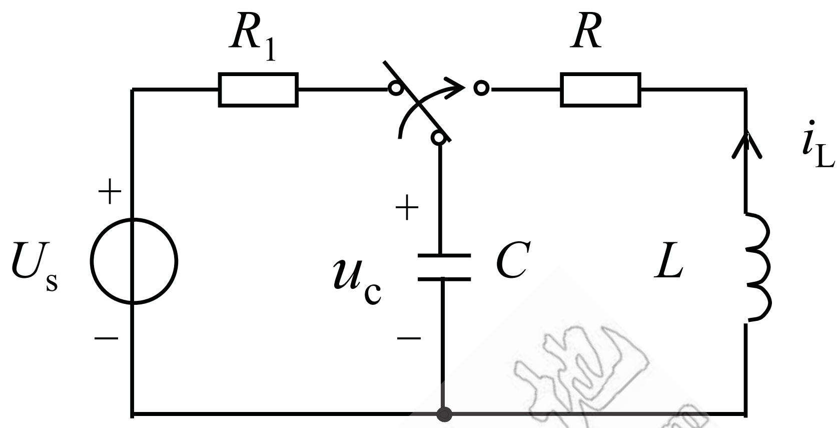
第十二章 二阶电路的时域分析

§12-1 二阶电路的零输入响应

二阶电路：



i_L为变量：

$$L\frac{di_L}{dt}+Ri_L+\frac{1}{C}\int i_L dt=0$$

$$LC\frac{d^2i_L}{dt^2}+RC\frac{di_L}{dt}+i_L=0$$

u_c为变量：

$$LC\frac{d^2u_c}{dt^2}+RC\frac{du_c}{dt}+u_c=0$$

特征方程： $LCp^2 + RCp + 1 = 0$

特征根： $p_{1,2} = -\dfrac{R}{2L} \pm \sqrt{(\dfrac{R}{2L})^2 - \dfrac{1}{LC}}$

$$u_c = K_1 e^{P_1 t} + K_2 e^{P_2 t}$$

K_1、K_2为待定常数

初值： $u_c(0_+) = u_c(0_-) = U_S = U_0$

$$\left.\frac{du_c(t)}{dt}\right|_{t=0_+} = \frac{1}{C} i_L(0_+) = 0$$

$$u_c = K_1 e^{P_1 t} + K_2 e^{P_2 t}$$

$$\begin{cases} K_1 + K_2 = U_0 \\ K_1 p_1 + K_2 p_2 = 0 \end{cases}$$

得 $K_1 = \dfrac{p_2 U_0}{p_2 - p_1}$ $K_2 = \dfrac{-p_1 U_0}{p_2 - p_1}$

$$u_c = \frac{U_0}{p_2 - p_1}[p_2 e^{p_1 t} - p_1 e^{p_2 t}] \quad t \ge 0$$

1、 $R > 2\sqrt{\frac{L}{C}}$ $\quad p_{1,2} = -\frac{R}{2L} \pm \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}}$

p_1, p_2 为两个不等实根，且为负

① $u_c = \frac{U_0}{p_2 - p_1}[p_2 e^{p_1 t} - p_1 e^{p_2 t}] \quad t \geq 0$

$p_2 < p_1 \qquad e^{p_2 t} < e^{p_1 t}$

$p_2 e^{p_1 t} < p_2 e^{p_2 t} < p_1 e^{p_2 t} \qquad p_2 e^{p_1 t} < p_1 e^{p_2 t}$

