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The stable non-Gaussian asset allocation: a comparison with the classical Gaussian approach

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Abstract

This paper analyzes the asset allocation problem of an investor who can invest in equity and cash when there is time variation in expected returns on the equity. The solution methodology is multistage stochastic asset allocation problem with decision rules. The uncertainty is modeled using economic scenarios with Gaussian and stable Paretian non-Gaussian innovations. The optimal allocations under these alternative hypotheses are compared. Our calculations suggest that the asset allocations may be up to 26% different depending on the objective function and risk aversion level of the investor. The certainty equivalent return can be improved up to 0.7% by switching to the stable Paretian model. An investor can earn up to eight times as much return on the unit of risk he bears by applying the stable model. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Strategic investment planning is the allocation of portfolio across broad asset classes such as bonds, stocks, cash and real estate considering the legal and policy constraints facing an institution or individual. Empirical evidence by Culp et al. (1997) suggests that asset allocation is the most important factor in determining investment performance.

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Most of the early models in this field are either myopic or represent deterministic formulations of multiperiod problems. An investor that has iso-elastic utility function chooses the same investment proportions independent of the investment horizon [1] if the market is frictionless and the returns are independent over time. However, myopic models cannot capture long-term investment goals in the presence of transaction costs. There is considerable evidence of predictability in asset returns [2] and the myopic models do not take this empirical finding into account. These models tend to produce high portfolio turnovers and opportunistic asset trades.

Recent papers analyze the effects of asset return predictability on asset allocation decision of long-term investors. These papers investigate how the investor's horizon or the uncertainty of the estimated parameters affect the allocation decision. [3] There has been a growing interest in the development of multiperiod stochastic models for asset and liability management (ALM). Kusy and Ziemba (1986) developed a multiperiod stochastic linear programming model for Vancouver City Savings Credit Union. Another successful application of multistage stochastic programming is the Russell–Yasuda Kasai model by Carino et al. (1994). The investment strategy suggested by the model resulted in extra income of $79 million during the first two years of its application (1991 and 1992). Boender (1997) reports the success of a hybrid simulation/optimization scenario model for ALM of pension funds in the Netherlands.

There are various approaches to modeling the predictability of asset returns. Wilkie (1986, 1995) suggests using ARMA model for each variable of interest in a cascade structure rather than a multivariate model. Mulvey (1996) describes an economic projection model that uses stochastic differential equations in a similar cascade framework. Hodrick (1992) compares the statistical properties of using univariate ordinary least-squares approach with vector autoregression (VAR) and suggests that VAR is a better method. Boender et al. (1998) extend VAR model to a vector error correction model (VECM) which additionally takes economic regime changes and long term equilibria into account. They report that the VECM improves the explanatory power of the specification.

Most of these models assume that the variables or the innovations of these variables follow normal distribution or the continuous time counterpart, Brownian motion. In response to the empirical evidence about the heavy tail, high peak and possible skewness in financial data, Fama (1965) and Mandelbrot (1963, 1967) propose stable Paretian distribution [4] as an alternative model. Among the alternative non-Gaussian distributions in the literature, stable distribution has unique characteristics that make it an ideal candidate. The stable laws are the only possible limit distributions for properly normalized and centered sums of independent identically distributed random variables (Embrechts

[1] Merton (1969) and Samuelson (1969) show that a constant relative risk aversion investor chooses the same investment proportions independent of the investment horizon if the market is frictionless and the returns are independent over time.

[2] See for example Hodrick (1992), Bekaert and Hodrick (1992), Kandel and Staumbaugh (1996), and Brandt (1999).

[3] See for example Brennan et al. (1997), Kandel and Staumbaugh (1996), Brandt (1999), Barberis (2000), and Ait-Sahalia et al. (2001).

[4] We will call it stable distribution from now on.

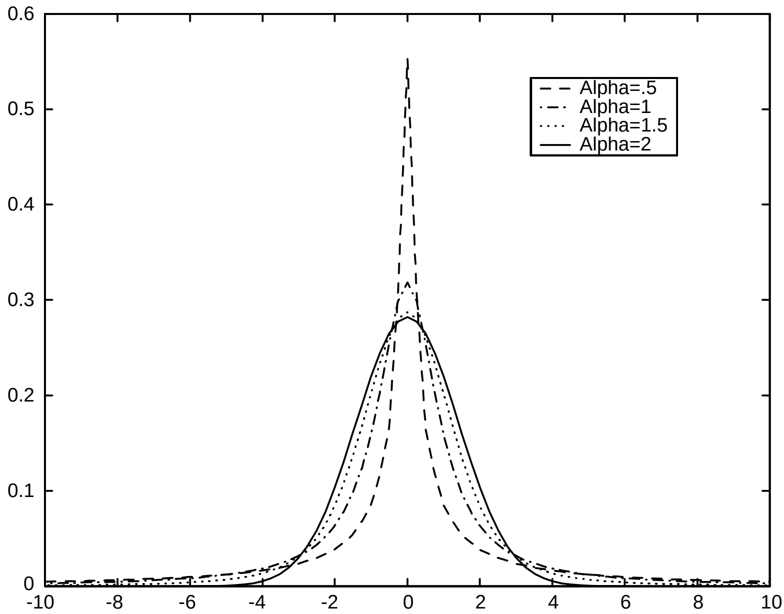

Fig. 1. Probability density functions for standard symmetric stable random variables: varying alpha.

et al., 1997; Rachev and Mittnik, 2000). If a financial variable can be regarded as the result of many microscopic effects, then it can be described by a stable law. Stable distributions are leptokurtotic. When compared to normal distribution, they typically have fatter tails and higher peak around the center. Fig. 1 depicts the probability density function for various indices of stability: $\alpha = 2$, i.e. Gaussian distribution, $\alpha = 1.5, 1$, and 1.5. The smaller the index of stability, the fatter the tails are. Fig. 2 shows the flexibility of stable distribution in modeling various levels of skewness. Due to these flexibilities, stable model fits the empirical distribution of the financial data better (see Mittnik et al., 2000). Gaussian distribution is a special case of stable distribution. In fact, it is the only distribution in the stable family with a finite second moment. Although autoregressive conditional heteroskedastic models driven by normally distributed innovations imply unconditional distributions that possess heavier tails, there is still considerable kurtosis unexplained by this model. Mittnik et al. (2000) present empirical evidence favoring stable hypothesis over the normal assumption as a model for unconditional, homoskedastic conditional, and heteroskedastic conditional distributions of several asset return series.

In this paper we analyze the multistage asset allocation problem of an investor under the Gaussian and stable returns scenarios. We use stochastic programming with decision rules to solve the allocation problem. Our model captures uncertainty by a branching event tree. Each node of the tree represents a joint outcome of all the random variables at that decision stage. Each path through the event tree represents a 'scenario'. The major advantage of stochastic programming is that it permits a very rich description of

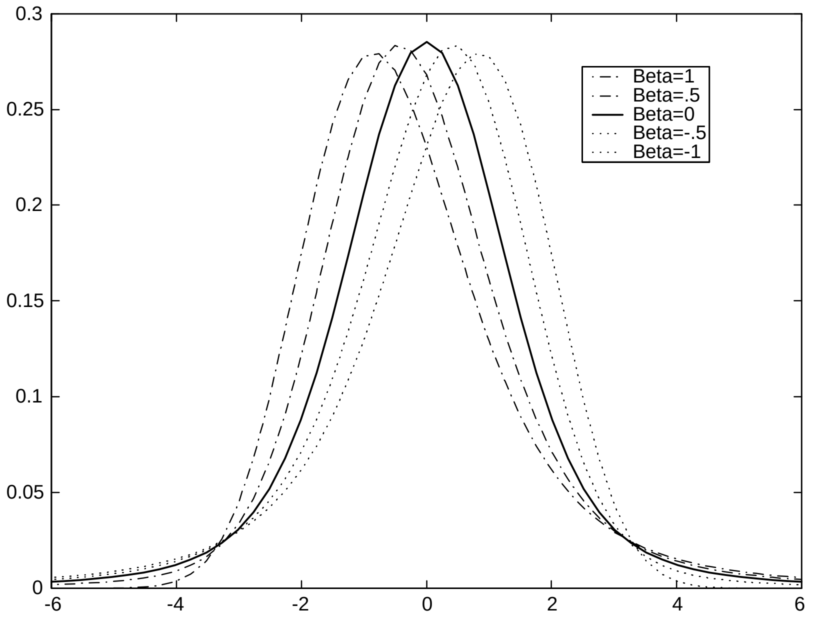

Fig. 2. Probability density functions for standard skewed stable random variables: varying beta for $\alpha = 1.5$.

the state of world at each node by easily accommodating a large number of random variables. This characteristic leads to several important practical applications of the model such as Carino et al. (1994), Zenios (1993a, b), and Mulvey (1996). Stochastic programming with decision rules simplifies the computation of a solution by imposing the use of an allocation rule such as fix-mix at each decision stage. Some commercial applications of this approach include Boender et al. (1998) and Berger and Mulvey (1998).

The investor is assumed to invest in two assets—cash and an equity portfolio. While we do not model any liabilities, it is relatively straightforward to generalize the model to include them. [5] The variables that predict the return on the equity portfolio are dividend yield on the equity portfolio, Treasury bill rate, Treasury bond yield, and inflation. The investor updates his portfolio at every decision stage using the fix-mix allocation rule. Although fix-mix rule is the optimal strategy under certain conditions, [6] it is still widely used by financial practitioners. Fixed-mix strategy maintains a fixed exposure to equity market by requiring the purchase of stocks as they fall in value, and the sale of stocks as they rise in value (Perold and Sharpe, 1988).

[5] Boender et al. (1998) analyzes a similar model that takes liabilities into consideration in the Gaussian context.

[6] A constant relative risk aversion investor chooses the same investment proportions independent of the investment horizon if the market is frictionless and the returns are independent over time.

We consider several alternative objective functions for the investor. In the first three objective functions, the investor is presumed to be trading off between the mean return and some measure of risk of the portfolio. The first objective function uses value-at-risk (VaR) as the risk measure. VaR is a popular way of evaluating the exposure to market risk (see Duffie and Pan, 1997 for a review). It is the highest possible loss over a certain period of time at a given confidence level. Recently, there have been some criticisms[7] of VaR regarding its use of probability of loss as opposed to the magnitude of loss and its lack of sub-additivity (see Artzner et al., 1998; Basak and Shapiro, 2001). Artzner et al. (1998) suggest using conditional value-at-risk (CVaR), which is the expected loss on a portfolio given the return is less than VaR. CVaR is gaining popularity among researchers as well as practitioners due to its desirable properties.[8] The second risk measure considered in the analysis is CVaR. The third risk measure we use is an analog of variance which assigns less importance to extreme observations: a power of mean absolute deviation. When the power is equal to two, this measure is equal to variance. The final objective function we analyze is the classical power utility function.

Our computational results suggest that the significance of the asset allocation and certainty equivalent return differences between Gaussian and stable returns models depend on the objective function of the investor. We find that if the investor has very high or low risk aversion, then the normal and stable scenarios result in similar asset allocations for all the objective functions analyzed. However, when the risk aversion level is between the two cases, the two distributional assumptions may result in considerably different asset allocations depending on the objective function and the risk aversion level of the decision maker. The investor may reduce his equity allocation up to 26%. The differences in the allocations are more pronounced for investors that use VaR or CVaR to evaluate market risk. Since these measures pay more attention to the tail of the distribution, preserving the heavy tails with the use of stable model makes an important difference for the investor. For instance, an investor who trades-off between the mean return and CVaR of the portfolio will reduce his equity allocation from 62% to 36% if he switches to the stable model and this will improve his objective function by 0.7%. This implies that the investor can increase his certainty equivalent return by 0.7%. This investor can earn twice as much return on the unit of risk he bears by applying the stable model. Since stable economic scenarios extreme events more realistically, they suggest more conservative asset allocations. Ortobelli et al. (1999) report similar observations in their single period asset allocation model.

