

# Functional Encryption: Origins and Recent Developments

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**Abstract.** In this talk, we will present the notion of functional encryption and recent progress in the area. We will begin by describing the concept and origins of functional encryption. Next, we will describe intuitively why current bilinear map based constructions appear to be “stuck” with boolean formula type functionality even in the public index setting. Finally, we will see some very recent work that uses multilinear forms to move beyond these barriers and achieve functionality for any circuit.

## Overview

Encryption is a method to encode data such that it can only be understood by a recipient that holds a certain private key object. The traditional notion of public key encryption [10,11,21,13] is that a data owner will encrypt data to the public key of a specific targeted user to create a ciphertext. Later, a user possessing the corresponding private key can decrypt the ciphertext to obtain the original data. Ingrained in this notion is that: (1) *Encryption is a method to target to a specific user.* (2) *Decryption is an all or nothing operation; either a ciphertext is fully decrypted and the original data is recovered or else it fails and nothing is learned.*

Functional encryption is a new vision of encryption that moves past these barriers. In a Functional Encryption system what a user learns from decryption is determined by a function of the encrypted data and the user’s secret key descriptor (as issued by some authority). Briefly, in a functional encryption system with functionality  $F(\cdot, \cdot)$  a user is issued a secret key  $sk_k$  for value  $k$  by some authority. Suppose that a ciphertext  $ct$  is the encryption of data  $x$ . The user can apply their secret key to learn  $F(k, x)$ .

Functional encryption for expressive functionalities open up a wide variety of applications. For instance, one might determine access to encrypted data based on an arbitrary policy over a user’s credentials. Another possibility is that encrypted data could consist of images and a user’s private key of their headshot.

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The user would be able to view the image only if their face appeared in it as determined by some vision recognition algorithm. Moreover, the functionality could blur out parts of the image not immediately surrounding the user's body. In a medical research environment, one could consider encrypting a large database containing medical histories of patients coupled with DNA sequencing. Later, if a researcher is granted permission to test a correlation between a certain type of cancer and genotype they could be given a secret key that divulges the correlation and nothing else.

## Origins of Functional Encryption

The origins of functional encryption can be traced to the concept of Attribute-Based Encryption (ABE) [23] proposed by Sahai and Waters. In a (Key-Policy) ABE scheme a ciphertext contains a hidden message as well as (unhidden) metadata or attributes. A user's private key is associated with a formula  $\phi$ . A user can decrypt a given ciphertext and recover the hidden message if and only if the formula is satisfied when its values are assigned according to the metadata. A technical lynchpin was the concept that any secure system must be *collusion resistant*. Suppose an attacker obtains multiple secret keys, e.g.,  $sk_k, sk_{k'}$ . In particular, the attacker should not be able to combine two private keys to decrypt ciphertexts that neither private key was authorized for.

While Attribute-Based Encryption moves beyond the notion of encrypting to a particular user, decryption is still an all or nothing proposition. In subsequent works [6,16] the concept evolved to hide the metadata. The notion of Functional Encryption first appeared in presentation slides prepared by Sahai and Waters in 2008 [24] and was described during talks given by both authors. Significant conceptual work was done while both Sahai and Waters were researchers at IPAM for the 2006 Securing Cyberspace program. The term functional encryption first appeared in a published research paper by Lewko et. al. [17]. Finally, a definitional framework for functional encryption was put forward by Boneh, Sahai, and Waters [5] where they put forward both simulation and indistinguishability definitions.<sup>1</sup> The above work was influenced by concepts such as Identity-Based Encryption (IBE) [26,4,9] and Anonymous IBE [3,1].

## Achieving Stronger Functionality

Over the past several years there has been significant research activity on a variety of directions in functional encryption including proofs of adaptive security [17,19], revocation of secret keys [2,22], policies across multiple authorities [7,8,18], and investigation of definitions [5,20]. *Arguably, the most important question is what functionality can we achieve.* For several years the strongest form of expression we had was boolean formulas<sup>2</sup> in ABE cryptosystems. While

<sup>1</sup> Concurrently, with [5] and subsequent to discussions stemming from [24], O'Neill [20] also put forward general definitions for functional encryption.

<sup>2</sup> Technically, one can obtain span programs.

boolean formula ABE systems give rise to several interesting applications, they are still a far cry from being able to express access control in the form of any program or circuit.

In this talk we will first explore the techniques that give rise to ABE systems for boolean formulas. Our starting point will be the “Key-Policy” ABE system of Goyal et. al. [15]. We will see how they use bilinear maps as the primary mechanism for decryption blended with interpolation in the exponent techniques. Together these give the boolean formula functionality and the needed protection against collusion attacks. We also give insight into the difficulty of obtaining stronger functionality using bilinear maps by arguing why such constructions are “stuck” at the level of boolean formulas. Intuitively, the bilinear map mechanism is “used up” in pairing the ciphertext with the secret key to prevent collusions between different users. However, this leaves natural larger fanout generalizations of GPSW to so called backtracking attacks.

We will next describe some very recent progress [25] that obtains Attribute-Based Encryption for circuits. Obtaining ABE for circuits is a major jump in that circuits can express any program of fixed running time. The new result is obtained by applying the recent work of Garg, Gentry, and Halevi [12] which describes some approximation of groups with multilinear maps. The new ABE crypto leverages these multilinear forms to create a new “move forward and shift” technique for decryption that replaces and subsumes the prior methods. Independently, Gorbunov, Vaikuntanathan and Wee [14] obtained the same result under the Learning with Error (LWE) assumption. They create a set of novel and elegant techniques to combat the backtracking issue. We refer the reader to the introduction of [25] for further discussion of backtracking attacks and how these are circumvented by new techniques.

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