任一时刻u_c>0。非振荡放电过程。过阻尼状态。

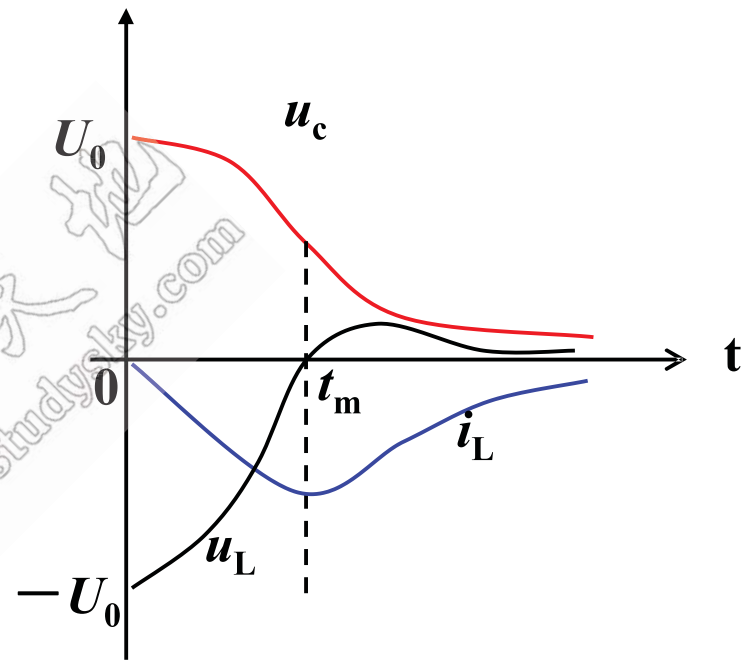
② 电流

$$i_L = C\frac{du_c}{dt} = \frac{Cp_1p_2U_0}{p_2-p_1}[e^{p_1t}-e^{p_2t}]$$

$$= \frac{U_0}{L(p_2-p_1)}[e^{p_1t}-e^{p_2t}] \quad t \geq 0$$

$$p_2 < p_1 \quad e^{p_2t} < e^{p_1t}$$

$$\therefore \quad i_L < 0$$



i_L在t_m处，有一极值。

③ $$u_L = L\frac{di_L}{dt} = \frac{U_0}{p_2-p_1}[p_1e^{p_1t}-p_2e^{p_2t}]$$

2、 $R < 2\sqrt{\frac{L}{C}}$ $\qquad$ $p_{1,2} = -\frac{R}{2L} \pm \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}}$

$p_1 p_2$ 为一对共轭复根。

令 $\frac{R}{2L} = \alpha$ $\qquad$ $\sqrt{\frac{1}{LC}} = \omega_0$

$$\sqrt{\frac{1}{LC} - (\frac{R}{2L})^2} = \sqrt{\omega_0^2 - \alpha^2} = \omega$$

则 $p_{1,2} = -\alpha \pm j\omega = -\omega_0 \angle \mp \varphi$

$$\varphi = tg^{-1}\frac{\omega}{\alpha}$$

$$u_c = \frac{U_0}{p_2 - p_1}[p_2 e^{p_1 t} - p_1 e^{p_2 t}]$$

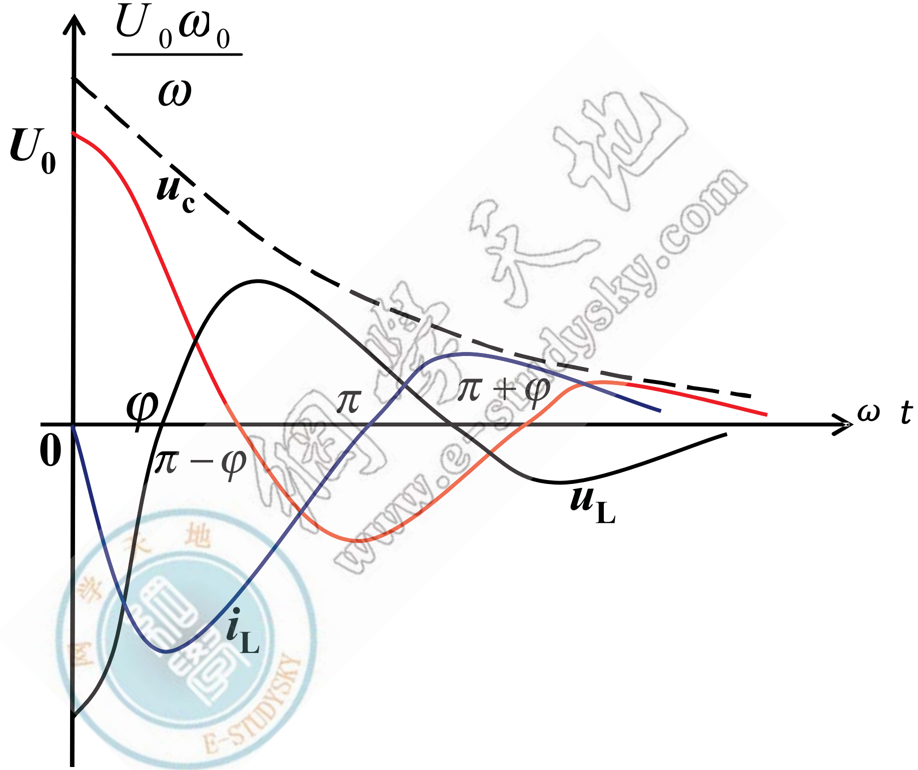
$$= \frac{U_0}{-2j\omega}[-\omega_0 e^{j\varphi} e^{(-\alpha + j\omega)t} + \omega_0 e^{-j\varphi} e^{(-\alpha - j\omega)t}]$$

