

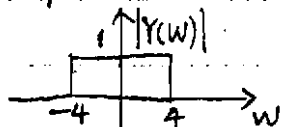
西南交通大学 2002 年硕士研究生信号与系统入学考试试题参考答案 450#

### 一. 选择题

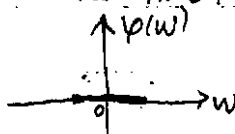
1. A. 2. C. 3. B. 4. D. 5. C

### 二. 画图题

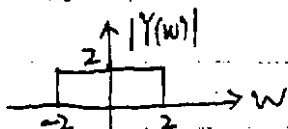
(1)  $Y(t) = X(2t) \xrightarrow{FT} \frac{1}{2} X(\frac{w}{2})$



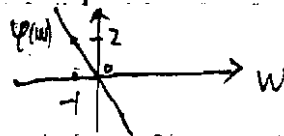
相位频谱不变仍为  $\varphi(w) = 0$



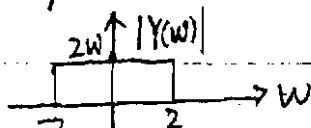
(2)  $X(t-2) \xrightarrow{FT} X(w) e^{-j2w}$



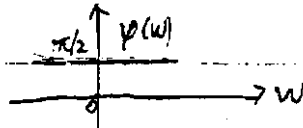
$\varphi(w) = -2w$



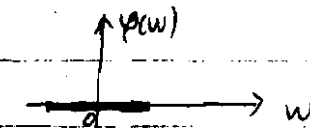
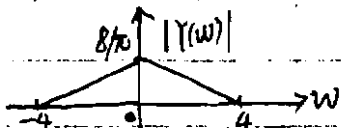
(3)  $Y(t) = X'(t) \xrightarrow{FT} jw X(w)$



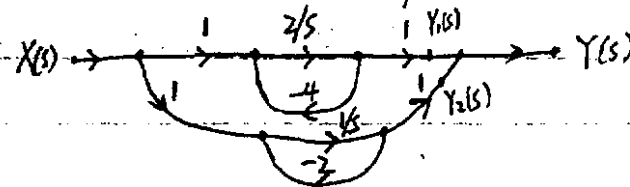
$e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} = j$  故  $\varphi(w) = \frac{\pi}{2}$



(4)  $Y(t) = X^2(t) = X(t) \cdot X(t) \xrightarrow{FT} \frac{1}{2\pi} X(w) * X(w)$   $\varphi(w) = 0$



### 三. (1) 信号流图:

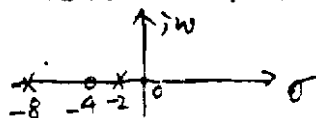


(2)  $Y_1(s) = [X(s) - 4Y_1(s)] \frac{2}{5} \Rightarrow Y_1(s) = \frac{2}{s+8} X(s)$

$Y_2(s) = [X(s) - 2Y_2(s)] \frac{1}{5} \Rightarrow Y_2(s) = \frac{1}{s+2} X(s)$

$\therefore H(s) = \frac{Y(s)}{X(s)} = \frac{Y_1(s) + Y_2(s)}{X(s)} = \frac{2}{s+8} + \frac{1}{s+2} = \frac{3s+12}{(s+8)(s+2)}$   $\sigma 7-2$

零点 -4, 极点 -8, -2.



零、极点图

$\therefore H(s)$  的极点全部都在  $w$  轴的左平面

$\therefore$  该系统渐近稳定

(3)  $H(s) = \frac{3s+12}{(s+8)(s+2)} = \frac{3s+12}{s^2+10s+16} = \frac{Y(s)}{X(s)}$

$$(s^2 + 10s + 16)Y(s) = (3s + 12)X(s)$$

该系统的微分方程为:  $Y''(t) + 10Y'(t) + 16Y(t) = 3X'(t) + 12X(t)$

(4) 零状态响应  $Y(s) = H(s) \cdot X(s) = \frac{3s+12}{(s+8)(s+2)(s+3)} = \frac{A}{s+8} + \frac{B}{s+2} + \frac{C}{s+3}$

其中:  $A = Y(s) \cdot (s+8) \big|_{s=-8} = -2/5$

$B = Y(s) \cdot (s+2) \big|_{s=-2} = 1$

$C = Y(s) \cdot (s+3) \big|_{s=-3} = -3/5$

故:  $Y(s) = -\frac{2}{5} \frac{1}{s+8} + \frac{1}{s+2} - \frac{3}{5} \frac{1}{s+3} \quad \sigma > -2$

$y_f(t) = (-\frac{2}{5}e^{-8t} + e^{-2t} - \frac{3}{5}e^{-3t})u(t)$

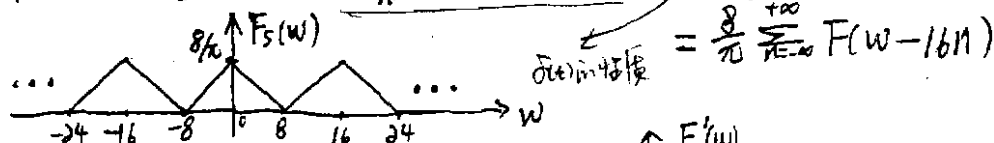
其中: 自由响应分量为:  $(-\frac{2}{5}e^{-8t} + e^{-2t})u(t)$  受迫响应分量为:  $-\frac{3}{5}e^{-3t}u(t)$

暂态响应分量为:  $(-\frac{2}{5}e^{-8t} + e^{-2t} - \frac{3}{5}e^{-3t})u(t)$  稳态响应分量为: 0

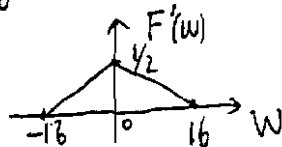
II. (1)  $W_s \geq 2W_m = 16 \quad f_s \geq \frac{W_s}{2\pi} = \frac{8}{\pi} \quad T_s \leq \frac{2\pi}{W_s} = \frac{1}{f_s} = \frac{\pi}{8}$

(2)  $f_s(t) = f(t) \cdot \delta_T(t) = f(t) \cdot \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$  时域周期信号

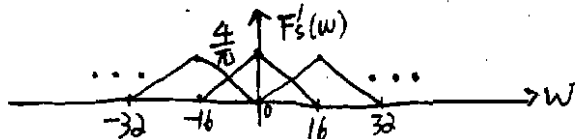
$F_s(w) = \frac{1}{2\pi} F(w) * W_s \sum_{n=-\infty}^{+\infty} \delta(w - nW_s) = \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} F(w - nW_s)$



(3)  $f'(t) = f(2t) \xrightarrow{FT} \frac{1}{2} F(\frac{w}{2})$



同理有:  $F_s'(w) = \frac{8}{\pi} \sum_{n=-\infty}^{+\infty} F'(\frac{1}{2}(w - 16n)) = \frac{4}{\pi} \sum_{n=-\infty}^{+\infty} F[\frac{1}{2}(w - 16n)]$

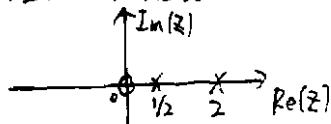


五. (1)  $y(n-1) - \frac{1}{2}y(n) + y(n+1) = x(n)$  — (差分方程)

对两边作z变换:  $z^{-1}Y(z) - \frac{1}{2}Y(z) + zY(z) = X(z)$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{z^{-1} - \frac{1}{2} + z} = \frac{2z}{2z^2 - 5z + 2} = \frac{z}{(z-2)(z-\frac{1}{2})} \quad |z| > 2$

