自动控制原理 Automatic Control Theory

西南交通大学

电气工程学院

Chapter 2 Mathematical Model of Systems

2.1 Introduction

Mathematic Models 数学模型:

Descriptions of the behavior of a system using mathematics.

利用数学工具对系统行为进行的描述

***Control System Mathematical Models 控制系统数学模型*:**

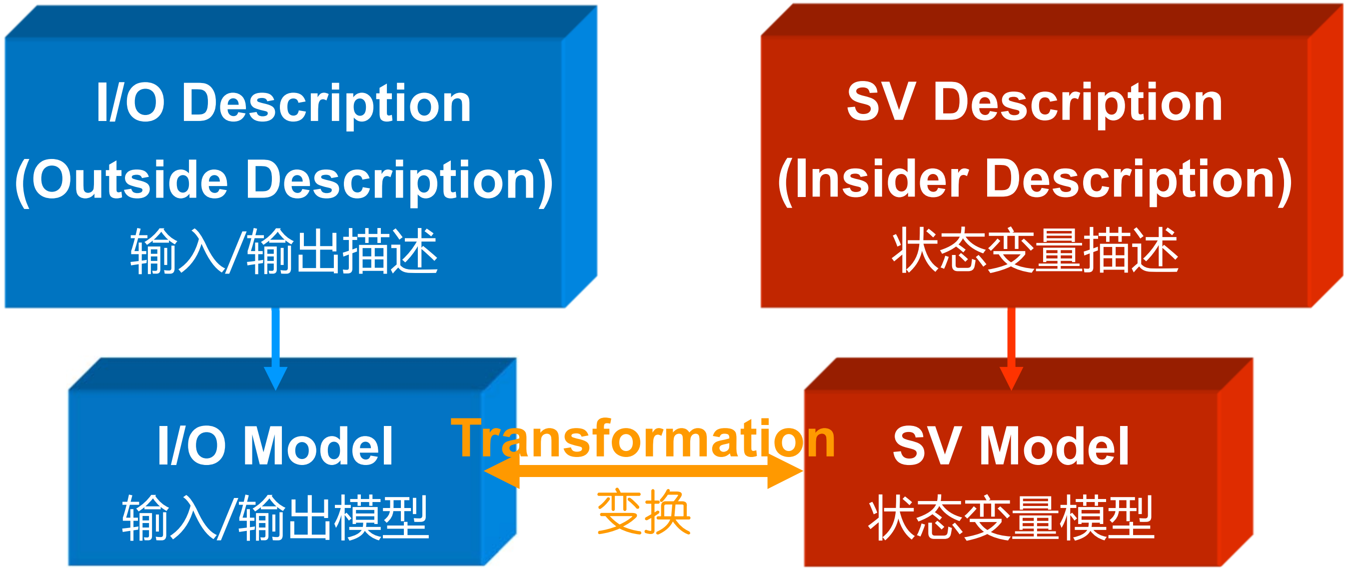
We use the quantitative mathematic models to design and analyze the physical control system.

采用定量数学模型分析和设计控制系统

2.1 Introduction

Linear Control System Description

线性控制系统的描述方法



Linear Control System Mathematical Model

线性控制系统的数学模型

2.1 Introduction

Linear Control System I/O Model

线性控制系统的输入/输出模型

I/O Model

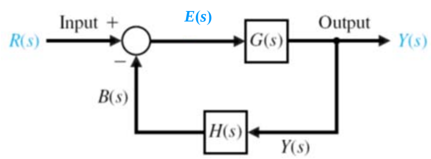
- **时域模型** → **Differential Equation**
- **复、频域模型** → **Transfer Function**
- **图示模型** → **Block Diagram Signal Flow Graph**

$$a_n\frac{d^n y(t)}{dt^n}+a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}}+\cdots+a_1\frac{dy(t)}{dt}+a_0 y(t)$$

$$=b_m\frac{d^m r(t)}{dt^m}+b_{m-1}\frac{d^{m-1}r(t)}{dt^{m-1}}+\cdots+b_1\frac{dr(t)}{dt}+b_0 r(t)$$

$a_i(i=0,1,\cdots,n)$、$b_j(j=0,1,\cdots,m)$均为实数

$$G(s)=\frac{b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{a_n s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0}=\frac{M(s)}{N(s)}$$



2.1 Introduction

How to build a mathematical model of control system

- Analysis Method 分析法
- Experimental Method 实验法

The Rationality of Mathematical Model：

Tradeoff between of *Simplicity* Model and *Accuracy* of the results.在模型的***简化性***和分析结果的***准确性***之间，作*折衷*考虑.

***The Characteristics of the Linear System*：**

- Superposition 叠加性 （Additivity）
- Homogeneity 齐次性

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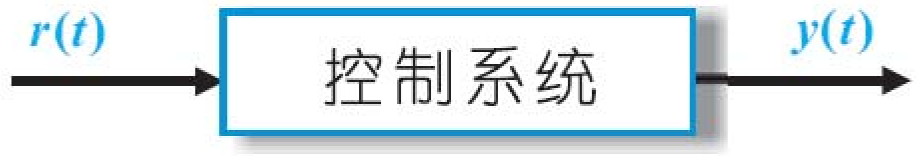
2.2 Input Output Models

Linear Time-invariant System



An input-output model is a mathematical model of a system which is expressed by input-output signal or its transformation.

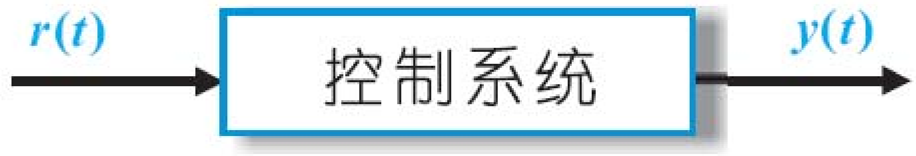
输入-输出模型：用系统的输入、输出信号或其变换式所表示的数学模型；

When I/O signal is：

- $r(t)$, $y(t)$ —Differential Equation 微分方程
- $R(s)$, $Y(s)$ — Transfer Function 传递函数
- $R(j\omega)$, $Y(j\omega)$ — Transfer function in frequency domain频率特性

2.2 Input Output Models

Linear Time-invariant System



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输入-输出模型：用系统的输入、输出信号或其变换式所表示的数学模型；

Why LTI system is important：

- "Linear systems are important, because we can solve them"

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2.2.1 Differential Equations 微分方程（组）

2.2.1.1 Differential equations of linear, time-invariant systems

用微分方程（组）描述线性时不变系统

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_m \frac{d^m r(t)}{dt^m} + b_{m-1} \frac{d^{m-1} r(t)}{dt^{m-1}} + \cdots + b_1 \frac{dr(t)}{dt} + b_0 r(t) \quad (2.1)$$

$a_i(i=0,1,\cdots,n)$、$b_j(j=0,1,\cdots,m)$均为实数

The general procedure to establish the differential equations of a system:

1. Ascertain the input and output signals $r(t)$ $y(t)$;
2. Write the differential equation of every part;
3. Eliminate the interim variables and gain the relationship of input-output;
4. Standardize the differential equation;

2.2.1 Differential Equations

2.2.1.2 Linear approximations of physical systems

物理系统的线性近似

Consider the excitation variable $x(t)$ and a response variable $y(t)$, the relation ship of the two variables is non-linear as

$$y(t) = g(x(t)) \qquad (2.2)$$

If the function $y=g(x(t))$ is continuous and differentiable at the operating point x_0 , then its Taylor series expansion is

$$y = g(x) = g(x_0) + \left.\frac{dg}{dx}\right|_{x=x_0} \frac{(x-x_0)}{1!} + \left.\frac{d^2 g}{dx^2}\right|_{x=x_0} \frac{(x-x_0)^2}{2!} + \cdots \qquad (2.3)$$

2.2.1 Differential Equations

When the range $\Delta x = x - x_0$ is small, we can neglect higher-order terms and have

$$g(x) - g(x_0) = y - y_0 = \left.\frac{dg}{dx}\right|_{x=x_0} \frac{(x-x_0)}{1!}$$

$$\Delta y = K\Delta x, \tag{2.4}$$

where

$$\Delta y = y - y_0 = y - g(x_0) \quad \Delta x = x - x_0 \quad K = \left.\frac{dg}{dx}\right|_{x=x_0}$$

This kind of Linear approximation is based on the small signal analytical theory.