$$= \frac{U_0 \omega_0}{\omega} e^{-\alpha t} [\frac{e^{j(\omega t + \varphi)} - e^{-j(\omega t + \varphi)}}{2j}]$$

$$= \frac{U_0 \omega_0}{\omega} e^{-\alpha t} \sin(\omega t + \varphi)$$

$$i_L = -\frac{U_0}{\omega L} e^{-\alpha t} \sin \omega t$$

$$u_L = \frac{U_0 \omega_0}{\omega} e^{-\alpha t} \sin(\omega t - \varphi)$$



α 为衰减常数。暂态过程为衰减振荡。欠阻尼状态。

特殊情况：当$R=0$时

$$\alpha = 0 \qquad \omega_0 = \omega = \frac{1}{\sqrt{LC}} \qquad \varphi = tg^{-1}\frac{\omega}{\alpha} = \frac{\pi}{2}$$

$$u_c = U_0 \sin(\omega t + \frac{\pi}{2})$$

$$i_L = -\frac{U_0}{\sqrt{\frac{L}{C}}}\sin \omega t$$

$$u_L = -U_0 \sin(\omega t - \frac{\pi}{2}) = -u_c$$

等幅振荡。无阻尼状态。

3、 $R = 2\sqrt{\dfrac{L}{C}}$

$$p_1 = p_2 = p = -\frac{R}{2L} = -\alpha$$ **重根**

$$u_c = \frac{U_0}{p_2 - p_1}[p_2 e^{p_1 t} - p_1 e^{p_2 t}]$$

设p_2为变量，p_1为定值

$$u_c = U_0 \lim_{p_2 \to p_1} \frac{e^{p_1 t} - p_1 t e^{p_2 t}}{1}$$

$$= U_0[e^{p_1 t} - p_1 t e^{p_2 t}] = U_0[1 + \alpha t]e^{-\alpha t}$$

$$i_L = C\frac{du_c}{dt} = -\frac{U_0}{L}te^{-\alpha t}$$

$$u_L = L\frac{di_L}{dt} = -U_0(1-\alpha t)e^{-\alpha t}$$

临介阻尼状态。

二阶电路分下列三种情况：

1） $p_1 \neq p_2$ （不相等实根）

零输入响应 $= K_1 e^{p_1 t} + K_2 e^{p_2 t}$

K_1、K_2为待定系数

2） $p_1 = \overset{*}{p_2}$ （共轭）

设 $p_1 = -\alpha + j\omega$

$p_2 = -\alpha - j\omega$

零输入响应 $= Ke^{-\alpha t}\sin(\omega t+\varphi)$

或

$= e^{-\alpha t}(K_1\sin\omega t + K_2\cos\omega t)$

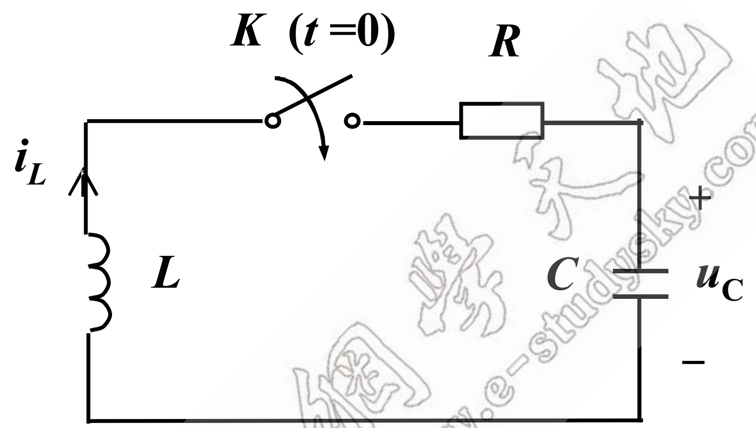
K,φ 或者K_1、K_2为待定系数

3) $p_1 = p_2 = p$ (重根)

零输入响应 $= (K_1 + K_2 t)e^{pt}$

K_1、K_2为待定系数

例1　求t≥0的u_C和i_L。已知$L=0.1\text{H}$，$R=2\Omega$，$C=0.02\text{F}$，$u_C(0_-)=30\text{V}$。