(2) 零点 0, 极点: 2,  $\frac{1}{2}$



∴ 该系统为因果系统,  $|z| > 2$  收敛域没有包含单位圆

∴ 该系统不稳定

(3)  $X(n] = (\frac{1}{3})^n u(n) \xrightarrow{ZT} \frac{z}{z - \frac{1}{3}}$

$Y(z) = H(z)X(z) = \frac{z}{(z-2)(z-\frac{1}{2})(z-\frac{1}{3})}$

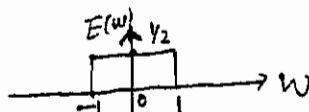
$$\frac{Y(z)}{z} = \frac{z}{(z-2)(z-\frac{1}{2})(z-\frac{1}{3})} = \frac{A}{z-2} + \frac{B}{z-\frac{1}{2}} + \frac{C}{z-\frac{1}{3}}$$

其中:  $A = \frac{Y(z)}{z} \cdot (z-2) \Big|_{z=2} = \frac{4}{5}$      $B = \frac{Y(z)}{z} \cdot (z-\frac{1}{2}) \Big|_{z=\frac{1}{2}} = -2$      $C = \frac{Y(z)}{z} \cdot (z-\frac{1}{3}) \Big|_{z=\frac{1}{3}} = \frac{6}{5}$

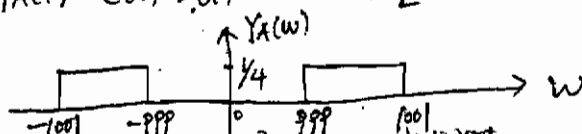
$$\therefore Y(z) = \frac{4}{5} \frac{z}{z-2} - 2 \frac{z}{z-\frac{1}{2}} + \frac{6}{5} \frac{z}{z-\frac{1}{3}}$$

$$Y(n) = \left[ \frac{4}{5}(2)^n - 2\left(\frac{1}{2}\right)^n + \frac{6}{5}\left(\frac{1}{3}\right)^n \right] u(n)$$

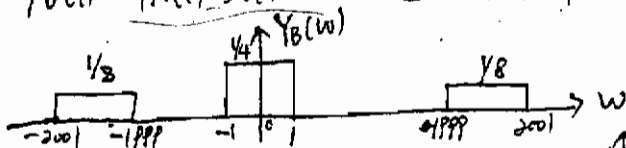
六. (1)  $e(t) = \frac{\sin t}{2\pi t} \xrightarrow{FT} \frac{1}{2} G_2(\omega)$



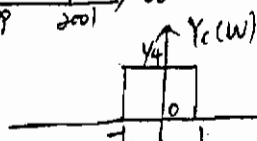
A 复:  $Y_A(t) = e(t) \cdot s(t) \xrightarrow{FT} \frac{1}{2} [E(\omega-1000) + E(\omega+1000)]$



B 复:  $Y_B(t) = Y_A(t) \cdot s(t) \xrightarrow{FT} \frac{1}{2} E(\omega) + \frac{1}{4} E(\omega-2000) + \frac{1}{4} E(\omega+2000)$



C 复:  $Y_C(t) \xrightarrow{FT} \frac{1}{2} E(\omega)$



(2)  $R(\omega) = Y_C(\omega) = \frac{1}{2} E(\omega) = \frac{1}{4} G_2(\omega) \quad \therefore r(t) = \frac{\sin t}{4\pi t}$

## 西南交通大学 2003 年硕士研究生 信号与系统 入学考试 试题参考答案 45#

## . 选择题

1 c, 2 b, 3 d, 4 a, 5 d, 6 a, 7 b, 8 a, 9 a, 10 c.

$$Y(t) = Y_x(t) + Y_f(t)$$

零输入响应:  $r^2 + 3r + 2 = 0 \quad r_1 = -1, r_2 = -2$

$$Y_x(t) = (C_1 e^{-t} + C_2 e^{-2t}) u(t) \quad Y(0) = 3, Y'(0) = 4$$

$$\begin{cases} C_1 + C_2 = 3 \\ -C_1 - 2C_2 = 4 \end{cases} \Rightarrow \begin{cases} C_1 = -7 \\ C_2 = 10 \end{cases} \quad Y_x(t) = (-7e^{-t} + 10e^{-2t}) u(t)$$

零状态响应:  $s^2 Y(s) + 3s Y(s) + 2Y(s) = X(s) = \frac{4}{s+2}$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{4}{(s+1)(s+2)^2} = 4 \left[ \frac{A}{s+1} + \frac{B}{(s+2)^2} + \frac{C}{(s+2)} \right]$$

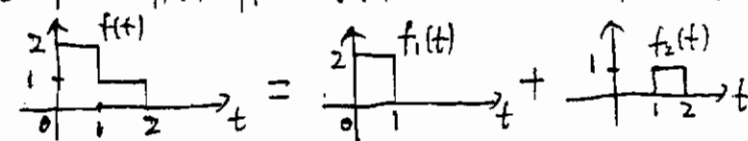
其中:  $4A = H(s) \cdot (s+1) \big|_{s=-1} = 4$      $4B = H(s) (s+2)^2 \big|_{s=-2} = -4$

$$4C = [H(s) \cdot (s+2)^2]' \big|_{s=-2} = -4$$

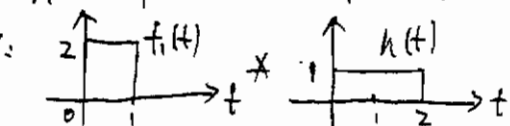
$$\therefore H(s) = \frac{4}{s+1} - \frac{4}{(s+2)^2} - \frac{4}{s+2}$$

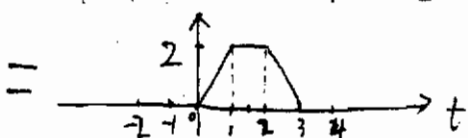
$$Y_f(t) = (4e^{-t} - 4e^{-2t} - 4te^{-2t}) u(t)$$

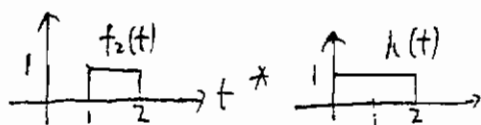
故  $Y(t) = Y_x(t) + Y_f(t) = (14e^{-t} - 11e^{-2t} - 4te^{-2t}) u(t)$

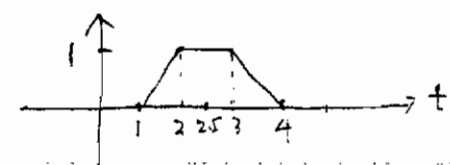
三. 

$$Y_f(t) = f(t) * h(t) = [f_1(t) + f_2(t)] * h(t) = f_1(t) * h(t) + f_2(t) * h(t)$$

其中: 

= 



= 

中心点:  $f_1(t)$  为 0.5,  $h(t)$  为 1 故  $1+0.5=1.5$

上底宽度:  $|1-2|=1$

下底宽度:  $1+2=3$

高度:  $f_1(t)$  高 2,  $h(t)$  高 1,  $f_1(t)$  宽度为 1. 故  $2 \times 1 \times 1 = 2$

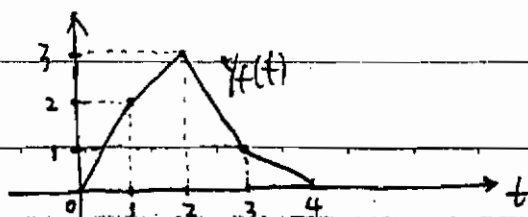
中心点:  $1.5+1=2.5$

上底宽度:  $|1-2|=1$

下底宽度:  $1+2=3$

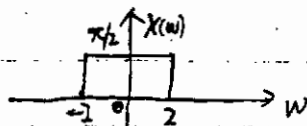
高度:  $1 \times 1 \times 1 = 1$

∴  $Y_f(t)$  的图:

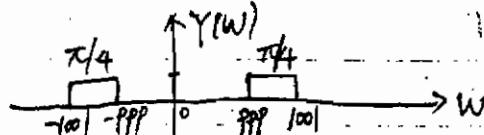
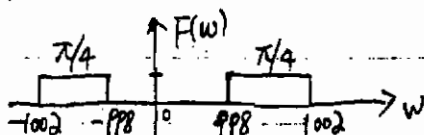


四  $X(t) = Sa(2t)$

$$X(\omega) = \frac{\pi}{2} G_4(\omega)$$



$$F(\omega) = X(\omega) * \frac{1}{2\pi} [\pi \delta(\omega - 1000) + \pi \delta(\omega + 1000)] = \frac{\pi}{4} [G_4(\omega - 1000) + G_4(\omega + 1000)]$$



$$Y(\omega) = \frac{\pi}{4} [G_2(\omega - 1000) + G_2(\omega + 1000)]$$

五 (1)

$$H(s) = \frac{H_1(s)}{1 - H_1(s)G(s)} = \frac{1/s+3}{1 - k/s+3} = \frac{1}{s+3-k}$$

(2) 系统稳定则极点必须在  $s$  轴左边，故  $3-k > 0$  得:  $k < 3$

$$(3) k=1 \quad H(s) = \frac{1}{s+2} \quad h(t) = e^{-2t} u(t)$$

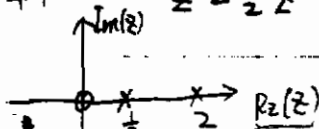
$$(4) H(s) = \frac{1}{s+2} \quad H(j\omega) = \frac{1}{2(j\omega+2)}$$

$$六 (1) Y(n-2) - \frac{5}{2} Y(n-1) + Y(n) = -\frac{3}{2} X(n-1)$$

$$z^{-2} Y(z) - \frac{5}{2} z^{-1} Y(z) + Y(z) = -\frac{3}{2} z^{-1} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-\frac{3}{2} z^{-1}}{z^{-2} - \frac{5}{2} z^{-1} + 1} = \frac{-\frac{3}{2} z}{z^2 - \frac{5}{2} z + 1} = \frac{-\frac{3}{2} z}{(z-2)(z-\frac{1}{2})} \quad |z| > 2$$

零点 0, 极点 2, 1/2.



$$(2) H(z) = z \left( \frac{-\frac{3}{2} A}{z-2} + \frac{-\frac{3}{2} B}{z-\frac{1}{2}} \right)$$

$$\text{其中: } A = \frac{1}{z-\frac{1}{2}} \Big|_{z=2} = \frac{2}{3}, \quad B = \frac{1}{z-2} \Big|_{z=\frac{1}{2}} = -\frac{2}{5} \times -\frac{2}{3}$$

$$\text{故 } H(z) = \frac{-z}{z-2} + \frac{z}{z-\frac{1}{2}} \quad h(t) = -2^n u(n) + (\frac{1}{2})^n u(n)$$

(3) ∵  $|z| > 2$  不包含单位圆，故该系统不稳定。

当  $\frac{1}{2} < |z| < 2$  时，收敛域包含单位圆，此时系统稳定。

$$h(t) = \frac{3}{5} (\frac{1}{2})^n u(n) + (2)^n u(-n-1) \quad \text{为因果}$$

$|z| > \frac{1}{2}$

收敛域  $|z| < 2$

$$t: (1) \quad sY_c(s) + Y_c(s) = X_c(s)$$

$$H_1(s) = \frac{Y_c(s)}{X_c(s)} = \frac{1}{s+1}$$

$$(2) \quad Y_c(s) = H_1(s)X_c(s) = \frac{1}{s+1} \quad \text{to} \quad y_c(t) = e^{-t}u(t)$$

$$(3) \quad \begin{aligned} Y(n) &= y_c(t) \cdot \sum_{k=-\infty}^{+\infty} \delta(t-kT) \\ &= e^{-t}u(t) \cdot \sum_{k=-\infty}^{+\infty} \delta(t-kT) \\ &= e^{-nT} \underbrace{u(nT)}_{n \geq 0} \end{aligned}$$

$$(4) \quad \begin{aligned} Y_0(n) &= Y(n) * h_2(n) = e^{-nT}u(n) * [\delta(n) - e^{-T}\delta(n-1)] \\ &= e^{-nT}u(n) - e^{-T}e^{-(n-1)T}u(n-1) \\ &= e^{-nT}u(n) - e^{-nT}u(n-1) = e^{-nT}\delta(n) \end{aligned}$$

$$Y_0(s) = Y(s) \cdot h_2(s)$$

$$h_2(s) \cdot Y(s) = h_2(n) * Y(n)$$

↓  
频域相乘

↓  
时域卷积

一. 选择题

1b, 2b, 3a, 4b, 5a, 6b, 7d, 8c, 9b, 10a

二. (1)  $y''(t) + 4y'(t) + 3y(t) = x(t)$

$$s^2 Y(s) + 4sY(s) + 3Y(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 + 4s + 3} = \frac{1}{(s+1)(s+3)} = \frac{\frac{1}{2}}{s+1} - \frac{\frac{1}{2}}{s+3}$$

$$h(t) = (\frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t})u(t)$$

(2) 零输入响应:  $r^2 + 4r + 3 = 0$   $r_1 = -1, r_2 = -3$

$$y_{zi}(t) = C_1 e^{-t} u(t) + C_2 e^{-3t} u(t) \quad \text{代入 } y(0^-) = 1, y'(0^-) = 1$$

$$\begin{cases} C_1 + C_2 = 1 \\ -C_1 - 3C_2 = 1 \end{cases} \Rightarrow \begin{cases} C_1 = 2 \\ C_2 = -1 \end{cases}$$

$$\text{故: } y_{zi}(t) = 2e^{-t} u(t) - e^{-3t} u(t)$$

零状态响应:  $Y_{zs}(s) = H(s) \cdot X(s) = \frac{1}{(s+1)(s+2)(s+3)} = \frac{\frac{1}{2}}{s+1} + \frac{-1}{s+2} + \frac{\frac{1}{2}}{s+3}$

$$\text{故: } y_{zs}(t) = \frac{1}{2}e^{-t} u(t) - e^{-2t} u(t) + \frac{1}{2}e^{-3t} u(t)$$

$$y(t) = y_{zi}(t) + y_{zs}(t) = \frac{5}{2}e^{-t} u(t) - e^{-2t} u(t) - \frac{1}{2}e^{-3t} u(t)$$

三. (1)  $H(s) = H_0 \frac{(s-z_1)}{(s-z_2)(s-z_3)}$   $H_0 = 1$ , 零极点, 极零点, 2.

$$\text{故 } H(s) = \frac{1}{(s+1)(s-2)} = \frac{-1/3}{s+1} + \frac{1/3}{s-2}$$

$$(2) h(t) = -1/3 e^{-t} u(t) + 1/3 e^{2t} u(t)$$

(3)  $\because$  极点 2 位于  $s$  轴右侧, 故该系统不稳定.