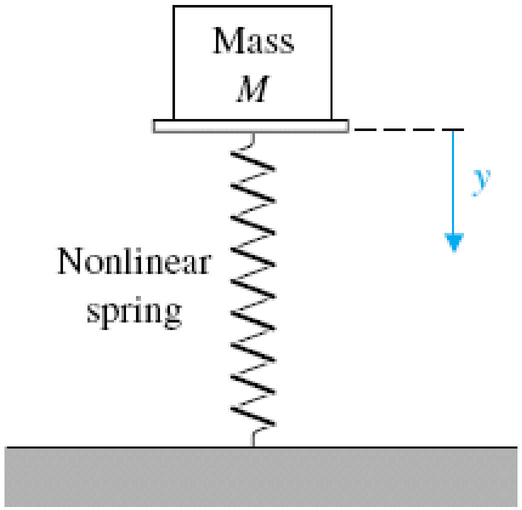
此类线性近似是基于小信号分析理论

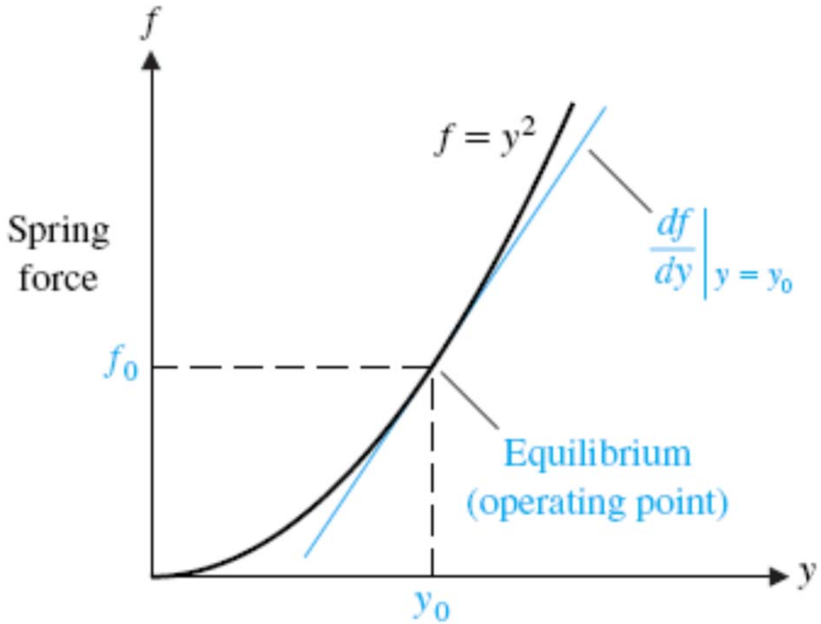
2.2.1 Differential Equations

Consider a mass M on a nonlinear spring with $f = y^2$. The normal operation point is the equilibrium position, thus $f_0 = Mg$. The linear model for small deviation is as follows

$$\Delta f = m\Delta y$$

where $m = \left.\frac{df}{dy}\right|_{y_0}$





2.2.1 Differential Equations

If the dependent variable y depends upon several excitation variables, the relationship is written as

$$y = g(x_1, x_2, \cdots, x_n)$$

The Taylor series expansion about the operating point is useful for a linear approximation to the nonlinear function. When the higher-order terms are neglected, the linear approximation can be written as

$$y = g(x_{1_0}, x_{2_0}, \cdots, x_{n_0}) + \left.\frac{\partial g}{\partial x_1}\right|_{x=x_0} (x_1 - x_{1_0})$$

$$+ \left.\frac{\partial g}{\partial x_2}\right|_{x=x_0} (x_2 - x_{2_0}) + \cdots + \left.\frac{\partial g}{\partial x_n}\right|_{x=x_0} (x_n - x_{n_0})$$

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2.2.2 Transfer Functions 传递函数

2.2.2.1 Transfer function of linear, time-invariant systems

For a linear , time-invariant system

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_1 \frac{dy(t)}{dt} + a_0 y(t)$$
$$= b_m \frac{d^m r(t)}{dt^m} + b_{m-1} \frac{d^{m-1} r(t)}{dt^{m-1}} + \cdots + b_1 \frac{dr(t)}{dt} + b_0 r(t) \qquad (2.1)$$

$a_i (i = 0,1,\cdots,n)$、$b_j (j == 0,1,\cdots,m)$均为实数

Taking the Laplace transform on both sides, we have

$$(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0) Y(s)$$
$$= (b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0) R(s) \qquad (2.5)$$

where $Y(s) = \mathcal{L}[y(t)],\ R(s) = \mathcal{L}[r(t)]$

2.2.2 Transfer Functions

The ratio of the Laplace transform of the *output variable* to the Laplace transform of the *input variable*.

$$\boxed{G(s) \overset{def}{=} \left.\frac{Y(s)}{R(s)}\right|_{\text{零初始条件}} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}} \quad (2.6)$$

传递函数反映系统“零初始状态”响应的传递关系；表明了系统数学模型的阶次n，它表征着系统的固有特性，与输入$r(t)$的形式无关;

传递函数是研究线性系统动态特性的重要工具。利用Laplace变换给出的传递函数$G(s)$是最常见的形式；

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2.2.2 Transfer Functions

Common expression of transfer function ：

- **Rational function expression有理分式函数表示形式**

$$G(s)=\frac{b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{a_n s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0}=\frac{M(s)}{N(s)} \qquad (2.6)$$

$M(s)$ — Numerator Polynomial 分子多项式

$N(s)$ — Denominator Polynomial 分母多项式

$N(s)$ — *Characteristic Polynomial 特征多项式*

2.2.2 Transfer Functions

Common expression of transfer function ：

- **Time Constant Expression 时间常数表示形式**

$$G(s)=\frac{b_0}{a_0}\,\frac{d_m s^m+d_{m-1}s^{m-1}+\cdots+d_1 s+1}{c_n s^n+c_{n-1}s^{n-1}+\cdots+c_1 s+1}=K\frac{\prod_{i=1}^{m}(T_i s+1)}{\prod_{j=1}^{n}(\tau_j s+1)} \qquad (2.7)$$

where, K—System Gain 系统增益或传递系数

T_i, τ_j— Time Constant 时间常数

When s=0, thus $G(s)= b_0/a_0= K$

从微分方程的角度看，此时相当于所有的导数项都为零。

K—系统处于静态时，输出与输入的比值

2.2.2 Transfer Functions

Common expression of transfer function ：

- **Zeros Poles Expression** 零极点表示形式

$$G(s) = \frac{b_m}{a_n} \frac{s^m + h_{m-1}s^{m-1} + \cdots + h_1 s + h_0}{s^n + l_{n-1}s^{n-1} + \cdots + l_1 s + l_0} = K_g \frac{\prod_{i=1}^{m}(s + z_i)}{\prod_{j=1}^{n}(s + p_j)} \qquad (2.8)$$

where, -z_i—Zeros 系统零点　　-p_j—Poles系统极点

K_g— Root locus Gain　根轨迹增益

系统传递函数的极点就是系统的特征根。

零点和极点的数值完全取决于系统的结构参数。

2.2.2 Transfer Functions

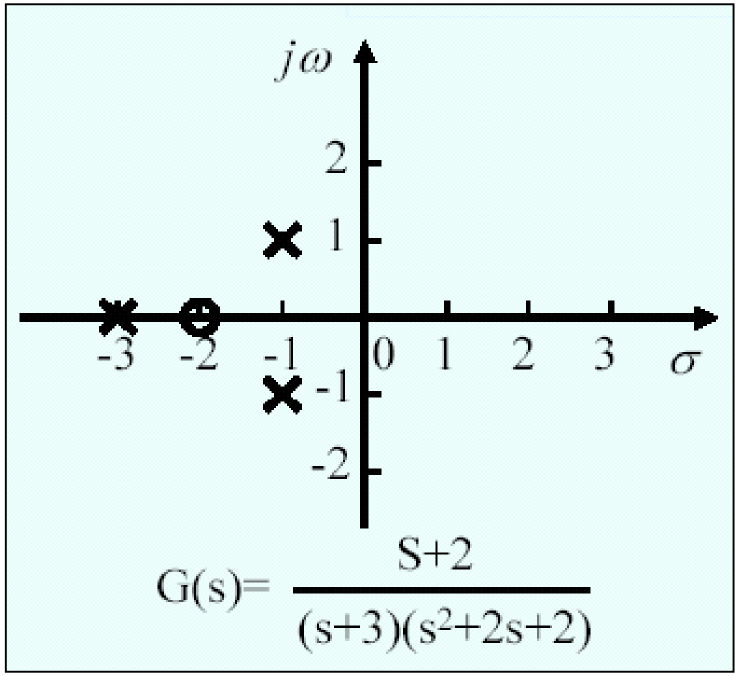
Common expression of transfer function ：