解：$i_L(0_+)=i_L(0_-)=0$　　$u_C(0_+)=u_C(0_-)=30V$

$$LC\frac{d^2u_C}{dt^2}+RC\frac{du_C}{dt}+u_C=0$$

$$\frac{d^2u_C}{dt^2}+20\frac{du_C}{dt}+500u_C=0$$

$$p^2+20p+500=0$$

$$p_{1,2}=-10\pm j20$$

$$u_C(t)=Ke^{-10t}\sin(20t+\varphi)\quad \text{V}$$

$$i_L=C\frac{du_C}{dt}=C[-10Ke^{-10t}\sin(20t+\varphi)$$

$$+20Ke^{-10t}\cos(20t+\varphi)]\quad \text{A}$$

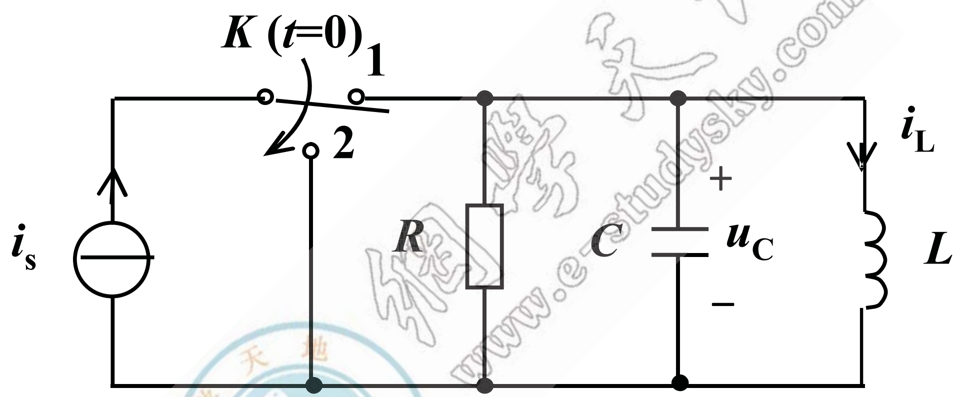
代入初始值 $\begin{cases} 0 = -10K\sin\varphi + 20K\cos\varphi \\ 30 = K\sin\varphi \end{cases}$

解得 $\begin{cases} K = 33.54 \\ \varphi = 63.435° \end{cases}$

$$u_C(t) = 33.54e^{-10t}\sin(20t + 63.435°)V \qquad t \ge 0$$

$$i_L = C\frac{du_C}{dt} = -15e^{-10t}\sin 20t \quad A \qquad t \ge 0$$

例2 已知 $i_S = 3A, R = 2\Omega, L = \frac{1}{6}H, C = 0.01F$，电路原来处于稳态，$t=0$时开关由位置1换到位置2，求$t \geqslant 0$的$u_C$和$i_L$。



