* Making Change Problem.

1 dollar = 100 cents

1 quarter = 25 cents

1 dimes = 10 cents

1 nickels = 5 cents.

1 Pennies = 1 cent.

$$\boxed{\begin{array}{l} C[i,0] = 0. \\ 1.\ \text{If } i=1 \text{ then } C[i,j] = 1 + C[1, j-d_i] \\ 2.\ \text{If } j<d_i \text{ then } C[i,j] = C[i-1,j] \\ 3.\ \text{otherwise } C[i][j] = \min(C[i-1,j],\ 1 + C[i, j-d_i]) \end{array}}$$

Ex. Solve making change problem for $d_1 = 1$, $d_2 = 4$, $d_3 = 6$, $n = 3$ and $N = 8$ units.

Sol^n :→ Here, $d_1 = 1$, $d_2 = 4$, $d_3 = 6$, $n = 3$, $N = 8$ units.

So, we have to create a table having 3 rows and column ranging from 0 to 8.

Initially $C[i,0] = 0$ $\therefore C[1,0] = C[2,0] = C[3,0] = 0.$

j →

		0	1	2	3	4	5	6	7	8
i = 1	$d_1 = 1$	0	1	2	3	4	5	6	7	8
i = 2	$d_2 = 4$	0	1	2	3	1	2	3	4	2
i = 3	$d_3 = 6$	0	1	2	3	1	2	1	2	2

⇒ $C[1,1]$ with $i=1$, $j=1$, $d_1 = 1$.

As $i = 1$

Formula used : $C[1,1] = 1 + C[1, j-d_1]$

$= 1 + C[1,0]$

$= 1 + 0 = 1$

$\boxed{\therefore C[1,1] = 1}$

$\Rightarrow C[1,2]$ with $i=1$, $j=2$, $d_1=1$.

As $i=1$

$$C[1,2] = 1 + C[1, j-d_1]$$
$$= 1 + C[1,1]$$
$$= 1 + 1 = 2$$

$$\boxed{\therefore C[1,2] = 2}$$

$\Rightarrow C[1,3]$ with $i=1$, $j=3$, $d_1=1$

As $i=1$

$$C[1,3] = 1 + C[1, j-d_1]$$
$$= 1 + C[1,2]$$
$$= 1 + 2 = 3$$

$$\boxed{\therefore C[1,3] = 3}$$

$\Rightarrow C[1,4]$ with $i=1$, $j=4$, $d_1=1$.

As $i=1$

$$C[1,4] = 1 + C[1, j-d_1]$$
$$= 1 + C[1,3]$$
$$= 1 + 3 = 4$$

$$\boxed{\therefore C[1,4] = 4}$$

$\Rightarrow C[1,5]$ with $i=1$, $j=5$, $d_1=1$.

As $i=1$

$$C[1,5] = 1 + C[1, j-d_1]$$
$$= 1 + C[1,4]$$
$$= 1 + 4 = 5$$

$$\boxed{\therefore C[1,5] = 5}$$

$\Rightarrow C[1,6]$ with $i=1$, $j=6$, $d_1=1$

As $i=1$

$$C[1,6] = 1 + C[1, j-d_1]$$
$$= 1 + C[1,5]$$
$$= 1 + 5 = 6$$

$$\boxed{\therefore C[1,6] = 6}$$

$\Rightarrow$ $C[1,7]$ with $i=1$, $j=7$, $d_1=1$.

As $i=1$

$$C[1,7] = 1 + C[1, j-d_1]$$
$$= 1 + C[1, 6]$$
$$= 1 + 6 = 7$$

$$\boxed{\therefore C[1,7] = 7}$$

$\Rightarrow$ $C[1,8]$ with $i=1$, $j=8$, $d_1=1$.

As $i=1$

$$C[1,8] = 1 + C[1, j-d_1]$$
$$= 1 + C[1, 7]$$
$$= 1 + 7 = 8$$

$$\boxed{\therefore C[1,8] = 8}$$

- Now, let's move on to the second row.

$\Rightarrow$ $C[2,1]$ with $i=2$, $j=1$, $d_2=4$

As $j<d_2$ i.e $1<4$

Formula used:

$$C[2,1] = C[i-1, j]$$
$$= C[1,1]$$
$$= 1.$$

$$\boxed{\therefore C[2,1] = 1}$$

$\Rightarrow$ $C[2,2]$ with $i=2$, $j=2$, $d_2=4$.

As $j<d_2$ i.e. $2<4$.

$$C[2,2] = C[i-1, j]$$
$$= C[1,2]$$
$$= 2$$

$$\boxed{\therefore C[2,2] = 2}$$

$\Rightarrow C[2,3]$ with $i=2$, $j=3$, $d_2=4$.

As $j<d_2$ i.e. $3<4$.

$C[2,3] = C[i-1,j]$

$= C[1,3]$

$= 3.$

$\boxed{\therefore C[2,3] = 3}$

$\Rightarrow C[2,4]$ with $i=2$, $j=4$, $d_2=4$.

As $i \neq 1$ and $j \not< d_2$.

$C[2,4] = \min(C[i-1,j], 1 + C[i, j-d_i])$

$= \min(C[1,4], 1 + C[2,0])$

$= \min(4, 1+0)$

$= \min(4,1) = 1.$

$\boxed{\therefore C[2,4] = 1}$

$\Rightarrow C[2,5]$ with $i=2$, $j=5$ and $d_2=4$.

