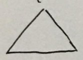
1. If α is tableau refutable, then α is *unsatisfiable.*
2. If α is *unsatisfiable*, then there is a tableau refutation of α.
3. Let Σ be finite set of propositions and $\wedge\Sigma$ the conjunction of its members. Prove that for any proposition α the following are equivalent:
    - (a) $\Sigma \models \alpha$.
    - (b) $\models \wedge\Sigma \rightarrow \alpha$.
    - (c) $\Sigma \vdash \alpha$
    - (d) $\vdash \wedge\Sigma \rightarrow \alpha$.
4. Suppose Σ is a finite set of propositions. Show that every CST from Σ is finite.
5. Prove $\{A \vee \neg B, B \vee \neg C, C \vee \neg D\} \models (D \rightarrow A)$.
6. Define CST with premises.
7. Prove that all axioms are valid in Hilbert proof system.
8. Is $((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)) \rightarrow (\alpha \rightarrow (\beta \rightarrow \gamma))$ valid? Otherwise, construct a truth valuation which make it false.

1. ∵ α 是 tableau refutable.

∴ $T\alpha$ is contradictory.



假设 α is satisfiable.

则存在 $V(\alpha)=T$.

∴ V agree with root $T\alpha$.

∴ there is a noncontradictory path.

∴矛盾

∴ If α is tableau refutable, then α is unsatisfiable.

2. 证 ∵ α 是 unsatisfiable

∴ $V(\alpha)=F$.

假设不存在 tableau refutation.

则 $T\alpha$ 是 noncontradictory 的

∴ V agree with every entry. on that path.

∴ $V(\alpha)=T$. 矛盾.

∴ If α is unsatisfiable, then there is a tableau refutation of α.

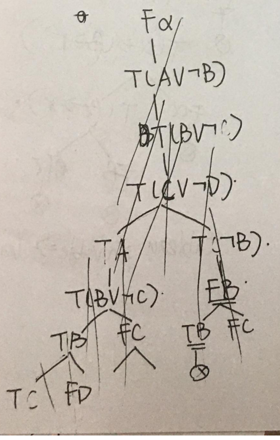
3. According to theorem 9. $a \Rightarrow c$
According to theorem b. $c \Rightarrow a$
$\therefore$ $a = c$.
同理 $b = d$.
只需证 $a = b$ 或 $c = d$.
$\because$ $\Sigma \models \alpha$. 即 α is a consequence of a set Σ of premises
$\therefore$ $V(\alpha) = T$.
$V(\wedge\Sigma) = T$.
$\therefore$ $V(\wedge\Sigma \rightarrow \alpha) = T$.
$\therefore$ $\models \wedge\Sigma \rightarrow \alpha$.
$\therefore a \Rightarrow b$.

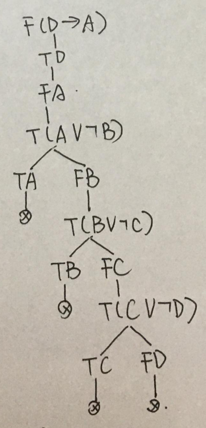
$\because$ $\models \wedge\Sigma \rightarrow \alpha$.
$\therefore$ $V(\wedge\Sigma \rightarrow \alpha) = T$.
if. $V(\wedge\Sigma) = T$ $V(\alpha) = T$.
$\because \wedge\Sigma = T$ $\therefore$ $V(\tau) = T$ $(\tau \in \Sigma)$
$\because$ $V(\tau) = T$ $\Rightarrow V(\alpha) = T$.
$\therefore$ $\Sigma \models \alpha$.
if $V(\wedge\Sigma) = F$ $V(\alpha) = T / F$.
$V(\tau) = F$ $\tau \in \Sigma$ $\therefore$ $\Sigma \models \alpha$.

$\therefore b \Rightarrow \alpha$
$\therefore a = b$.
$\therefore a = b = c = d$.