- **Zeros Poles Expression 零极点表示形式**

Zeros and Poles on S-plane

Where × represents Poles

O represents Zeros



2.2.2 Transfer Functions

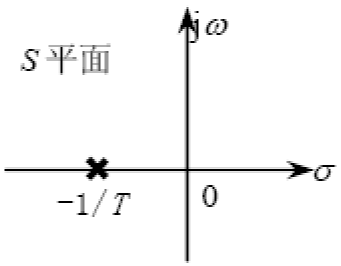
2.2.2.2 Transfer function of Typical Components

1. Inertial component 惯性环节

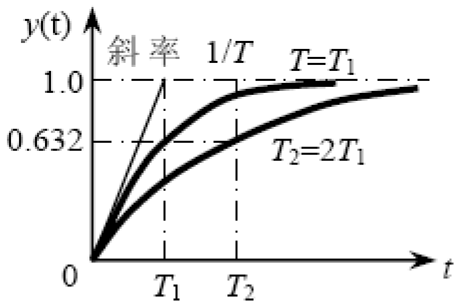
$$T\frac{dy(t)}{dt}+y(t)=r(t) \qquad (2.9)$$

$$G(s)=\frac{1}{Ts+1} \qquad (2.10)$$

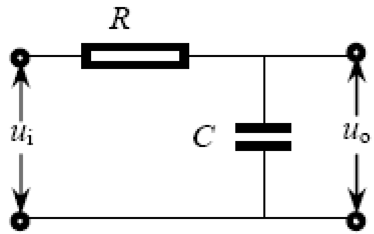
where, T—Time Constant



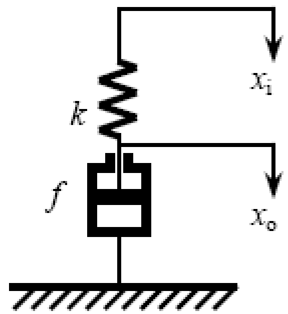
(a)零极点分布



(b)单位阶跃响应曲线



(c)RC 电路 $\frac{U_o(s)}{U_i(s)}=\frac{1}{Ts+1}$



(d)弹簧-阻尼器系统

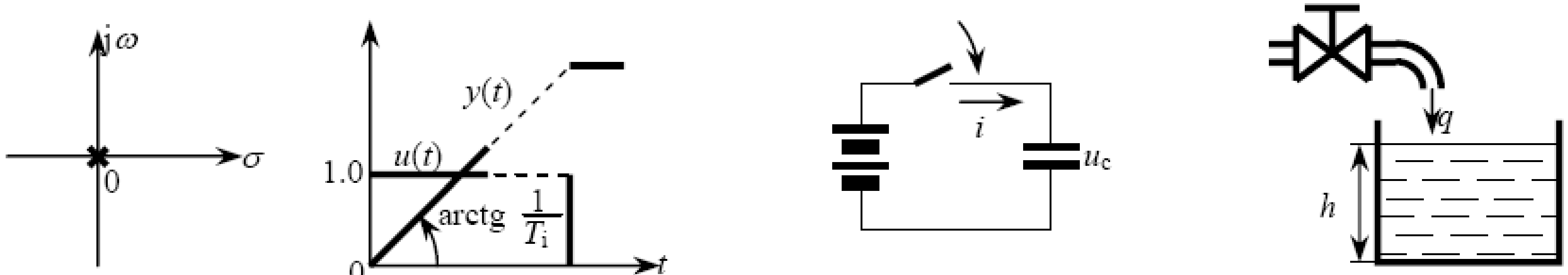
2.2.2 Transfer Functions

2. Integral Component积分环节

$$T_i \frac{dy(t)}{dt} = r(t) \qquad (2.11)$$

$$G(s) = \frac{1}{T_i s} \qquad (2.12)$$

where, T_i— Time Constant



(a)零极点分布 (b)单位阶跃响应曲线 (c)电容器充电$\frac{U_c(s)}{I(s)} = \frac{1}{Cs}$ (d)水箱$\frac{H(s)}{Q(s)} = \frac{1}{Cs}$

2.2.2 Transfer Functions

3. Oscillatory component 振荡环节

$$T^2\frac{d^2y(t)}{dt^2}+2T\zeta\frac{dy(t)}{dt}+y(t)=r(t), 0<\zeta<1 \tag{2.13}$$

$$G(s)=\frac{1}{T^2s^2+2\zeta Ts+1}, 0<\zeta<1 \tag{2.14}$$

令 $\omega_n=\frac{1}{T}$ $\quad G(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ (2.15)

Where ζ is ***Damping Ration*** *阻尼比*

ω_n is ***Nature Frequency*** *无阻尼比自然振荡频率*

T is ***Time Constant*** *时间常数*

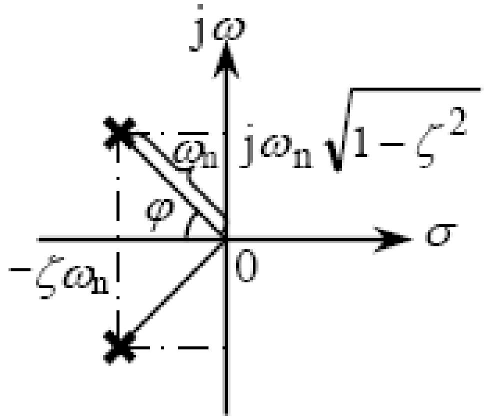
2.2.2 Transfer Functions

$$G(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

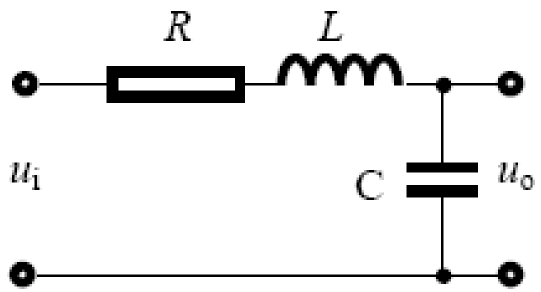
$$=\frac{\omega_n^2}{(s+p_1)(s+p_2)}$$

Characteristic Roots

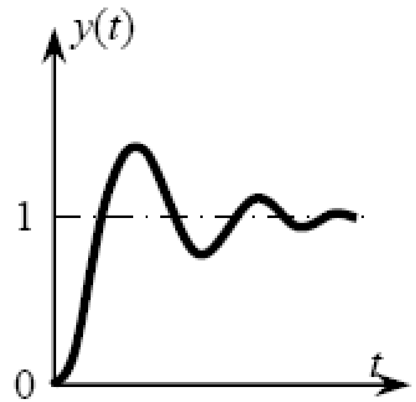
$$-p_{1,2}=-\zeta\omega_n\pm j\omega_n\sqrt{1-\zeta^2}$$



(a)零极点分布



(c)RLC 串联电路



(b)单位阶跃响应曲线

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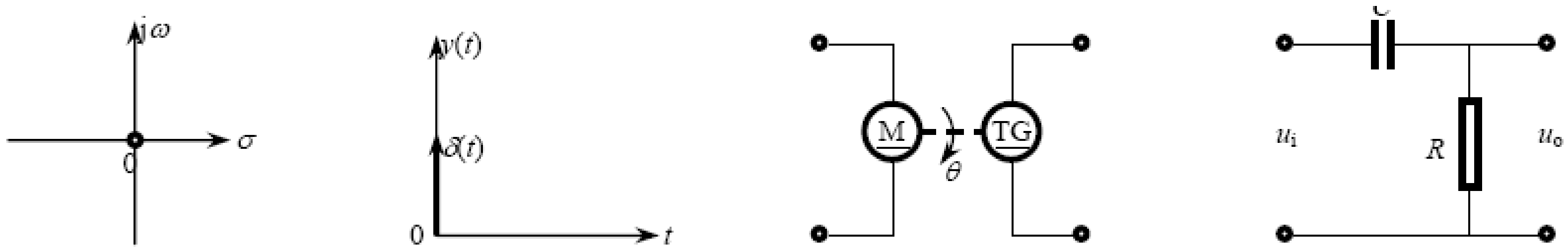
2.2.2 Transfer Functions