Section 2 reviews the literature on multistage stochastic ALM programming with decision rules, scenario generation, and stable distribution. Our model is set up in Section 3 with the discussion of the scenario generation and asset allocation modules. The computational results are reported in Section 4. Section 5 concludes.

[7] See Section 3.1 for further criticism of VaR.

[8] See for example Bucay and Rosen (1999), Embrechts et al. (1999), and Rockafellar and Ursayev (2000).

2. Literature review

2.1. Multistage stochastic ALM programming with decision rules

The asset allocation problem is discretized into n-stages across the planning horizon, and investments are made using a decision rule, e.g. fixed mix, at the beginning of each time period. The use of this approach hinges on discovering policies that are intuitive and that will produce superior results. Decision rules may lead to non-convexities and highly nonlinear functions. Some decision rules used in the literature are fixed mix, buy-and-hold, life cycle mix (Berger and Mulvey, 1998), constant proportional portfolio insurance (Perold and Sharpe, 1988), and target wealth path tracking (Mulvey and Ziemba, 1998).

Boender (1997) and Boender et al. (1998) describe an ALM model designed for Dutch pension funds. Their goal is to find efficient frontiers of initial asset allocations which minimize the value of downside risk for certain given values of average contribution rates. The scenarios are generated across the time horizon of interest. The management selects a funding policy, an indexation policy of the earned pension rights, and an investment decision rule. These strategies are simulated against generated scenarios. Then, the objective function of the optimization problem is a completely specified simulation model except for the initial asset mix. The hybrid simulation/optimization model requires the following three steps:

(1) Randomly generate initial asset mixes, simulate them, and evaluate their contribution rates and downside risks.
(2) Select the best performing initial asset mixes that are located at a minimal critical distance from each other.
(3) Use a local search algorithm to identify the optimal initial asset mix.

Maranas et al. (1997) adopt another approach to stochastic programming with decision rules. They determine the optimal parameters of the decision rule by means of a global optimization algorithm. They propose a dynamically balanced investment policy which is specified by the following parameters:

w_0: initial dollar wealth,

r_{it}^s: percentage return of asset $i \in \{1,2,\ldots,I\}$ in time period $t \in \{1,2,\ldots,T\}$ under scenario $s \in \{1,2,\ldots,S\}$,

p^s: probability of occurrence of scenario s

The decision variables are:

w_t^s: dollar wealth at time t in scenario s,

λ_i: fraction of wealth invested in asset category i (note that it is constant over time).

The model is a multiperiod extension of mean-variance method. The multi-period efficient frontier is obtained by varying $\beta (0 \leqslant \beta \leqslant 1)$. The formulation is as follows:

$$\max_{\lambda_i, w_t^s} \beta \, \mathrm{mean}(w_T) - (1-\beta)\, \mathrm{var}(w_T)$$

subject to

$$w_T^s = w_0 \prod_{t=1}^{T} \left[\sum_{i=1}^{I} (1 + r_{it}^s) \lambda_i \right], \quad s = 1, \ldots, S \tag{1}$$

$$\sum_{i=1}^{I} \lambda_i = 1, \tag{2}$$

$$0 \leqslant \lambda_i \leqslant 1, \quad i = 1, \ldots, I.$$

The wealth accumulation is governed by (1). When (1) is substituted into the objective function, we get a nonconvex multivariable polynomial function in λ_i involving multiple local minima. A global optimization tool which obtains the above efficient frontier has been developed.

There are other models constructed using similar methodologies. Berger and Mulvey (1998) describe Home Account Advisor^{TM} which assists individual investors in ALM using decision rules. Sweeney et al. (1998) applies a simulation/optimization scenario approach to optimal insurance asset allocation in a multi-currency environment.

2.2. Scenario generation

A scenario gives a single set of outcomes for the random variables in the model over the planning horizon. A representative set of scenarios describes the possible future economic environment. Traditional quantitative forecasting methods extrapolate new ideas about future developments based on the knowledge of the past and present. However, the economic environment may change invalidating the past assumptions. Hence, subjective beliefs of the management has become an essential part of scenario building. See Bunn and Salo (1993) for a review of qualitative scenario generation techniques.

The earlier ALM models used few number of independent scenarios to describe uncertainty. Recent models have become more sophisticated in scenario generation methods. There are two distinct lines of literature on scenario generation. One line of research has been developed in the operations research community. In an early model, Wilkie (1986, 1995) suggests using a cascade structure rather than a multivariate model, in which each variable could affect each of the others. Mulvey (1996) describes an economic projection designed model for Towers Perrin using stochastic differential equations in a similar cascade framework. First the Treasury yield curve, and then government bond returns, price and wage inflation, and large cap returns are generated. Lastly, returns on primary asset categories such as small cap stock and corporate bonds are projected. An alternative approach is suggested by Dert (1998). Dert uses vector autoregression (VAR) methodology to generate scenarios for a pension plan. He creates future price inflation, wage inflation, stock returns, bond returns, cash return and real estate returns that are consistent with historical patterns in means, standard deviations, autocorrelations and cross correlations between state variables. A Markov model is used in determining future development of each individual participating in the pension plan. Carino et al. (1998) also employ VAR in generating scenarios for the Yasuda Kasai model. VAR may sometimes diverge from long-term equilibrium. Boender et al. (1998)

extend VAR model to a vector error correction model (VECM) which additionally takes economic regime changes and long-term equilibria into account. They report that the VECM improves the explanatory power of the model. Their VECM incorporates regime changes as well as long-run equilibrium.

The literature on asset return predictability in finance has evolved separately from the operations research literature. The goal is usually to predict the return on a stock or bond index using VAR methodology. There are a number of variables used to predict stock returns in various studies. Brennan et al. (1997) use Treasury bill rate, Treasury bond rate and dividend yield as state variables in their model. Brandt (1999) uses lagged excess return on NYSE index over Treasury bill rate as a state variable in addition to dividend yield, default spread and term spread. In most of the models, once the parameters of the VAR are estimated, they are assumed as certain. The effects of parameter uncertainty on optimal portfolio choice have been addressed by Kandel and Staumbaugh (1996) and Barberis (2000), among others.

Although this literature does not utilize inflation as a possible predictor, there is a large body of macro-finance research on the relationship between stock returns and inflation. Fisher hypothesis states that expected rates of return consist of real return plus the expected rate of inflation, and the real return does not move systematically with the rate of inflation. However, the empirical analysis of the post-war return on common stocks and the rate of inflation suggests that there is a negative relation between returns and both anticipated rates of inflation and unanticipated changes in the rate of inflation. Nelson (1976) reports that the regression coefficient on both the contemporaneous inflation and one month lagged inflation are negative and statistically significant at 5% level for the period of 1953–1971 (see Barnes et al., 1999 for more recent supportive results). This evidence suggests that inflation can also be utilized as a predictor of stock market performance. [9]

2.3. Stable distribution

There are several important reasons for modeling financial variables using stable distributions. The stable law is supported by a generalized central limit theorem (Embrechts et al., 1997; Rachev and Mittnik, 2000). Stable distributions are leptokurtotic. Since they can accommodate the fat tails and asymmetry, they fit empirical distribution of the financial data better.

Any distribution in the domain of attraction of a specified stable distribution will have properties which are close to the ones of stable distribution. Even if the observed data does not exactly follow the ideal distribution specified by the modeler, in principle, the resulting decision is not affected.

Each stable distribution has an index of stability which remains the same regardless of the sampling interval adopted. The index of stability can be regarded as an overall parameter that can be employed in inference and decision making. However, we should note that for some financial data empirical analysis shows that the index of stability increases as the sampling interval increases.

[9] This is already done in the operations research based scenario generation literature.

It is possible to check whether a distribution is in the domain of attraction of a stable distribution or not by examining the tails of the distribution. The tails dictate the properties of the distribution.

This section describes the properties of stable distribution and addresses the estimation issues.

2.3.1. Description of stable distribution

If the sums of linear functions of independent identically distributed (i.i.d.) random variables belong to the same family of distributions, the family is called stable. Formally, a random variable r has stable distribution if for any $a>0$ and $b>0$ there exists constants $c>0$ and $d\in R$ such that

$$ar_1+br_2\overset{\mathrm{d}}{=}cr+d, \tag{3}$$

where r_1 and r_2 are independent copies of r, and $\overset{\mathrm{d}}{=}$ denotes equality in distribution. The distribution is described by the following parameters: $\alpha\in(0,2]$ (index of stability), $\beta\in[-1,1]$ (skewness parameter), $\mu\in R$ (location parameter), and $\sigma\in[0,\infty)$ (scale parameter). The variable is then represented as $r\sim S_{\alpha,\beta}(\mu,\sigma)$. Gaussian distribution is actually a special case of stable distribution when $\alpha=2$, $\beta=0$. The smaller the stability index is, the stronger the leptokurtic nature of the distribution becomes, i.e. with higher peak and fatter tails. If the skewness parameter is equal to zero, as in the case of Gaussian distribution, the distribution is symmetric. When $\beta>0$ ($\beta<0$), the distribution is skewed to the right (left). If $\beta=0$ and $\mu=0$, then the stable random variable is called symmetric α-stable ($S\alpha S$). The scale parameter generalizes the definition of standard deviation. The stable analog of variance is variation, v_α, which is given by σ^α.

Stable distributions generally do not have closed form expressions for density and distribution functions. They are more conveniently described by characteristic functions. The characteristic function of random variable r, $\Phi_r(\theta)=\mathrm{E}[\exp(\mathrm{i}r\theta)]$, is given by

$$\begin{aligned}\Phi_r(\theta)&=\exp\left\{-\sigma^\alpha|\theta|^\alpha\left(1-\mathrm{i}\beta\,\mathrm{sign}(\theta)\tan\frac{\pi\alpha}{2}\right)+\mathrm{i}\mu\theta\right\}\quad \text{if } \alpha\neq 1\\ &=\exp\left\{-\sigma|\theta|\left(1-\mathrm{i}\beta\frac{2}{\pi}\,\mathrm{sign}(\theta)\ln\theta\right)+\mathrm{i}\mu\theta\right\}\quad \text{if } \alpha=1.\end{aligned} \tag{4}$$

The pth absolute moment of r, $\mathrm{E}|X|^p=\int_0^\infty P(|X|^p>y)\,\mathrm{d}y$, is finite if $0<p<\alpha$, and infinite otherwise. Hence, when $\alpha<1$ the first moment is infinite, and when $\alpha<2$ the second moment is infinite. The only stable distribution that has finite first and second moments is the Gaussian distribution.

In models that use financial data, it is generally assumed that $\alpha\in(1,2]$. There are several reasons for this:

(1) When $\alpha>1$, the first moment of the distribution is finite. It is convenient to be able to speak of expected returns.
(2) Empirical studies support this parameterization.

(3) Although the empirical distributions of the financial data sometimes depart from normality, the deviation is not ‘too much’.

In scenario generation, one may need to use multivariate stable distributions. The extension to the multivariate case is nontrivial. Although most of the literature concentrates on the univariate case, recently some new results have become available. See for example Samorodnitsky and Taqqu (1994), and Rachev and Mittnik (2000).

