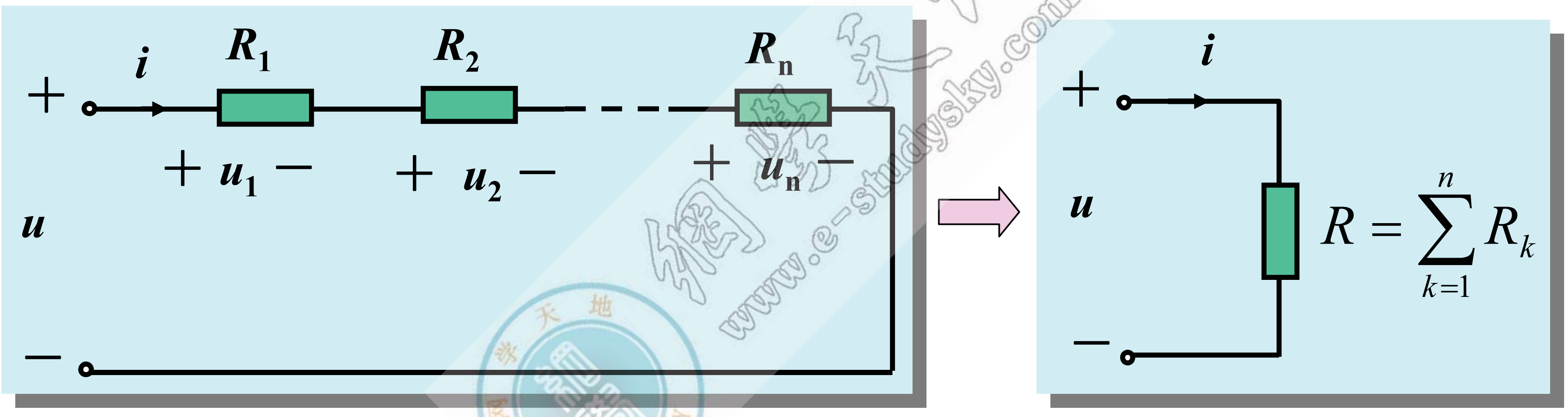
第二章　电阻电路的等效变换

§2-1 电阻的串联、并联

一、电阻的串联



$$KVL\quad u=u_1+u_2+\cdots+u_n=\sum_{k=1}^{n}u_k$$

所以
$$u=R_1i+R_2i+\cdots+R_ni$$
$$=(R_1+R_2+\cdots+R_n)i=Ri$$

R:等效电阻、输入电阻

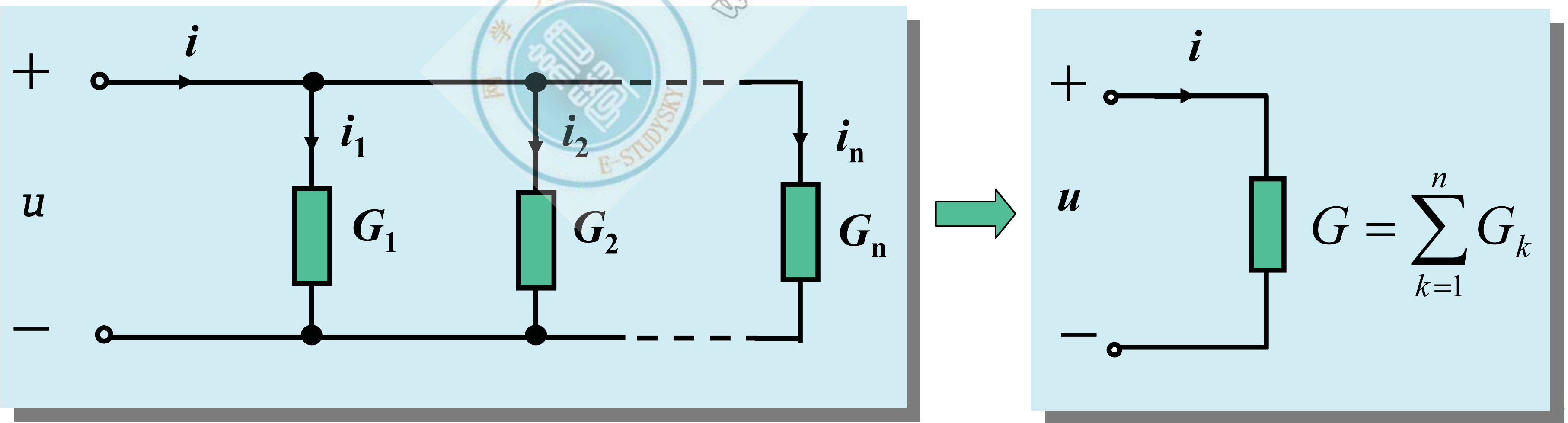
分压：$u_k = R_k i = \dfrac{R_k}{R} u$

电路吸收的总功率：

$$p = ui = (u_1 + u_2 + \cdots + u_n) i$$

$$= p_1 + p_2 + \cdots + p_n = \sum_{k=1}^{n} p_k$$

二、电阻的并联



KCL $$i = i_1 + i_2 + \cdots + i_n = \sum_{k=1}^{n} i_k$$

$$i = (G_1 + G_2 + \cdots + G_n)u = Gu$$

$$G = G_1 + G_2 + \cdots + G_n = \sum_{k=1}^{n} G_k$$

G：等效电导、输入电导

分流： $$i_k = G_k u = \frac{G_k}{G} i$$

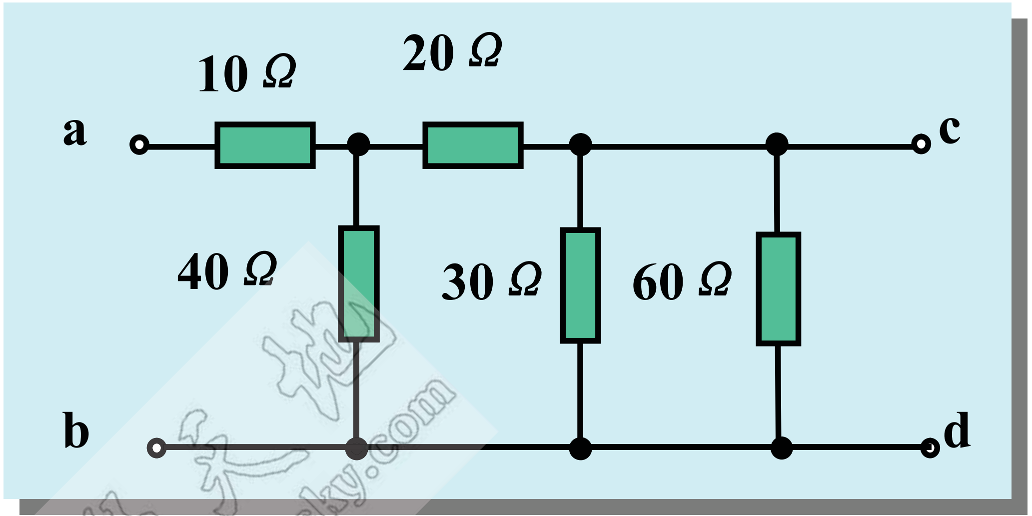
电路吸收的总功率： $$p = ui = (i_1 + i_2 + \cdots + i_n)u$$

$$= p_1 + p_2 + \cdots + p_n = \sum_{k=1}^{n} p_k$$

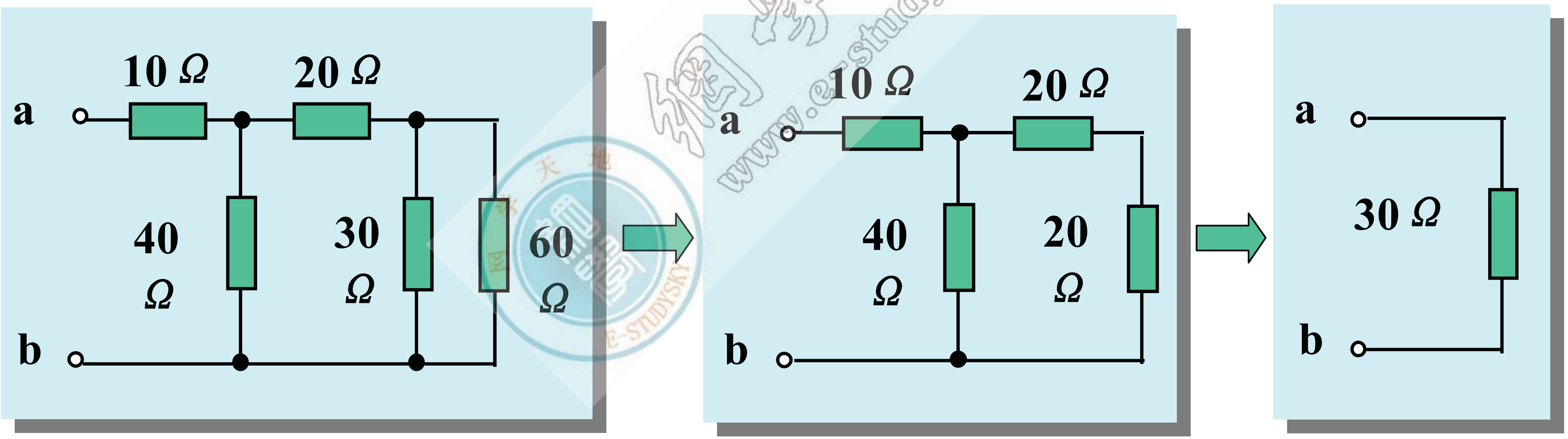
例2—1 电路如图。求：

（1）R_{ab}

（2）R_{cd}

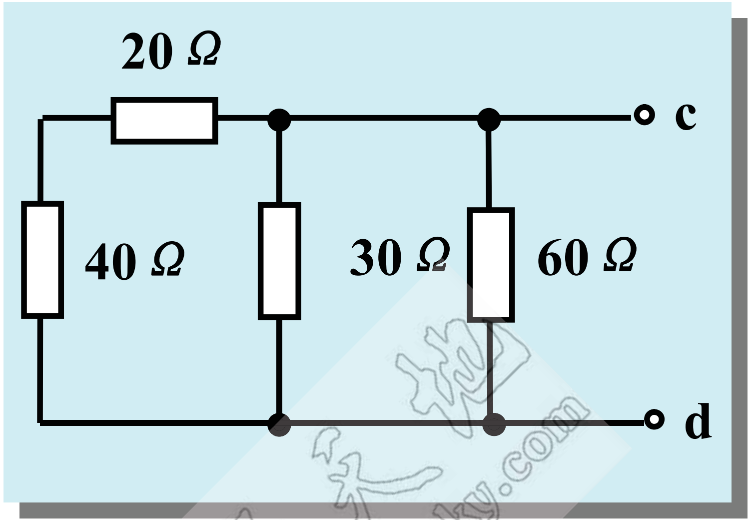


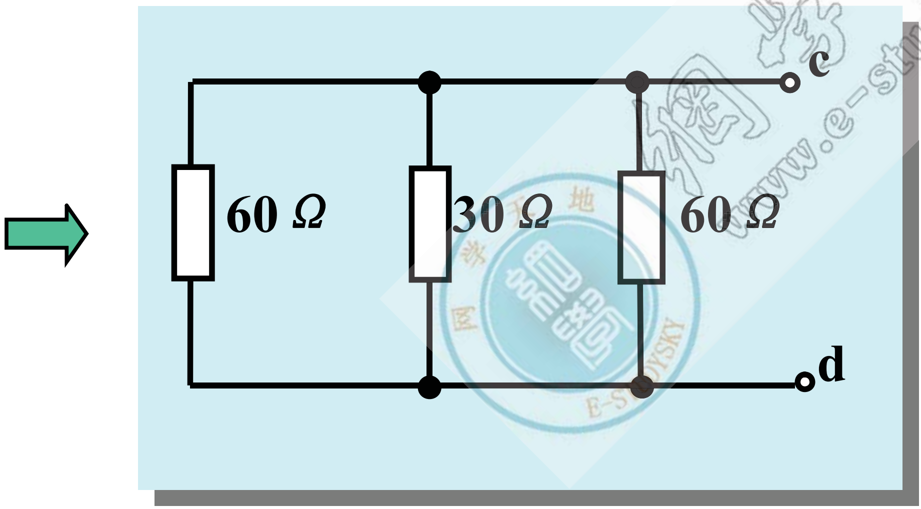
解：（1）求解R_{ab}

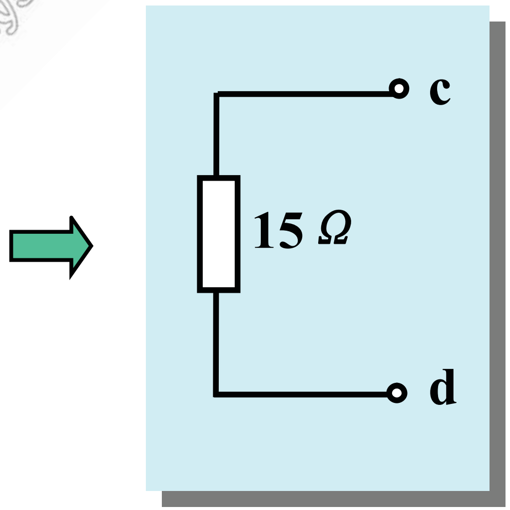


$$\therefore R_{ab} = 30\Omega$$

(2) 求R_{cd}

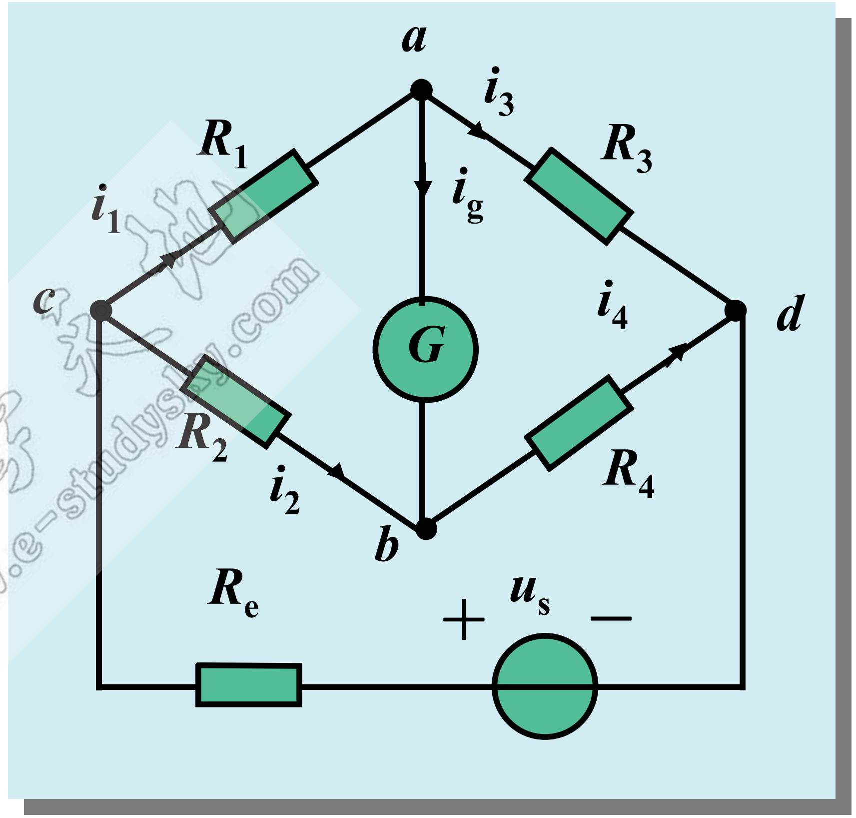






$$\therefore R_{cd} = 15\Omega$$

例2—2 求惠斯通电桥的平衡条件



解：电桥平衡时

$$i_g = 0, i_1 = i_3, i_2 = i_4$$

另外 $u_{ab} = 0$

所以 $u_{ca} = u_{cb}$

即 $R_1 i_1 = R_2 i_2$

$u_{ad} = u_{bd}$ 即 $R_3 i_3 = R_4 i_4$

故电桥平衡的条件：$\dfrac{R_1}{R_3} = \dfrac{R_2}{R_4}$ 即 $R_1 R_4 = R_2 R_3$

§2-2 电阻的三角形（Δ）联接与星形（Y）联接

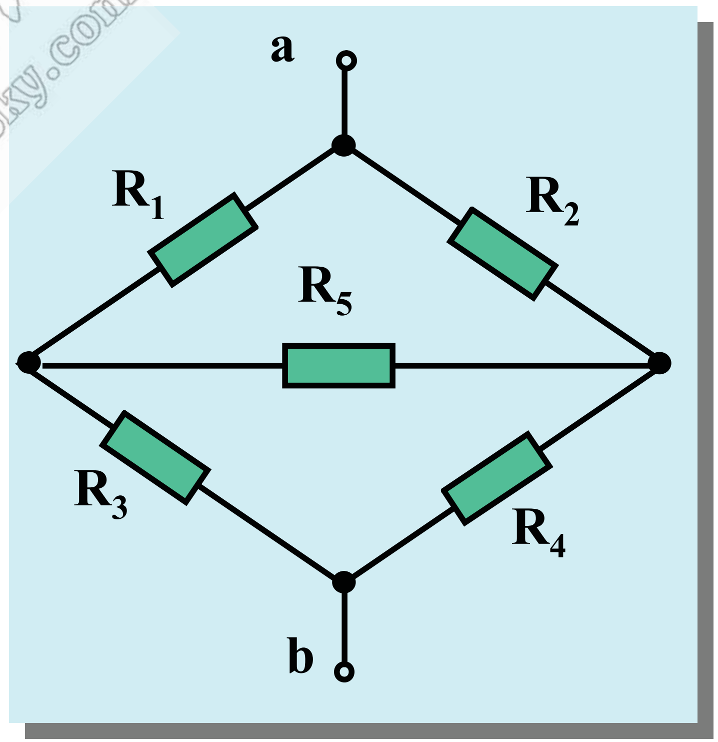
一、电阻的三角形（Δ）与星形（Y）联接

三角形（Δ）联接:

如 $R_1R_2R_5$、 $R_3R_4R_5$

星形（Y）联接:

如 $R_1R_5R_3$、 $R_2R_5R_4$



二、Δ联接与Y联接的等效变换

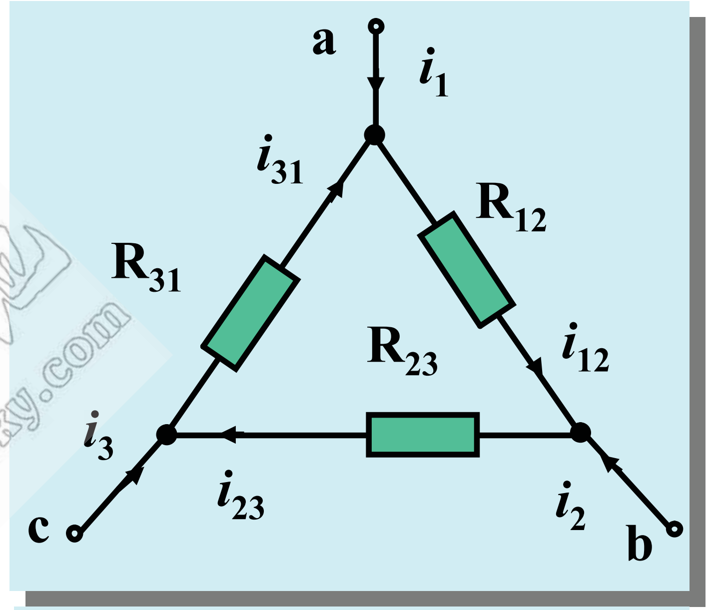
$Y \rightarrow \Delta$

已知R_1、R_2、R_3求R_{12}、R_{23}、R_{31}

根据KCL $i_1 = i_{12} - i_{31} = \dfrac{u_{ab}}{R_{12}} - \dfrac{u_{ca}}{R_{31}}$

$$i_2 = i_{23} - i_{12} = \frac{u_{bc}}{R_{23}} - \frac{u_{ab}}{R_{12}}$$

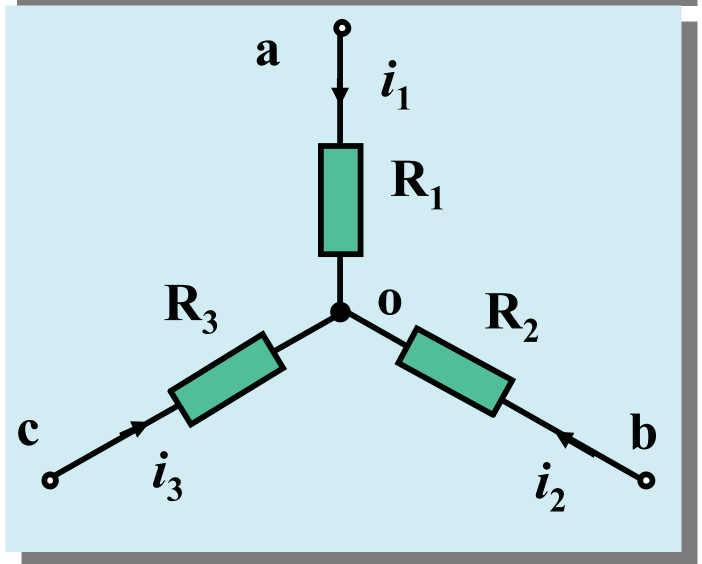
$$i_3 = i_{31} - i_{23} = \frac{u_{ca}}{R_{31}} - \frac{u_{bc}}{R_{23}}$$



根据KVL $u_{ab} = R_1 i_1 - R_2 i_2$

$$u_{bc} = R_2 i_2 - R_3 i_3$$

$$u_{ca} = R_3 i_3 - R_1 i_1 = -(u_{ab} + u_{bc})$$



另根据KCL $i_1 + i_2 + i_3 = 0$

$$i_1 = \frac{u_{ab}}{\dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}} - \frac{u_{ca}}{\dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}}$$

$$i_2 = \frac{u_{bc}}{\dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}} - \frac{u_{ab}}{\dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}}$$

$$i_3 = \frac{u_{ca}}{\dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}} - \frac{u_{bc}}{\dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}}$$

$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_{31} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

同理

$$R_1 = \frac{R_{31}R_{12}}{R_{12} + R_{23} + R_{31}}$$

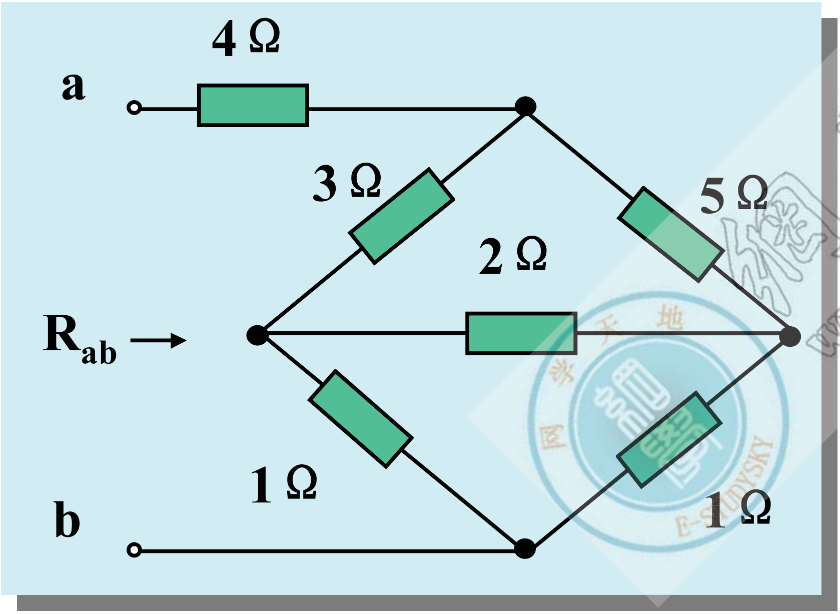
$$R_2 = \frac{R_{12}R_{23}}{R_{12} + R_{23} + R_{31}}$$

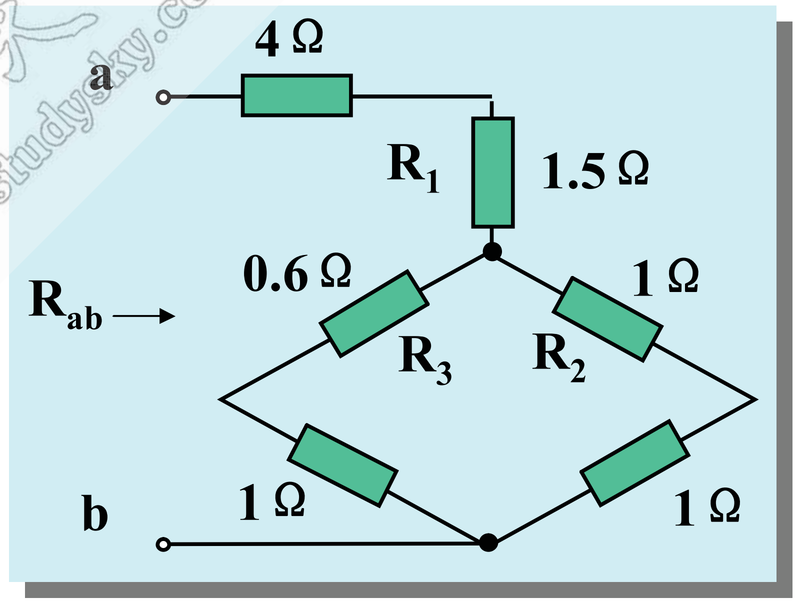
$$R_3 = \frac{R_{23}R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_{12}=R_{23}=R_{31}=R_{\Delta}\text{时，}R_1=R_2=R_3=R_Y$$

且 $$R_Y=\frac{1}{3}R_{\Delta}$$

例2—3：求R_{ab}。



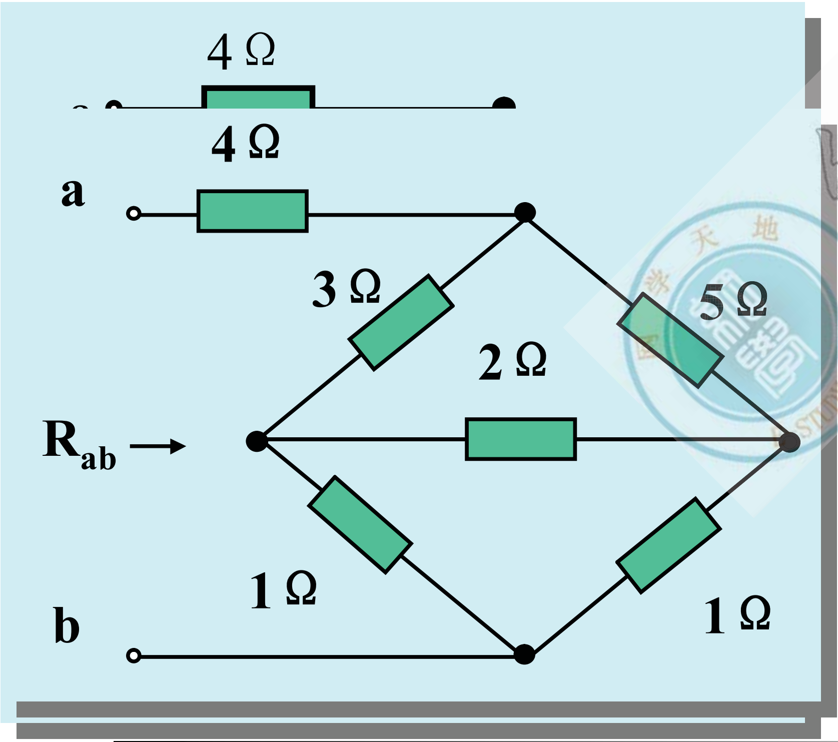


解：$R_1=\dfrac{3\times5}{3+5+2}=1.5\Omega$ $\qquad R_2=\dfrac{2\times5}{3+5+2}=1\Omega$

$$R_3 = \frac{2\times 3}{3+5+2} = 0.6\Omega$$

$$R_{ab} = 4 + 1.5 + \frac{2\times 1.6}{2+1.6} = 5.5 + 0.89 = 6.39\Omega$$

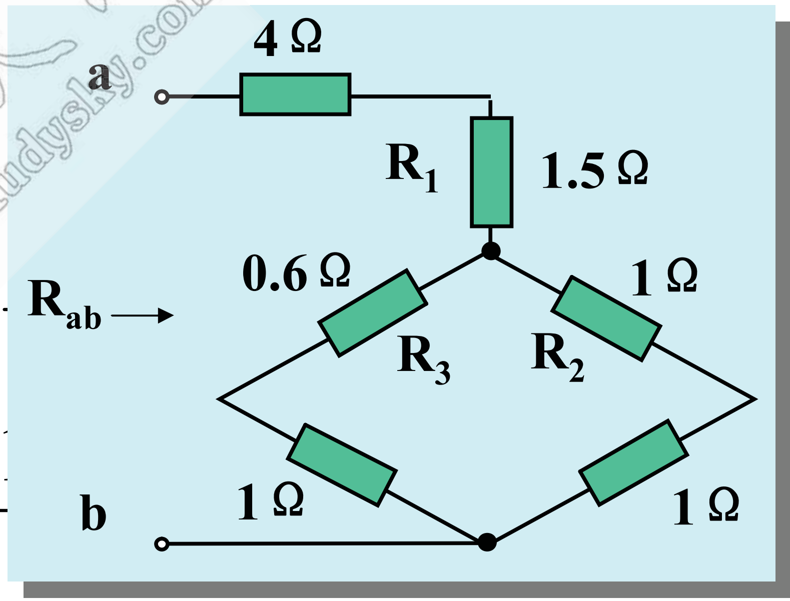
另解 Y → Δ 变换



$R_1 =$

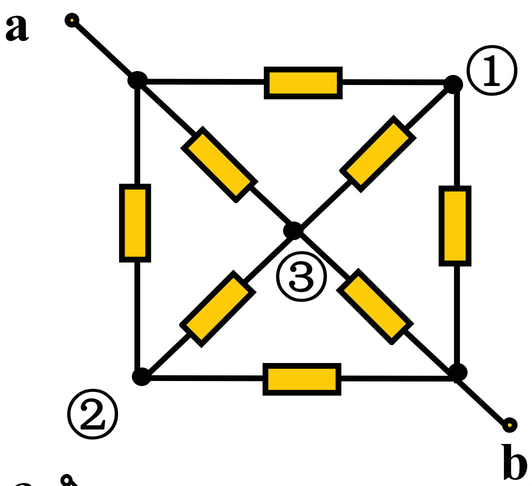
$R_2 =$

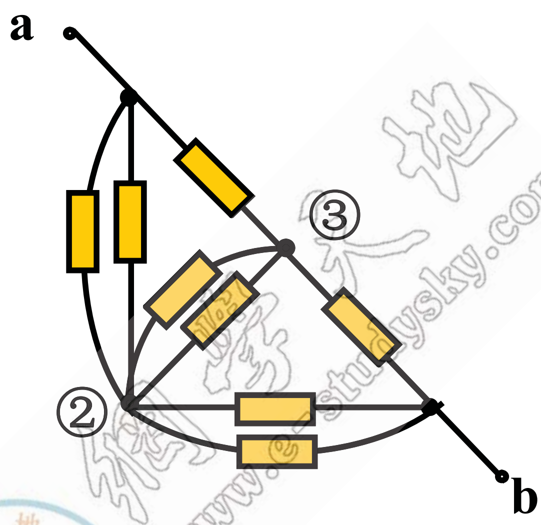
$R_3 =$

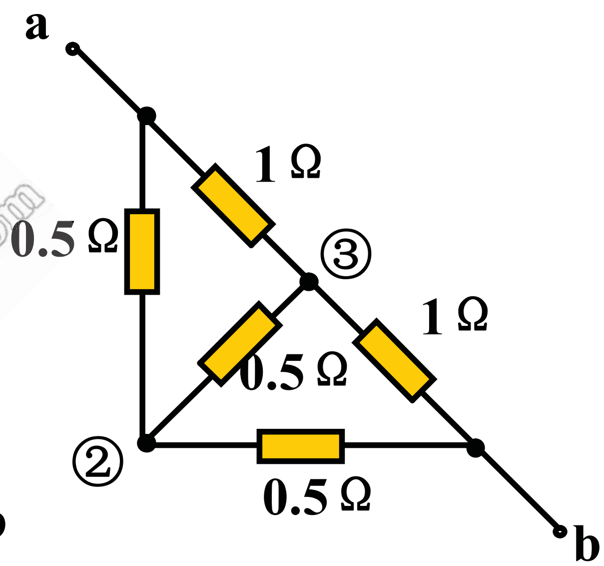


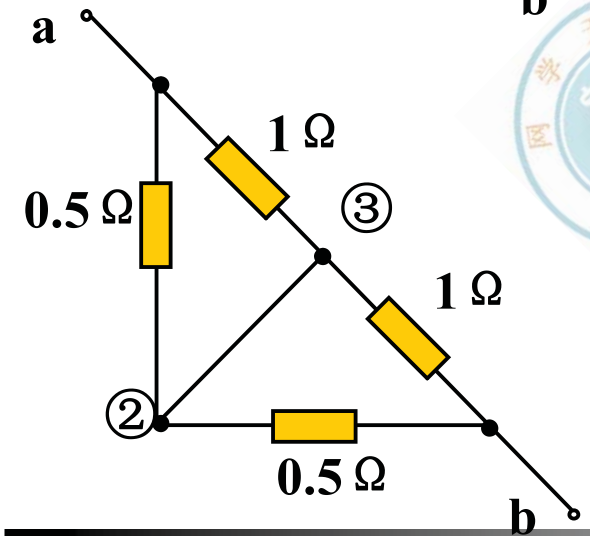
$$R_{ab} = 4 + \frac{5.5\times 4.224}{5.5+4.224} = 6.39\Omega$$

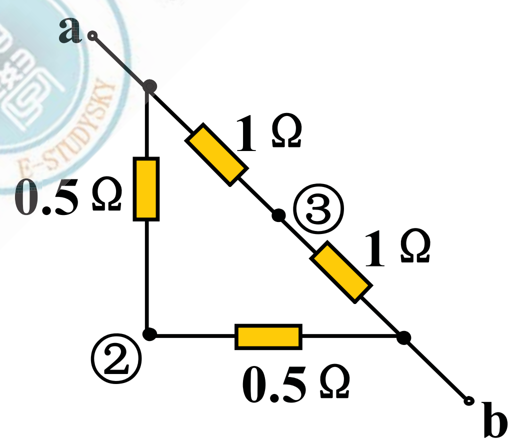
例2—4 电路如图，各电阻的阻值均为1Ω。试求ab间的等效电阻。





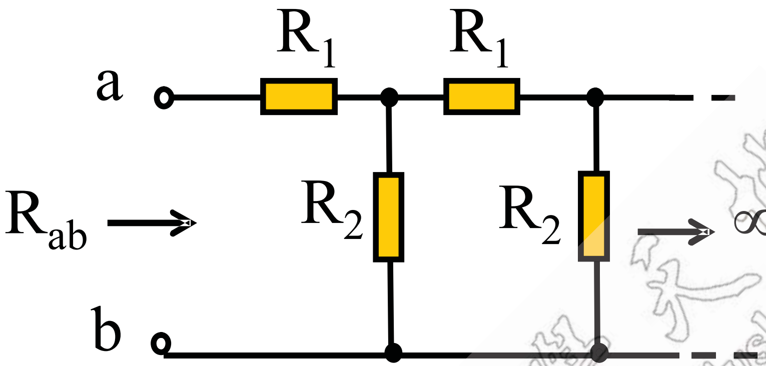


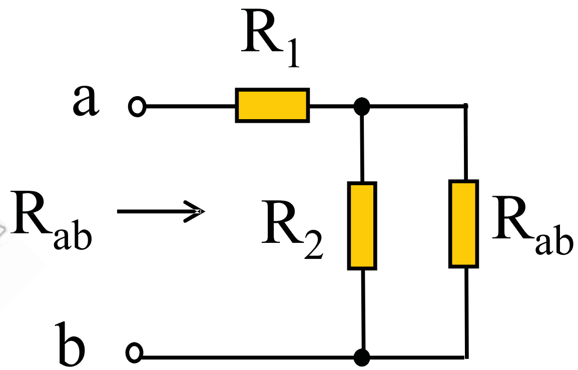




$$R_{ab}=\frac{2}{3}\Omega$$

例2—5 图示电路为一个无限链形网络，每个环节由R_1与R_2组成，求输入电阻R_{ab}。





解：$R_{ab}=R_1+\dfrac{R_2R_{ab}}{R_2+R_{ab}}$，$R_{ab}{}^2-R_1R_{ab}-R_1R_2=0$

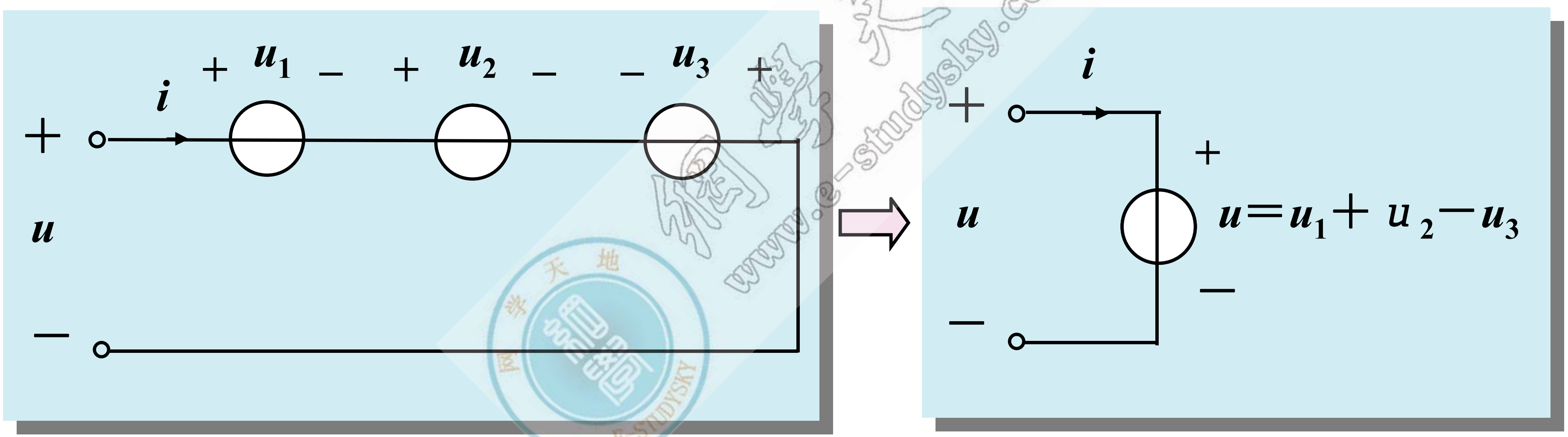
$$R_{ab}=\frac{R_1\pm\sqrt{R_1{}^2+4R_1R_2}}{2}$$

由于$R_{ab}>0$, 所以 $R_{ab}=\dfrac{R_1+\sqrt{R_1{}^2+4R_1R_2}}{2}$

§2-3 电源的串联、并联

一、电压源的串联与并联

电压源的串联:

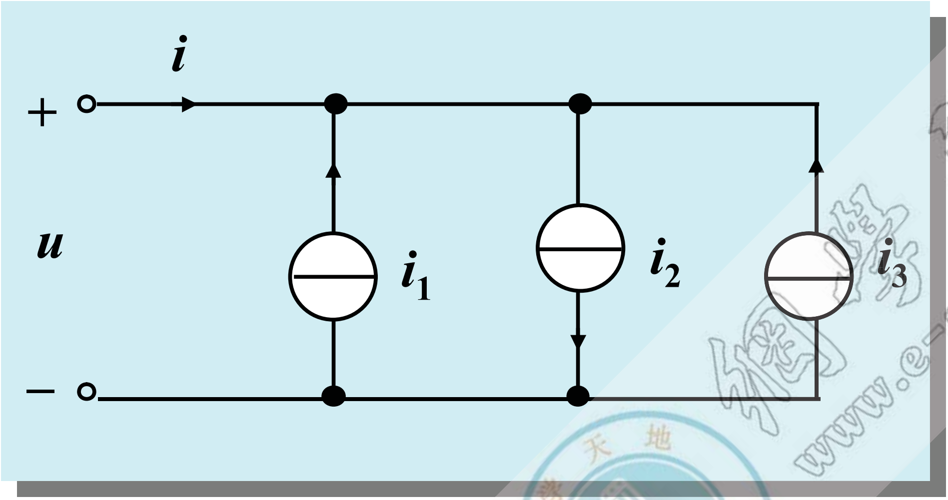


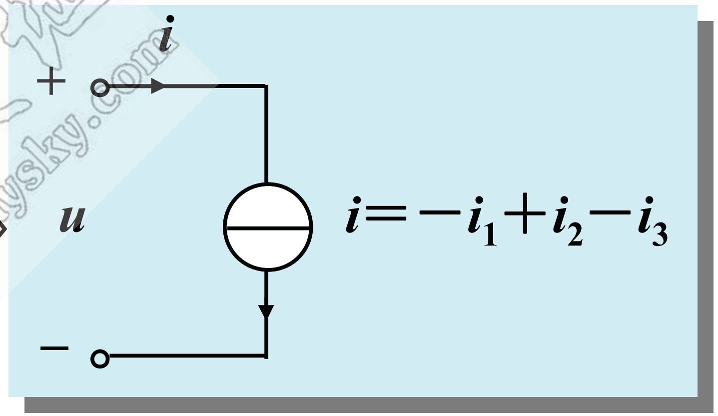
根据KVL $u = u_1 + u_2 - u_3$

电压源的并联:大小相等、方向相同

二、电流源的并联与串联

电流源的并联：

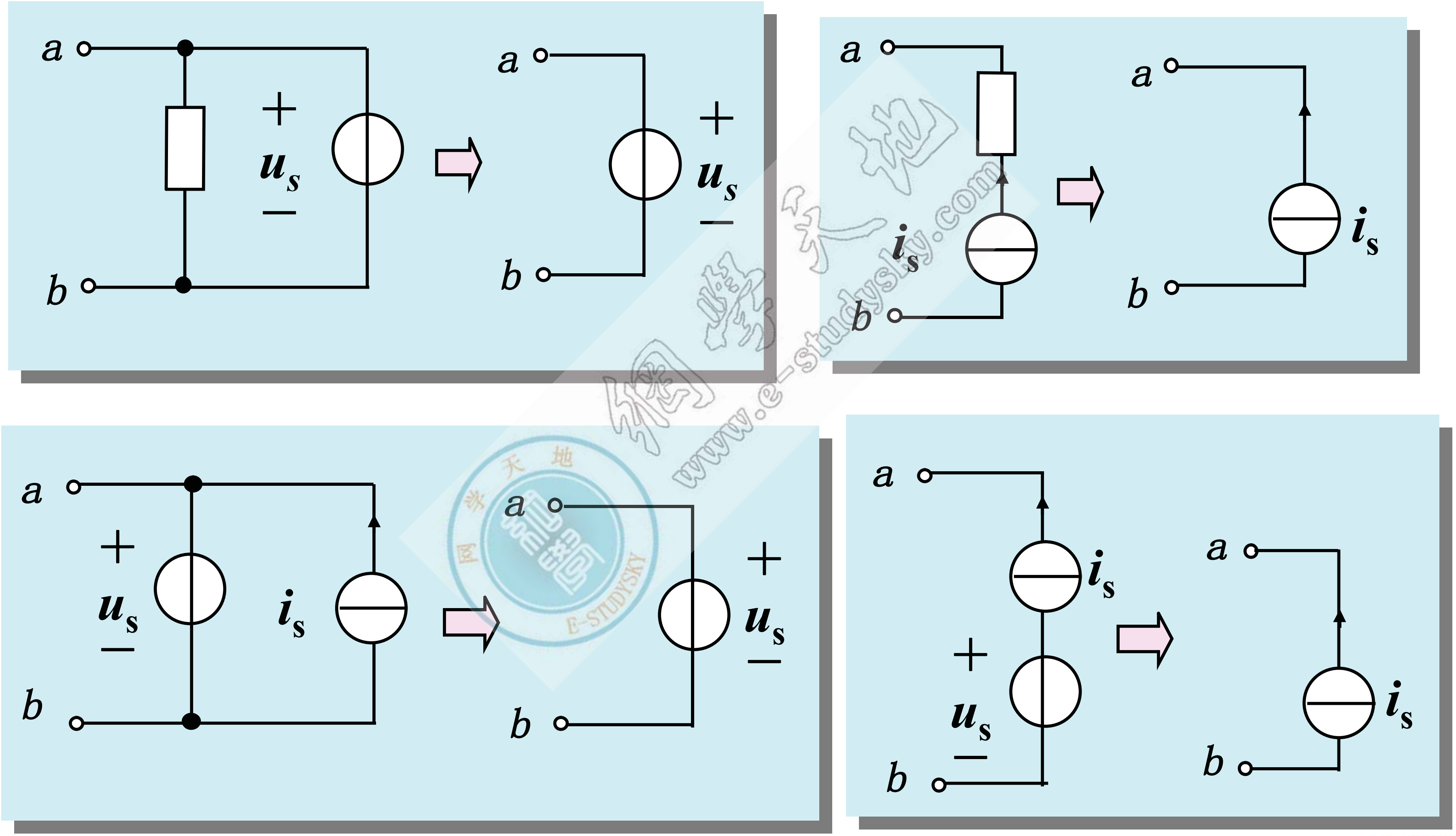




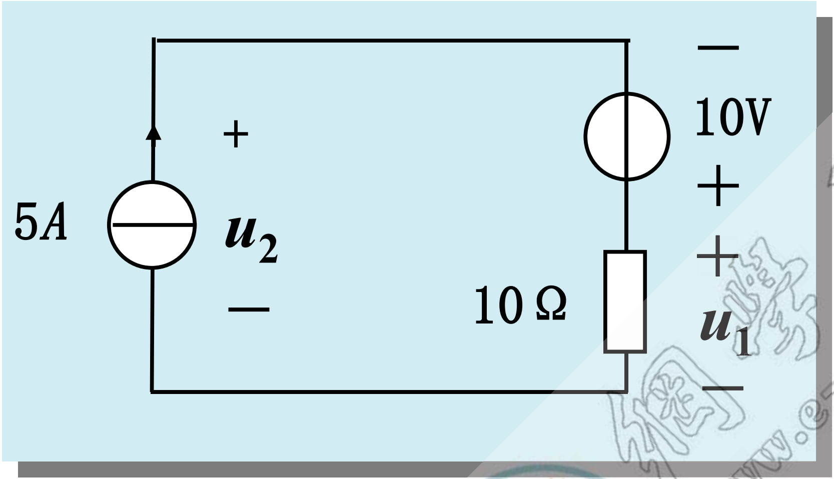
根据KCL $\quad i = -i_1 + i_2 - i_3$

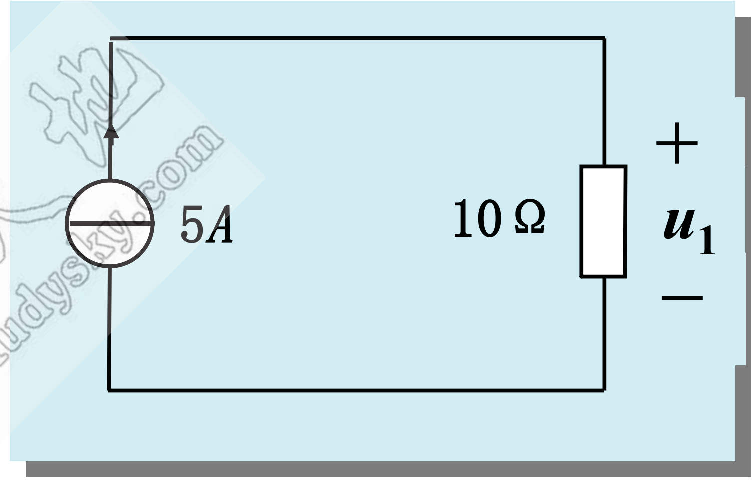
电流源的串联:大小相等、方向相同

对外电路而言：



例2-6 求电阻和电流源上的电压。



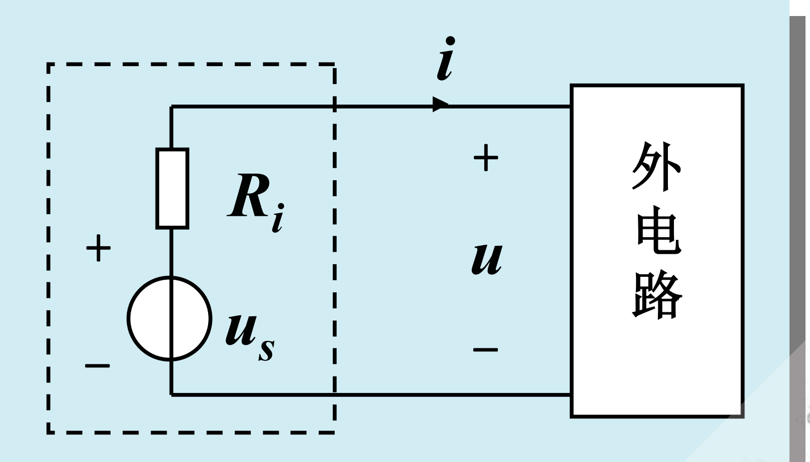


解：

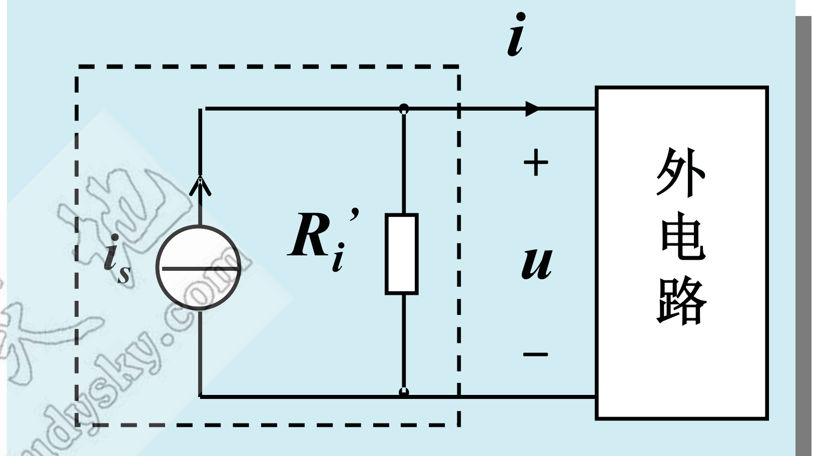
$$u_1 = 5\times 10 = 50V$$

$$u_2 = -10 + u_1 = -10 + 50 = 40V$$

§2-4 电源的等效变换



(a)

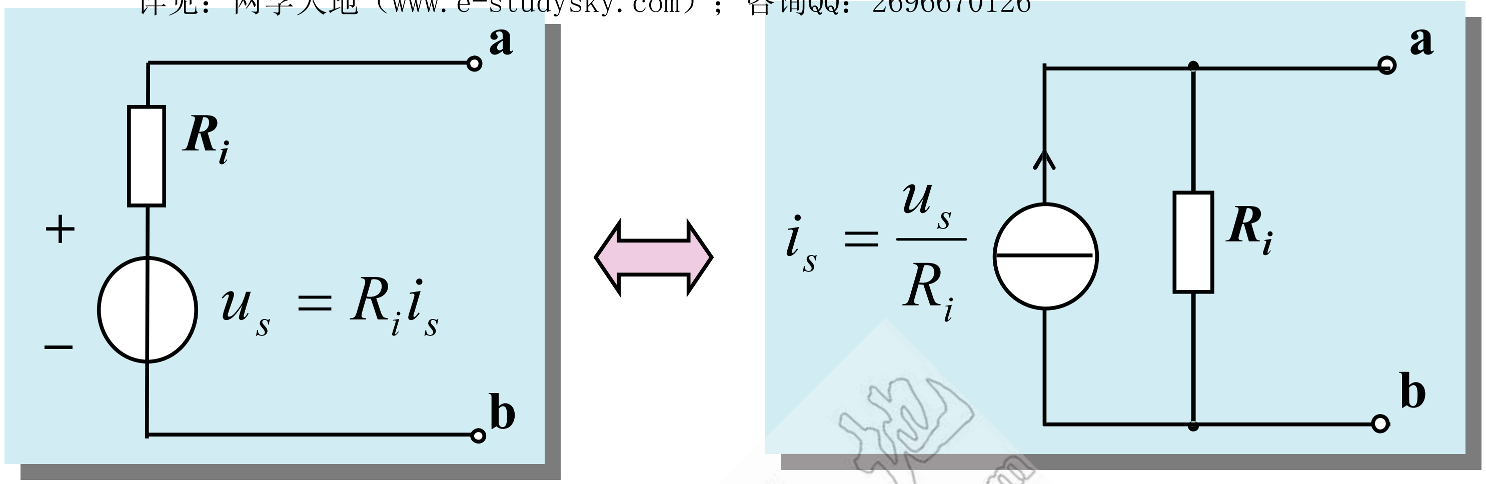


(b)