$$(4) H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 - s - 2}$$

$$(s^2 - s - 2)Y(s) = X(s)$$

$$\text{微分方程为: } y''(t) - y'(t) - 2y(t) = x(t)$$

四. (1) 据图可得:  $Y(n) = X(n) + \frac{3}{4}Y(n-1) - \frac{1}{8}Y(n-2)$

$$\therefore Y(n) - \frac{3}{4}Y(n-1) + \frac{1}{8}Y(n-2) = X(n)$$

$$(2) Y(z) - \frac{3}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}Y(z) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} = \frac{z^2}{z^2 - \frac{3}{4}z + \frac{1}{8}} = \frac{z^2}{(z-\frac{1}{2})(z-\frac{1}{4})} \quad |z| > \frac{1}{2}$$

$$(3) H(z) = z \left[ \frac{A}{z-\frac{1}{2}} + \frac{B}{z-\frac{1}{4}} \right] = \frac{2z}{z-\frac{1}{2}} - \frac{z}{z-\frac{1}{4}} \quad |z| > \frac{1}{2}$$

$$h(n) = 2 \left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{4}\right)^n u(n)$$

$$4) Y(z) = H(z)X(z) = \frac{z^3}{(z-\frac{1}{2})^2(z-\frac{1}{4})} = z \left[ \frac{A}{(z-\frac{1}{2})^2} + \frac{B}{(z-\frac{1}{2})} + \frac{C}{z-\frac{1}{4}} \right]$$

$$\text{其中: } A = \left. \frac{z^3}{z-\frac{1}{4}} \right|_{z=\frac{1}{2}} = 2 \quad C = \left. \frac{z^3}{(z-\frac{1}{2})^2} \right|_{z=\frac{1}{4}} = 4$$

$$B = \left( \frac{z^3}{z-\frac{1}{4}} \right)' \bigg|_{z=\frac{1}{2}} = -4$$

$$\therefore Y(z) = 2n \left( \frac{1}{2} \right)^{n-1} U(n) - 4 \left( \frac{1}{2} \right)^n U(n) + 4 \left( \frac{1}{4} \right)^n U(n)$$

$$5. (1) Y(z) + 0.2z^{-1}Y(z) - 0.24z^{-2}Y(z) = X(z) + z^{-1}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{1+0.2z^{-1}-0.24z^{-2}} = \frac{z^2+z}{z^2+0.2z-0.24} = \frac{z(z+1)}{(z-0.4)(z+0.6)} \quad |z| > 0.6$$

零点: 0, -1. 极点: 0.4, -0.6

$$(2) H(z) = z \left[ \frac{A}{z-0.4} + \frac{B}{z+0.6} \right]$$

$$\text{其中 } A = \left. \frac{z+1}{z+0.6} \right|_{z=0.4} = 1.4$$

$$B = \left. \frac{z+1}{z-0.4} \right|_{z=-0.6} = -0.4$$

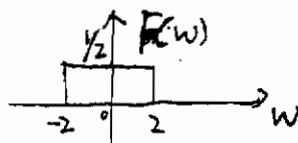
$$H(z) = \frac{1.4z}{z-0.4} - \frac{0.4z}{z+0.6}$$

$$h(n) = 1.4(0.4)^n U(n) - 0.4(-0.6)^n U(n)$$

(3)  $|z| > 0.6$  包含单位圆. 故该系统稳定.

$$6. (1) f(t) = \frac{\sin 2t}{2\pi t} = \frac{1}{\pi} \text{Sa}(2t)$$

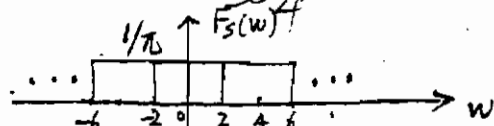
$$F(\omega) = \frac{1}{2} G_4(\omega)$$



$$(2) \omega_m = 2 \quad \omega_s = 4 \quad f_s = \frac{2}{\pi} \quad T_s = \frac{\pi}{2}$$

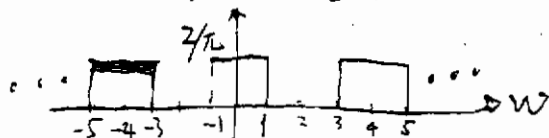
$$(3) f_s = f(t) \cdot \delta_T(t) = f(t) \cdot \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$$

$$F_s(\omega) = \frac{1}{2\pi} F(\omega) * \omega_s \sum_{n=-\infty}^{+\infty} \delta(\omega - n\omega_s) = \frac{2}{\pi} \sum_{n=-\infty}^{+\infty} F(\omega - 4n) = \frac{1}{\pi} \sum_{n=-\infty}^{+\infty} G_4(\omega - 4n)$$



$$(4) f(t/2) \xrightarrow{FT} 2F(2\omega) = G_2(\omega) \quad F_s(\omega) = \frac{1}{2\pi} G_4(\omega) * 4 \sum_{n=-\infty}^{+\infty} \delta(\omega - 4n)$$

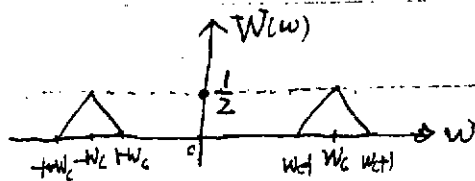
$$\text{同理有: } F_s'(\omega) = \frac{2}{\pi} \sum_{n=-\infty}^{+\infty} G_2(\omega - 4n) = \frac{2}{\pi} G_2(\omega - 4n)$$





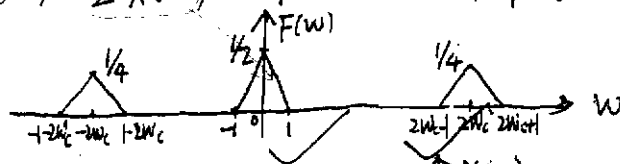
$$1) w(t) = x(t) \cdot \cos w_c t$$

$$W(w) = \frac{1}{2} [X(w - w_c) + X(w + w_c)]$$



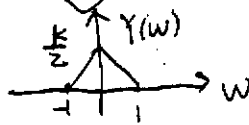
$$2) f(t) = w(t) \cdot \cos w_c t$$

$$F(w) = \frac{1}{2} X(w) + \frac{1}{4} X(w - 2w_c) + \frac{1}{4} X(w + 2w_c)$$



$$3) Y(w) = \frac{1}{2} X(w)$$

$$= \frac{k}{2} X(w) = X(w)$$



4) 据(3)可知,  $k=2$ ,  $1 \leq w_c \leq 2w_c - 1$

· 选择题

b, 2a, 3b, 4c, 5a, 6a, 7a, 8c, 9d, 10b

(1)  $y(n+1) - \frac{26}{5}y(n) + y(n+1) = x(n)$

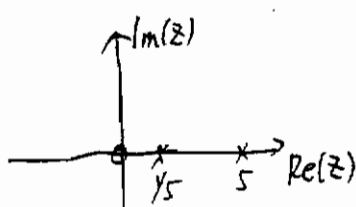
$z^1 Y(z) - \frac{26}{5}Y(z) + zY(z) = X(z)$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{z^1 - \frac{26}{5} + z} = \frac{5z}{5z^2 - 26z + 5}$

(2)  $H(z) = \frac{5z}{(5z-1)(z-5)} \quad 5 > |z| > \frac{1}{5}$

零复: 0

极复:  $\frac{1}{5}, 5$



(3)  $\because$  收敛域  $\frac{1}{5} < |z| < 5$  为双边序列。

$\therefore$  该系统不是因果系统。因果系统收敛域。

(4)  $H(z) = \frac{5z}{(5z-1)(z-5)} = -\frac{5}{24} \frac{z}{z-1/5} + \frac{5}{24} \frac{z}{z-5}$