If R is a stable d-dimensional stable vector, then any linear combination of the components of R is also a stable random variable. However, the converse is true under certain conditions (Samorodnitsky and Taqqu, 1994). The characteristic function of R is given by

$$\begin{aligned}\Phi_Y(\theta) &= \exp\left\{-\int_{S_d} |\theta^T s| \left(1 - \mathrm{i}\,\mathrm{sign}(\theta^T s)\tan\frac{\pi\alpha}{2}\right)\Gamma(\mathrm{d}s) + \mathrm{i}\theta^T\mu\right\} \quad \text{if } \alpha \neq 1\\ &= \exp\left\{-\int_{S_d} |\theta^T s| \left(1 + \mathrm{i}\frac{2}{\pi}\,\mathrm{sign}(\theta^T s)\ln|\theta^T s|\right)\Gamma(\mathrm{d}s) + \mathrm{i}\theta^T\mu\right\}\\ &\quad \text{if } \alpha = 1, \end{aligned} \tag{5}$$

where Γ is the spectral measure which replaces the scale and skewness parameters that enter the description of the univariate stable distribution. It is a bounded nonnegative measure on the unit sphere S_d, and $s \in S_d$ is the integrand unit vector. The index of stability is again α, and μ is the vector of locations.

In some financial applications, one needs to model the dependence between variables. Stable distributions have infinite second moment: covariance is not defined. However, Gaussian subordinated $S\alpha S$ can be used to model dependence between stable variables.[10] Gaussian subordinated $S\alpha S$ is defined as follows: Let $X \tilde{} N(0, 2\sigma^2)$, and $A \tilde{} S_{\alpha/2,0}(1, c)$, X and A being independent. Then one can generate $Z = A^{1/2}X \tilde{} S_{\alpha,0}(0, \sigma^*)$, where $c = (\sigma^{*2}/\sigma^2)[\cos(\pi\alpha/4)]^{2/\alpha}$.

The ‘truncated’ covariance matrix can be used to capture the dependence by leaving out the very extreme events. Let Σ be the truncated covariance matrix, it can be estimated by exponential smoothing[11] as follows:

$$c^2_{j,t+1|t} = (1-\lambda)\sum_{i=1}^{\infty} \lambda^i R^2_{j|t-i} \tag{6}$$

is the diagonal element of the truncated covariance matrix, and

$$c^2_{jk,t+1|t} = (1-\lambda)\sum_{i=1}^{\infty} \lambda^i R_{j|t-i} R_{k|t-i}, \tag{7}$$

where λ is the smoothing parameter, measures the truncated covariance between j and k. Hence, $\Sigma = \{c_{ik}\}$ and $c_{jj} = 2\sigma_j^2$. Suppose the truncation points are x and $-x$, then

[10] See Rachev et al. (1999, 2000) for more detailed discussions.

[11] See Morgan Stanley’s Riskmetrics (Morgan Guaranty Trust Company, 1996) for a discussion of exponential smoothing.

$R_{j|t-i}$ is defined as follows:

$$R_{j|t-i} = \begin{cases} r_t^j & \text{if } r_t^j \text{ is in } (-x, x), \\ -x & \text{if } r_t^j < -x, \\ x & \text{if } r_t^j > x. \end{cases} \tag{8}$$

Rachev et al. (2000) use this methodology for modeling the credit risk of a portfolio of fixed income securities.

2.3.2. Financial modeling and estimation

Financial modeling involves information on past market movements. In such cases, it is not the unconditional return distribution which is of interest, but the conditional distribution, which is conditioned on information contained in past return data, or a more general information set. The class of autoregressive moving average (ARMA) models is a natural candidate for conditioning on the past of a return series. These models have the property that the conditional distribution is homoskedastic. In view of the fact that financial markets frequently exhibit volatility clusters, the homoskedasticity assumption may be too restrictive. As a consequence, conditional heteroskedastic models, such as Engle's (1982) autoregressive conditional heteroskedastic (ARCH) models and the generalization (GARCH) of Bollerslev (1986), possibly in combination with an ARMA model, referred to as an ARMA–GARCH model, are now common in empirical finance. It turns out that ARCH-type models driven by normally distributed innovations imply unconditional distributions which themselves possess heavier tails. Thus, in this respect, ARCH models and stable distributions can be viewed as competing hypotheses.

We utilize stable law to describe the fundamental 'building blocks', i.e. innovations, that derive the asset return process. Along this line, Mittnik et al. (2000) present empirical evidence favoring stable hypothesis over the normal assumption as a model for unconditional, homoskedastic conditional, and heteroskedastic conditional distributions of several asset return series.

Maximum likelihood estimation: We use an approximate conditional maximum likelihood (ML) estimation procedure to estimate the parameters of the stable distribution (Mittnik et al., 1996). The unconditional ML estimate of $\theta = (\alpha, \beta, \mu, \sigma)$ is obtained by maximizing the logarithm of the likelihood function

$$L(\theta) = \prod_{t=1}^{T} S_{\alpha,\beta}\left(\frac{r_t - \mu}{\sigma}\right)\sigma^{-1}. \tag{9}$$

The estimation of all stable models is approximate in the sense that the stable density function, $S_{\alpha,\beta}(\mu, \sigma)$, is approximated via fast Fourier transformation (FFT) of the stable characteristic function,

$$\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}tx}\, \mathrm{d}H(x) = \begin{cases} \exp\{-\sigma^{\alpha}|t|^{\alpha}[1 - \mathrm{i}\beta\,\mathrm{sign}(t)\tan\frac{\pi\alpha}{2}] + \mathrm{i}\mu t\} & \text{if } \alpha \neq 1, \\ \exp\{-\sigma|t|[1 + \mathrm{i}\beta\frac{2}{\pi}\,\mathrm{sign}(t)\ln|t|] + \mathrm{i}\mu t\} & \text{if } \alpha = 1, \end{cases} \tag{10}$$

where H is the distribution function corresponding to $S_{\alpha,\beta}(\mu, \sigma)$.

This ML estimation method essentially follows that of DuMouchel (1973), but differs in that the stable density is approximated numerically by an FFT of the characteristic function rather than some series expansion. As DuMouchel shows, the resulting estimates are consistent and asymptotically normal with the asymptotic covariance matrix of $T^{1/2}(\hat{\theta} - \theta_0)$ being given by the inverse of the Fisher information matrix. The standard errors of the estimates are obtained by evaluating the Fisher information matrix at the ML point estimates. For details on stable ML estimation, see Mittnik et al. (1996), Rachev and Mittnik (2000), and Paulauskas and Rachev (1999).

Comparison of estimation methods: When the residuals of the ARMA model have Gaussian distribution, least-squares (LS) estimation is equivalent to condition ML estimation. Furthermore, Whittle estimator is asymptotically equivalent to LS and conditional ML estimation methods. However, when the innovations have stable distribution, the properties of conventional estimation methods may change due to the infinite variance property. In the stable case, ML estimates are still consistent and asymptotically normal (DuMouchel, 1973); LS and Whittle estimates are consistent but they are not asymptotically normal. The LS and Whittle estimates have infinite variance limits with a convergence rate that is faster than that of the Gaussian case (Mikosch et al., 1995). When $\alpha < 2$, Mikosch (1998) suggests using the classical confidence bands and test regions based on L^2 in a conservative sense.

Calder and Davis (1998) compare LS, least absolute deviation (LAD), and ML methods for the estimation of ARMA model with stable innovations. Their simulations reveal that the difference between the estimates of the three methods is insignificant when the index of stability of the residuals is 1.75. However, when $\alpha = 1$ or 0.75, they report that the LAD and ML estimation procedures are superior to LS estimation.

ML estimation has desirable properties in both the Gaussian and stable setting, but it is computationally very demanding. Since the variables of interest in this paper have indices of stability greater than 1.5, nonlinear LS estimation method has been utilized in this study. Our parameter estimates are consistent, but they are not asymptotically normal. However, due to the high index of stability, the parameter estimates are comparable to those that would be achieved if ML estimation were to be used.

3. Model setup

3.1. Asset allocation model

The dynamic asset allocation approach used in this study is very similar to that of Boender[12] (1997). A number of alternative initial asset allocations are generated. These allocations are then simulated into the future by using the economic scenarios, which are generated under the Gaussian and stable assumptions for the innovations of the time series models. While the initial allocations are simulated, the asset allocation

[12] See Section 2.1 for a brief review.

is updated every month according to fixed mix decision rule.[13] In general, fixed mix strategy requires the purchase of stocks as they fall in value, and the sale of stocks as they rise in value. Fixed mix strategy does not have much downside protection, and tends to do very well in flat but oscillating markets. However, it tends to do relatively poorly in bullish markets (Perold and Sharpe, 1988).

Once the initial asset allocations are simulated, the risk and reward corresponding to these initial allocations at the end of the horizon (such as one year) are calculated. The decision-maker, then, chooses the initial asset allocation (fixed mix proportions) today that results in the best risk-reward combination at the end of the horizon of interest for the given decision rule.

The initial asset allocation that is selected by the decision-maker depends on the assumptions made about the innovations of the economic scenarios. The economic scenarios driven by stable innovations result in a different risk-reward profile than the economic scenarios driven by Gaussian innovations.

While we follow the general structure of Boender (1997), the risk and reward measures used are different. The reward measure considered in this study is the mean compound portfolio return of initial allocation $i \in \{1, 2, \ldots, I\}$ at the final date:

$$\mathrm{E}[\acute{R}_T^i] = \frac{1}{S} \sum_{s=1}^{S} \acute{R}_{s,T}^i, \tag{11}$$

where $\acute{R}_{s,T}^i$ is compound return of allocation i in time period of 1 through T under scenario $s \in \{1, 2, \ldots, S\}$. It is calculated as

$$\acute{R}_{s,T}^i = \prod_{t=1}^{T} (1 + R_{s,t}^i) - 1, \tag{12}$$

where $R_{s,t}^i$ is the return of the portfolio i under scenario $s \in \{1, 2, \ldots, S\}$ in time period $t \in \{1, 2, \ldots, T\}$:

$$R_{s,t}^i = \sum_{j=1}^{J} w_j^i r_{jst}, \tag{13}$$

where r_{jst} is the percentage return of asset $j \in \{1, 2, \ldots, J\}$ under scenario s in time period t, and w_j^i is the proportion of funds[14] of portfolio i invested in asset j.

We consider several different objective functions that an investor might be interested in maximizing. Value-at-risk is a popular risk measure in applied finance (see Duffie and Pan, 1997 for a review). Value-at-risk (VaR) is an estimate of the level of loss on a portfolio which is expected to be equaled or exceeded with a given probability over a certain time interval. Although it was first used for short horizons of up to one month, it is currently being used for longer horizons of up to several years (see, for instance, Gupta et al., 2000; Panning, 1999). If $f(\acute{R}_T^i)$ is the return-loss distribution for

[13] Perold and Sharpe (1988) suggest constant proportion portfolio insurance as an alternative strategy. In this strategy, one sells stocks as they fall in value and buy stocks as they rise in value.

[14] Fix mix rule requires that w_j^i does not depend on time.

the final compound return on a portfolio, then 99% VaR is defined as the 1% lower quantile of the return distribution.[15]

Currently, VaR is mostly used to evaluate the exposure of a portfolio to market risk. Some recent articles analyze using VaR as an ex ante market risk control measure in the asset allocation decision (Huisman et al., 1999; Alexander and Baptista, 2000; Basak and Shapiro, 2001). If an investor seeks a trade-off between the mean final return and the 99% VaR of the final return, then the objective function can be expressed as

$$U(\acute{R}_T^i) = \mathrm{E}[\acute{R}_T^i] - c\,\mathrm{VaR}_{99\%}(\acute{R}_T^i), \tag{14}$$

where c is a measure of risk aversion. This approach is related to the safety first approach first advocated by Roy (1952).

