15/10/2020

$$E(w) = \sum_{n=1}^{N} \underline{r_n} \, \underline{(y_n - w^T \phi_n)^2}$$

$$\sum_{n=1}^{N} (y_n - w^T \phi_n)^2$$

$$\sum r_n \left[y_n^2 - 2 w^T \phi_n y_n + (w^T \phi_n)^2 \right] = \underbrace{\sum r_n y_n^2}_{1} - 2 \underbrace{\sum r_n w^T \phi_n y_n}_{2} + \underbrace{\sum r_n (w^T \phi_n)^2}_{3}$$

$\sum r_n y_n^2 =$

$r_1 y_1 y_1 + r_2 y_2 y_2 + \dots + r_n y_n y_n$

$\rightarrow \sum y_n^2 =$

$\sum r_n y_n = 1^T R y$

$$R = \begin{bmatrix} r_1 & & \\ & \ddots & \\ & & r_n \end{bmatrix} \rightarrow$$

$y^T y R$

$N\times 1 \leftrightarrow N\times N$

$$\begin{bmatrix} r_1 & 0 & \dots & 0 \\ 0 & r_2 & \dots & 0 \\ 0 & \dots & & r_n \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} r_1 y_1 \\ \vdots \\ r_n y_n \end{bmatrix}$$

$$\underbrace{[1 \; 1 \; 1 \; 1]}_{y} \begin{bmatrix} r_1 y_1 \\ r_n y_n \end{bmatrix} = \sum r_n y_n$$

$$\sum r_n y_n = 1^T R y \qquad 1_p$$

$$\underbrace{[y_1 \dots y_n]}_{y^T} \begin{bmatrix} r_1 y_1 \\ r_n y_n \end{bmatrix} = \underline{\sum r_n y_n y_n}$$

① $\sum r_n y_n^2 = \underline{y^T R y}$

② $\sum r_n \underbrace{w^T \phi_n}_{} y_n = \sum r_n \underbrace{y_n}_{} \underbrace{w^T \phi_n}_{} = \underline{y^T R \Phi w}$

$\sum y_n \underbrace{w^T \phi_n}_{}$

$\sum y_n \underbrace{\phi_n^T w}_{}$

$$\Phi = \begin{bmatrix} \phi_1^T \\ \phi_2^T \\ \phi_n^T \end{bmatrix}$$

$$\underline{\Phi w} = \begin{bmatrix} \phi_1^T \\ \phi_2^T \\ \vdots \\ \phi_n^T \end{bmatrix} w = \begin{bmatrix} \phi_1^T w \\ \phi_2^T w \\ \vdots \\ \phi_n^T w \end{bmatrix}$$

$$y^T \Phi w = [y_1 \dots y_n] \begin{bmatrix} \phi_1^T w \\ \vdots \\ \phi_n^T w \end{bmatrix} = \sum y_n \phi_n^T w$$

$$\underbrace{[y_1 r_1 \dots y_n r_n]}_{y^T R} \begin{bmatrix} \phi_1^T w \\ \vdots \\ \phi_n^T w \end{bmatrix} = \qquad y^T R * \Phi w$$

$$R y = \begin{bmatrix} r_1 y_1 \\ \\ r_n y_n \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$(Ry)^T = y^T R^T = \underline{y^T R}$$

$\phi(x_n) = \phi_n$ $\quad \phi : R^{D+1} \to R^{M+1}$

$\phi_1, \phi_2 \dots \phi_n$ $\quad \phi_n = \begin{bmatrix} \phi_0(x_n) \\ \phi_1(x_n) \\ \\ \phi_n(x_n) \end{bmatrix} R^{(M+1)\times 1}$ $\quad \Phi = \begin{bmatrix} \phi_1^T \\ \phi_2^T \\ \\ \phi_n^T \end{bmatrix}$

$$X = \begin{bmatrix} x_1^T \\ x_2^T \\ \vdots \\ x_n^T \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & x_{12} & x_{13} & \dots & x_{1D} \\ 1 & x_{21} & x_{22} & x_{23} & \dots & x_{2D} \\ \\ 1 & x_{n1} & x_{n2} & & \dots & x_{nD} \end{bmatrix}$$

$\phi_1 \phi_1^T$

(3) $\sum r_n (w^T \phi_n)^2 = \sum_{n=1}^{N} r_n (w^T \phi_n)(\phi_n^T w)$

$= w^T \left(\sum r_n \phi_n \phi_n^T\right) w$

$\sum \phi_n \phi_n^T$

$\sum_{n=1}^{N} w^T \phi_n = w^T \sum \phi_n$

$w^T \phi_1 + w^T \phi_2 + w^T \phi_3 + \dots + w^T \phi_N$

$\sum \phi_n \phi_n^T \to \phi_1 \phi_1^T$

$[\ ][\ ] = [\ ]$

$[\ ][\ ] + [\ ][\ ] + [\ ][\ ] + \dots$

$[\ ]([\ ] + [\ ] + [\ ] + [\ ] +)$

$\sum \phi_n \phi_n^T$

$\underbrace{[\phi_1\ \phi_2 \dots \phi_n]}_{\Phi^T} \underbrace{\begin{bmatrix} \phi_1^T \\ \phi_2^T \\ \vdots \\ \phi_n^T \end{bmatrix}}_{\Phi}$

$\sum x_n x_n^T$

$\sum r_n \phi_n$

$r_1\phi_1 + r_2\phi_2 + \dots + r_n\phi_n =$

$[r_1 \dots r_n] \begin{bmatrix} \phi_1^T \\ \vdots \\ \phi_n^T \end{bmatrix}$ $\quad \underbrace{[\phi_1 \dots \phi_n]}_{\Phi^T} \begin{bmatrix} r_1 \\ \vdots \\ r_n \end{bmatrix}$

$\phi_1 r_1 + \phi_2 r_2 + \dots + \phi_n r_n$

$\sum \phi_n \phi_n^T = \Phi^T \Phi$

$\sum r_n \phi_n \phi_n^T = \Phi^T R \Phi$

$$\left([\phi_1 \dots \phi_n] \begin{bmatrix} r_1 \\ \vdots \\ r_n \end{bmatrix} \right)^T = [r_1 \dots r_n] \underbrace{\begin{bmatrix} \phi_1^T \\ \vdots \\ \phi_n^T \end{bmatrix}}_{\Phi}$$

R

$$R\Phi = \begin{bmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 \\ 0 & 0 & \ddots & \\ 0 & 0 & 0 & r_n \end{bmatrix} \begin{bmatrix} \phi_1^T \\ \phi_2^T \\ \vdots \\ \phi_n^T \end{bmatrix} = \begin{bmatrix} r_1\phi_1^T \\ r_2\phi_2^T \\ \vdots \\ r_n\phi_n^T \end{bmatrix}_{N\times(M+1)}$$

$1_{M+1}^T R\Phi = \sum r_n \phi_n$ $\quad \sum r_n \phi_n \phi_n^T$

$$\underline{\sum r_n \phi_n \phi_n^T}$$

$$= \underline{\Phi^T R \Phi}$$

$$\underline{\Phi^T} \quad \underline{R\Phi}$$

$$[\phi_1 \phi_2 \cdots \phi_n] \begin{bmatrix} r_1 \phi_1^T \\ r_2 \phi_2^T \\ r_n \phi_n^T \end{bmatrix} = r_1 \phi_1 \phi_1^T + r_2 \phi_2 \phi_2^T + \cdots + r_n \phi_n \phi_n^T$$

$$= \sum r_n \phi_n \phi_n^T$$

③ $\sum r_n (w^T \phi_n)^2 = w^T \left(\sum r_n \phi_n \phi_n^T\right) w = w^T \Phi^T R \Phi w$

$$\sum r_n (y_n - w^T \phi_n)^2 = y^T R y - 2 y^T R \Phi w + w^T \Phi R \Phi w$$

$$= \underline{y^T R y} - y^T R \Phi w - w^T \Phi^T R y + w^T \Phi^T R \Phi w$$

$$= y^T [R y - R \Phi w] - w^T \Phi^T (R y - R \Phi w) = \underbrace{(y - \Phi w)^T R (y - \Phi w)}$$

$$= (y^T - w^T \Phi^T)(R y - R \Phi w)$$

$$= \underline{(y - \Phi w)^T R (y - \Phi w)}$$

$$y^T x = \sum y_i x_i = x^T y$$

$$y^T x = 5$$

$$(y^T x)^T = (x^T y) = (5)^T = 5$$