一、 选择题(每题4分, 共24分)

D, B, A, C, C, D

二、 填空题(每题4分，共24分)

$\frac{1-e^{-5}}{5} \approx 0.19865,\ \ 0.24,\ \ 0.85,$

$\frac{\ln 2}{3}$(或$\frac{\log 2}{3}$) $\approx 0.231049,\ \ -3,\ \ 64$

三.1 (13分)

(1)

$$1 = \iint f(x,y)dxdy = a\int_0^1 xdx\int_0^x ydy = \frac{a}{2}\int_0^1 x^3dx = \frac{a}{8} \Rightarrow a = 8$$

.....3分

(2)

$$P\{Y > X^2\} = \iint_D f(x,y)dxdy$$

$$= 8\int_0^1 xdx\int_{x^2}^x ydy = 4\int_0^1 x(x^2-x^4)dx = 1-\frac{2}{3} = \frac{1}{3}$$

.....5分

(3) 当$0 < y < 1$时，

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)dx = \int_y^1 8xydx = 4y(1-y^2)$$

$$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} = \begin{cases} \frac{2x}{1-y^2}, & y < x < 1; \\ 0, & x \leqslant y\text{或}x \geqslant 1 \end{cases}$$

.....5分

三.2:(12分)

(1)

由$P\{X=Y\} = 0 \Rightarrow P\{X=0,Y=0\} = P\{X=1,Y=1\} = 0$

及$P\{Y=0\} = P\{X=0,Y=0\} + P\{X=1,Y=0\} = 0.3 \Rightarrow P\{X=1,Y=0\} = 0.3$

$P\{Y=1\} = P\{X=0,Y=1\} + P\{X=1,Y=1\} = 0.3 \Rightarrow P\{X=0,Y=1\} = 0.3$得

$X\backslash Y$	-1	0	1
0	0.1	0	0.3
1	0.3	0.3	0

.....6分

(2)

$cov(X, XY) = E(X^2Y) - E(X)E(XY)$

$E(X) = 0.6$

$E(X^2Y) = -0.3$

$E(XY) = -0.3$

$cov(X, XY) = -0.3 - 0.6 \times (-0.3) = -0.12$

.....6分

三.3:(15分)

(1)

令$E(X) = \overline{X}$

因$E(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_0^1 x\frac{\alpha}{1-\alpha}x^{\frac{\alpha}{1-\alpha}-1}dx = \alpha$

代入上式得$\alpha = \overline{X}$

矩估计量为$\hat{\alpha}_1 = \overline{X}$

.....5分

(2)

设$x_1, x_2, \cdots, x_n$是样本观测值，知$0 < x_i < 1$, 所以

$L(\alpha) = (\frac{\alpha}{1-\alpha})^n(x_1x_2\cdots x_n)^{\frac{\alpha}{1-\alpha}-1}$

$\ln L(\alpha) = n(\ln\alpha - \ln(1-\alpha)) + (\frac{\alpha}{1-\alpha} - 1)\ln(x_1x_2\cdots x_n)$

$\frac{d\ln L(\alpha)}{d\alpha} = n(\frac{1}{\alpha} + \frac{1}{1-\alpha}) + \frac{1}{(1-\alpha)^2}\ln(x_1x_2\cdots x_n) = 0$

得最大似然估计值$\hat{\alpha}_2 = \frac{n}{n - \ln(x_1x_2\cdots x_n)}$

最大似然估计量$\hat{\alpha}_2 = \frac{n}{n - \ln(X_1X_2\cdots X_n)}$

.....5分

(3) 因

$$F(x) = x^{\frac{\alpha}{1-\alpha}}, (0 < x < 1)$$

$$F_{(n)}(x) = [F(x)]^n = x^{\frac{n\alpha}{1-\alpha}}, (0 < x < 1)$$

$$f_{(n)}(x) = \frac{n\alpha}{1-\alpha}x^{\frac{n\alpha}{1-\alpha}-1}, (0 < x < 1)$$

$$E(\hat{\alpha}_3) = \int_0^1 x\frac{n\alpha}{1-\alpha}x^{\frac{n\alpha}{1-\alpha}-1}dx = \frac{\frac{n\alpha}{1-\alpha}}{\frac{n\alpha}{1-\alpha}+1} = \frac{n\alpha}{(n-1)\alpha+1}$$

.....5分

三.4：(12分)

(1)

$n=5, \alpha=1-0.9=0.1, \overline{x}=1.26, s^2=0.023, s=0.15166$

σ未知时，求μ的置信区间，根据

$$\frac{\overline{X}-\mu}{S/\sqrt{n}} \sim t(n-1)$$

$$P\left\{-t_{0.05}(4) \leqslant \frac{\overline{X}-\mu}{S/\sqrt{5}} \leqslant t_{0.05}(4)\right\}=0.9$$

得μ的置信水平为0.9的置信区间是

$$\left(\overline{X}-t_{0.05}(4)S/\sqrt{5},\ \ \overline{X}-t_{0.05}(4)S/\sqrt{5}\right)$$

代入样本观测值得

$$(1.26-2.13\times 0.15166/\sqrt{5},\ \ 1.26-2.13\times 0.15166/\sqrt{5})$$

$$(1.1155,\ \ 1.4045)$$

.....6分

(2)

$n=5, \alpha=0.05, \overline{x}=1.26, s^2=0.023$

1, 建立原假设和备择假设$H_0: \sigma=0.5; \quad H_1: \sigma \neq 0.5$

2, 选择检验统计量$Y=\dfrac{(n-1)S^2}{0.5^2}=16S^2$, 在$H_0$成立的条件下$Y \sim \chi^2(4)$

3, 确定H_0的拒绝域: 由$P\{Y<\chi^2_{0.975}(4)$或$Y>\chi^2_{0.025}(4)\}=0.05$ 得拒绝域为$\{y:\ y<\chi^2_{0.975}(4)=0.5$或$y>\chi^2_{0.025}(4)=11.1\}$

4, 代入样本观测值得$y=16s^2=0.368$, 在拒绝域内，应拒绝原假设，认为不稳定.

.....6分