4. $\because$ CST from Σ is formed by putting $T\alpha$ at the end of noncontradictory path not containing it
while path containing $T\alpha$ extending. , an entry could be split into two branches.
the degree of every path decrease for each reduction.
So $d(P_{m+1}) < d(P_m)$ where P_{m+1} is an extension of P_m.
So every path is finite and the CST has finite branches.
So the CST is finite.

5.



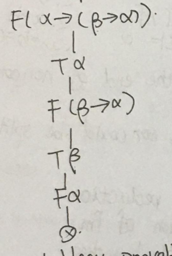


$\therefore \{A \vee \neg B, B \vee \neg C, C \vee \neg D\} \models (D \rightarrow A)$.

6. CST with premises: ~~let τ_0 be the unique atomic tableau with R at its root.~~

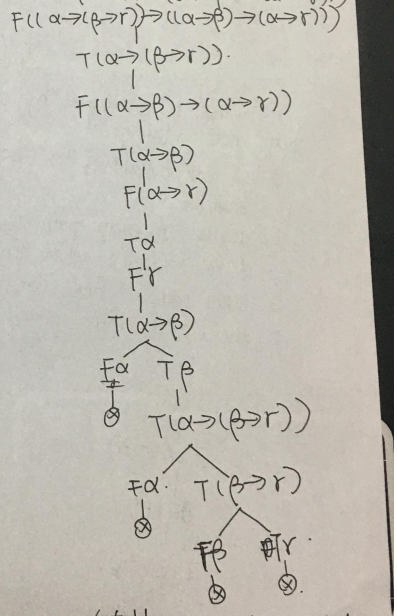
List the elements of $\overset{\partial}{\Sigma}$ as α_m, $m \in N$, and revise the definition of the CST by simply add one step to the definition of τ_{m+1}.
If our new construction has produced τ_m, we let τ'_{m+1} be the next tableau that would be defined by the standard CST procedure.
We now add on $T\alpha_m$ to the end of every non contradictory path in τ'_{m+1} that does not already contain $T\alpha$ to form our new τ_{m+1}.

7. ① $(\alpha \to (\beta \to \alpha))$.



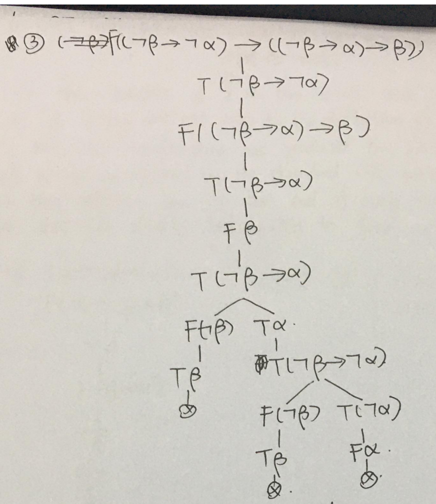
∴ is tableau provable.
∴ is valid.

② $((\alpha \to (\beta \to \gamma)) \to ((\alpha \to \beta) \to (\alpha \to \gamma)))$



∴ tableau provable ⇒ valid.

③



∴ tableau provable ⇒ valid.

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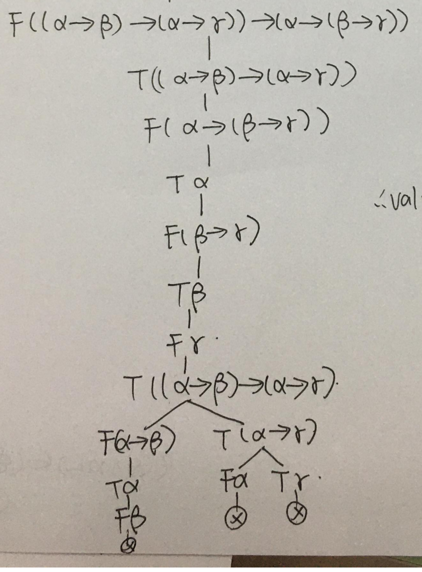

∴valid.