As $i \neq 1$ and $j > d_2$

$C[2,5] = \min(C[i-1,j], 1 + C[i, j-d_i])$

$= \min(C[1,5], 1 + C[2,1])$

$= \min(5, 1+1)$

$= \min(5, 2) = 2.$

$\boxed{\therefore C[2,5] = 2}$

$\Rightarrow C[2,6]$ with $i=2$, $j=6$, $d_2=4$.

As $i \neq 1$ and $j > d_2$

$C[2,6] = \min(C[i-1,j], 1 + C[i, j-d_i])$

$= \min(C[1,6], 1 + C[2,2])$

$= \min(6, 1+2)$

$= \min(6,3) = 3$

$\boxed{\therefore C[2,6] = 3}$

$\Rightarrow$ $C[2,7]$ with $i=2$, $j=7$, $d_2=4$.

As $i \neq 1$ and $j > d_2$ i.e $7>4$.

$$C[2,7] = \min(C[i-1,j], 1 + C[i, j-d_i])$$
$$= \min(C[1,7], 1 + C[2,3])$$
$$= \min(7, 1+3)$$
$$= \min(7,4) = 4.$$

$\boxed{\therefore C[2,7] = 4}$

$\Rightarrow$ $C[2,8]$ with $i=2$, $j=8$, $d_2=4$.

As $i \neq 1$ and $j > d_2$

$$C[2,8] = \min(C[i-1,j], 1 + C[i, j-d_i])$$
$$= \min(C[1,8], 1 + C[2,4])$$
$$= \min(8, 1+1)$$
$$= \min(8, 2) = 2.$$

$\boxed{\therefore C[2,8] = 2}$

- Now, let's compute third row.

$\Rightarrow$ $C[3,1]$ with $i=3$, $j=1$, $d_3=6$.

As $i \neq 1$ and $j < d_3$ i.e. $1<6$.

$$C[3,1] = C[i-1,j]$$
$$= C[2,1]$$
$$= 1$$

$\boxed{\therefore C[3,1] = 1}$

$\Rightarrow$ $C[3,2]$ with $i=3$, $j=2$, $d_3=6$.

As $i \neq 1$ and $j<d_3$

$$C[3,2] = C[i-1,j]$$
$$= C[2,2]$$
$$= 2$$

$\boxed{\therefore C[3,2] = 2}$

$\Rightarrow$ $C[3,3]$ with $i=3$, $j=3$, $d_3=6$.

As $i \neq 1$, $j < d_3$.

$C[3,3] = C[i-1,j]$

$= C[2,3]$

$= 3$

$\boxed{\therefore C[3,3] = 3}$

$\Rightarrow$ $C[3,4]$ with $i=3$, $j=4$, $d_3=6$.

As $i \neq 1$, $j < d_3$

$C[3,4] = C[i-1,j]$

$= C[2,4]$

$= 1$

$\boxed{\therefore C[3,4] = 1}$

$\Rightarrow$ $C[3,5]$ with $i=3$, $j=5$, $d_3=6$

As $i \neq 1$, $j < d_3$

$C[3,5] = C[i-1,j]$

$= C[2,5]$

$= 2$

$\boxed{\therefore C[3,5] = 2}$

$\Rightarrow$ $C[3,6]$ with $i=3$, $j=6$, $d_3=6$

As $i \neq 1$, $j \not< d_3$

$C[3,6] = \min(C[i-1,j], 1 + C[i, j-d_i])$

$= \min(C[2,6], 1 + C[3,0])$

$= \min(3, 1+0)$

$= \min(3, 1)$

$= 1.$

$\boxed{\therefore C[3,6] = 1}$

$\Rightarrow$ $C[3,7]$ with $i=3$, $j=7$, $d_3=6$.

As $i \neq 1$, $j > d_3$.

$C[3,7] = \min(C[i-1,j], 1 + C[i, j-d_i])$

$= \min(C[2,7], 1 + C[3,1])$

$= \min(4, 1+1)$

$= \min(4, 2) = 2.$

$\boxed{\therefore C[3,7] = 2}$

$\Rightarrow$ $C[3,8]$ with $i=3, j=8, d_3=6$

As $i \neq 1$, $j > d_3$.

$C[3,8] = \min(C[i-1,j], 1 + C[i, j-d_i])$

$= \min(C[2,8], 1 + C[3,2])$

$= \min(2, 1+2)$

$= \min(2,3) = 2.$

$\boxed{\therefore C[3,8] = 2.}$

$\rightarrow$ The value $C[n,N]$ i.e. $C[3,8] = 2$ represents the minimum number of coins required to get the sum for 8 units. Hence we require 2 coins for getting 8 units. The coins are:

$$\begin{array}{rl} 4 \text{ units} & = 1 \text{ coin} \\ + \; 4 \text{ units} & = 1 \text{ coin} \\ \hline 8 \text{ units} & = 2 \text{ coins.} \end{array}$$

* Algorithm of Making Change Problem

```
Algorithm Number-of-Coins (N)
{
    for (i <- 1 to n) do

  for (i <- 1 to N) do
        C[i, 0] <- 0.

 For (i <- 1 to n) do
 {
        For (i <- 1 to N) do
        {
            if (i=1 && j < d[i]) then
                C[i][j] = infinity;
            else if (i=1) then
                C[i][j] = 1 + C[1, j - di];
            else if (j < d[i]) then
                C[i][j] = C[i-1, j];
            else
                C[i][j] = min (C[i-1, j], 1+C[i, j-di]);
        }
 }

  return C[n, N]
```