4. Derivative component 微分环节

$$y(t) = T_d \frac{dr(t)}{dt} \tag{2.16}$$

$$G(s) = T_d s \tag{2.17}$$

理想微分环节的传递函数不是真有理分式，工程实现较为困难，工程上常采用具有惯性环节的微分环节



(a)零极点分布　(b)单位阶跃响应曲线　(c)安装在电动机轴上的测速发电机　(d)RC 电路

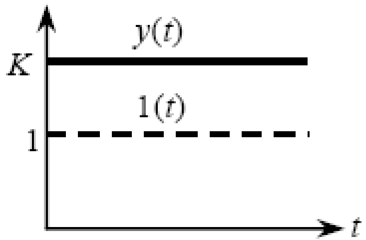
2.2.2 Transfer Functions

5. Proportional component 比例环节

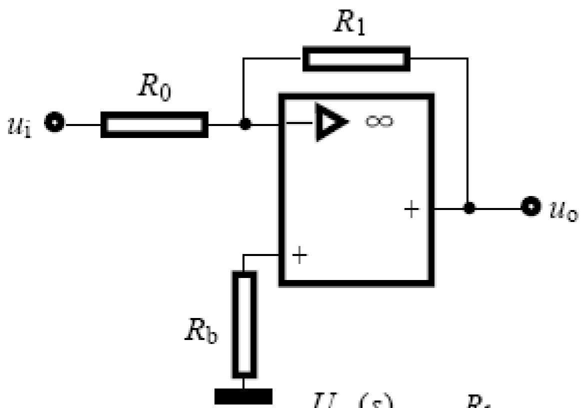
$$G(s) = K_p \tag{2.18}$$

$$y(t) = K_p r(t) \tag{2.19}$$

Where, K_p — Gain



(a)单位阶跃响应曲线



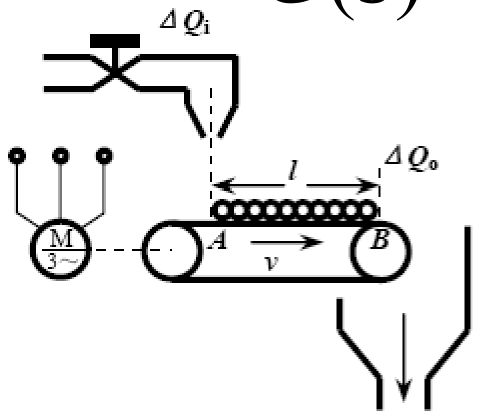
(b)比例调节器 $\dfrac{U_o(s)}{U_i(s)} = -\dfrac{R_1}{R_0}$

2.2.2 Transfer Functions

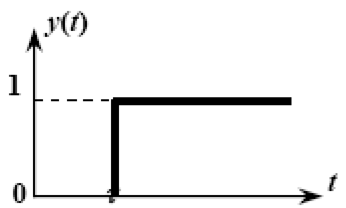
6. Time delayed component 时滞环节

$$y(t) = r(t-\tau)1(t-\tau) \qquad (2.20)$$

$$G(s) = e^{-\tau s} \qquad (2.21)$$



(a)带式运输机系统



(b)单位阶跃响应曲线

实际控制系统的传递函数均可视为上述典型环节的某种组合,因此熟悉和掌握典型环节对于分析研究系统是很基本的,也是很重要的

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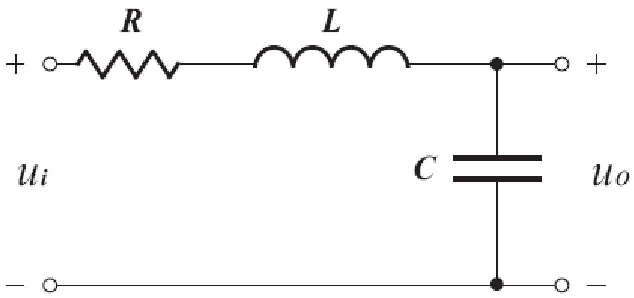
2.2.2 Transfer Functions

Review：

Determine the transfer function of the system



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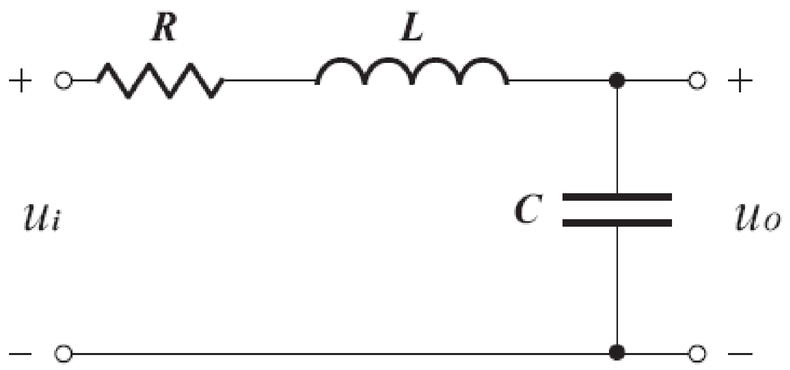
2.2.2 Transfer Functions

2.2.2.3 Transfer function of the circuit

<E2.1> Determine the transfer function of the circuit.



$$Z(s)=V(s)/I(s)$$
$$Z_L(s)=Ls,$$
$$Z_C(s)=1/Cs,$$
$$Z_R(s)=R$$

$$G(s)=\frac{U_o(s)}{U_i(s)}=\frac{Z_C(s)}{Z_L(s)+Z_R(s)+Z_C(s)}$$
$$=\frac{1/Cs}{Ls+R+1/Cs}=\frac{1}{LCs^2+RCs+1}$$

2.2.3 Transfer Functions in Frequency Domain

Transfer Functions in Frequency Domain:

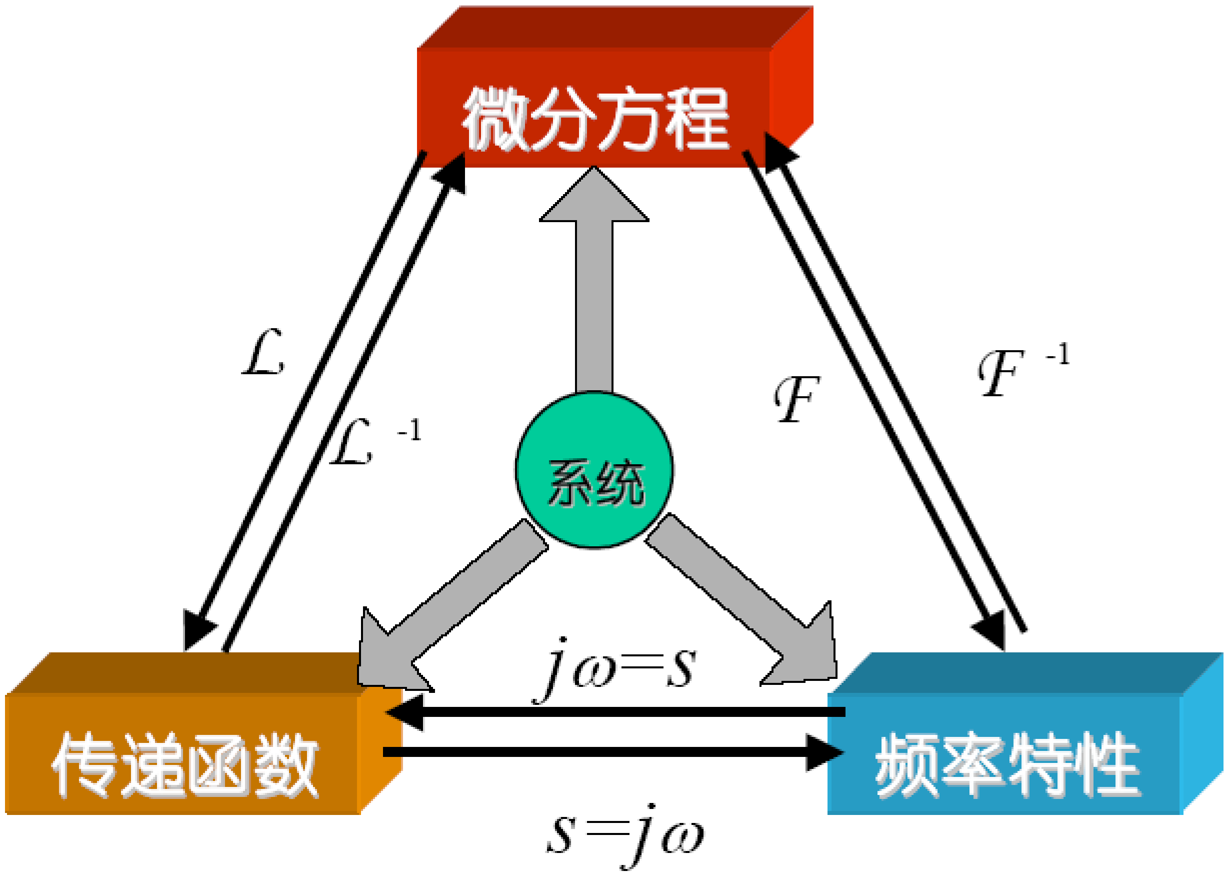
The ratio of the Fourier transform of the output signal to the Fourier transform of the input signal where the input is a *sinusoid*.