$h[n] = -\frac{5}{24} (\frac{1}{5})^n u[n] - \frac{5}{24} (5)^n u[-n-1]$

02年六考题与此题类似。

三 (1). 由图可知: 1 零复; -2, -1 极复。

$H(s) = k \cdot \frac{(s-1)}{(s+1)(s+2)}$

$\therefore h(0^+) = 2$ . 由初值定理得:

$\lim_{s \rightarrow \infty} sH(s) = \lim_{s \rightarrow \infty} \frac{ks(s-1)}{(s+1)(s+2)} = k = 2$

故:  $H(s) = \frac{2(s-1)}{(s+1)(s+2)}$

(2)  $H(s) = \frac{-4}{s+1} + \frac{6}{s+2}$

$h(t) = (-4e^{-t} + 6e^{-2t})u(t)$

(3)  $\because$  系统两极复者在  $s$  轴左边, 故该系统为渐近稳定系统。

(4)  $H(s) = \frac{2(s-1)}{(s+1)(s+2)} = \frac{Y(s)}{X(s)}$

$(s^2 + 3s + 2)Y(s) = (2s - 2)X(s)$

微分方程:  $y''(t) + 3y'(t) + 2y(t) = 2x'(t) - 2x(t)$

四 (1) 由图可得:  $Y(n) = X(n) - 0.1Y(n-1) + 0.12Y(n-2)$

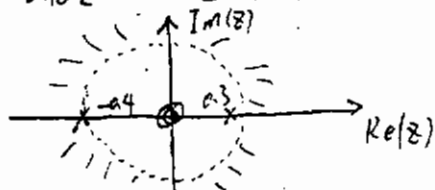
$$Y(n) + 0.1Y(n-1) - 0.12Y(n-2) = X(n)$$

$$(2) Y(z) + 0.1z^{-1}Y(z) - 0.12z^{-2}Y(z) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 + 0.1z^{-1} - 0.12z^{-2}} = \frac{z^2}{z^2 + 0.1z - 0.12} = \frac{z^2}{(z-0.3)(z+0.4)} \quad |z| > 0.4$$

0.3 = 阶零

0.3, -0.4 = 阶极



$$(3) H(z) = \frac{z^2}{(z-0.3)(z+0.4)} = \frac{3}{7} \frac{z}{z-0.3} + \frac{4}{7} \frac{z}{z+0.4}$$

$$h(n) = \left[ \frac{3}{7}(0.3)^n + \frac{4}{7}(-0.4)^n \right] u(n)$$

(4) 收敛域  $|z| > 0.4$  包括了单位圆, 故系统渐近稳定:

$$\text{五. (1)} H(s) = \frac{G(s)}{1 - G(s)E(s)} = \frac{3s}{s^2 + 2s - 3} = \frac{3s}{(s+3)(s-1)}$$

(2)  $\because$  极点 1 位于  $s$  轴右边, 故该系统不稳定.

$$(3) Y(s) = F(s)H(s)$$

$$= \frac{3s}{(s+2)(s+3)(s-1)} = \frac{2}{s+2} + \frac{-\frac{9}{4}}{s+3} + \frac{\frac{1}{4}}{s-1}$$

$$y_{zs}(t) = \left( 2e^{-2t} - \frac{9}{4}e^{-3t} + \frac{1}{4}e^t \right) u(t)$$

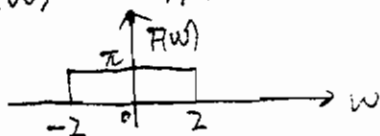
$$(4) H(s) = \frac{3s}{s^2 + 2s - 3} \quad y^2 + 2y - 3 = 0. \quad r_1 = -3, r_2 = 1$$

$$\begin{cases} c_1 + c_2 = 1 \\ -3c_1 + c_2 = 2 \end{cases} \Rightarrow \begin{cases} c_1 = -1/4 \\ c_2 = 5/4 \end{cases}$$

$$y_{zi}(t) = \left( -\frac{1}{4}e^{-3t} + \frac{5}{4}e^t \right) u(t)$$

$$\text{六 (1)} f(t) = \frac{\sin 2t}{t} = 2 \frac{\sin 2t}{2t} = 2 \text{Sa}(2t)$$

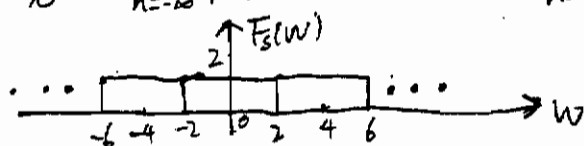
$$F(\omega) = \pi G_4(\omega) \quad // \text{注: } \text{Sa}(bt) \xrightarrow{\text{FT}} \frac{\pi}{b} G_{2b}(\omega)$$



$$2) f_s(t) = f(t) \cdot \delta_T(t) \quad W_m = 2 \quad W_s = 4 \quad T_s \leq \frac{2\pi}{W_s} = \frac{\pi}{2}$$

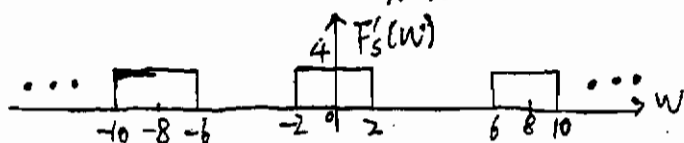
$$3) F_s(\omega) = \frac{1}{2\pi} F(\omega) * W_s \sum_{n=-\infty}^{+\infty} \delta(\omega - nW_s)$$

$$= \frac{2}{\pi} \cdot \sum_{n=-\infty}^{+\infty} F(\omega - 4n) = \cancel{\frac{2}{\pi} \cdot \sum_{n=-\infty}^{+\infty} F(\omega - 4n)} = 2 \sum_{n=-\infty}^{+\infty} G_4(\omega - 4n)$$

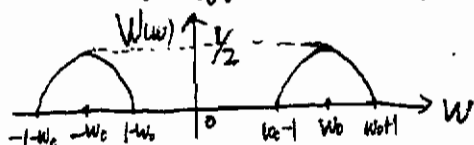


$$4) \text{ 以 } \frac{T_s}{2} \text{ 抽样, 即 } T_s' = \frac{\pi}{4} \quad W_s' = 8 \quad F_s'(\omega) = \frac{1}{2\pi} F(\omega) * W_s' \sum_{n=-\infty}^{+\infty} \delta(\omega - nW_s')$$

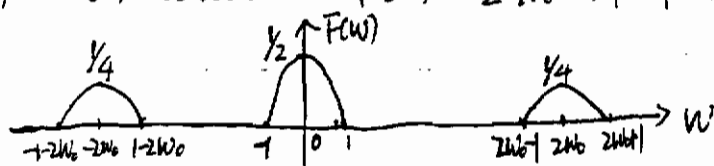
$$\text{同理有: } F_s'(\omega) = 4 \sum_{n=-\infty}^{+\infty} G_4(\omega - 8n) = \frac{1}{2\pi} \cdot 2G_4(\omega) * 8 \sum_{n=-\infty}^{+\infty} \delta(\omega - 8n) = 4G_4(\omega - 8n)$$