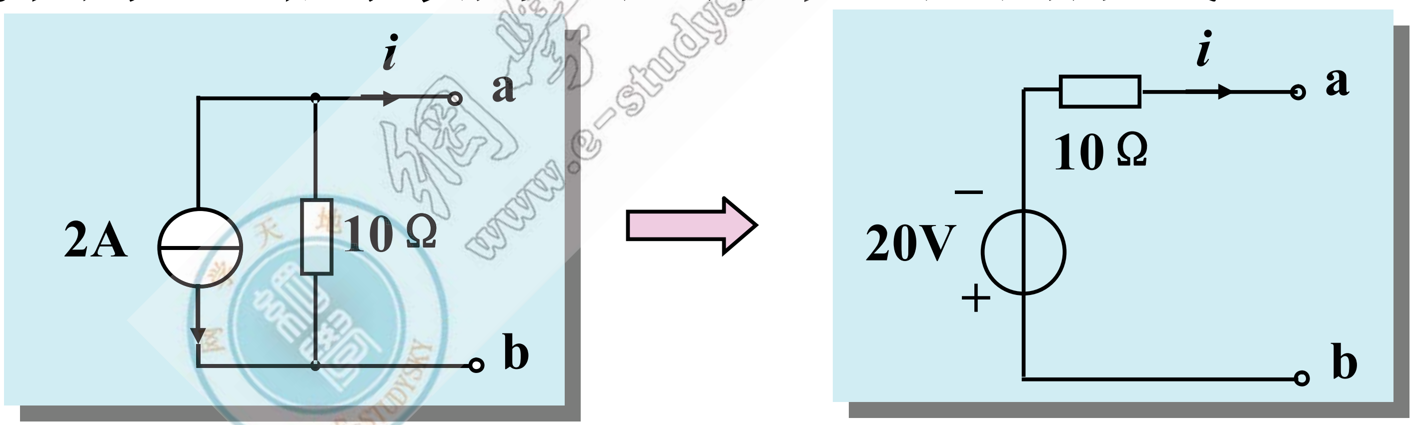
对图(a) $u = u_s - R_i i$ 即 $i = \dfrac{u_s}{R_i} - \dfrac{1}{R_i}u$

对图(b) $i = i_s - \dfrac{1}{R_i'}u$

故 $i_s = \dfrac{u_s}{R_i}, R_i' = R_i$



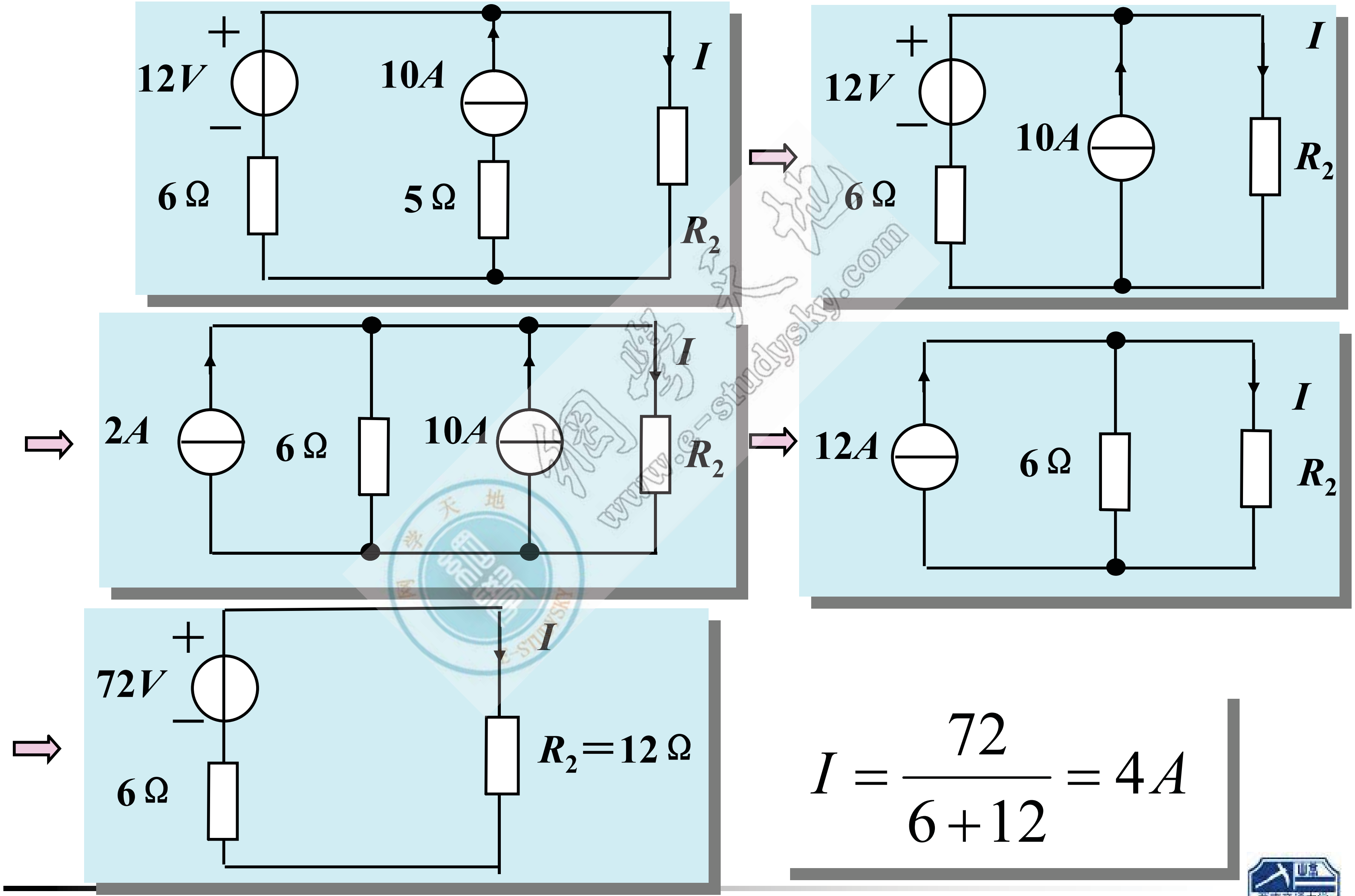
例2-7 将图示电路等效为电压源串电阻的形式。



检查方法： 等效变换前后两电路的开路电压应相等。
等效变换前后两电路的短路电流应相等。

注意： 理想电压源和理想电流源不能进行等效变换。

例2-8 用电源等效变换法求流过负载的电流I。



$$I=\frac{72}{6+12}=4A$$