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Frame-Based Calculus of solving Arithmetic Multi-Step Addition and Subtraction word problems

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Abstract—**computer-assisted instruction systems have been broadly applied to help students solving math word problems. However, these systems offer little help to students encountering multi-step arithmetic word problems that do not be stored in the predesigned database. MSWPAS is a computer simulation system of people's arithmetic multi-step addition and subtraction word problems behavior, which composed of MSWPAS-NP and MSWPAS-CP. In MSMPAS-NP, the natural language of word problems are processed and reflected into frames, and in MSMPAS-CP, calculus is performed based on those frames. This paper focuses on MSMPAS-CP. Psychological theoretical foundation and knowledge representation of MSMPAS-CP are presented at first. A method of frame-based calculus for solving multi-step addition and subtraction word problems is proposed, and then the architecture of system and how it works are discussed. Finally, an experiment is carried out, which shows that the method of frame-based calculus can effectively solve multi-step word problems.**

Keywords—Arithmetic Word Problem; Problem Solving; Frame-based Calculus; Intelligent Tutoring System

I. INTRODUCTION

The ability of mathematical problem solving has been attention of education around the world. In the USA, the National Council for Teachers of Mathematics (NCTM) identifies in its Agenda for Action (NCTM, 1989) that problem solving as the central focus of the mathematics curriculum in elementary schools. And meanwhile, many other countries have concerned about training students' math ability of problem solving. The ability refers to applying mathematics to real-world problem situations. Solving arithmetic word problems is a main part of the mathematical problem solving.

But many students have trouble in solving arithmetic word problems because it requires both text-comprehension's ability and problem-solving ability [2, 3, 4]. Therefore, many computer-assisted instruction systems are developed to promote the students' ability of solving word problems. ARITHPRO is a computer simulation of children's arithmetic word-problem solving behavior, which is compared with children's behavior of solving word problems by deleting or changing linguistic and mathematical knowledge, then infer the errors of children [5]. MathCAL diagnoses stages at which errors occurred during solving word problems for targeted help [6]. LIM-G comprehends the natural language of one-step geometry word problems, and then represents the telegraphic and diagrammatic representations of the problem [7]. These systems focus on solving one-step word problems or help students to solve the pre-designed problems in the database, but have little help to solve multi-step arithmetic word problems students encountering.

MSWPAS is proposed to solve arithmetic multi-step addition and subtraction word problems. MSWPAS solves word problems with (1) comprehending the natural language of problems and constructing problem frames in MSWPAS-NP [8], (2) calculating the problem frames in MSWPAS-CP. This paper will focus on how MSWPAS-CP works. Other sections are arranged as follows: researches on cognitive process of solving word problems will be presented in section two. Section three will describe the category of the addition and subtraction word problems. The knowledge in MSWPAS-CP will be presented in section four. Section five will introduce the realization of frame-based calculus of word problems, and section six will be the experiments and conclusions.

II. COGNITIVE PROCESS OF SOLVING WORD PROBLEMS

A. multi-step word problems solving and problem solving

The process of solving one-step word problems can be regarded as a matching process with the schema. But multi-step word problems are much more complicated, and often can not be solved easily. Solving multi-step word problems requires planning, establishing sub-goals, generating new facts, solving every sub-goal, and finally the problem will be solved [9]. Therefore, solving multi-step word problems is process of problem solving. Problem-solving process contains stages as follows: problem representation, planning, trying to solve and assessment [10].

B. Problem-solving Process

1) Comprehension and Representation

The first step of solving problems is representation and comprehension, which refer to translating the problem text, especially the relation statements. Every word problem is composed of a series of propositions: assignment proposition, relation proposition, question proposition and some contextual information [11]. Schemas are constructed in the mental mind for representing the word problems after

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comprehension. The schemas include four slots: object, number, specification and role [12]. Those slots represent semantic relations of the problems. Solving word problems are based on these schemas.

