In textbook, compactness theorem of proposition logic is proved based on König lemma. Here
prove inversely. It means that they are equivalent.

Exercises

(1) ~~0001~~. 2 10*0
1100 ~~1100~~. 0110. 0100

1 1 1 2 3 6.
01*01=0001. 10*11=0110.

1. Design a circuit for multiply with two two bits input and four bits output. For example,
have $1 * 1 = 1, 10 * 11 = 110$. 2个2位的输入.

2. Brown, Jones, and Smith are suspected of a crime. They testify as follows:

   (a) Brown: Jones is guilty and Smith is innocent.

   (b) Jones: If Brown is guilty then so is Smith.

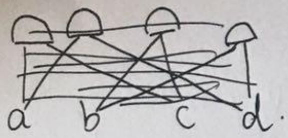
   (c) Smith: I'm innocent, but at least one of the others is guilty.

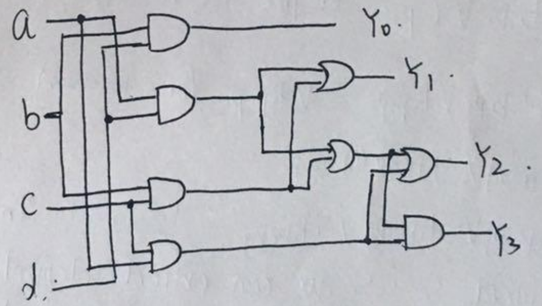
   Represent their testimonies with propositions and show who would be the criminal.

3. Every set S can be (totally) ordered. (*Hint*: Use proposition to represent partial order and dichotomy, then try to apply compactness theorem.)

4. Ex 7/p46.

1. ab. cd.





2. ~~B : B is the criminal.~~

JG : ~~Joh~~ Jones is guilty.

SG : Smith is guilty.

~~BG SG~~: If

$BG \to SG$: Brown is guilty, then so is Smith.

Brown : $JG \land \neg SG$.

Jones : $\cancel{BG \to SG}$ $(BG \land SG) \lor (\neg BG \land SG) \lor (\neg BG \land \neg SG)$

Smith : $\neg SG \land ((BG \land \neg JG) \lor (\neg BG \land JG) \lor (BG \land JG))$.

~~If B is guilty then S is guilty., So B is innocent~~

J is the criminal.

3 Given a set S. if its every finit subset is can be ordered, then the set itself can be ordered

~~$P_{i,j}$ 表示 a_i 比 a_j, $i,j \in S$~~. $a_i, a_j \in S$.

~~$\therefore P_{1,1} \vee P_{2,1}$~~

P_{a_i,a_j} 表示 $a_i R a_j$. $a_i, a_j \in S$.

~~$\therefore P_{a_1,a_1} \vee P_{a_1,a_2} \cdots \vee P_{a_1,a_i} \cdots \vee P_{a_i,a_i}$ 表示 S中至少有2个表示可比较~~.

$\begin{cases} P_{a_1,a_1} \vee P_{a_1,a_2} \cdots \vee P_{a_1,a_i} \\ P_{a_2,a_1} \vee P_{a_2,a_2} \cdots \vee P_{a_2,a_i} \\ P_{a_i,a_1} \vee P_{a_i,a_2} \cdots \vee P_{a_i,a_j} \end{cases}$ 表示S中的所有元素都至少与一个元素有关.

取出 S中的一个子集 S_0 对于任何 S_0 can be ordered. 上述的 positions 是可满足的., 所以对于 S来说也是可满足的, 所以 S can be ordered.

Let elements of the order be $\{p_n \mid n \in N\}$. Consider propositions $R p_i p_j$, $A p_i$, $B p_i$ and $C p_i$ for $i,j \in N$. Think of $R p_i p_j$ as saying that $p_i < p_j$. Think of $A p_i$ as saying that p_i is in chain A and similarily for $B p_i$ and $C p_i$; the order has width 3

Each of A, B, C is a chain ; every element is in A, B, or C

①. $A p_i \vee B p_i \vee C p_i$ ~~for~~ $1 \le i \le n$ ~~for all i c~~ for all $p_i \in S$.

②. $R p_1 p_i \vee R p_2 p_i \vee R p_3 p_i \cdots \vee R p_i p_i$ ~~for k = A~~. for all $p_i \in A/B/C$.

③. 最多有3对不可比较的元素

$\neg R p_i p_j \vee \neg R p_m p_n \vee \neg R p_x p_y$. $p_i, p_j, p_m, p_n, p_x, p_y \in S$.

for any subset $S_0 \subseteq S$, we can extract elements p_i from it and construct a set S_1 ~~which des~~. For every finite subset of width at most three can be divided into three chains. S_1 is satisfiable. So must be satisfiable

According to compactness theorem, S is satisfiable means an infinite partial order of width at most 3 can be divided into three chains