$$G(j\omega) = G(s)\,|_{s=j\omega} \qquad (2.22)$$

频率特性：

在正弦输入信号作用下，系统输出的稳态分量与输入量的复数之比，用$G(j\omega)$表示

2.2.4 Relationship of I/O Models



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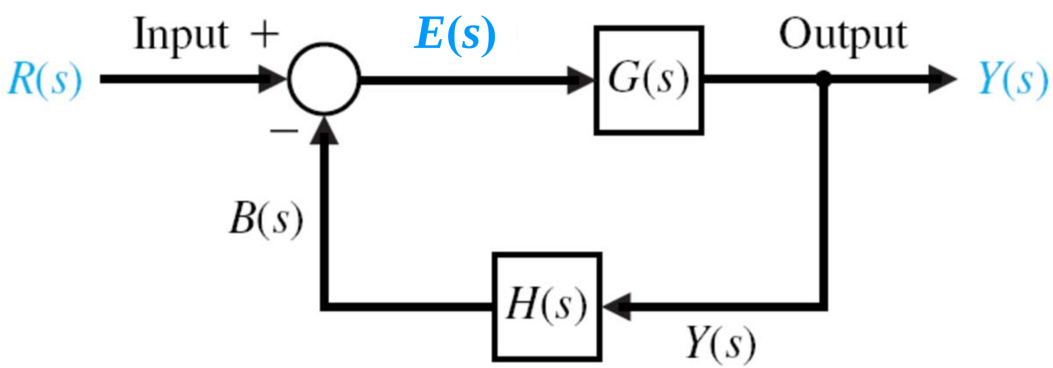
2.3 Block Diagram Models

2.3.1 Block Diagram

Block Diagram: Unidirectional, operational blocks that represent the transfer functions of the elements of the system.

系统结构图：用单方向功能方框组成的一种结构图，这些方框代表了系统元件的传递函数。



2.3.2 Equivalent transformations of block diagrams

1.Transformation Principle 原则

Keep the same relationship between the input and output signals after transformation.

变换前后，输入、输出及其相互关系不变。

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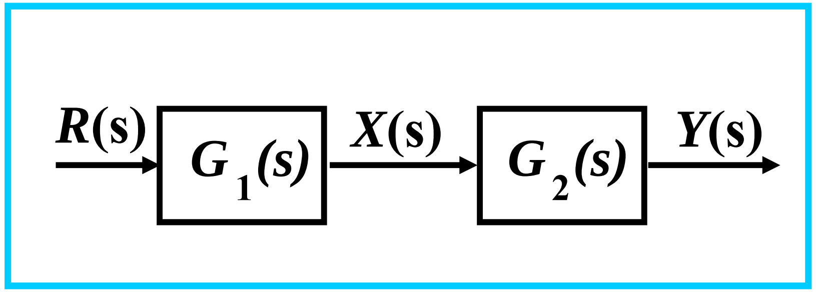
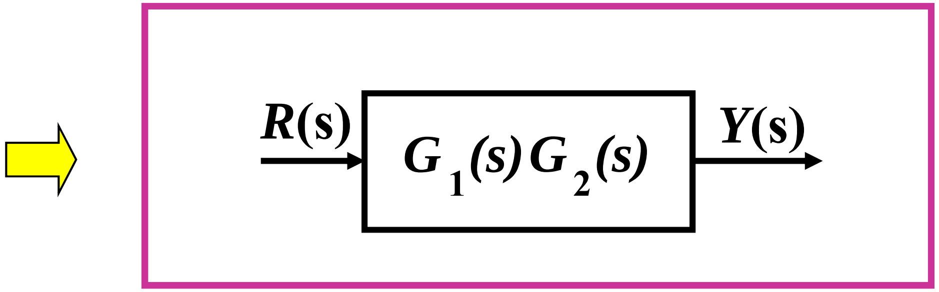
2.3.2 Equivalent transformations of block diagrams