2) Planing

During the process of problem solving, especially the complicated problem solving, different strategies would be chosen depended on the problems and solvers' knowledge. The general problem-solving strategy is Means-end Analysis: Given the present and desired objects, find a difference between them. Next find an operator relevant to the difference; determine wehter the operator can be applied to the present object. If so, apply it. If not, describe the objects which it would apply and transform the present object into an object of that kind—a new "desired object". Take the new object thus obtained as the present object and repeat the process.

3) Trying to Solve and Assessment

After planning, every goal of the problem will be implemented step by step. During the process of solving problem, domain knowledge is required. If incomplete or wrong knowledge exists or related knowledge doesn't exist, the result will be fault. Finally, the result is assessed. As to solving mathematical problems, checking the calculus of the number is performed.

III. CATEGORY OF ADDITION AND SUBTRACTION WORD PROBLEMS

Pedagogy and cognitive psychology have different categories on word problems. In this paper, addition and subtraction word problems are classified according to the cognitive classification. One-step addition and subtraction problems are classified into 14 types according to the semantic relations among quantities in the problems [13]. From the perspective of computer solving, one-step addition and subtraction word problems are classified into 8 types, following as table 1.

TABLE I. A LIST OF ONE-STEP ADDITION AND SUBTRACTION WORD PROBLEMS CATEGORY

Major category	Subcategory	
Change	Result unknow	transferIn
		transferOut
	Change unknow	
Combine	Superset unknow	
	Subset unknow	
Compare	Difference unknow	
	Compared quality unknow	Big compared unknow
		Small compared unknow

Multi-step addition-subtraction word problems can be regarded as assembled by some one-step addition-subtraction word problems. For example, "it takes one dolor for a pencil. It is one dolor cheaper than a knife. How much does it take for one pencil and one knife?" This problem is composed of one assignment proposition: "it takes one dolor for a pencil", and two relation propositions: "it is one dolor cheaper than a knife", "how much does it take for one pencil and one knife", which can be regarded as composed of a compare problem and a combine problem. To solve it, knowledge of comparison problem and combinatorial problem, and the strategy of general problem solving such as Means-end Analysis are required. Therefore, multi-step addition-subtraction word problems can be solved if the knowledge of one-step word problems and the strategy of Means-end Analysis are available.

IV. KNOWLEDGE AND KNOWLEDGE REPRESENTATION

In MSWPAS-CP, the word problems have been transformed into frames, so problem solving is based on those frames, and domain knowledge of arithmetic one-step addition and subtraction word problem is stored in knowledge base. The details of the representation are described in the following.

A. Word Problem Representation

In order to solve multi-step addition and subtraction word problems, we represent word problem with frames, which contains the whole semantic information of problems. The frame described in document [12] is only fitted to part of one step addition and subtraction word problems in Chinese textbooks, and not suitable to solve multi-step problems because so much information is held in specification slot, which makes difficult for automatic solving problems. So in this paper, a frame with five slots is constructed, which can hold necessary semantic relation of problems in Chinese textbooks, and facilitate automatically solving multi-step addition and subtraction word problems. These slots are object slot, quality slot, specification slot, time slot and role slot. Object slot holds the kind of objects that one sentence of problem contains, or a verb only if sentence expresses action of range, amplitude, frequency etc. Quality slot holds the number of object, or null if quality of object is desired. Specification slot holds the owner of object, subject of action, or location where object locates. Time slot holds the order of transfer action's occurrence or start set and result set of transfer action. Time slot in assignment frame of transfer action's start set equals the order of transfer action's occurrence, and time slot of end set is plus one. Role slot holds a relational term that identifies the type of frame. The value of it will be EVALUATION, MORE, LESS, SUPERSET, TRANSFERIN, TRANSFEROUT. For example, "Joe had two apples. Mom gave him some apples, and then Dad gave him three apples. Now Joe had seven apples. How many apples did Mom give him?" The frames of this problem follow as Fig. 1.

