        }
44
45
        // 输出最终矩阵
46
        for (int i = 1; i <= n; ++i) {
47
            for (int j = 1; j <= m; ++j) {
                cout << a[i][j] << " ";</pre>
48
49
            }
            cout << "\n";</pre>
50
        }
51
52
53
        return 0;
54
55
```

Java

```
import java.io.*;
 2
    import java.util.*;
 3
4
    public class Main {
 5
        public static void main(String[] args) {
            Scanner in = new Scanner(System.in);
 6
 7
            PrintWriter out = new PrintWriter(System.out);
8
            // 读取矩阵大小 n, m 和查询次数 q
9
10
           int n = in.nextInt(), m = in.nextInt(), q = in.nextInt();
11
12
            // 定义矩阵 a (存储初始值) 和 差分数组 b
13
           int[][] a = new int[n + 1][m + 1];
           int[][] b = new int[n + 1][m + 1];
14
15
16
            // 读取初始矩阵值
```

```
17
            for (int i = 1; i \le n; i++) {
18
                for (int j = 1; j <= m; j++) {
19
                    a[i][j] = in.nextInt();
20
                }
            }
21
22
23
            // 计算初始差分数组
            for (int i = 1; i \le n; i++) {
24
25
                for (int j = 1; j <= m; j++) {
                    b[i][j] = a[i][j] - a[i - 1][j] - a[i][j - 1] + a[i - 1][j]
26
    - 1];
27
                }
            }
28
29
            // 处理 q 次增量修改
30
31
            while (q-- > 0) {
32
                int x1 = in.nextInt(), y1 = in.nextInt(), x2 = in.nextInt(), y2
    = in.nextInt();
33
                int c = in.nextInt();
34
                b[x1][y1] += c;
35
                b[x2 + 1][y1] -= c;
36
                b[x1][y2 + 1] -= c;
37
                b[x2 + 1][y2 + 1] += c;
38
            }
39
            // 还原前缀和得到最终矩阵
40
            for (int i = 1; i \le n; i++) {
41
42
                for (int j = 1; j <= m; j++) {
43
                    a[i][j] = b[i][j] + a[i - 1][j] + a[i][j - 1] - a[i - 1][j]
    - 1];
44
                }
45
            }
46
            // 输出最终矩阵
47
48
            for (int i = 1; i \le n; i++) {
49
                for (int j = 1; j <= m; j++) {
                    out.print(a[i][j] + " ");
50
51
52
                out.println();
53
            }
54
            out.flush();
55
56
            in.close();
57
        }
58
    }
59
```

• Python

```
9
        b = [[0] * (m + 2) for _ in range(n + 2)]
10
11
        for i in range(1, n + 1):
12
            row = list(map(int, sys.stdin.readline().split()))
13
            for j in range(1, m + 1):
14
                a[i][j] = row[j - 1]
15
        # 计算初始差分数组
16
17
        for i in range(1, n + 1):
18
            for j in range(1, m + 1):
                b[i][j] = a[i][j] - a[i - 1][j] - a[i][j - 1] + a[i - 1][j - 1]
19
20
        # 处理 q 次修改
21
22
        for _ in range(q):
23
            x1, y1, x2, y2, c = map(int, sys.stdin.readline().split())
24
            b[x1][y1] += c
25
            b[x2 + 1][y1] -= c
26
            b[x1][y2 + 1] -= c
27
            b[x2 + 1][y2 + 1] += c
28
        # 还原前缀和得到最终矩阵
29
30
        for i in range(1, n + 1):
31
            for j in range(1, m + 1):
32
                a[i][j] = b[i][j] + a[i - 1][j] + a[i][j - 1] - a[i - 1][j - 1]
33
        # 输出最终矩阵
34
35
        for i in range(1, n + 1):
            print(" ".join(map(str, a[i][1:m+1])))
36
37
38
    if __name__ == "__main__":
39
        main()
40
```

棋盘

C++

```
#include <iostream>
 2
    using namespace std;
 3
 4
    const int N = 2005;
    int a[N][N], b[N][N];
 5
 6
 7
    int main() {
 8
        ios::sync_with_stdio(false);
 9
        cin.tie(0);
10
11
        int n, m;
12
        cin >> n >> m ;
13
        // 处理 q 次增量修改
14
15
        while (m--) {
16
            int x1, y1, x2, y2, c;
17
            cin >> x1 >> y1 >> x2 >> y2;
18
            b[x1][y1] += 1;
19
            b[x2 + 1][y1] = 1;
20
            b[x1][y2 + 1] -= 1;
```

```
21
             b[x2 + 1][y2 + 1] += 1;
22
        }
23
        // 还原前缀和得到最终矩阵
24
25
        for (int i = 1; i <= n; ++i) {
26
             for (int j = 1; j <= n; ++j) {
27
                 a[i][j] = b[i][j] + a[i - 1][j] + a[i][j - 1] - a[i - 1][j -
    1];
28
                 if(a[i][j]\%2==1){
29
                   cout<<1;</pre>
30
                 }else{
31
                   cout<<0;
32
33
             }
             cout<<"\n";</pre>
34
35
        }
36
37
        return 0;
38
   }
39
```

Java

```
import java.util.Scanner;
 2
 3
    public class Main {
        static final int N = 2005;
 4
 5
        static int[][] a = new int[N][N], b = new int[N][N];
 6
 7
        public static void main(String[] args) {
 8
            Scanner scanner = new Scanner(System.in);
 9
            int n = scanner.nextInt();
10
            int m = scanner.nextInt();
11
12
            while (m-- > 0) {
13
                int x1 = scanner.nextInt();
14
                int y1 = scanner.nextInt();
15
                int x2 = scanner.nextInt();
16
                int y2 = scanner.nextInt();
17
                b[x1][y1] += 1;
18
                b[x2 + 1][y1] = 1;
19
                b[x1][y2 + 1] = 1;
20
                b[x2 + 1][y2 + 1] += 1;
            }
21
22
            // 还原前缀和得到最终矩阵
23
24
            for (int i = 1; i <= n; ++i) {
25
                 for (int j = 1; j <= n; ++j) {
26
                     a[i][j] = b[i][j] + a[i - 1][j] + a[i][j - 1] - a[i - 1][j]
    - 1];
27
                     System.out.print(a[i][j] \% 2 == 1 ? 1 : 0);
                }
28
29
                System.out.println();
30
            }
31
32
            scanner.close();
33
        }
```

```
34 | }
35 |
```

• Python

```
import sys
 2
 3
    N = 2005
 4
    a = [[0] * N for _ in range(N)]
 5
    b = [[0] * N for _ in range(N)]
 6
 7
    def main():
 8
        input = sys.stdin.read
9
        data = input().split()
10
        index = 0
11
        n, m = int(data[index]), int(data[index + 1])
12
13
        index += 2
14
15
        for _ in range(m):
            x1, y1, x2, y2 = map(int, data[index:index + 4])
16
17
            index += 4
            b[x1][y1] += 1
18
19
            b[x2 + 1][y1] -= 1
20
            b[x1][y2 + 1] -= 1
21
            b[x2 + 1][y2 + 1] += 1
22
23
        # 还原前缀和得到最终矩阵
24
        output = []
25
        for i in range(1, n + 1):
26
            row = []
27
            for j in range(1, n + 1):
28
                a[i][j] = b[i][j] + a[i - 1][j] + a[i][j - 1] - a[i - 1][j - 1]
29
                row.append('1' if a[i][j] % 2 == 1 else '0')
            output.append("".join(row))
30
31
32
        print("\n".join(output))
33
    if __name__ == "__main__":
34
35
        main()
36
```