解：$i_L(0_+) = i_L(0_-) = i_s = 3\ \text{A}$

$$u_C(0_+) = u_C(0_-) = 0$$

$$LC\frac{d^2 i_L}{dt^2}+\frac{L}{R}\frac{di_L}{dt}+i_L=0$$

$$p^2+50p+600=0$$

解得 $p_1=-20$ ，$p_2=-30$

$$i_L=K_1e^{-20t}+K_2e^{-30t} \qquad t\geq 0$$

$$u_C=L\frac{di_L}{dt}=L[-20K_1e^{-20t}-30K_2e^{-30t}] \qquad t\geq 0$$

$$\begin{cases} 3 = K_1 + K_2 \\ 0 = -20K_1 - 30K_2 \end{cases}$$

$$\begin{cases} K_1 = 9 \\ K_2 = -6 \end{cases}$$

所以 $i_L = 9e^{-20t} - 6e^{-30t}\ \mathrm{A} \quad t \geq 0$

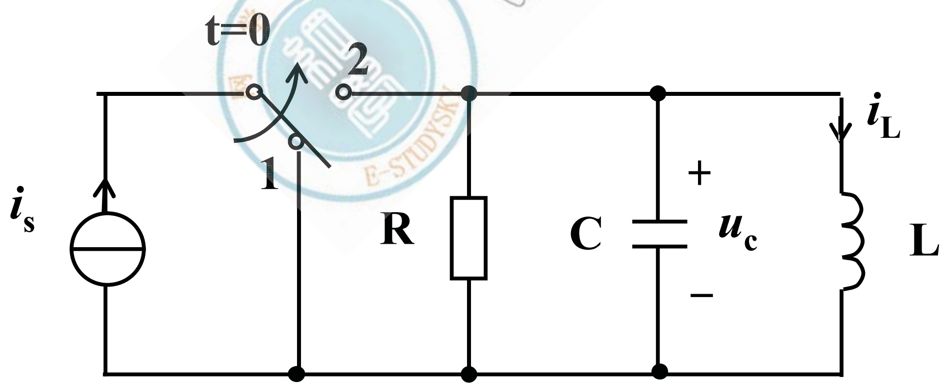
$$u_C = L\frac{di_L}{dt} = -30e^{-20t} + 30e^{-30t}\ \mathrm{V} \quad t \geq 0$$

§12-2 二阶电路的零状态响应和全响应

一、零状态响应：

零状态网络 $[\,u_c(0_-)=0, i_L(0_-)=0\,]$ 对外加激励产生的响应。

例3：$t<0$时电路处于稳态，求$t\geqslant 0$时的电感电流。



解： $$LC\frac{d^2 i_L}{dt^2}+\frac{L}{R}\frac{di_L}{dt}+i_L=i_s$$

$$i_L(t)=i_{Lp}(t)+i_{Lh}(t)$$

$i_{Lp}(t)$ 取决于激励的形式

$i_{Lh}(t)$ 其形式与零输入响应相同

1) $p_1 \neq p_2$（不相等实根）

$$i_{Lh}(t)=K_1 e^{p_1 t}+K_2 e^{p_2 t}$$

2） $p_1 = \overset{*}{p}_2$ （共轭）

设 $p_1 = -\alpha + j\omega$

$$i_{Lh}(t) = Ke^{-\alpha t}\sin(\omega t + \varphi)$$

或

$$= e^{-\alpha t}(K_1 \sin \omega t + K_2 \cos \omega t)$$

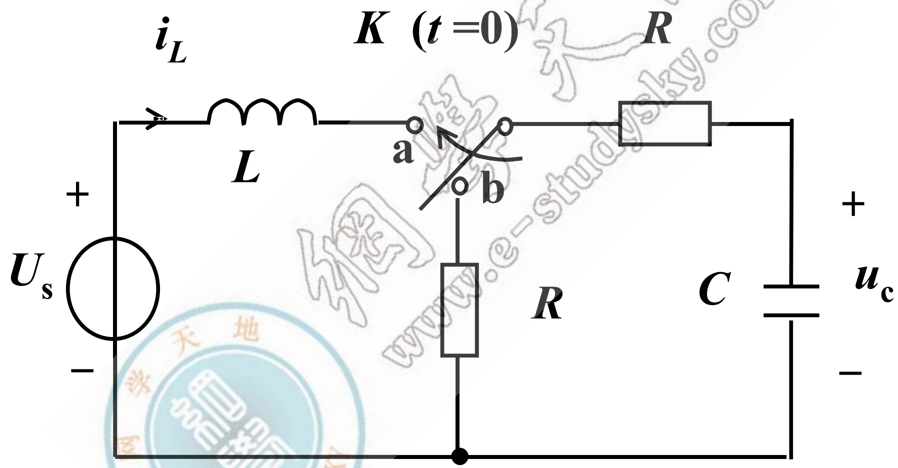
3） $p_1 = p_2 = p$ （重根）

$$i_{Lh}(t) = (K_1 + K_2 t)e^{pt}$$

注意：零初值代入i_L而非i_{Lh}

例4：图示电路，t<0时电路处于稳态。t=0时开关K由位置b换到位置a。求t≥0的u_C和i_L。已知

$$U_s = 4V, L = 1H, C = 1F, R = 2\Omega。$$



解： $u_c(0_+) = u_c(0_-) = 0$ $\qquad i_L(0_+) = i_L(0_-) = 0$

$$LC\frac{d^2u_c}{dt^2}+RC\frac{du_c}{dt}+u_c=U_s$$

$$p^2+2p+1=0 \qquad p_{1,2}=-1$$

$$u_{ch}(t)=(K_1+K_2t)e^{-t} \qquad u_{cp}(t)=4V$$

$$u_c(t)=(K_1+K_2t)e^{-t}+4$$

代入初值 $u_c(0_+)=0$

$$\left.\frac{du_c}{dt}\right|_{0_+}=\frac{i_L(0_+)}{C}=0$$

$$\begin{cases} 0 = K_1 + 4 \\ 0 = K_2 - K_1 \end{cases} \qquad \begin{cases} K_1 = -4 \\ K_2 = -4 \end{cases}$$