Name: frame1 Object: apple Quantity: 2 Specification: Joe Time; 0 Role: evaluation	Name: frame2 Object: apple Quantity: 3 Specification: Joe&&Dad Time; 1 Role:transferIn	Name: frame3 Object: apple Quantity: 7 Specification: Joe Time; 2 Role: evaluation	Name: goal1 Object: apple Quantity: Specification: Joe&&Mom Time; 0 Role: transferIn

Known conditions Goal

Figure 1. Problem Frames

B. *Domain Knowledge Representation*

The domain knowledge of multi-step addition and subtraction word problems is composed of knowledge of one-step addition and subtraction word problems, which is represented with production rules. For example, one rule's formal description is depicted as Fig. 2. In MSWPAS-CP, there are eight rules in knowledge base. Related frames of goal and how to find them will be described as follows.

IF the goal's role slot is SUPERSET
THEN add quantity slot value of related assignment frames

IF the goal's role slot is ASSIGNMENT and related relation frame's role slot is MORE
THEN add quantity slot value of related relation frame and assignment frame

Figure 2. Example of production rules

V. SYSTEM ARCHITECTURE

MSWPAS-CP is composed of six components: goal stack, fact base, frame identification, working memory, rule base and executive controlling. The diagram of MSWPAS-CP architecture is depicted as Fig. 3.

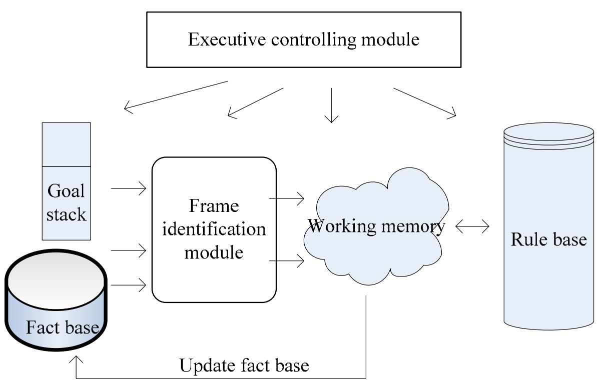

Figure 3. Example of production rules

A. *Function of Each Module*

1) *Fact base*
Fact base stores frames of problem, including conditional frames and new generating frames. After a rule is triggered, fact base will be updated, the related relation frame will be deleted from fact base, and goal frame which has been completed will be added to it.

2) *Goal stack*
Goal stack stores the goal of problem and sub-goals generated during solving procedure. Data structure is implemented with stack, which means last in, first out.

3) *Frame identification module*
This module is responsible to identify and select frames from fact base, which are related to the current goal, and copy them to working memory. One word problem is composed by some assignment propositions and relation propositions. Generally, some of them can be calculated together, but some can not. For example, "Joe had five apples. Mom gave Mike two apples". These two propositions can not be calculated because they are not interrelated. During solving word problems, the related propositions must be identified. The related frames are identified by slot values. Object slot, specification slot, time slot and role slot decide whether these frames are interrelated. When the related frames is not found, it is a gap between known conditions and desired, these frames that are not found become sub-goals.

The problem mentioned above is used to illustrate how to find the related propositions and how to generate sub-goals. The goal of problem is "How many apples did Mom give him". It is a TRANSFERIN frame, specification slot contains "Mom" and "Joe", time slot is 0. Therefore, the related frames of it must be assignment frames, and object slot is "apple", specification slot is "Joe", time slot equals the corresponding slot, that is 0 (start set) or equals value plus one, that is 1 (result set). If assignment frame, which object slot is apple, specification slot is Joe and time slot is 1, is not found in the current fact base, it will become the new current sub-goal. Now, the current goal is assignment frame, and the related frames are relation frame and assignment frames related to the relation frame. The related relation frame must coincide with the following conditions: object slot is "apple", specification slot contains "Joe", time slot is 1, and role slot does not equal "Evaluation". If the related relation frame is not found, this problem is no solution. Otherwise, the next is to find the related assignment frame related to it.

4) *Working memory*
Working memory simulates people's short term memory, which stores the current goal and assignment frames and relation frame related to it. Working memory will be reset whenever the goal is changed.