Artzner et al. (1999, 1998) point out that VaR has some undesirable properties such as lack of sub-addivity, non-convexity and non-smoothness. Due to the lack of sub-additivity, VaR of a portfolio with two instruments may be greater than the sum of individual VaRs of these two instruments. Basak and Shapiro (2001) indicate that an investor who uses VaR as the risk measure is concerned with the probability of loss rather than its magnitude and they suggest controlling the first moment of the loss distribution as a better approach.[16] (Further criticisms of VaR can be found in Danielsson et al., 1992; Garman, 1997). Artzner et al. (1998, 1999) suggest conditional value-at-risk[17] (CVaR) as an alternative measure of losses. CVaR is the expected loss on a portfolio given the loss is less than VaR. We define 99% CVaR as

$$\mathrm{CVaR}_{99\%} = \mathrm{E}[\acute{R}_T^i \,|\, \acute{R}_T^i < \mathrm{VaR}_{99\%}(\acute{R}_T^i)]. \tag{15}$$

Since CVaR has the desirable properties such as sub-additivity and convexity, it is gaining popularity among researchers and practitioners (Bucay and Rosen, 1999; Embrechts et al., 1999; Rockafellar and Ursayev, 2000). When the return-loss distribution is normal, using CVaR or VaR result in the same optimal portfolio. Numerical experiments indicate that the minimization of CVaR leads to near optimal solutions in terms of VaR and CVaR is more conservative than VaR (Rockafellar and Ursayev, 2000).

The objective function that trades-off between the mean final return and the 99% CVaR of the final return can be expressed as

$$U(\acute{R}_T^i) = E[\acute{R}_T^i] - c\,\mathrm{CVaR}_{99\%}(\acute{R}_T^i), \tag{16}$$

where c is a measure of risk aversion.

We also use an analog of the mean-variance criterion as a possible objective function for the investor. If the index of stability of a random variable is less then 2, then the second moment is not well defined. Hence, we need to resort to other measures of risk. Konno and Yamazaki (1991) advocate mean absolute deviation as an alternative to standard deviation. Mean absolute deviation accords less importance to outliers,

[15] Use of 99% VaR is suggested by the Basle Committee.

[16] They define a risk measure similar to CVaR: limited expected losses (LEL) when losses occur. This measure penalizes both the probability of a loss and a high expected loss given there is a loss.

[17] It is also called mean shortfall, mean excess loss or tail VaR.

it is computationally easier to calculate, and it can be used to model the asymmetric perception of risk around the mean return. They show that if the returns are multivariate normally distributed, then the two measures are essentially the same.[18] Although this measure is well defined for both normal and stable distributions, we do not use it in the current analysis since it leads to trivial allocations.[19] We consider the following risk measure which gives importance to outliers less than variance does and more than MAD does:

$$\mathrm{MD}(\acute{R}_T^i) = \frac{1}{S}\sum_{s=1}^{S} |\acute{R}_{s,T}^i - \mathrm{E}[\acute{R}_T^i]|^r, \quad \text{where } 1 < r < 2. \tag{17}$$

Notice that when $r=2$, the above risk measure becomes the variance. In the analysis, we use those values of $r < 2$ for which $\mathrm{MD}(\acute{R}_T^i)$ is finite, such as $r=1.5$. The objective function defined over the mean final return and this new risk measure is

$$U(\acute{R}_T^i) = E[\acute{R}_T^i] - c\,\mathrm{MD}(\acute{R}_T^i), \tag{18}$$

where c is the coefficient of risk aversion.

This utility functional is consistent with stochastic dominance rules, namely every risk averse investor should choose a portfolio that maximizes this utility functional for some $r > 1$ (see Ortobelli et al., 1999).

Finally, we consider a more classical objective function, power utility of final wealth, which has constant relative risk aversion. It is calculated as follows:

$$U(W^i) = \frac{1}{S}\sum_{s=1}^{S} \frac{1}{(1-\gamma)}(W_s^i)^{(1-\gamma)}, \quad \gamma > -1, \tag{19}$$

where γ is the coefficient of relative risk aversion,[20] and W_s^i is the final wealth. Assuming that the initial wealth is 1, we compute the final wealth as follows:

$$W_s^i = 1 \cdot (1 + \acute{R}_{s,T}^i). \tag{20}$$

3.2. Scenario generation

The portfolio we analyze is composed of Treasury bill and S&P 500. The monthly return on Treasury bill is assumed to be constant at 6% annualized rate of return. The main challenge is predicting the return scenarios for S&P 500. We follow the ALM scenario generation literature in modeling the nominal stock return. Since real stock

[18] Konno and Yamazaki (1991) compare efficient frontiers for NIKKEI 225 index generated by standard deviation and mean absolute deviation risk measures. They report that the difference of the optimal portfolio generated by the two risk measures is at most 10%. They suggest that this difference can be mainly attributed to the non-normality of the data.

[19] Since we use one risk-free asset and one risky asset in the analysis, the resulting asset allocation is trivial: 100% in the risk asset, 100% in the risk-free asset, or every allocation is optimal (see Ortobelli et al., 1999).

[20] Note that $U(W^i)$ is finite if $(1-\gamma) < 2$ or $\gamma > -1$.

return is an ex post variable, i.e. it is known only after the inflation is announced on the third week of the following month, it is more interesting to model the nominal asset return for forecasting purposes (similarly, Ait-Sahalia et al., 2001 model nominal excess returns on S&P index).

We use logarithm of S&P 500 Price Index, dividend yield on S&P 500, 3-month Treasury bill rate, 10-year Treasury bond yield, and Consumer Price Index (CPI) to predict the stock return (see the appendix for the sources and definitions). We find that all the variables exhibit unit root behavior in the levels and stationary behavior in the first differences during the period of 1965–1999. Hence, we use a vector error correction (VEC) framework to model the time series behavior of these variables. The system is estimated in first differences and the variables respond to departure form the long-term relationship through the co-integrating vector. Note that the first differences of two variables have clear interpretations: first difference of logarithm of S&P price index is the capital gains to the stock and the first difference of logarithm of CPI is the inflation rate.

Recent studies have found that many financial time series contain stochastic trends and some series contain common stochastic trends so that they are co-integrated. It is generally accepted that interest rates and Treasury bill rate in particular are well described as processes integrated of order one (Hall et al., 1992). For instance, Campbell and Shiller (1987) conclude that 20-year Treasury bond yield and 1-month Treasury bill rate are integrated of order one, and they report tests supportive of the view that 20-year Treasury bond yield and 1-month Treasury bill rate are cointegrated with the cointegrating vector of $[-1\ 1]$ over their analysis period of 1959–1983. Campbell (1991), Hodrick (1992), and Bekaert and Hodrick (1992) quasi-difference the Treasury bill rate in order to make is stationary. Balduzzi and Lynch (1999) deflate it by inflation and use real Treasury bill rate, which is again stationary. Our approach is to first difference the Treasury bill rate and to estimate it within a VEC framework.

There is no consensus in the literature on stationarity of dividend yield. Campbell and Shiller (1988) report unit root tests for log dividends, prices, and the log dividend–price ratio. They cannot reject the existence of unit root for nominal and real log Cowles/S&P price indices, and nominal log dividend. The unit root null is rejected for the log dividend–price ratio and real log dividend. They point out the inconsistency in the results of the unit root test since it cannot be true that the log dividend–price ratio and the log real dividend are stationary while the log real price has a unit root. Despite this anomaly, they conclude that log dividend–price ratio is stationary for the annual 1871–1986 data of Cowles/S&P indices. Campbell and Shiller (1987) investigate stock price and dividend series in the present value framework. They report that there is some evidence for the existence of cointegration between stock prices and dividends. However, Phillips and Ouliaris (1988) apply a test for cointegration based on principal component methods, and cannot reject the null of no cointegration between stock prices and dividends. More recently, Lee (1996) investigates the co-movements of log of earnings, stock prices and dividends. He finds that the three series are cointegrated with a single cointegrating vector.

The asset return predictability literature recognizes the high persistence of dividend yield, however the general tendency is to model it as stationary. Hodrick (1992) notes that the dividend yields are highly persistent (with a first order autoregressive coefficient estimate of 0.981) and this might negate validity of the VAR analysis which assumes stationarity of the regressors. He reports the deterioration of some of the test statistics in the presence of unit root and postpones the analysis of this issue to a later study. Ait-Sahalia et al. (2001) report a first order autocorrelation estimate of 0.99 for the logarithm of dividend yield, but they continue to treat it as a stationary variable. Bekaert and Hodrick (1992) note that dividend yields and nominal interest rates are highly persistent and might invalidate the asymptotic distribution theory. They suggest that if the results on predictability of asset returns are not spurious, the quasi-differenced variables should continue to explain returns. The reported results are qualitatively similar if quasi-differenced dividend yield and quasi-differenced nominal interest rate are used. Given this evidence, they continue to use VAR in levels for further analysis, and subsequently blame near non-stationarity of VAR for the large standard errors of the parameter estimates.

Morgan Stanley's LongRun argues that VEC captures the long term relationships better. They consider random walk with zero drift as the benchmark model for US equity index and compare VEC and random walk with forward premium to it. They evaluate these three models using the selection criteria of mean absolute error and proportion of correct direction of the forecasts over a 12-month horizon. They report that VEC model performs the best among them. Boender et al. (1998) compare VAR model to a VEC model which takes economic regime changes and long term equilibria into account. They report that the VEC model improves the explanatory power of the regression.

3.2.1. Time series analysis

Monthly data from 2/1965 through 12/1999 are used for the estimation of the time series models. The economic variables are modeled using Box–Jenkins methodology. The standard Gaussian Box–Jenkins techniques carry over to the stable setting with some possible changes. There are two criteria that we used in the model selection:

(1) Autocorrelation function (ACF) and Partial Autocorrelation function (PACF) were used to determine the order of the autoregressive and moving average terms and to detect the significance of the serial correlation of the residuals. Adler et al. (1998) compare stable, Cauchy and Gaussian limits in the construction of confidence interval for ACF and PACF. Their simulations show that when $\alpha \geq 1.7$, Gaussian and Cauchy limits are better than stable limits. However, for $\alpha < 1.7$, while Gaussian limits still perform in the acceptable range, Cauchy and stable limits are better than the Gaussian limits. Mikosch (1998) suggests using the classical confidence bands and test regions based on L^2 theory in a conservative sense when a variable has stable distribution with $\alpha < 2$. Drawing upon these results, we used Gaussian limits for the variables with indices of stability greater than 1.7 and conservative Gaussian limits for the variables with indices of stability less than 1.7.[21]

[21] The typical confidence limit of 1.96 was used for change in Treasury bond rate, change in dividend yield, and change in S&P index level, and confidence limit of 2.57 was used for change in CPI level and change in Treasury bill rate.

(2) Akaike information criteria was used to trade of between extra explanatory power and parsimonious parameter selection. It is valid in both the Gaussian and the stable setting (Adler et al., 1998).

We do not model the time varying volatility of the economic variables. Fitting ARMA–GARCH models may reduce the kurtosis in the residuals. However, Balke and Fomby (1994) show that even after estimating GARCH models, significant excess kurtosis and/or skewness still remains. Mittnik et al. (2000) present empirical evidence favoring stable hypothesis over the normal assumption as a model for ARMA–GARCH residuals. We postpone modeling the time varying volatility in the generation of economic scenarios to a further paper.

Augmented Dickey Fuller and Phillips Perron tests for unit root cannot reject the existence of unit root for S&P price index, dividend yield, CPI, Treasury bill rate, and Treasury bond yield. The system is estimated in first differences and the variables respond to departure form the long-term relationship through the single co-integrating equation. Akaike information criteria suggests the use of a second order VAR. When a second order VAR is used, there is no significant serial correlation in the residuals.

The estimated long-run equilibrium relationship between the variables and the standard errors of the parameter estimates is as follows:

$$\begin{aligned}&\mathrm{CPI}_{t-1} - \underset{(0.039)}{0.793} * \mathrm{SPI}_{t-1} - \underset{(0.137)}{0.497} * \mathrm{Divy}_{t-1} + \underset{(0.107)}{0.484} * \mathrm{Tbill}_{t-1} - \underset{(0.162)}{0.884} * \mathrm{Tbond}_{t-1}\\ &\quad +0.070 = 0.\end{aligned}$$

The variables that respond to a deviation from the long-run equilibrium relationship are change in S&P 500 index level and inflation. The other variables do not have statistically significant coefficients for the co-integration equation (see Table 1).

The sign and significance of parameter estimates reported in Table 1 are similar to the ones reported previously in the literature. Our VEC model estimates a statistically significant negative effect of lagged inflation on stock returns (capital gains only). Nelson (1976) also reports that the regression coefficient one month lagged inflation is negative and statistically significant at 5% level for the period of 1953–1971. The estimated coefficient of one period lagged change in Treasury bill rate is negative and insignificant, whereas that of two period lagged change in Treasury bill rate is negative and statistically significant. Hodrick (1992) also reports a statistically significant negative coefficient on lagged quasi-differenced Treasury bill rate.

We find a positive and significant coefficient for the affect of inflation on change in Treasury bill rate. Similarly, Barnes et al. (1999) report that there is strong evidence that inflation has a strong and positive effect on short term nominal interest rates.

The estimated indices of stability[22] for the first differences of S&P price index, dividend yield, CPI, Treasury bill, and Treasury bond rates are 1.807, 1.833, 1.480,

[22] The approximate MLE method explained in Section 2.3.2 was used to estimate these parameters.

Table 1
Second-order vector error correction model of S&P index and the predictors

Regressors	Dependent variables				
	d(SPI)	d(Divy)	d(Tbill)	d(Tbond)	d(CPI)
Coint. Eq.	0.063*	−0.024*	0.038*	−0.002*	0.004*
	(0.023)	(0.024)	(0.027)	(0.016)	(0.001)
Constant	0.020*	−0.012*	−0.001	−0.004	0.002*
	(0.004)	(0.004)	(0.006)	(0.004)	(0.0002)
$d(\text{SPI})_{-1}$	−0.305*	−0.247*	0.157	0.117	0
	(0.101)	(0.104)	(0.149)	(0.088)	(0.005)
$d(\text{SPI})_{-2}$	−0.113	−0.186	0.117	0.129	0.003
	(0.101)	(0.104)	(0.149)	(0.088)	(0.005)
$d(\text{Divy})_{-1}$	−0.334*	−0.188	0.041	0.060	0
	(0.100)	(0.103)	(0.147)	(0.087)	(0.005)
$d(\text{Divy})_{-2}$	−0.089	−0.217*	0.104	0.084	0.006
	(0.098)	(0.101)	(0.145)	(0.085)	(0.005)
$d(\text{Tbill})_{-1}$	−0.042	0.025	0.288*	0.006	0
	(0.040)	(0.041)	(0.059)	(0.035)	(0.002)
$d(\text{Tbill})_{-2}$	−0.080*	0.084*	−0.106	0.020	0.004
	(0.040)	0.041	(0.058)	(0.035)	(0.002)
$d(\text{Tbond})_{-1}$	−0.218*	0.311*	0.297*	0.369*	0.010*
	0.070	(0.072)	(0.103)	(0.060)	(0.004)
$d(\text{Tbond})_{-2}$	0.079	−0.031	−0.057	−0.219*	−0.001
	(0.074)	(0.076)	(0.109)	(0.064)	(0.004)
$d(\text{CPI})_{-1}$	−2.884*	2.504*	3.210*^{T}	1.268	0.366*
	(0.904)	(0.928)	(1.318)	(0.783)	(0.049)
$d(\text{CPI})_{-2}$	−0.011	0.285	−3.181*^{T}	−0.304	0.232*
	(0.090)	(0.922)	(1.320)	(0.778)	(0.048)
Adj. R^2	0.130	0.151	0.174	0.139	0.512

Standard errors are in parentheses.

1.544, 1.886, respectively. Since the indices of stability are lower for change in CPI and Treasury bill, we use the more conservative critical value of 2.57 for these variables (see Adler et al., 1998; Mikosch, 1998). This does not change the statistically significant regressors for the inflation regression but it makes two regressors insignificant for the Treasury bill equation(marked by the superscript T).

The adjusted R^2 of the stock return regression is considerably higher than the values reported in previous studies that utilize VAR methodology. We can compare the explanatory power of various regression specifications for our data set. When we estimate a second order VAR[23] for real return on S&P 500, change in dividend yield, real Treasury bill rate and real Treasury bond rate, the adjusted R^2 in that regression is 0.045 and the R^2 is 0.064, which is similar to values reported in other studies using VAR such as 0.057 in Hodrick (1992). When inflation is added to this regression, it has a statistically significant coefficient and the adjusted R^2 improves to 0.119. The explanatory power the regression is further increased to 0.130 when the variables are

[23] We do not report the regressions here. The results are available upon request.

Table 2
The estimated normal and stable parameters for the innovations

Innovations of	Normal dist.		Stable dist.			
	μ	σ	α	β	μ	σ
S&P capital yield (d(SPI))	0	0.039	1.811	− 0.294	0	0.024
Change in dividend yield (d(Divy))	0	0.040	1.851	− 0.029	0	0.023
Price inflation (d(CPI))	0	0.002	1.747	0.212	0	0.001
Change in Treasury bill (d(Tbill))	0	0.057	1.611	0.025	0	0.031
Change in Treasury bond (d(Tbond))	0	0.033	2.000	0	0	0.033

modeled in nominal terms within a VEC framework as above. Our analysis confirms the finding by Boender et al. (1998) that using a VECM that takes long term equilibrium into account improves the explanatory power of the model.

3.2.2. Simulation of future scenarios

Future economic scenarios are simulated at monthly intervals. One set of scenarios is generated by assuming that the residuals of the variables are iid normal and another set of scenarios is generated by assuming that the residuals are iid stable. The estimated normal and stable parameters [24] for the innovations of the time series models are given in Table 2. All of the innovations except for that of change in Treasury bond have indices of stability estimates less than 2. This indicates that these variables have fatter tails in comparison to the Gaussian distribution. The variables are slightly skewed. This flexibility of stable distribution is more useful for significantly skewed financial variables, such as corporate bonds. The variable with the lowest index of stability and hence the fattest tails is the residuals of change in Treasury bill rate. Fig. 3 depicts comparison of empirical probability density function, the stable fit and the normal fit.

Rachev and Mittnik (2000) compare the empirical fit of several fat-tailed distributions to daily returns on S&P 500. The best fit in the tails of the distribution is achieved by log-stable and Student-t models. A major drawback of Student-t distribution is its lack of stability with respect to summation, i.e. a portfolio of Student-t distributed asset returns does not have Student-t distribution. It is not supported by a central limit theorem. Student-t distribution is a symmetric distribution and it cannot capture the possible skewness in financial data.

In the Gaussian setting, the correlation between residuals of d(Tbill) and d(Tbond) is 0.56, and the correlation between residuals of d(SPI) and d(Divy) is −0.88. All the other correlations are smaller than 0.16 in absolute value (see Table 3). Rachev et al. (2000) use the 'truncated covariance' idea explained in Section 2.3.1 for modeling credit risk using correlated daily data. This approximation does not work well in our

[24] Stable parameters are estimated using maximum likelihood estimation method. See Section 2.3.2.

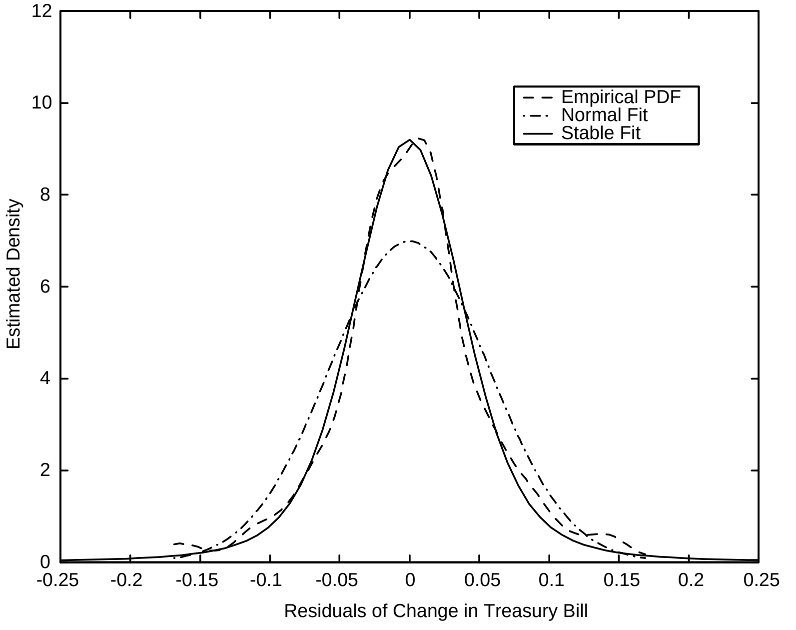

Fig. 3. Comparison of normal and stable fit for residuals of change in treasury Bill.

Table 3
Correlation of the innovations to the variables

	d(SPI)	d(Divy)	d(CPI)	d(Tbill)	d(Tbond)
d(SPI)	1	− 0.8838	0.0086	0.1198	0.0874
d(Divy)		1	− 0.0002	− 0.1129	− 0.1192
d(CPI)			1	0.1365	0.1596
d(Tbill)				1	0.562
d(Tbond)					1

problem due to the limited number of data available at the monthly level.[25] Hence, the simulated residuals are modeled as independent Gaussian or stable.

The scenarios have a tree structure. At each stage (month) we generate n possible scenarios. For each scenario, we randomly draw from the innovation distribution of each economic variable, feed it back to the time series model and generate a possible observation for the variable. Once the dividend yield and the price index are generated,

[25] This problem can be overcome either by using more historical data at the monthly level or estimating the model with higher frequency data and aggregating over time. Recent studies suggest using higher frequency data to infer information about lower frequency data. Nijman and Palm (1990) report predictability gains offered by more frequent sampling using standard ARIMA-type models. Andersen et al. (1999) shows the significant improvement in terms of volatility forecasting errors when daily or hourly data is used to forecast monthly volatility.

the corresponding monthly return on S&P 500 under scenario s is calculated as follows:

$$r_{st=}\frac{\mathrm{SPI}_{st}+\mathrm{Divy}_{st}*\mathrm{SPI}_{st}}{\mathrm{SPI}_{s(t-1)}}-1. \tag{21}$$

The first part is the capital gains and the second part is the dividend gains during the holding period.

At the next stage, n new offspring scenarios are generated from the parent scenarios. This continues until the final time of interest. If the horizon of interest is T months, then n^T alternative economic scenarios are generated.

In this study, the horizon of interest is 3 quarters and 2 scenarios are generated for each month, so 512 possible economic scenarios are considered. The 9-month scenario tree is repeated 100 times.

The generally used measure of central tendency, arithmetic mean, is very sensitive to outliers. Median, trimmed mean, and Hodges–Lehman estimator are some more robust measures of central tendency (see Hampel et al., 1986 for these as well as other robust estimators of the location parameter). When the underlying distribution is Gaussian, the arithmetic mean converges to the sample mean at the rate of $n^{0.5}$, where n is the sample size. However, the rate of convergence in the case of stable distribution is $n^{(1-1/\alpha)}$, $1 \leqslant \alpha \leqslant 2$ (Rachev, 1991). For instance, when $\alpha = 1.5$, the rate of convergence is $n^{0.333}$, which is considerably slower than the Gaussian convergence rate. If the sample size is not very large, then the trimmed sample mean is a more accurate measure of the central tendency of the data in the context of stable model.

Intuitively, since Gaussian distribution has exponentially decaying thin tails, the scenarios generated using it may generate few, if at all, extreme observations. However, stable distribution has power decaying fat tails and the scenarios generated using this distribution may generate more extreme observations. Since the number of scenarios is not very large, these extreme scenarios influence the mean greatly and trimmed mean is a more robust estimator in this context.

When the monthly returns generated under the stable distribution assumption are not trimmed, the mean annualized return on S&P 500 in 100 repetitions of the 9 month scenario tree is 9.86. This is considerably higher than the mean annualized return of 8.97 in Gaussian scenarios. In order to decide on reasonable trimming levels, we used the historical data. The highest and lowest monthly return on S&P 500 during the analysis period are 17%[26] and −21%,[27] respectively. When we trim any monthly return three times above or below these bounds, we get a mean annualized return of 9.14. Since the stable sample mean is very close to the Gaussian sample mean for this trimming level, we report results using this trimming level. When we trim any monthly return four times above or below these bounds, we get a mean annualized return of 8.72. The results for this trimming level give more pronounced asset allocation differences[28] and they are not reported here.

[26] This occurs on October 1974.

[27] This occurs on November 1987.

[28] The reason for this is that stable scenarios not only have fat tails, but they also have lower mean return on S&P 500 and lower equity premium. Therefore, S&P 500 is even less attractive as an investment option.

Table 4
Annualized return scenarios on S&P 500

	Mean	1%	5%	25%	75%	95%	99%
Normal scenarios	8.97	− 114.90	− 80.84	− 29.69	46.31	103.62	146.13
Stable scenarios	9.14	− 140.86	− 81.15	− 26.47	44.58	101.35	159.94

The mean and several quantiles of annualized return on S&P 500 under the Gaussian and stable distribution model scenarios are reported in Table 4. It should be noted that the S&P 500 returns generated by stable scenarios have fatter tails than those of Gaussian scenarios. Hence, stable scenarios consider more extreme scenarios than Gaussian scenarios do. Khindanova et al. (2001) report similar observations in their paper where they compute VaR employing Gaussian and stable distributed daily returns. They state that 5% percentile of normal and stable distribution are very close, but the 1% percentile of stable distribution is greater than that of the Gaussian.

We have not investigated the forecasting power of our model. Khindanova et al. (2001) report the superior forecasting power of stable models as compared to normal in VaR modeling. Hence, we also expect stable distribution to have superior forecasting performance in our context. Back-testing results will be reported in a later study.

4. Computational results

The asset allocation problem has been solved for several different objective functions. We report the optimal allocations under the stable and normal scenarios. We compare objective function values, certainty equivalent returns or wealths, and the expected return-risk ratios corresponding to these allocations. The comparison of the objective functions intends to capture the increase in the overall 'satisfaction' of the investor. Certainty equivalent return/wealth is the risk-free rate of return that makes the investor indifferent between receiving it for sure and having the opportunity to invest his money at the risky rate up to the horizon. Certainty equivalent return/wealth comparison is used as a way of gauging the economic importance of the allocation differences in more concrete monetary terms. Sharpe ratio[29] (Sharpe, 1966) is a tool for comparing the risk-adjusted performance of mutual funds. We employ a similar idea to evaluate the risk-adjusted expected return differences between the optimal allocations achieved under the stable and normal returns models. We define the *risk-adjusted expected return* as expected return divided by the relevant risk measure. It gives us another tool for assessing the performance of the asset allocations.

Significant differences in asset allocation are observed when one uses risk measures such as VaR and CVaR. These risk measures concentrate on the tail of the return distribution. Since the difference between the stable model and Gaussian models are

[29] Sharpe ratio is calculated as $\mathrm{E}[r_p] - r_f/\sigma_p$.

Table 5
Optimal allocations and percentage change in objective function value when VaR is the risk measure ($T=3$ quarters)

c	Normal scenarios			Stable scenarios			% Change in OFV
	S&P 500	OFV[a]	RAER[b]	S&P 500	OFV	RAER	
0.003	100%	0.0508	—	100%	0.0508	—	0
0.020	74	0.0479	0.34	52	0.0481	0.53	0.45
0.025	60	0.0475	0.44	38	0.0477	0.87	0.45
0.030	44	0.0474	0.69	24	0.0475	2.55	0.39
0.050	0	0.0482	—	0	0.0482	—	0

[a]OFV stands for objective function value. It is $U(\acute{R}_T^i) = \mathrm{E}[\acute{R}_T^i] - c\,\mathrm{VaR}_{99\%}(\acute{R}_T^i)$ for this table.
[b]RAER $= \mathrm{E}[\acute{R}_T^i]/\mathrm{VaR}_{99\%}(\acute{R}_T^i)$.

more pronounced in the tails, which one of these models is used makes an important difference in the allocations.

If the investor's risk aversion, which is measured by the trade-off constant c or by the coefficient of relative risk aversion γ, is very low or very high then the Gaussian and stable scenarios result in similar asset allocations . The intuitive explanation for this is that the investor who has very low risk aversion does not mind the risk very much. Therefore, his decision does not change when the extreme events are modeled more realistically. Similarly, the investor who has very high risk aversion, is already scared away from the risky asset. The fatter tails do not affect his decision much either. This behavior has been observed for all of the objective functions analyzed in this study.

An investor, who trades-off between the mean return on his portfolio and VaR of the portfolio, allocates 60% of his wealth to S&P 500 if he uses normal scenarios, but he will put only 38% in S&P 500 if he uses stable scenarios. The fact that stable scenarios model the extreme events more realistically, results in stable investor putting less in the risky asset than the Gaussian investor does. This results in 0.45% increase in the value of the objective function. [30] This also implies that the investor can improve the certainty equivalent return [31] by 0.45% if he switches to stable modeling. The risk-adjusted expected return for 60% S&P 500–40% Treasury bill allocation is 0.44, whereas the investor achieves a ratio of 0.87 if he invests in the optimal allocation of 38% S&P 500–62% Treasury bill. The investor can earn almost twice as much return on the unit of risk he bears if he applies the stable model rather than the normal model (see Table 5).

[30] Since Gaussian distribution is a special case of stable distribution, the stable model encompasses the Gaussian model. Therefore, the comparison of the objective function values, certainty equivalent returns and return-risk ratios are made under the assumption that stable is the correct model. See Pierides and Zenios (1998) for a similar approach.

[31] When return is constant at r%, VaR of the return is also r. So, $\mathrm{E}[r] - c\,\mathrm{VaR}_{99\%} = (1-c)r$. Then the percentage increase in the certainty equivalent return is equal to percentage increase in the value of the objective function.

Table 6
Optimal allocations and percentage change in objective function value when CVaR is the risk measure ($T = 3$ quarters)

c	Normal scenarios			Stable scenarios			% Change in OFV
	S&P 500	OFV[a]	RAER[b]	S&P 500	OFV	RAER	
0.003	100%	0.0507	—	100%	0.0507	—	0
0.018	74	0.0475	0.27	48	0.0478	0.47	0.62
0.022	62	0.0472	0.34	36	0.0475	0.71	0.70
0.030	46	0.0470	0.50	18	0.0473	4.07	0.76
0.040	0	0.0477	—	0	0.0477	—	0

[a]The objective function is $U(\acute{R}_T^i) = \mathrm{E}[\acute{R}_T^i] - c\,\mathrm{CVaR}_{99\%}(\acute{R}_T^i)$.
[b]$\mathrm{RAER} = \mathrm{E}[\acute{R}_T^i]/\mathrm{CVaR}_{99\%}(\acute{R}_T^i)$.

Huisman et al. (1999) also point out that the use of a fatter tailed distribution with more degrees of freedom is better able to capture the risk-return trade off in VaR context. They analyze an asset allocation problem in which the investor maximizes the expected return subject to a VaR constraint. They report a 2.86% decrease in investment to S&P 500 when Student-*t* distribution is used instead of normal distribution. Since stable distribution allows for skewness and fatter tails than that of Student-*t*, we find more pronounced differences in the allocations.

An investor that uses CVaR to measure the risk of his portfolio will reduce his stock holdings from 62% to 36% and improves his objective function by 0.7% by adopting the stable model. For his level of risk aversion, the investor can increase his certainty equivalent return [32] by 0.7%. The risk adjusted expected return of the Gaussian optimal allocation is 0.33 and that of stable optimal allocation is 0.71. If the investor utilizes the stable model rather than the normal model, he can receive more than twice the risk-adjusted return he used to earn (see Table 6).

Another objective function we consider is an analog of mean-variance criterion: mean-MD criterion. [33] This risk metric does not concentrate on extreme events, therefore the allocation is less sensitive to the tails. The investor who has very low or very high risk aversion, does not gain much from using the stable model. However, the stable model makes a difference for the investors in the middle. Table 7 depicts that an investor who would invest 60% in S&P 500 if he were to use normal scenarios, will put only 50% in S&P 500 if he uses stable scenarios. We also report the percentage improvement in the objective function [34] if one uses stable model as opposed to

[32] Since the CVaR corresponding to certainty equivalent return is zero, the certainty equivalent return is equal to the utility of return. Hence, the percentage improvement in the utility of return is equivalent to the percentage improvement in the certainty equivalent return.

[33] When we use the asymmetric counterpart that penalizes only negative deviations from the mean, the results are very similar. This is possible due to the fact that the return on S&P 500 index is not very skewed.

[34] Since the MD corresponding to certainty equivalent return is zero, the certainty equivalent return is equal to the utility of return. Hence, the percentage improvement in the utility of return is equivalent to the percentage improvement in the certainty equivalent return.

Table 7
Optimal allocations and percentage change in objective function value when MD is the risk measure ($T = 3$ quarters)

c	Normal scenarios			Stable scenarios			% Change in OFV
	S&P 500	OFV[a]	RAER[b]	S&P 500	OFV	RAER	
0.01	100%	0.05114	—	100%	0.05114	—	0.00
0.10	76	0.04870	2.21	64	0.04879	2.84	0.17
0.15	60	0.04797	3.12	50	0.04804	4.08	0.15
0.22	44	0.04730	4.90	36	0.04736	6.55	0.12
1.00	4	0.04604	—	4	0.04604	—	0.00

[a]The objective function is $U(\acute{R}_T^i) = \mathrm{E}[\acute{R}_T^i] - c\,\mathrm{MD}(\acute{R}_T^i)$.
[b]$\mathrm{RAER} = \mathrm{E}[\acute{R}_T^i]/\mathrm{MD}(\acute{R}_T^i)$.

Gaussian model. The increase in certainty equivalent return is 0.15% for this investor. If the investor employs the stable model rather than the normal model, he receives return-risk ratio of 4.09 as opposed to 3.12, which is 31% improvement in risk-adjusted expected return.

Ortobelli et al. (1999) also analyze the asset allocation decision of an investor with the objective of maximizing the mean-MAD criterion. There are two assets in their economy: a risk-free asset and a risky asset, such S&P 500 or DAX 30. Their model is different from ours since it is a single period allocation model using unconditional daily return distributions. They estimate the index of stability of 1.708 for daily returns on S&P 500 and report that an investor who puts 58.4% of his money in S&P 500 would put 24% less than that if he were to use stable distribution as the model for returns. The estimated index of stability for DAX 30 is 1.823, which implies that its distribution has thinner tails than S&P 500. In this case, the investor who allocates 54.7% of his wealth to DAX 30 would reduce his exposure to 48.8%. One conclusion from this analysis is that the allocation differences are more pronounced for assets with fatter tails. Since the estimated index of stability for monthly capital gains on S&P 500 is 1.807, we observe more modest allocation differences as in the DAX case above. The potential benefits of using stable distribution should be higher for more volatile stock indices such as NASDAQ or Wilshire 5000.

For an investor that maximizes the power utility of his final wealth, the difference in the asset allocation is again less pronounced. Power utility pays attention to the whole distribution rather than concentrating on the extreme events, therefore the allocation is less sensitive to the tails. Our computational results suggest that an investor who would invest 66% in S&P 500 if he were to use normal scenarios, will put only 54% in S&P 500 if he uses stable scenarios. Table 8 depicts the change in the utility[35] if the investor uses stable scenarios rather than Gaussian scenarios. The improvement in

[35] Note that the utility value becomes negative when $\gamma > 1$. Although negative utility does not make much sense, it can be made positive by monotonic transformations.

Table 8
Optimal allocations and percentage change in utility when power utility is used ($T = 3$ quarters)

γ	Normal scenarios		Stable scenarios		% Change in utility
	S&P 500	Utility	S&P 500	Utility	
0.05	100%	1.1036	100%	1.1036	0
0.4	78	1.7153	64	1.7154	0.008
0.6	66	2.5479	54	2.5480	0.005
0.8	56	5.0473	46	5.0474	0.002
1[a]	50	0.0468	40	0.0469	0.235
2	30	− 0.9550	24	− 0.9549	0.005
10	8	− 0.0740	6	− 0.0740	0

[a]Note that when $\gamma = 1$ the power utility function reduces to logarithmic utility function.

Table 9
Comparison of certainty equivalent wealth achieved from normal and stable scenarios ($T = 3$ quarters)

γ	Normal scenarios		Stable scenarios		% Change in CEFW
	S&P 500	CEFW	S&P 500	CEFW	
0.1	100%	1.05109	100%	1.05109	0
0.4	78	1.04913	64	1.04926	0.00014
0.6	66	1.04857	54	1.04870	0.00014
0.8	56	1.04820	46	1.04830	0.00010
1	50	1.04789	40	1.04800	0.00011
2	30	1.04714	24	1.04719	0.00005
10	8	1.04623	6	1.04623	0

the utility is modest, it can be as large as 0.235% depending on the risk aversion level of the investor.

The comparison can also be made in terms of certainty equivalent final wealth (CEFW). CEFW is the amount of wealth that leaves the investor indifferent between receiving it for sure at the horizon, and having his current wealth today and the opportunity to invest it up to the horizon. It is defined formally as

$$\frac{1}{(1-\gamma)}(\mathrm{CEFW}^i)^{(1-\gamma)} = \mathrm{E}[u(W^i)]. \tag{22}$$

Table 9 reports the improvement in CEFW if an investor uses stable scenarios rather than Gaussian scenarios. The computations show a 1.4 basis point improvement in the certainty equivalent wealth of the investor who would allocate 66% to S&P 500. The difference, being modest, could get larger or smaller depending on the risk aversion level of the decision maker. Tokat et al. (2002) perform a similar analysis using a scenario generation method based on Mulvey (1996). The computational results in that study show more pronounced allocation, utility and CEFW differences.

Table 10
Optimal allocations and percentage change in objective function value when VaR is the risk measure ($T=3$ quarters)

c	Normal scenarios			Stable scenarios			% Change in OFV
	S&P 500	OFV[a]	RAER[b]	S&P 500	OFV	RAER	
0.020	74%	0.04821	0.37	52%	0.04823	0.59	0.05
0.025	60	0.04774	0.48	38	0.04780	0.98	0.13
0.035	44	0.04743	0.76	24	0.04755	3.16	0.17

[a]The objective function is $U(\acute{R}^i_T) = \mathrm{E}[\acute{R}^i_T] - c\,\mathrm{VaR}_{99\%}(\acute{R}^i_T)$.
[b]RAER $= \mathrm{E}[\acute{R}^i_T]/\mathrm{VaR}_{99\%}(\acute{R}^i_T)$.

Table 11
Optimal allocations and percentage change in objective function value when CVaR is the risk measure ($T=3$ quarters)

c	Normal scenarios			Stable scenarios			% Change in OFV
	S&P 500	OFV[a]	RAER[b]	S&P 500	OFV	RAER	
0.018	74%	0.04797	0.30	48%	0.04780	0.53	0.07
0.022	62	0.04751	0.38	36	0.04760	0.82	0.20
0.030	46	0.04692	0.56	18	0.04732	6.32	0.86

[a]The objective function is $U(\acute{R}^i_T) = \mathrm{E}[\acute{R}^i_T] - c\,\mathrm{CVaR}_{99\%}(\acute{R}^i_T)$.
[b]RAER $= \mathrm{E}[\acute{R}^i_T]/\mathrm{CVaR}_{99\%}(\acute{R}^i_T)$.

4.1. Stress test of the allocations

We evaluate the performance[36] of the optimal portfolios under another set of scenarios for the same holding period. This analysis intends to stress test the portfolio against unexpected risks during the planning period. We generate another 100 simulations of the scenario tree from the same time series model. We evaluate the optimal allocations achieved under the stable and Gaussian return scenarios in the previous section according to this new set of scenarios. Tables 10–13 show that the percentage improvement in the objective function value and certainty equivalent return/wealth decreases but still persists under this separate set of scenarios. The risk-adjusted expected return does not change significantly.

As in the previous section, the increase in the objective function and hence certainty equivalent return is more pronounced when risk measures that concentrate on the tails are used. The benefit of using stable distribution in terms of objective function value

[36] This analysis was suggested by the Editor, Stavros A. Zenios. Golub et al. (1997) designs a ‘portfolio analyzer’, which evaluates the performance of the portfolio under another set of scenarios for the same holding period.

Table 12
Optimal allocations and percentage change in objective function value when MD is the risk measure ($T = 3$ quarters)

c	Normal scenarios			Stable scenarios			% Change in OFV
	S&P 500	OFV[a]	RAER[b]	S&P 500	OFV	RAER	
0.10	76%	0.048873	2.33	64%	0.048870	3.01	− 0.01
0.15	60	0.048870	3.30	50	0.048081	4.31	0.02
0.22	44	0.047356	5.19	36	0.047375	6.95	0.04

[a] The objective function is $U(\acute{R}^i_T) = E[\acute{R}^i_T] - c\,\mathrm{MD}(\acute{R}^i_T)$.
[b] $\mathrm{RAER} = \mathrm{E}[\acute{R}^i_T]/\mathrm{MD}(\acute{R}^i_T)$.

Table 13
Optimal allocations and percentage change in utility when power utility is used ($T = 3$ quarters)

γ	Normal scenarios	Stable scenarios	% Change in utility	Change in CEFW
	S&P 500	S&P 500		
0.4	78%	64%	− 0.00008	0
0.6	66	54	0.00083	0.00002
0.8	56	46	0.00024	0.00001
10	50	40	0.04612	0.00002
2	30	24	0.00003	0

gets smaller if the objective function is not sensitive to the tails. The change in the objective function value becomes negative for two instances below.

5. Conclusion

Generating scenarios that realistically represent the future uncertainty is important for the validity of the results of asset allocation models. The assumption underlying most of the scenario generation models used in the literature is the normal distribution. The validity of normal distribution has been questioned in the finance and macroeconomics literature. The leptokurtotic and asymmetric nature of economic variables can be better captured by using stable distribution as opposed to normal distribution.

We have analyzed the effects of the distributional assumptions on optimal asset allocation. A multistage dynamic asset allocation model with decision rules has been set up. The optimal asset allocations found under normal and stable scenarios are compared. The analysis suggests that modeling the fat tails of the returns distribution may have significant effects on the asset allocation. Stable scenario modeling leads to equity asset allocations that are up to 26% less than those of normal scenario modeling. The certainty equivalent return costs of this difference may be up to 0.7%. An investor can earn up to eight times as much return on the unit of risk he bears by applying the stable model. We observe more significant differences in asset allocation and certainty equivalent return/wealth when we use risk measures that concentrate on the tails of the return distribution, such as VaR and CVaR.

We have used stable modeling for an equity index that is not as highly volatile as some other indices. We expect that the allocation and certainty equivalent returns differences will be greater for more volatile indices such as NASDAQ, Russell 2000 or Wilshire 5000. Ortobelli et al. (1999) observe such a behavior for several indices they compare.

Although data exhibit heavy tails, time varying volatility, and long range dependence, this study has only considered explicit modeling of heavy tails in the financial data. The conditional heteroskedastic models (ARMA–GARCH) utilizing stable distributions can be used to describe the time varying volatility along with the asymmetric and leptokurtic behavior. In addition to these, the long-range dependence can also be modeled if fractional-stable GARCH models are employed. These aspects of financial data will be considered in a later paper.

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Appendix Data

All of the data are monthly and they cover the period of 2/1965–12/1999.

Tbond: logarithm of 10-year Treasury constant maturity yield-averages of business (source: Federal Reserve Board),

Tbill: logarithm of 3-month Treasury bill rate, secondary market-averages of business days on a bank discount basis (source: Federal Reserve Board),

CPI: logarithm of seasonally adjusted monthly consumer price index (source: Federal Reserve Board),

SPI: logarithm of Standard&Poors 500 price index (source: Datastream),

Divy: logarithm of monthly dividend yield calculated from the annualized dividend yield (source: Datastream),

d(.): first difference operator.

References

Adler, R.J., Feldman, R.E., Gallagher, C., 1998. Analysing stable time series. In: Adler, R.J., Feldman, R.E., Taqqu, M.S. (Eds.), A Practical Guide to Heavy Tails. Birkhauser, US, pp. 133–158.

Ait-Sahalia, Y., Brandt, M.W., Wachter, J.A., 2001. Variable selection for portfolio choice/Discussion. Journal of Finance 56 (4), 1297–1355.

Alexander, G.J., Baptista, A.M., 2000. Economic implications of using a mean-VaR model for portfolio selection: a comparison with mean-variance analysis. Working Paper, University of Minnesota.

Andersen, T.G., Bollerslev, T., Lange, S., 1999. Forecasting financial market volatility: sample frequency vis-à-vis forecast horizon. Journal of Empirical Finance 6, 457–477.

Artzner, P., Delbaen, F., Eber, J.M., Heath, D., 1999. Coherent measures of risk. Mathematical Finance 9, 203–228.

Artzner, P., Delbaen, F., Eber, J.M., Heath, D., 1998. Thinking coherently. Risk 10, 68–71.

Balduzzi, P., Lynch, A.W., 1999. Transaction costs and predictability: some utility cost calculations Journal of Financial Economics 52, 47–78.

Balke, N.S., Fomby, T.B., 1994. Large shocks, small shocks, and economic fluctuations: outliers in macroeconomic time series Journal of Applied Econometrics 9, 181–200.

Barberis, N., 2000. Investing for the long run when returns are predictable. The Journal of Finance 55 (1), 225–264.

Barnes, M., Boyd, J.H., Smith, B.D., 1999. Theories of money, credit and aggregate economic activity. European Economic Review 43, 737–754.

Basak, S., Shapiro, A., 2001. Value-at-risk based risk management: optimal policies and asset prices Review of financial studies 14, 371–405.

Brandt, M.W., 1999. Estimating portfolio and consumption choice: a conditional Euler equations approach Journal of Finance 54, 1609–1646.

Bekaert, G., Hodrick, R.J., 1992. Characterizing predictable components in excess returns on equity and foreign exchange markets. Journal of Finance 47 (2), 467–509.

Berger, A.J., Mulvey, J.M., 1998. The home account advisor: asset and liability management for individual investors. In: Ziemba, W.T., Mulvey, J.M. (Eds.), Worldwide Asset and Liability Modeling. Cambridge University Press, Cambridge, UK, pp. 634–665.

Boender, G.C.E., 1997. A hybrid simulation/optimization scenario model for asset/liability management. European Journal of Operational Research 99, 126–135.

Boender, G.C.E., van Aalst, P., Heemskerk, F., 1998. Modeling and management of assets and liabilities of pension plans in the Netherlands. In: Ziemba, W.T., Mulvey, J.M. (Eds.), Worldwide Asset and Liability Modeling. Cambridge University Press, Cambridge, UK, pp. 561–580.

Bollerslev, T., 1986. Generalized autoregressive conditional homoskedasticity. Journal of Econometrics 52, 5–59.

Brennan, M.J., Schwartz, E.S, Lagnado, R., 1997. Strategic asset allocation. Journal of Economic Dynamics and Control 21, 1377–1403.

Bucay, N., Rosen, D., 1999. Credit risk of a bond portfolio: a case study ALGO Research Quarterly 2 (1), 9–29.

Bunn, D.W., Salo, A.A., 1993. Forecasting with scenarios. European Journal of Operational Research 68 (3), 291–303.

Calder, M., Davis, R.A., 1998. Inference for linear processes with stable noise. In: Adler, R.J., Feldman, R.E., Taqqu, M.S. (Eds.), A Practical Guide to Heavy Tails. Birkhauser, US, pp. 159–176.

Campbell, J.Y., 1991. A variance decomposition for stock returns. The Economic Journal 10 (405), 157–179.

Campbell, J.Y., Shiller, R.J., 1987. Cointegration and tests of present value models. The Journal of Political Economy 95 (5), 1062–1088.

Campbell, J.Y., Shiller, R.J., 1988. The dividend-price ratio and expectations of future dividends and discount factors. Review of Financial studies 1 (3), 195–228.

Carino, D.R., Kent, T., Myers, D.H., Stacy, C., Sylvanus, M., Turner, A.L., Watanabe, K., Ziemba, W.T., 1994. The Russell–Yasuda Kasai model: an asset/liability model for a Japanese insurance company using multistage stochastic programming Interfaces 24, 29–49.

Culp, C., Tanner, K., Mensink, R., 1997. Risk, returns and retirement. RISK 10 (10), 63–69.

Danielsson, J., Hartmann, P., de Vries, C., 1992. The cost of conservatism: extreme returns, value-at-risk, and the Basle multiplication factor Risk 11 (1), 101–103.

Dert, C.L., 1998. A dynamic model for asset liability management for defined benefit pension funds. In: Ziemba, W.T., Mulvey, J.M. (Eds.), Worldwide Asset and Liability Modeling. Cambridge University Press, Cambridge, UK, pp. 501–536.

Duffie, D., Pan, J., 1997. An overview of value at risk. Journal of Derivatives 4 (2), 7–49.
DuMouchel, W.H., 1973. On the asymptotic normality of the maximum likelihood estimate when sampling from a stable distribution. Annals of Statistics 1, 948–957.
Embrechts, P., Resnick, S., Samorodnitsky, R., 1999. Extreme value theory as a risk management tool. North American Actuarial Journal 3 (2), 30–41.
Embrechts, P., Kluppelberg, C., Mikosch, T., 1997. Modelling Extremal Events for Insurance and Finance. Springer, Berlin.
Engle, R., 1982. Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. Econometrica 50, 987–1007.
Fama, E., 1965. The behavior of stock market prices. Journal of Business 38, 34–105.
Garman, M.B., 1997. Taking VaR to pieces. Risk 10 (10), 70–71.
Golub, B., Holmer, M., McKendall, R., Pohlman, L., Zenios, S.A., 1997. Stochastic programming models for money management. European Journal of Operational Research 85, 282–296.
Gupta, F., Eric, S., Thambiah, Y., 2000. US corporate pension plans. Journal of Portfolio Management 26 (3), 67–72.
Hall, A.D., Anderson, H.M., Granger, C.W.J., 1992. A cointegration analysis of treasury bill yields. Review of Economics and Statistics 74 (1), 116–126.
Hampel, F.R., Ronchetti, E.M., Rousseeuw, P.J., Stahel, W.A., 1986. Robust Statistics: The Approach Based on Influence Functions. Wiley, New York.
Hodrick, R.J., 1992. Dividend yields and expected stock returns: alternative procedures for inference and measurement Review of Financial Studies 5 (3), 357–386.
Huisman, R., Koedijk, K.G., Pownall, R.A.J., 1999. Asset allocation in a value-at-risk framework. Working Paper, Eramus University, Rotterdam.
Kandel, S., Staumbaugh, R.F., 1996. On predictability of stock returns: an asset allocation perspective Journal of Finance 51, 385–424.
Khindanova, I, Rachev, S.T., Schwartz, E., 2001. Stable modeling of value-at-risk. Mathematical and Computer Modeling 34 (9–11), 1223–1259.
Konno, H., Yamazaki, H., 1991. Mean-absolute deviation portfolio optimization model and its applications to Tokyo Stock Market. Management Science 37 (5), 519–531.
Kusy, M.I., Ziemba, W.T., 1986. A bank asset and liability management model. Operations Research 35, 356–376.
Lee, B.S., 1996. Comovements of earnings, dividends, and stock prices. Journal of Empirical Finance 3 (4), 327–346.
Mandelbrot, B.B., 1963. The variation of certain speculative prices. Journal of Business 26, 394–419.
Mandelbrot, B.B., 1967. The variation of some other speculative prices. Journal of Business 40, 393–413.
Maranas, C.D., Androulakis, I.P., Floudas, C.A., Berger, A.J., Mulvey, J.M., 1997. Solving long-term financial planning problems via global optimization. Journal of Economic Dynamics and Control 21, 1405–1425.
Merton, R.C., 1969. Lifetime portfolio selection under uncertainty: the continuous-time case Review of Economics and Statistics 51 (3), 247–257.
Mikosch, T., 1998. Periodogram estimates from heavy tailed data. In: Adler, R.J., Feldman, R.E., Taqqu, M.S. (Eds.), A Practical Guide to Heavy Tails. Birkhauser, US, pp. 241–257.
Mikosch, T, Gadrich, T., Kluppelberg, C., Adler, R.J., 1995. Parameter estimation for ARMA models with infinite variance innovations. Annals of Statistics 23 (1), 305–326.
Mittnik, S., Rachev, S. T., Doganoglu, T., Chenyao, D., 1996. Maximum likelihood estimation of stable Paretian models. Working Paper, Christian Albrechts University, Kiel.
Mittnik, S., Paolella, M.S., Rachev, S.T., 2000. Diagnosing and treating the fat tails in financial returns data. Journal of Empirical Finance 7, 389–416.
Morgan Guaranty Trust Company, 1996. RiskMetrics Technical Document, 4th Edition. JP Morgan.
Mulvey, J.M., 1996. Generating scenarios for the Towers Perrin investment system. Interfaces 26, 1–15.
Mulvey, J.M., Ziemba, W.T., 1998. Asset and liability management systems for long-term investors: discussion of the issues. In: Ziemba, W.T., Mulvey, J.M. (eds.), Worldwide Asset and Liability Modeling. Cambridge University Press, Cambridge, UK, pp. 3–38.
Nelson, C.R., 1976. Inflation and rates of return on common stocks. Journal of Finance 31 (2), 471–483.

Nijman, T.E., Palm, F.C., 1990. Predictive accuracy gain from disaggregate sampling in ARIMA models. Journal of Business and Economic Statistics 8, 405–415.
Ortobelli, S.L., Rachev, S.T., Schwartz, E., 1999. The problem of optimal portfolio with stable distributed returns. Working Paper, UCLA.
Panning, W., 1999. The strategic uses of value at risk: long-term capital management for property/casualty insurers. North American Actuarial Journal 3 (2), 84–105.
Paulauskas, V., Rachev, S.T., 1999. Maximum likelihood estimators in regression models with infinite variance innovation. Working Paper, Vilnius University, Lithuania.
Perold, J.M., Sharpe, W.F., 1988. Dynamic strategies for asset allocation. Financial Analysts Journal 44 (1), 16–27.
Phillips, P.C.B., Ouliaris, S., 1988. Testing for cointegration using principal components methods. Journal of Economic Dynamics & Control 12 (2,3), 205–230.
Pierides, Y.A., Zenios, S.A., 1998. Measuring the risk of using the wrong model: a new approach. In: Operational Risk and Financial Institutions, Risk Publications, London, pp. 173–179.
Rachev, S.T., 1991. Probability Metrics and the Stability of Stochastic Models. Wiley, Chichester, New York.
Rachev, S.T., Mittnik, S., 2000. Stable Paretian Models in Finance. Wiley, New York.
Rachev, S.T., Racheva-Jotova, B., Hristov, B., I. Mandev, 1999. Software Package for Market Risk (VaR) Modeling for Stable Distributed Financial Distributed Returns Mercury 1.0—Documentation, Bravo Risk Management Group, Santa Barbara, USA.
Rachev, S.T., Schwartz, E., Khindanova, I., 2000. Stable modeling of credit risk. Working Paper, UCLA.
Rockafellar, R.T., Ursayev, S., 2000. Optimization of conditional value-at-risk. The Journal of Risk 2 (3), 21–41.
Roy, A.D., 1952. Safety-first and the holding of assets. Econometrica 20, 431–449.
Samorodnitsky, G., Taqqu, M.S., 1994. Stable Non-Gaussian Random Variables. Chapman & Hall, New York.
Samuelson, P.A., 1969. Lifetime portfolio selection by dynamic stochastic programming. Review of Economics and Statistics 51 (3), 239–246.
Sharpe, W.T., 1966. Mutual fund performance. Journal of Business 39 (1), 119–138.
Sweeney, J.C., Sonlin, S.M., Correnti, S., Williams, A.P., 1998. Optimal insurance asset allocation in a multi-currency environment. In: Ziemba, W.T., Mulvey, J.M. (Eds.), Worldwide Asset and Liability Modeling. Cambridge University Press, Cambridge, UK, pp. 341–368.
Tokat, Y., Rachev, S.T., Schwartz, E.S., 2002. Asset liability management: a review and some new results in the presence of heavy tails. In: Ziemba, W.T. (Ed.), Handbook of Heavy Tailed Distributions in Finance, Handbooks of Finance, in preparation.
Wilkie, A.D., 1986. A stochastic investment model for actuarial use. Transactions of the Faculty of Actuaries 39, 391–403.
Wilkie, A.D., 1995. More on a stochastic asset model for actuarial use. British Actuarial Journal 1 (V), 777–964.
Zenios, S.A., 1993a. Financial Optimization. Cambridge University Press, Cambridge, UK.
Zenios, S.A., 1993b. A model for portfolio management with mortgage-backed securities. Annals of Operations Research 43, 337–356.