$$u_c(t) = (-4 - 4t)e^{-t} + 4 \ \ V \quad t \geq 0$$

$$i_L(t) = C\frac{du_c(t)}{dt} = 4te^{-t} \ A \quad t \geq 0$$

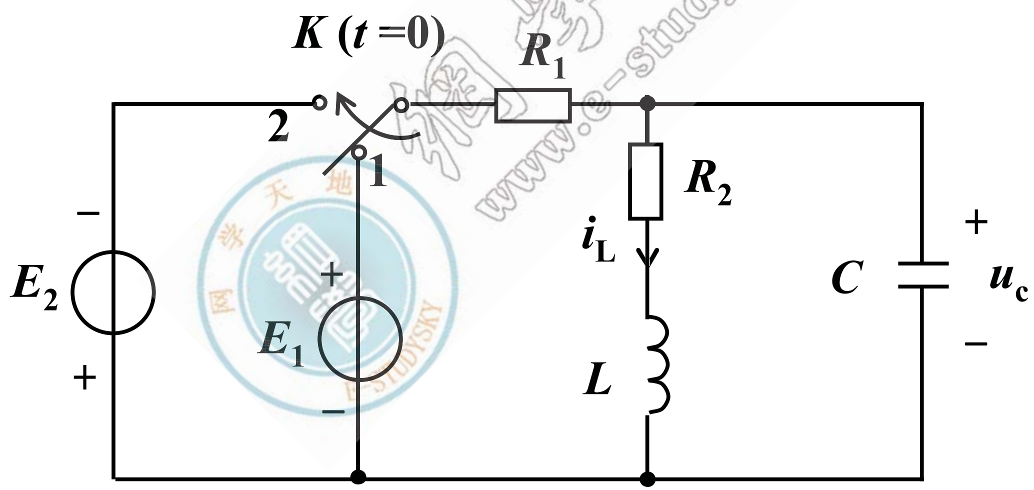
二、全响应

两种求法：

（1）全响应 = 零输入响应 + 零状态响应

（2）与零状态响应求法相同

例5：图示电路$t<0$时电路处于稳态，$t=0$时开关K由位置1换到位置2，求换位后电容电压的变化规律。其中$E_1=10V$，$E_2=20V$，$R_1=4\Omega$，$R_2=6\Omega$，$L=1H$，$C=0.25F$。



解：t<0时 $i_L(0_-)=\dfrac{E_1}{R_1+R_2}=1A$

$$u_c(0_-)=\frac{R_2}{R_1+R_2}E_1=6V$$

$$i_L(0_+)=i_L(0_-)=1A$$

$$u_c(0_+)=u_c(0_-)=6V$$

$$\left.\frac{du_c}{dt}\right|_{0_+}=\frac{1}{C}[-i_L(0_+)-\frac{u_c(0_+)+E_2}{R_1}]=-30$$

t ≥ 0时，依KCL得 $C\dfrac{du_c}{dt}+i_L+\dfrac{u_c+E_2}{R_1}=0$

即 $$i_L = -C\frac{du_c}{dt} - \frac{u_c + E_2}{R_1}$$

依KVL $$u_c - L\frac{di_L}{dt} - R_2 i_L = 0$$

$$\frac{d^2u_c}{dt^2} + 7\frac{du_c}{dt} + 10u_c = -120$$

$$p^2 + 7p + 10 = 0 \qquad p_{1,2} = -2, -5$$

$$u_{ch} = K_1e^{-2t} + K_2e^{-5t}$$

$$u_{cp} = -12V$$

$$u_c = u_{ch} + u_{cp} = K_1 e^{-2t} + K_2 e^{-5t} - 12$$

代入初始值

$$\begin{cases} K_1 + K_2 - 12 = 6 \\ -2K_1 - 5K_2 = -30 \end{cases}$$

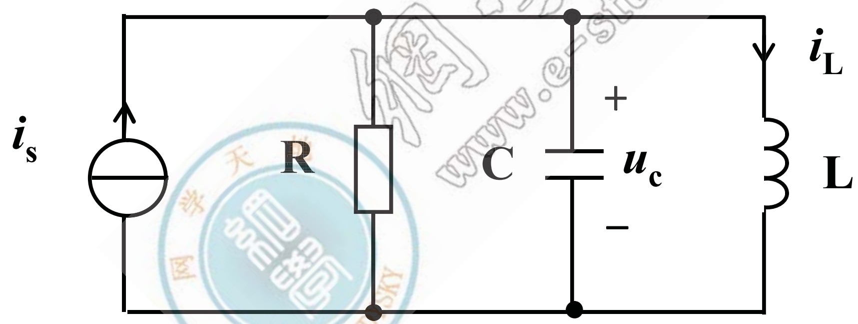
$$\begin{cases} K_1 = 20 \\ K_2 = -2 \end{cases}$$