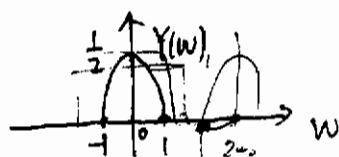
$$t. (1) w(t) = x(t) \cdot \cos w_0 t \quad W(\omega) = \frac{1}{2} [X(\omega - w_0) + X(\omega + w_0)]$$



$$(2) f(t) = w(t) \cdot \cos w_0 t \quad F(\omega) = \frac{1}{2} X(\omega) + \frac{1}{4} X(\omega - 2w_0) + \frac{1}{4} X(\omega + 2w_0)$$



$$(3) Y(\omega) = \frac{1}{2} X(\omega)$$



$$(4) \text{ 由(3)可知, } | \leq w_c \leq 2w_0 |$$

17

一. 选择题

1 b, 2 b, 3 c, 4 d, 5 d, 6 ~~a~~, 7 b, 8 a, 9 a, 10 d

二. (1)  $y''(t) + 5y'(t) + 6y(t) = x'(t)$

$$(s^2 + 5s + 6)Y(s) = sX(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s}{s^2 + 5s + 6} = \frac{-2}{s+2} + \frac{3}{s+3} \quad \sigma > -2$$

$$h(t) = -2e^{-2t}u(t) + 3e^{-3t}u(t)$$

(2)  $y_{zi}(t) = C_1 e^{-2t}u(t) + C_2 e^{-3t}u(t)$

代入  $y(0^-) = 1 \quad y'(0^-) = 2$

$$\begin{cases} C_1 + C_2 = 1 \\ -2C_1 - 3C_2 = 2 \end{cases} \Rightarrow \begin{cases} C_1 = 5 \\ C_2 = -4 \end{cases}$$

$$y_{zi}(t) = 5e^{-2t}u(t) - 4e^{-3t}u(t)$$

$$Y_{zs}(s) = H(s) \cdot X(s) = \frac{s}{(s+1)(s+2)(s+3)} = \frac{-\frac{1}{2}}{s+1} + \frac{2}{s+2} + \frac{-\frac{3}{2}}{s+3}$$

$$y_{zs}(t) = -\frac{1}{2}e^{-t}u(t) + 2e^{-2t}u(t) - \frac{3}{2}e^{-3t}u(t)$$

$$y(t) = y_{zi}(t) + y_{zs}(t)$$

$$= -\frac{1}{2}e^{-t}u(t) + 7e^{-2t}u(t) - \frac{11}{2}e^{-3t}u(t)$$

(3) 受迫响应分量:  $-\frac{1}{2}e^{-t}u(t)$

自然响应分量:  $7e^{-2t}u(t) - \frac{11}{2}e^{-3t}u(t)$

三. (1) 令  $H(z) = k \cdot \frac{z^2}{(z-\frac{1}{3})(z-\frac{1}{2})}$

$$h(0) = \lim_{z \rightarrow \infty} H(z) = k \quad \text{故 } k=1$$

$$H(z) = \frac{z^2}{(z-\frac{1}{3})(z-\frac{1}{2})} \quad |z| > \frac{1}{2}$$

(2)  $H(z) = z \cdot \left[ \frac{z}{(z-\frac{1}{3})(z-\frac{1}{2})} \right] = \frac{-2z}{z-\frac{1}{3}} + \frac{3z}{z-\frac{1}{2}}$

$$h(n) = -2\left(\frac{1}{3}\right)^n u(n) + 3\left(\frac{1}{2}\right)^n u(n)$$

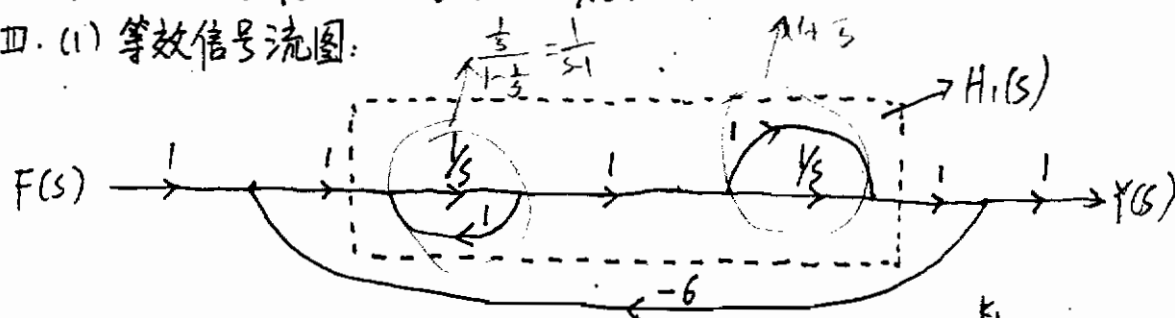
3) 收敛域  $|z| > \frac{1}{2}$  包含单位圆, 故该系统稳定.

$$(4) H(z) = \frac{Y(z)}{X(z)} = \frac{z^2}{(z - \frac{1}{3})(z - \frac{1}{2})}$$

$$(z^2 - \frac{5}{6}z + \frac{1}{6})Y(z) = z^2 X(z)$$

$$Y(n+2) - \frac{5}{6}Y(n+1) + \frac{1}{6}Y(n) = X(n+2)$$

四. (1) 等效信号流图:



令虚线框内的传递函数为  $H_1(s)$

$$H_1(s) = \frac{1}{s-1} \cdot \left[1 + \frac{1}{s}\right] = \frac{s+1}{s(s-1)}$$

$$H(s) = \frac{H_1(s)}{1+6H_1(s)} = \frac{s+1}{(s+2)(s+3)}$$

$$H(s) = \frac{k_1}{1 - k_2 k_1}$$

$$\sigma > -2 \quad \begin{array}{c} \times \times \\ -3 -2 \end{array} \quad \left| \begin{array}{c} 0 \\ 1 \end{array} \right. \rightarrow$$

$$(2) H(s) = Y(s)/F(s)$$

$$(s^2 + 5s + 6)Y(s) = (s+1)F(s)$$

$$Y''(t) + 5Y'(t) + 6Y(t) = f'(t) + f(t)$$

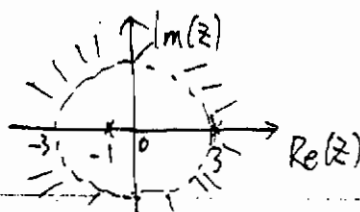
(3) 收敛域  $\sigma > -2$ , 包含  $j\omega$  轴, 故该系统稳定.  $\checkmark$

$$五 (1) Y(n+2) - 2Y(n+1) - 3Y(n) = X(n)$$

$$(z^2 - 2z - 3)Y(z) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{z^2 - 2z - 3} = \frac{1}{(z+1)(z-3)}$$

收敛域:  $|z| > 3$



(2) 令  $y_{zi}(n) = C_1(-1)^n u(n) + C_2(3)^n u(n)$

代入:  $y(0) = y(1) = 2$  得

$$\begin{cases} C_1 + C_2 = 2 \\ -C_1 + 3C_2 = 2 \end{cases} \Rightarrow \begin{cases} C_1 = 1 \\ C_2 = 1 \end{cases}$$

$\therefore y_{zi}(n) = (-1)^n u(n) + (3)^n u(n)$

$$Y_{zs}(z) = H(z) \cdot X(z)$$

$$= \frac{1}{(z+1)(z-3)} \cdot \frac{z}{z-2}$$

$$= z \left( \frac{1/2}{z+1} + \frac{1/4}{z-3} + \frac{-1/3}{z-2} \right)$$

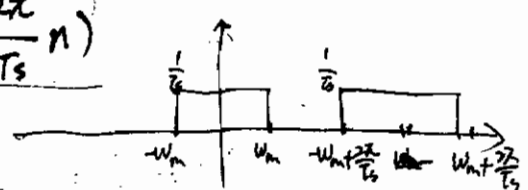
$\therefore y_{zs}(n) = \frac{1}{2}(-1)^n u(n) + \frac{1}{4}(3)^n u(n) - \frac{1}{3}(2)^n u(n)$