5) *Rule base*
Rule base is equivalent with people's long term memory, which stores the knowledge of solving one-step addition and subtraction word problems. It can be expanded when system want to solve more problems.

6) *Excecutive controlling module*
This module controls the whole modules and makes them coordinate with each other. It controls when to get goal, which goal is going to be solved currently, when to trigger rule base and update fact base.

B. *Implementation of MSWPAS-CP*

MSWPAS-CP implementation process is depicted as Fig. 4.

1) Push the goal into goal stack
2) Reset working memory
3) If goal stack is empty, program ends, otherwise gets the goal from goal stack

4) If there is a fact matching the current goal, turn to 2; otherwise If the goal is relation frame, turn to 6

5) Looking for relation frames related to assignment frame in fact base. If it is not found, no solution for this problem., otherwise copy it to working memory.

6) Looking for assignment frame related to relation frame and copy to woking memory

7) If the relation entities' number equals assignment frames' number plus one, turn to 9, otherwise turn to 10

8) If the relation entities' number equals assignment frames' number, turn to 9, otherwise turn to 10

9) To match rule's condition in rule base, add new generatin fact to fact base, delete related relation frame and turn to 2.

10) Push unfound assignment frame to stack goal, turn to 2

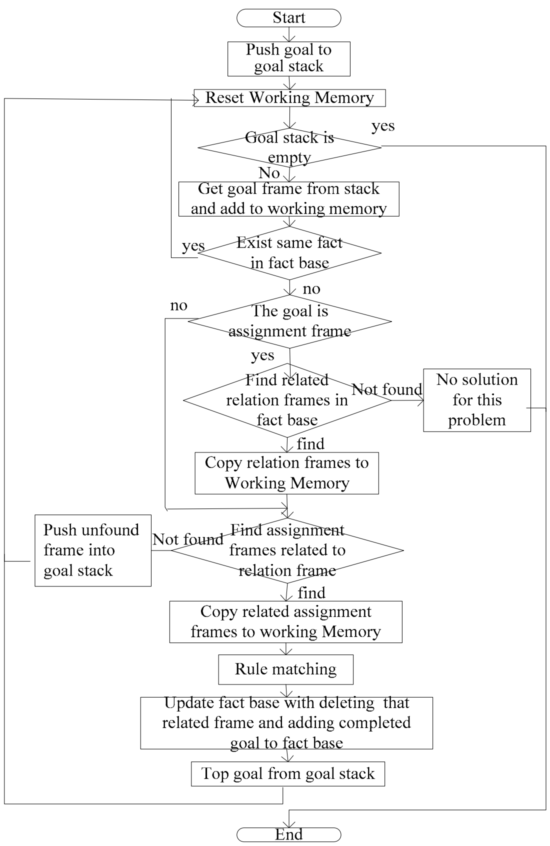

Figure 4. System implementation process

VI. EXPERIMENT AND DISCUSSION

MSWPAS-CP demo has been implemented with java program, and an experiment is carried out to assess whether MSWPAS-CP correctly solves multi-step addition and subtraction word problems in Chinese's elementary schools based on problem frames. The word problems are gathered from four publishers in Chinese (one book from People's Education Press, one book from Beijing Normal University Press, and two books from DONGBEI Normal University Press). These word problems which have been transformed to problem frames are correctly solved based on problem frames. That show the method of frame-base calculus can solve multi-step addition and subtraction word problems.

VII. CONCLUSON AND FUTURE WORK

In this study, MSWPAS-CP is constructed, which simulates the process of human's solving word problems, applied Means-end Analysis to generate new sub-goal, new facts and solve multi-step addition and subtraction word problem step by step. Word problems are represented with frames and domain knowledge is represented with rules. These new sub-goals and facts generated by system are important to counsel. The difference of generation order between system and students will be regarded as basis for diagnosis. At present, MSWPAS-CP can only solve addition and subtraction word problems. Extending knowledge base and semantic frame to solve multiplication and division word problems will be our future work.

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