$$u_c = 20e^{-2t} - 2e^{-5t} - 12\ V \quad t \geq 0$$

§12-3 二阶电路的阶跃响应和冲激响应

一、阶跃响应

例：电路如图。已知 $u_c(0_-)=0$，$i_L(0_-)=0$，$R=2\Omega$，$L=0.02\text{H}$，$C=1\text{F}$，$i_s=5\varepsilon(t)\text{A}$。求$i_L(t)$。



解：

$$LC\frac{d^2 i_L}{dt^2}+\frac{L}{R}\frac{di_L}{dt}+i_L=i_s$$

$$0.02p^2 + 0.01p + 1 = 0$$

$$p_{1,2} = -0.25 \pm j7.07$$

$$i_{Lh} = Ke^{-0.25t}\sin(7.07t + \varphi)$$

$$i_{Lp} = 5A$$

$$i_L = i_{Lh} + i_{Lp} = Ke^{-0.25t}\sin(7.07t + \varphi) + 5$$

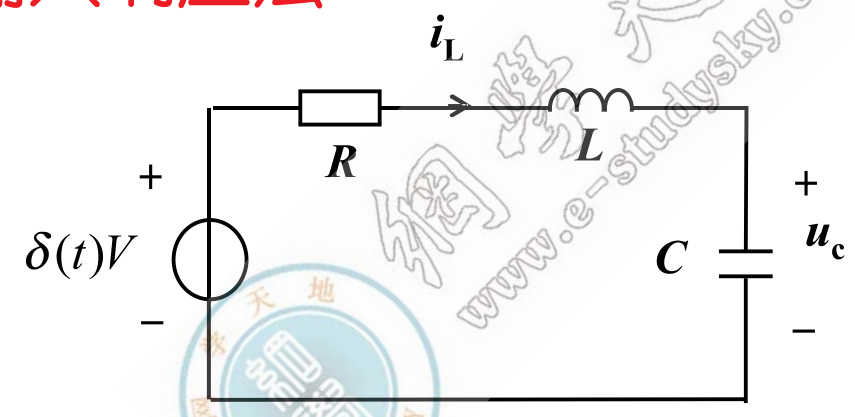
代入初始值 $\begin{cases} 0 = K\sin\varphi + 5 \\ 0 = -0.25K\sin\varphi + 7.07K\cos\varphi \end{cases}$

$$\begin{cases} K = -5 \\ \varphi = 90^\circ \end{cases}$$

$$i_L = [5 - 5e^{-0.25t}\cos 7.07t]\varepsilon(t) \qquad A$$

二、冲激响应

1、零输入响应法



$$LC\frac{d^2u_c}{dt^2} + RC\frac{du_c}{dt} + u_c = \delta(t)$$

$$LC\frac{d^2u_c}{dt^2}+RC\frac{du_c}{dt}+u_c=\delta(t)$$

$$LC\left[\frac{du_c}{dt}\bigg|_{t=0_+}-\frac{du_c}{dt}\bigg|_{t=0_-}\right]+RC[u_c(0_+)-u_c(0_-)]+\int_{0_-}^{0_+}u_c dt=1$$

$\int_{0_-}^{0_+}u_c dt$ ： u_c不可能为冲激函数

$$\therefore \quad \int_{0_-}^{0_+}u_c dt=0$$

$u_c(0_+)$ ： u_c也不可能在t=0时跳变（阶跃）

$$\therefore \quad u_c(0_+)=u_c(0_-)=0$$

$$LC\left[\left.\frac{du_c}{dt}\right|_{t=0_+}-\left.\frac{du_c}{dt}\right|_{t=0_-}\right]+RC[u_c(0_+)-u_c(0_-)]+\int_{0_-}^{0_+}u_c dt=1$$