(3) 因为收敛域  $|z| > 3$  不包含单位圆, 故该系统不稳定。

六. (1)  $f_s(t) = f(t) \cdot \delta_{Ts}(t)$

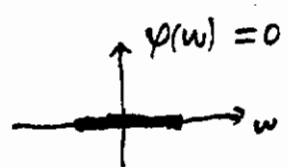
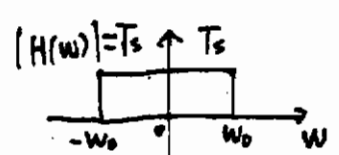
$$F_s(\omega) = \frac{1}{T_s} F(\omega) * \sum_{n=-\infty}^{+\infty} \delta(\omega - \frac{2\pi}{T_s} n)$$

$$= \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} F(\omega - \frac{2\pi}{T_s} n)$$



(2) 需要一个低通滤波器来无失真的恢复  $f(t)$

$$|H(\omega)| = \begin{cases} T_s & \omega_m \leq |\omega| \leq \frac{2\pi}{T_s} - \omega_m \\ 0 & \text{其它} \end{cases} \quad \varphi(\omega) = 0$$



(3) 满足(2)中条件必须满足奈奎斯特抽样定理:  $f_s \geq 2f_m$

我们取  $f_s = 2f_m$  有

$$|H(\omega)| = \begin{cases} \frac{\pi}{\omega_m} & |\omega| \leq \omega_m \\ 0 & \text{其它} \end{cases} \quad \varphi(\omega) = 0$$

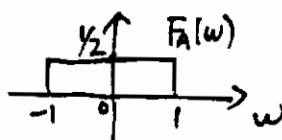
$$t. (1) X(t) = \frac{\sin t}{2\pi t} = \frac{1}{2\pi} \cdot \frac{\sin t}{t} = \frac{1}{2\pi} S_a(t)$$

由  $S_a(Bt) \xrightarrow{FT} \frac{\pi}{B} G_{2B}(\omega)$  得:

$$F(\omega) = \frac{1}{2\pi} \cdot \pi G_2(\omega)$$

$$= \frac{1}{2} G_2(\omega)$$

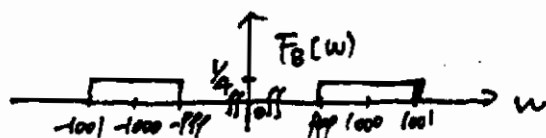
A 集:  $F_A(\omega) = F(\omega) = \frac{1}{2} G_2(\omega)$



B 集:  $f_B(t) = X(t) \cdot \cos 1000t \xrightarrow{FT} F_B(\omega) = \frac{1}{2} [F(\omega - 1000) + F(\omega + 1000)] = \frac{1}{4} [G_2(\omega - 1000) + G_2(\omega + 1000)]$

$$F_B(\omega) = \frac{1}{2\pi} F(\omega) * [\pi \delta(\omega - 1000) + \pi \delta(\omega + 1000)]$$

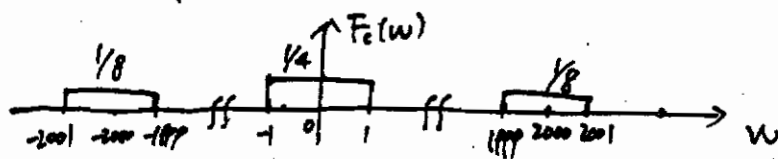
$$= \frac{1}{4} [G_2(\omega - 1000) + G_2(\omega + 1000)]$$



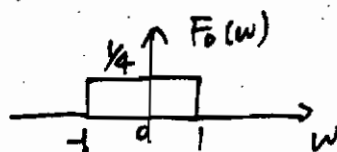
C 集:  $f_C(t) = f_B(t) \cdot \cos 1000t$

$$F_C(\omega) = \frac{1}{2\pi} F_B(\omega) * [\pi \delta(\omega - 1000) + \pi \delta(\omega + 1000)]$$

$$= \frac{1}{4} G_2(\omega) + \frac{1}{8} [G_2(\omega - 2000) + G_2(\omega + 2000)]$$



D 集:  $F_D(\omega) = \frac{1}{4} G_2(\omega)$



(2)  $Y(t) = \frac{\sin t}{4\pi t} = \frac{1}{2} X(t)$



# 西南交通大学2007年硕士研究生信号与系统试题参考答案 924#

## 一. 选择题

1c, 2b, 3b, 4b, 5d, 6A, 7b, 8a, 9d, 10b

二.  $f(t)$  全响应  $\rightarrow Y_1(t) = Y_{x1}(t) + Y_{f1}(t) = (2e^{-t} + \cos 2t)u(t)$  ①

$2f(t)$  全响应  $\rightarrow Y_2(t) = Y_{x2}(t) + Y_{f2}(t) = (e^{-t} + 2\cos 2t)u(t)$  ②

因为系统没有变, 故有  $Y_{x1}(t) = Y_{x2}(t)$ .  $Y_{f2}(t) = 2Y_{f1}(t)$  (由于传递函数不变, 输入信号加倍, 则输出信号也加倍)

②式-①式得:  $f(t)$  的零状态响应  $Y_f(t) = (\cos 2t - e^{-t})u(t)$  ③

把③代入①中得:  $f(t)$  的零输入响应  $Y_x(t) = 3e^{-t}u(t)$

故  $3f(t)$  全响应  $\rightarrow Y(t) = Y_x(t) + 3Y_f(t)$   
 $= 3\cos 2t u(t)$

三. (1)  $h(t) = \delta(t) - 4e^{-t}\cos t u(t) + 4e^{-t}\sin t u(t)$

$$H(s) = 1 - 4 \frac{s+1}{(s+1)^2+1} + 4 \frac{1}{(s+1)^2+1}$$

$$= \frac{s^2-2s+2}{s^2+2s+2} = \frac{(s-1)^2+1}{(s+1)^2+1}$$

零点:  $s = 1 \pm i$ , 极点  $s = -1 \pm i$

(2)  $H(j\omega) = H(s)|_{s=j\omega} = \frac{2-\omega^2-2j\omega}{2-\omega^2+2j\omega}$   
 $= \frac{\omega^4+4-8\omega^2-(8\omega-4\omega^3)j}{\omega^4+4}$

令:  $a = \frac{\omega^4+4-8\omega^2}{\omega^4+4}$

$b = \frac{-(8\omega-4\omega^3)}{\omega^4+4}$

$H(j\omega) = a + bj$

故  $|H(j\omega)| = \sqrt{a^2+b^2} = 1$

$\tan \varphi(j\omega) = \frac{b}{a}$

故  $\varphi(j\omega) = \arctan \frac{b}{a}$

(3) 因为两个极点:  $s_1 = -1+i$  和  $s_2 = -1-i$  都在  $j\omega$  左边, 该系统为线性因果系统, 故收敛域  $\sigma > -1$ , 包含  $j\omega$  轴. 故该系统为稳定系统.

