29/09/2020

1. $E\{X+Z\} = E\{X\} + E\{Z\}$

X
1
2
3

Z
0.1
-1
4
8.7
10
-10

$$E\{X\} = \sum_{\forall i} x_i P(X=x_i)$$

$$E\{X+Z\} = \sum_{\forall i}\sum_{\forall j} [x_i + z_j] P(X=x_i, Z=z_j)$$

$$P(X=x_i, Z=z_j) = P(X=x_i)P(Z=z_j)$$

Copulas

$P(X=x_i)$, $P(Z=z_j)$ $\longrightarrow$ $P(X=x_i, Z=z_j)$

$$\sum_i 5x_i = 5\sum_i x_i$$

$$E\{X+Z\} = \sum_{\forall i}\sum_{\forall j} [x_i + z_j] P(X=x_i) P(Z=z_j)$$

$$= \sum_{\forall i}\sum_{\forall j} x_i P(X=x_i) P(Z=z_j) + \sum_{\forall i}\sum_{\forall j} z_j P(X=x_i)P(Z=z_j)$$

$$= \sum_{\forall i} x_i P(X=x_i) \underbrace{\sum_{\forall j} P(Z=z_j)}_{=1} + \sum_{\forall j} z_j P(Z=z_j) \underbrace{\sum_{\forall i} P(X=x_i)}_{=1}$$

$$= \underbrace{\sum_{\forall i} x_i P(X=x_i)}_{} + \underbrace{\sum_{\forall j} z_j P(Z=z_j)}_{}$$

$$= E\{X\} + E\{Z\} \quad \square$$

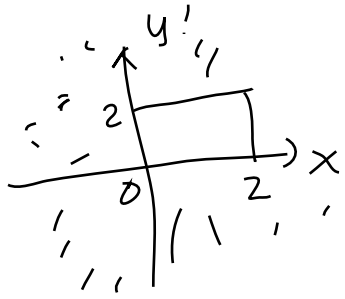
$$Var[X+Z] = Var[X] + Var[Z], \quad W = X+Z$$

$$Var[W] = E[(W-\mu_W)^2] = E[(W-E[W])^2]$$

$$= E\{W^2 - 2WE[W] + (E[W])^2\}$$

$$\vdots$$

4.



Using the properties of the joint pdf, we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x,y)\,dx\,dy = 1$$

$$\int_0^2\int_0^2 k(x+y)\,dx\,dy = 1 \Rightarrow k\int_0^2\int_0^2 (x+y)\,dx\,dy = 1$$

$$K = \frac{1}{\int_0^2\int_0^2 (x+y)\,dx\,dy}$$

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