$$\therefore \qquad LC\left.\frac{du_c}{dt}\right|_{t=0_+}=1$$

得初始条件：

$$u_c(0_+)=0$$

$$\left.\frac{du_c}{dt}\right|_{t=0_+}=\frac{1}{LC}$$

$$LCp^2 + RCp + 1 = 0$$

以特征根为不等实根为例 $p_1 \neq p_2$

$$u_c = K_1 e^{p_1 t} + K_2 e^{p_2 t}$$

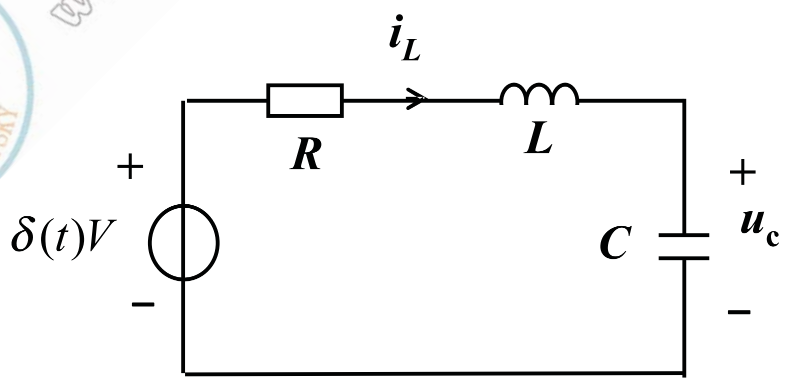
$$\frac{du_c}{dt} = K_1 p_1 e^{p_1 t} + K_2 p_2 e^{p_2 t}$$

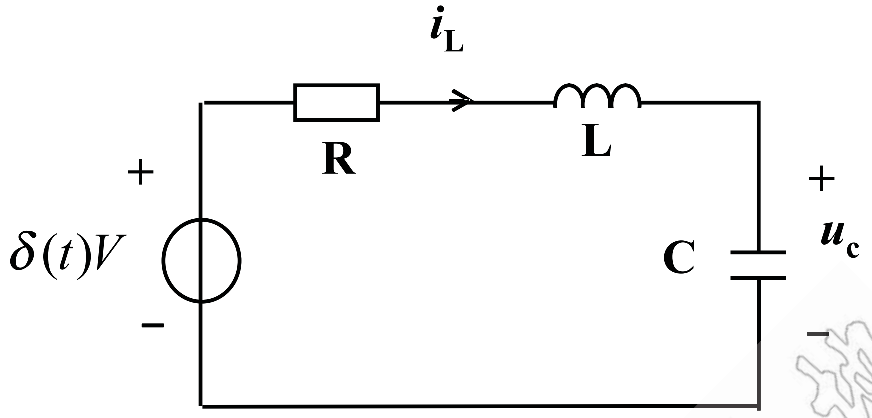
代入初值 $\begin{cases} K_1 + K_2 = 0 \\ K_1 p_1 + K_2 p_2 = \dfrac{1}{LC} \end{cases}$

$$\begin{cases} K_1 = \dfrac{-\frac{1}{LC}}{p_2 - p_1} \\ K_2 = \dfrac{\frac{1}{LC}}{p_2 - p_1} \end{cases}$$

$$u_c = \frac{-\frac{1}{LC}}{p_2 - p_1}\left[e^{p_1 t} - e^{p_2 t}\right] \cdot \varepsilon(t)$$

2、 $h(t) = \dfrac{ds(t)}{dt}$ 法





$$s(t)=(1+K_1e^{p_1t}+K_2e^{p_2t})\cdot\varepsilon(t)$$

代入零初始条件：

$$\begin{cases}1+K_1+K_2=0\\ K_1p_1+K_2p_2=0\end{cases}$$

$$\begin{cases}K_1=\dfrac{-p_2}{p_2-p_1}\\ K_2=\dfrac{p_1}{p_2-p_1}\end{cases}$$

$$s(t)=(1+\frac{-p_2}{p_2-p_1}e^{p_1t}+\frac{p_1}{p_2-p_1}e^{p_2t})\varepsilon(t)$$

$$u_c(t)=h(t)=\frac{ds(t)}{dt}=\frac{-p_2p_1}{p_2-p_1}(e^{p_1t}-e^{p_2t})\varepsilon(t)$$

而 $$p_1p_2=\frac{1}{LC}$$

$$\therefore \quad u_c(t)=h(t)=\frac{-\frac{1}{LC}}{p_2-p_1}(e^{p_1t}-e^{p_2t})\varepsilon(t)$$