IV. (1)  $y''(t) + 11y'(t) + 30y(t) = x'(t)$

两边作拉氏变换:  $(s^2 + 11s + 30)Y(s) = sX(s)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s}{s^2 + 11s + 30} = \frac{-5}{s+5} + \frac{6}{s+6} \quad \sigma > -5$$

$$h(t) = -5e^{-5t}u(t) + 6e^{-6t}u(t)$$

(2)  $y_{zi}(t) = C_1 e^{-5t}u(t) + C_2 e^{-6t}u(t)$

代入  $y(0^-) = 1 \quad y'(0^-) = 2$  得

$$\begin{cases} C_1 + C_2 = 1 \\ -5C_1 - 6C_2 = 2 \end{cases} \Rightarrow \begin{cases} C_1 = 8 \\ C_2 = -7 \end{cases}$$

故  $y_{zi}(t) = 8e^{-5t}u(t) - 7e^{-6t}u(t)$

$$Y_{zs}(s) = H(s) \cdot X(s) = \frac{s}{(s+5)(s+6)(s+2)}$$

$$= \frac{5/3}{s+5} + \frac{-3/2}{s+6} + \frac{-1/6}{s+2}$$

$$y_{zs}(t) = (-\frac{1}{6}e^{-2t} + \frac{5}{3}e^{-5t} - \frac{3}{2}e^{-6t}) \cdot u(t)$$

$$y(t) = y_{zi}(t) + y_{zs}(t)$$

$$= (-\frac{1}{6}e^{-2t} + \frac{29}{3}e^{-5t} - \frac{17}{2}e^{-6t})u(t)$$

(3) 受迫响应分量:  $-\frac{1}{6}e^{-2t}u(t)$

自然响应分量:  $(\frac{29}{3}e^{-5t} - \frac{17}{2}e^{-6t})u(t)$

五. (1) 由系统框图可知:

$$y(n) = x(n) + \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2)$$

故差分方程为:  $y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) \quad \textcircled{1}$

(2) 对①式两端作Z变换,

$$(1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2})Y(z) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^2}{z^2 - \frac{3}{4}z + \frac{1}{8}} = \frac{2z}{z - \frac{1}{2}} + \frac{-z}{z - \frac{1}{4}} \quad |z| > \frac{1}{2}$$

$$\therefore h(n) = 2(\frac{1}{2})^n u(n) - (\frac{1}{4})^n u(n)$$

$$(3) Y(z) = H(z) \cdot X(z)$$

$$= \left[ \frac{z^3}{(z-\frac{1}{2})^2(z-\frac{1}{4})} \right]$$

$$= z \left[ \frac{A_1}{z-\frac{1}{4}} + \frac{A_2}{(z-\frac{1}{2})^2} + \frac{A_3}{(z-\frac{1}{2})} \right]$$

$$\text{其中: } A_1 = \frac{z^2}{(z-\frac{1}{2})^2} \Big|_{z=\frac{1}{4}} = 1 \quad A_2 = \frac{z^2}{z-\frac{1}{4}} \Big|_{z=\frac{1}{2}} = 1$$

$$A_3 = \left( \frac{z^2}{z-\frac{1}{4}} \right)' \Big|_{z=\frac{1}{2}} = 0$$

$$\text{故 } Y(z) = \frac{z}{z-\frac{1}{4}} + \frac{z}{(z-\frac{1}{2})^2}$$

$$y(n) = \left(\frac{1}{4}\right)^n u(n) + n \left(\frac{1}{2}\right)^{n-1} u(n)$$

$$\text{六 (1) 令 } H(z) = k \cdot \frac{1}{(z+2)(z-1)}$$

$$\lim_{n \rightarrow \infty} h(n) = \lim_{z \rightarrow 1} \left( \frac{z-1}{z} \right) H(z) \quad \text{终值定理}$$

$$= \lim_{z \rightarrow 1} \frac{k}{z+2} = 3$$

$$\text{故: } k = 9$$

$$H(z) = \frac{9}{(z+2)(z-1)}$$

$$|z| > 2$$

$$(2) Y(z) = H(z) \cdot X(z)$$

$$(z^2 + z - 2)Y(z) = 9X(z)$$

$$\text{差分方程: } y(n+2) + y(n+1) - 2y(n) = 9x(n)$$

(3) 因为该系统为因果系统, 故  $|z| > 2$ , 收敛域不包含单位圆, 故该系统不稳定。

$$(4) \text{ 令: } y_{zi}(n) = C_1(-2)^n u(n) + C_2 u(n), \text{ 代入 } y(0)=1, y(1)=1 \text{ 得}$$

$$\begin{cases} C_1 + C_2 = 1 \\ -2C_1 + C_2 = 1 \end{cases} \Rightarrow \begin{cases} C_1 = 0 \\ C_2 = 1 \end{cases}$$

故  $Y_{zi}(n) = u(n)$

$Y(z) = H(z) \cdot X(z)$

$= \frac{P}{(z+2)(z-1)} \cdot \frac{z}{(z-\frac{1}{2})}$

$= z \left( \frac{-36/5}{z-\frac{1}{2}} + \frac{6/5}{z+2} + \frac{6}{z-1} \right)$

故  $Y_{zs}(n) = -36/5 (\frac{1}{2})^n u(n) + 6/5 (-2)^n u(n) + 6u(n)$

$Y_n(n) = Y_{zi}(n) + Y_{zs}(n)$

$= -36/5 \cdot (\frac{1}{2})^n u(n) + 6/5 (-2)^n u(n) + 7u(n)$

1. (1)  $X(t) = \left( \frac{\sin 30t}{\pi t} \right)^2 = \left( \frac{\sin 30t}{30t} \cdot \frac{30}{\pi} \right)^2 = \frac{P_{00}}{\pi^2} S_a^2 30t$

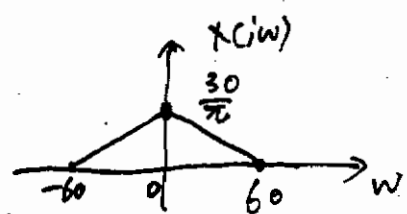
由:  $S_a Bt \xrightarrow{FT} \frac{\pi}{B} G_{2B}(w)$  得:

$S_a 30t \xrightarrow{FT} \frac{\pi}{30} G_{60}(w)$

$X_c(w) = \frac{1}{2\pi} \cdot \frac{P_{00}}{\pi^2} \left[ \frac{\pi}{30} G_{60}(w) * \frac{\pi}{30} G_{60}(w) \right]$

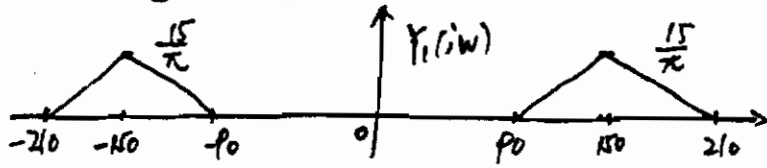
$= \begin{cases} \frac{1}{2\pi} w + \frac{30}{\pi} & -60 \leq w < 0 \\ -\frac{1}{2\pi} w + \frac{30}{\pi} & 0 \leq w \leq 60 \end{cases}$

$G_{60}(w) * G_{60}(w) = \begin{cases} -w+60 & w \in [-60, 0] \\ w+60 & w \in [0, 60] \end{cases}$



(2)  $Y_1(w) = \frac{1}{2\pi} X_c(w) * [\pi \delta(w-150) + \pi \delta(w+150)]$

$= \frac{1}{2} [X(w-150) + X(w+150)]$



$Y_2(w)$  是  $Y_1(w)$  通过带通滤波器后得到的频谱为:

