* Matrix Chain Multiplication

To construct $m[i,j]$ table, the formula is -

i) For $i=1$ to n set $m[i,i]=0$.

ii) For $i=2$ to n compute $m[i,j]$ using.

$$m[i,j] = \min\{m[i,k] + m[k+1,j] + P_{i-1}P_kP_j\}$$

with $i \leq k \leq j-1$.

Ex. Consider

Matrix	Dimension
A1	5×4
A2	4×6
A3	6×2
A4	2×7

Compute matrix chain order.

Sol^n :- Here, $P_0 = 5$, $P_1 = 4$, $P_2 = 6$, $P_3 = 2$, $P_4 = 7$

for all i, $1 \leq i \leq n$,

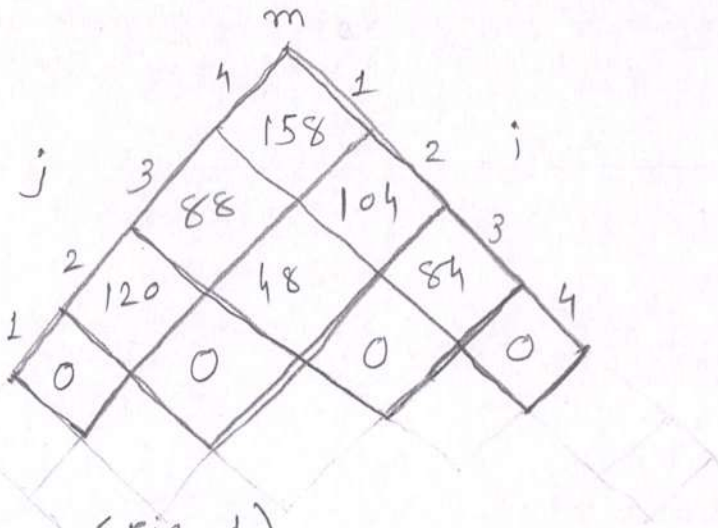
$m[i,i] = 0$.

Hence $m[1,1] = 0$

$m[2,2] = 0$

$m[3,3] = 0$

$m[4,4] = 0$



(Fig - 1)

Now, we will fill up the table horizontally from left to right, assumming $i \le K \le j-1$.

Let $i=1, j=2, K=1$.

$\Rightarrow m[1,2]$ Here $i=1, j=2, k=1$.

$m[1,2] = m[i,K] + m[K+1,j] + P_{i-1}P_K P_j$

$= m[1,1] + m[2,2] + P_0 P_1 P_2$

$= 0 + 0 + 5*4*6$.

$\boxed{m[1,2] = 120}$ When $K=1$.

$\Rightarrow m[2,3]$ Here, $i=2, j=3, K=2$

$m[2,3] = m[i,K] + m[K+1,j] + P_{i-1}P_K P_j$

$= m[2,2] + m[3,3] + P_1 P_2 P_3$

$= 0 + 0 + 4*6*2$

$\boxed{m[2,3] = 48.}$ When $k=2$

$\Rightarrow m[3,4]$ Here, $i=3, j=4, K=3$

$m[3,4] = m[i,k] + m[K+1,j] + P_{i-1}P_K P_j$.

$= m[3,3] + m[4,4] + P_2 P_3 P_4$

$= 0 + 0 + 6*2*7$

$\boxed{m[3,4] = 84}$ When $K=3$

$\Rightarrow$ $m[1,3]$ Here, $i=1, j=3$, $K=1$ or $K=2$.

$$m[1,3] = \min\{m[i,K] + m[K+1,j] + P_{i-1}P_K P_j\}$$

$$= \min \begin{cases} (m[1,1] + m[2,3] + P_0 P_1 P_3), \rightarrow K=1. \\ (m[1,2] + m[3,3] + P_0 P_2 P_3). \rightarrow K=2. \end{cases}$$

$$= \min \begin{cases} (0 + 48 + 5*4*2) \\ (120 + 0 + 5*6*2) \end{cases}$$

$$= \min \begin{cases} 48 + 40 \\ 120 + 60 \end{cases}$$

$$= \min \begin{cases} 88 & \text{when } K=1 \\ 180 & \text{when } K=2 \end{cases}$$

$\boxed{m[1,3] = 88}$ when $K=1$.

$\Rightarrow$ $m[2,4]$ Here, $i=2, j=4$, $K=2$ or $K=3$.

$$m[2,4] = \min\{m[i,K] + m[K+1,j] + P_{i-1}P_K P_j\}$$

$$= \min \begin{cases} (m[2,2] + m[3,4] + P_1 P_2 P_4), \rightarrow K=2. \\ (m[2,3] + m[4,4] + P_1 P_3 P_4), \rightarrow K=3. \end{cases}$$

$$= \min \begin{Bmatrix} (0 + 84 + 4*6*7) \\ (48 + 0 + 4*2*7) \end{Bmatrix}$$

$$= \min \begin{Bmatrix} 84 + 168 \\ 48 + 56 \end{Bmatrix}$$

$$= \min \begin{cases} 252 \\ 104 & \text{when } K=3 \end{cases}$$

$\boxed{m[2,4] = 104}$ when $K=3$

⇒ $m[1,4]$ Here, $i=1$, $j=4$, $K=1$ or $K=2$ or $K=3$.

$$m[1,4] = \min\{m[i,k] + m[k+1,j] + P_{i-1}P_kP_j\}$$

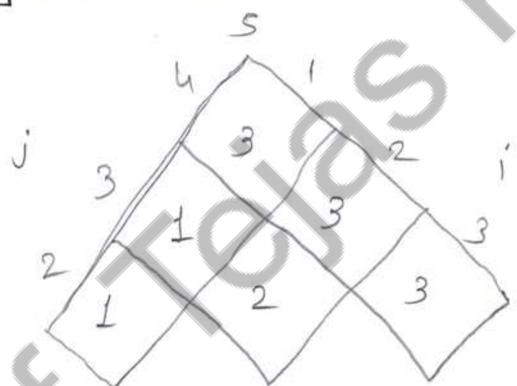
$$= \min \begin{cases} m[1,1] + m[2,4] + P_0P_1P_4 & \rightarrow K=1 \\ m[1,2] + m[3,4] + P_0P_2P_4 & \rightarrow K=2 \\ m[1,3] + m[4,4] + P_0P_3P_4 & \rightarrow K=3. \end{cases}$$

$$= \min \begin{cases} 0 + 104 + 5*4*7 \\ 120 + 84 + 5*6*7 \\ 88 + 0 + 5*2*7 \end{cases}$$

$$= \min \begin{cases} 104 + 140 \\ 204 + 210 \\ 88 + 70 \end{cases}$$

$$= \min \begin{cases} 244 \\ 414 \\ 158 \end{cases} \rightarrow \text{when } K=3.$$

$\boxed{m[1,4] = 158}$ when $K=3$.



(Fig - 2)

⇒ Finding Optimal Parenthsization Algorithm

Algorithm Print-Optimal-Parens (s, i, j)

1. if i == j
2. Print "A";
3. else Print "(" // opening Bracket
4. Print-Optimal-Parens (s, i, s[i,j])
5. Print-Optimal-Parens (s, s[i,j]+1, j)
6. Print ")" // Closing Bracket.

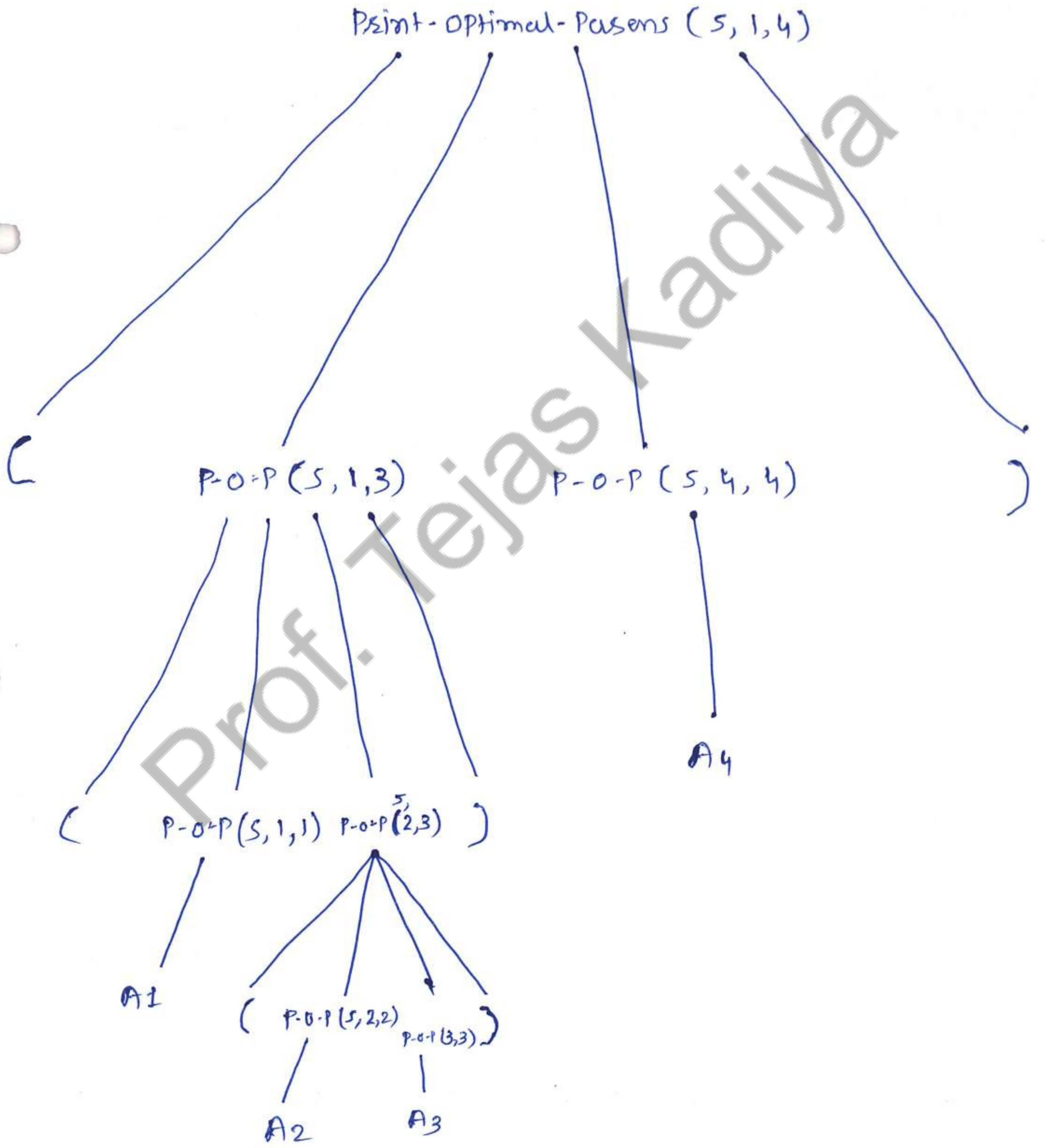
Using above algorithm we get the matrix chain order as

$\boxed{((A_1(A_2A_3))A_4)}$

⇒ Explanation of Optimal Parenthsization Algorithm

- In figure-2 we get the K value from which we can seperately differentiate the matrix chain order.

- Here, we have to starts from Print-Optimal-Parens (S, 1, 4) because we get the final value is m[1, 4].



→ So, Here we get

$$\boxed{((A1\,(A2\,A3))\,A4)}$$

⇒ Matrix Chain Multiplication Algorithm

Algorithm Matrix-Chain-Order (P)

```
n = P.length - 1
let m[1...n, 1....n] and s[1...n-1, 2...n] be new tables
for i = 1 to n
      m[i, i] = 0
for len = 2 to n        // len is the chain length
      for i = 1 to n - len + 1
            j = i + len - 1.
            m[i, j] = ∞
            for k = i to j - 1.
                  q = m[i, k] + m[k+1, j] + p(i-1), pk pj .
                  if q < m[i, j]
                  m[i, j] = q.
                  s[i, j] = k.
return m and s;
```