- The equivalent transformation of blocks in series

串联环节的等效变换

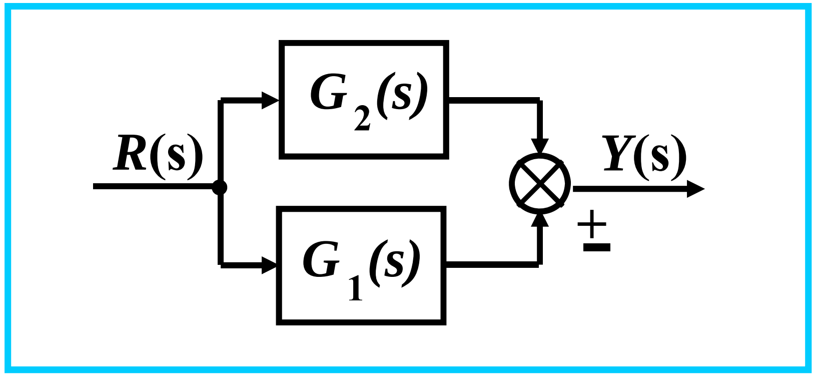


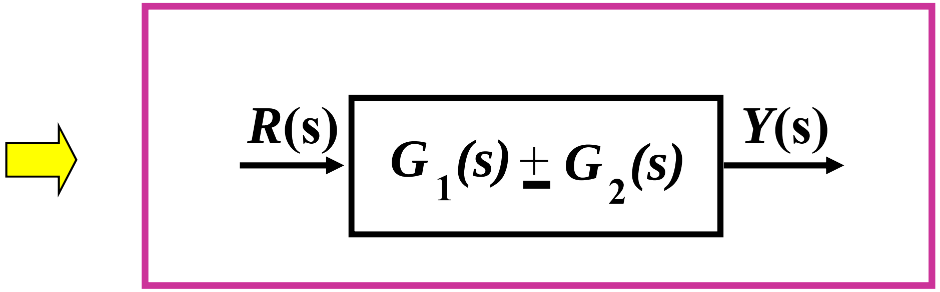


2.3.2 Equivalent transformations of block diagrams

- The equivalent transformation of blocks in parallel

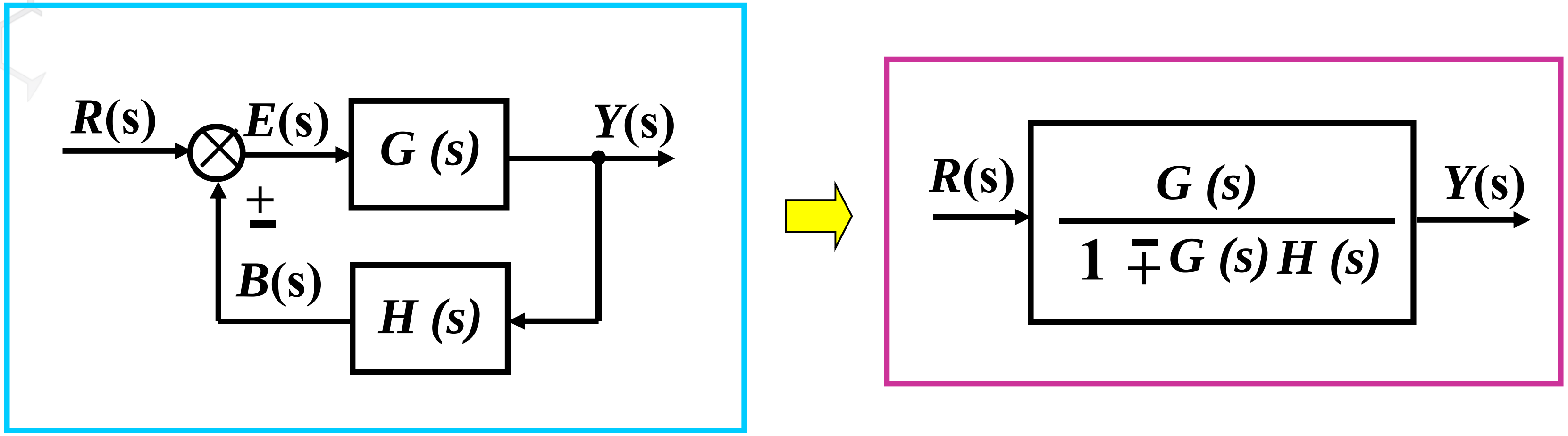
并联环节的等效变换





2.3.2 Equivalent transformations of block diagrams

- The equivalent transformation of feedback loop
  反馈环节的等效变换



$$B(s) = H(s)Y(s)$$

$$E(s) = R(s) \pm B(s) = R(s) \pm H(s)Y(s)$$

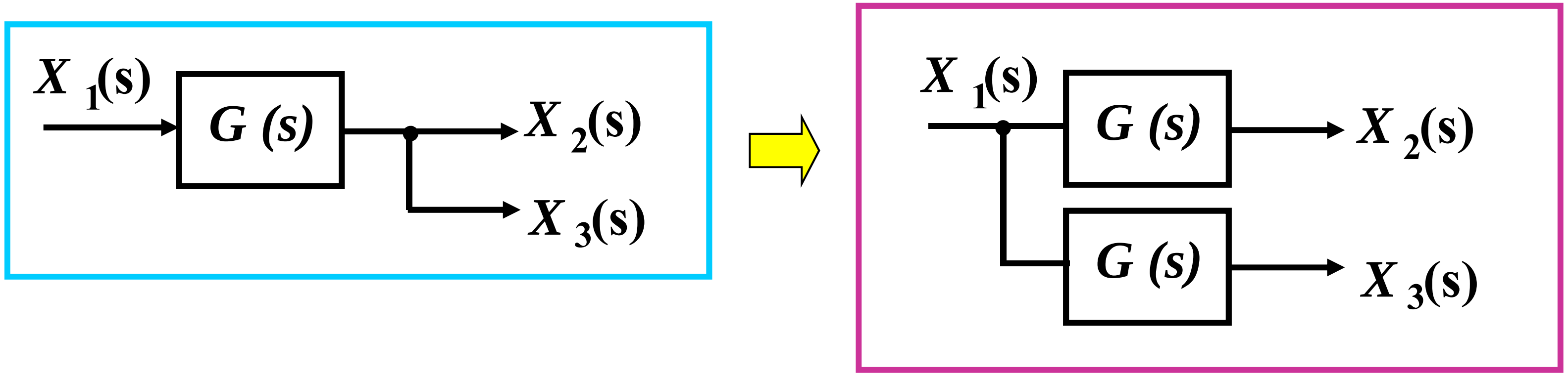
$$Y(s) = G(s)E(s) = G(s)[R(s) \pm H(s)Y(s)]$$

$$[1 \mp G(s)H(s)]Y(s) = G(s)R(s)$$

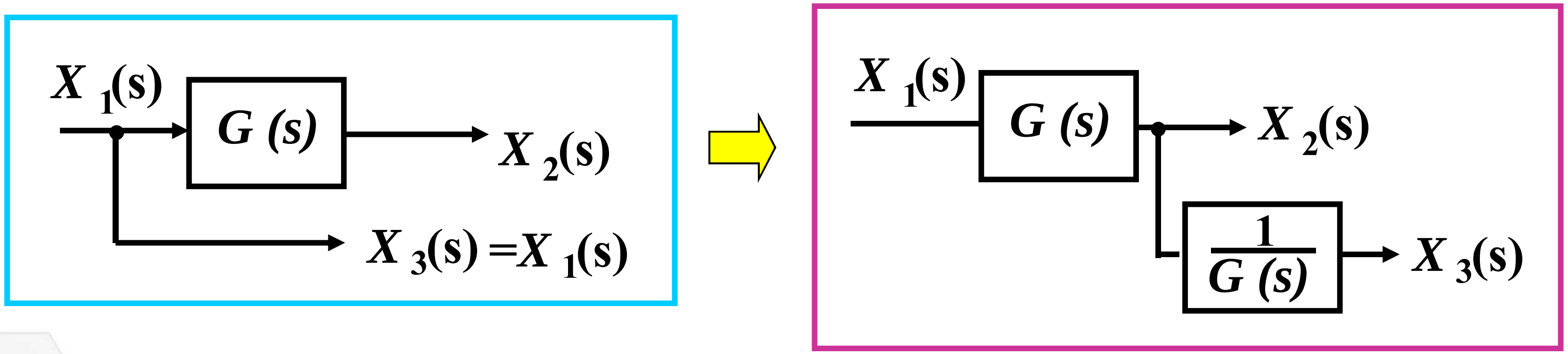
2.3.2 Equivalent transformations of block diagrams

- Moving pickoff points 分支点移动

Moving a pickoff point ahead of a block 分支点前移



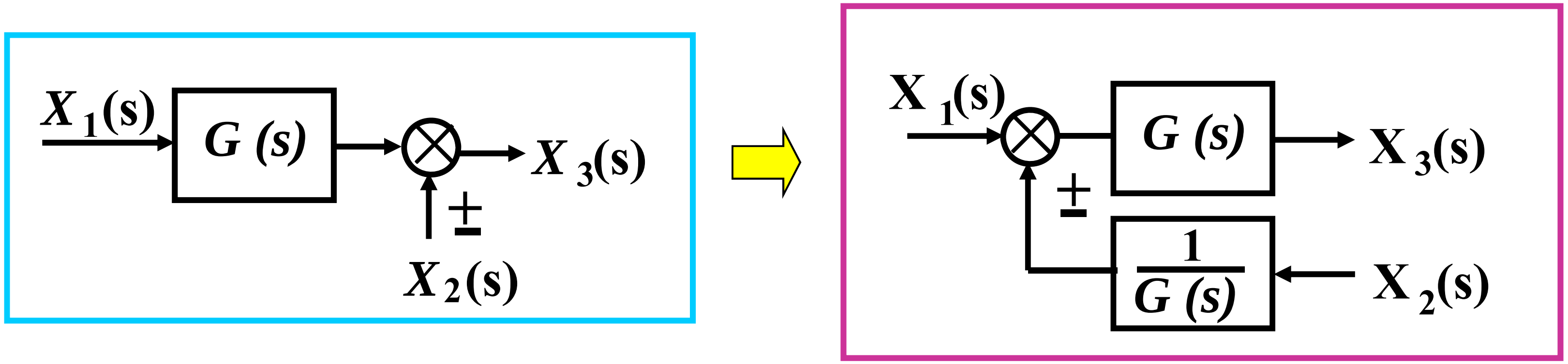
Moving a pickoff point behind of a block 分支点后移



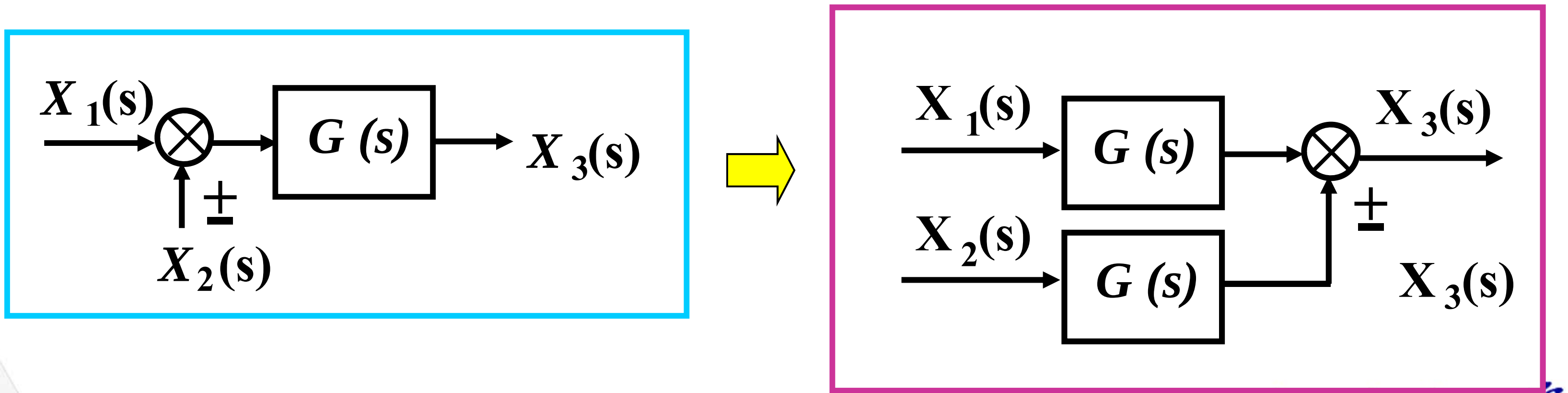
2.3.2 Equivalent transformations of block diagrams

- Moving summing points 相加点移动

Moving a summing point ahead of a block 相加点前移



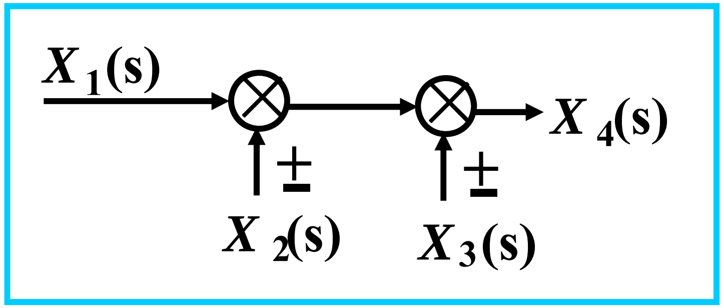
Moving a summing point behind of a block 相加点后移

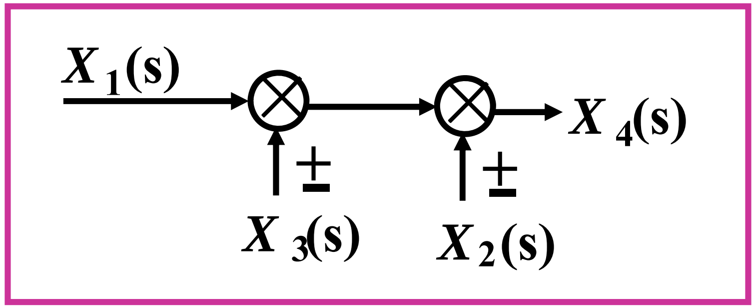


2.3.2 Equivalent transformations of block diagrams

- Exchanging summing points 相加点互换

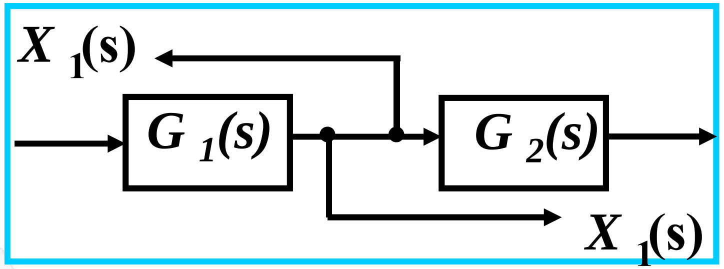
Exchanging a summing point ahead of another one

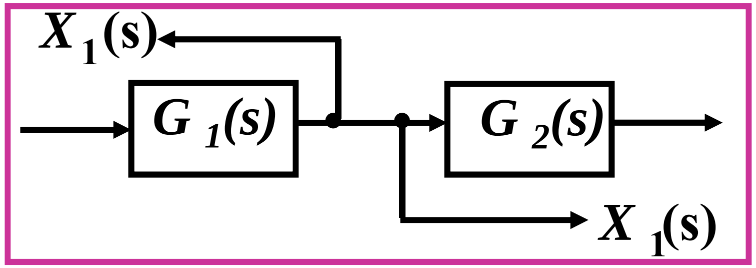




- Exchanging pickoff points 分支点互换

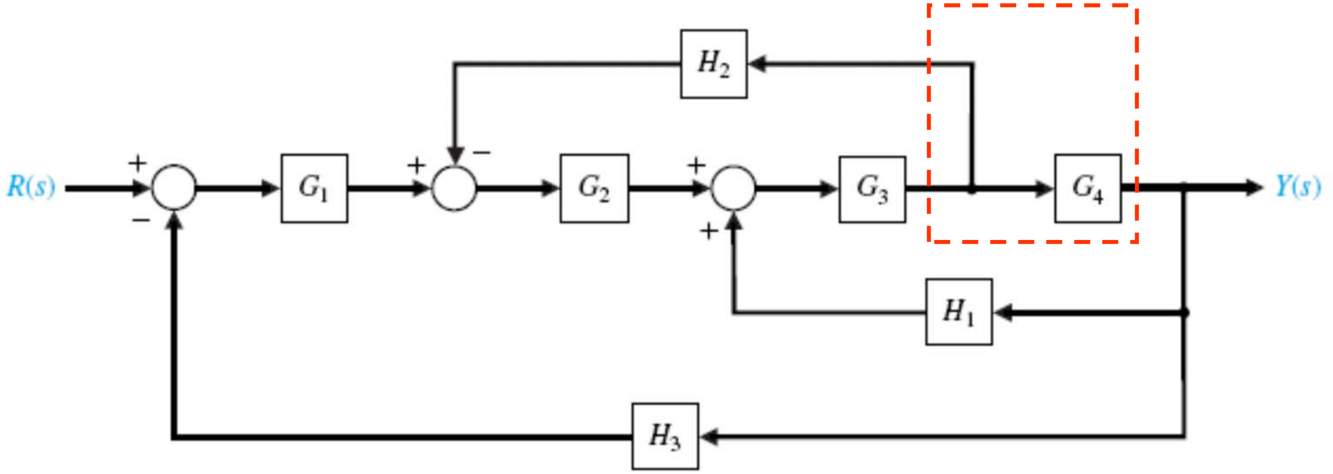
Exchanging a pickoff point ahead of another one

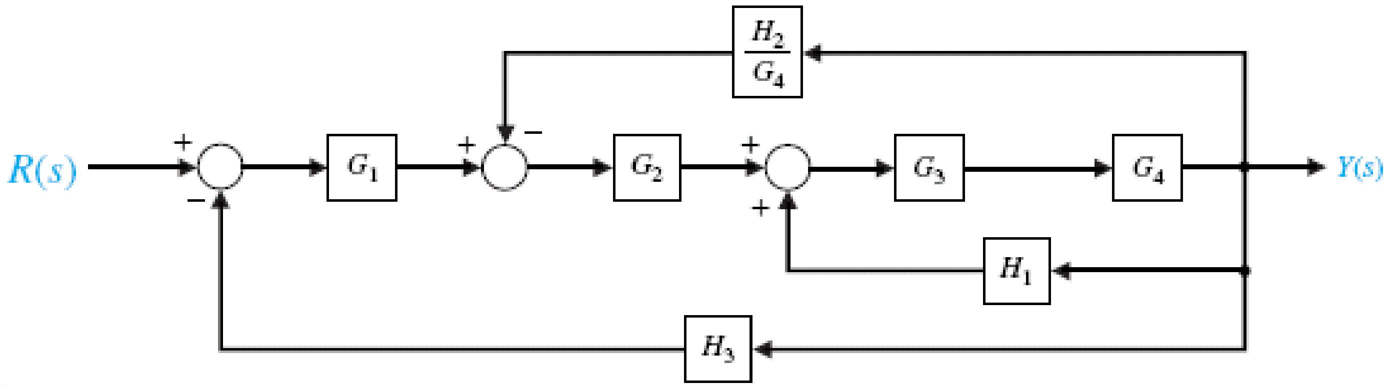




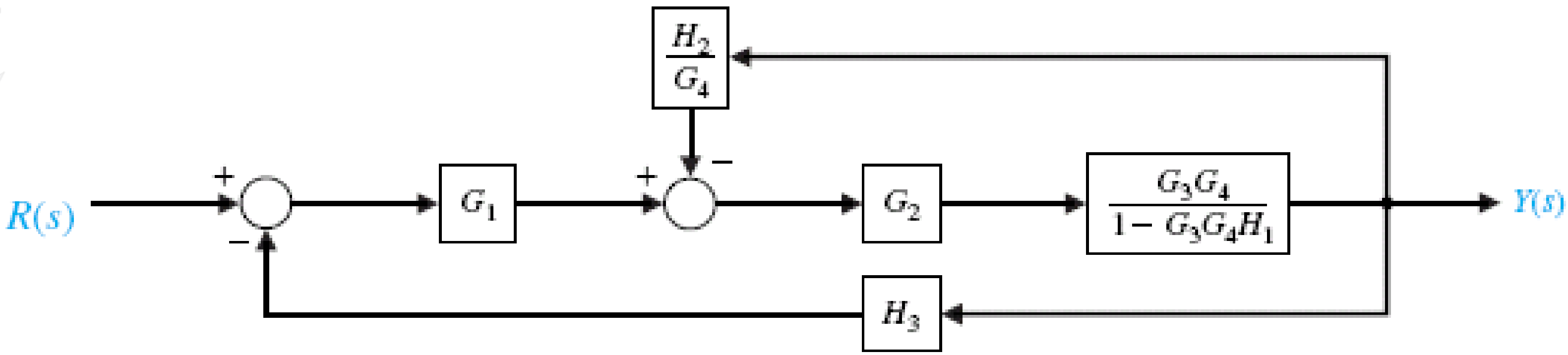
2.3.2 Equivalent transformations of block diagrams

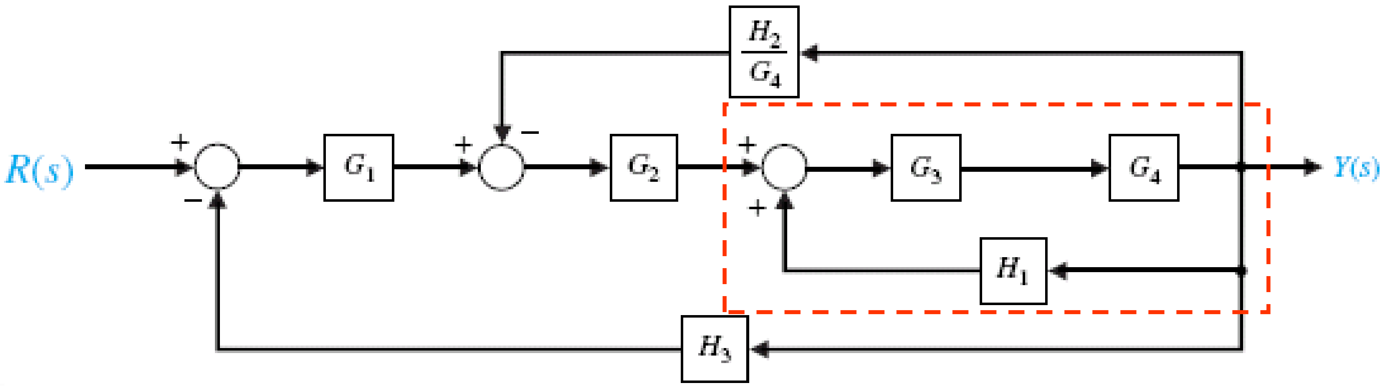
<E2.2> Block diagram reduction





2.3.2 Equivalent transformations of block diagrams





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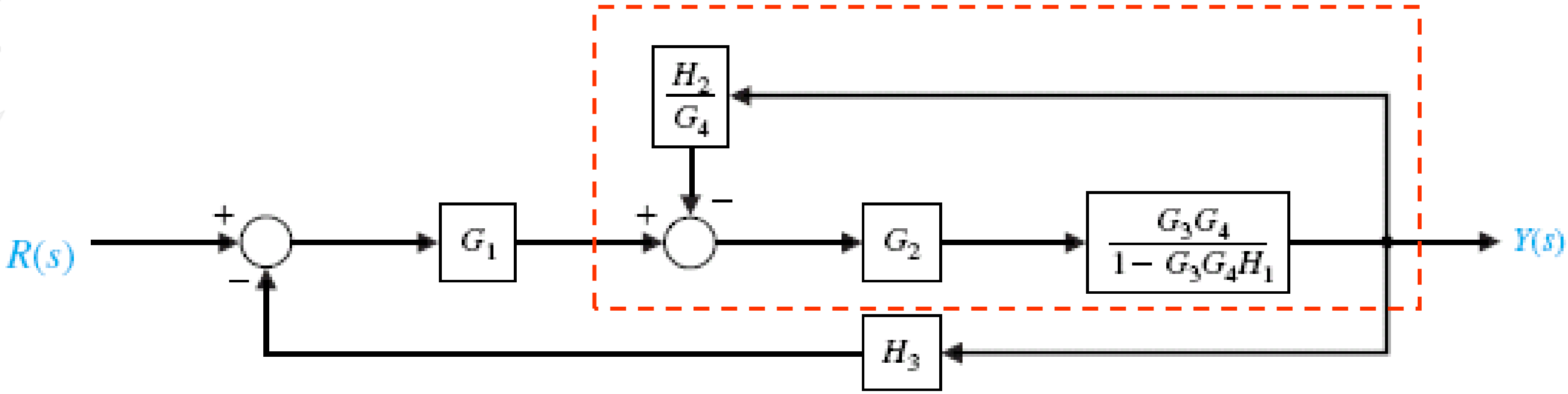
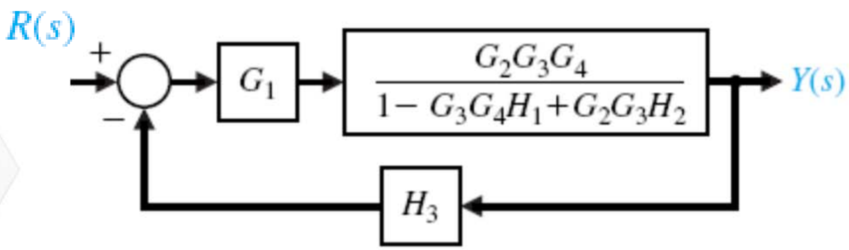
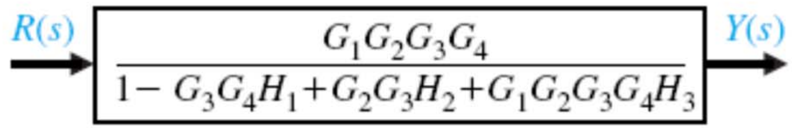
2.3.2 Equivalent transformations of block diagrams

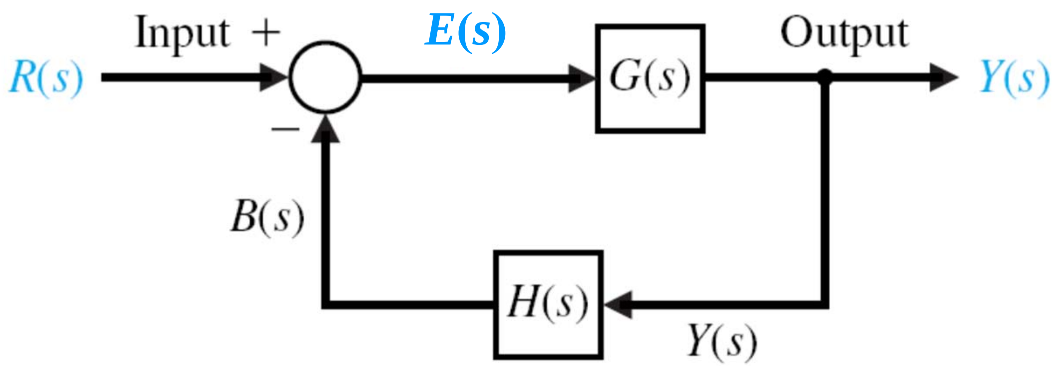






2.3.3 Transfer function of feedback control system

Negative feedback **control system block diagram**



Transfer function

$$B(s) = H(s)Y(s)$$

$$E(s) = R(s) - B(s) = R(s) - H(s)Y(s)$$

$$Y(s) = G(s)E(s) = G(s)[R(s) - H(s)Y(s)]$$

$$[1 + G(s)H(s)]Y(s) = G(s)R(s)$$

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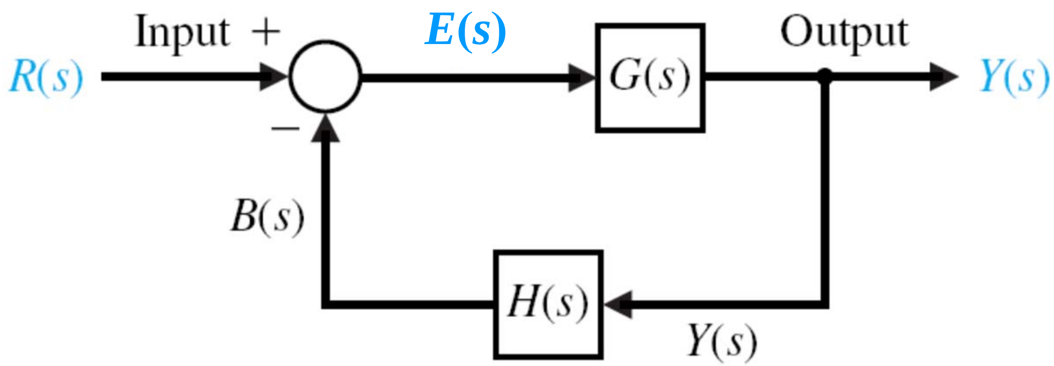
2.3.3 Transfer function of feedback control system

Negative feedback control system block diagram



$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$ **Closed-loop Transfer Function**

$$\frac{Y(s)}{E(s)} = G(s)$$ **Forward Transfer Function**

$$\frac{B(s)}{E(s)} = G(s)H(s)$$ **Loop Transfer Function**

返回

2.4 State Variable Model

***State of a system*:** A set of minimum numbers such that the knowledge of these numbers and the input function will, with the equations describing the dynamics, provide the future state of the system.

状态: 表示系统的一组变量，只要知道了这组变量的当前值，知道了输入激励和系统动态方程，就可以完全确定的未来状态；

2.4 State Variable Model

State Variable: The set of minimum numbers of variables that describe the system dynamics.

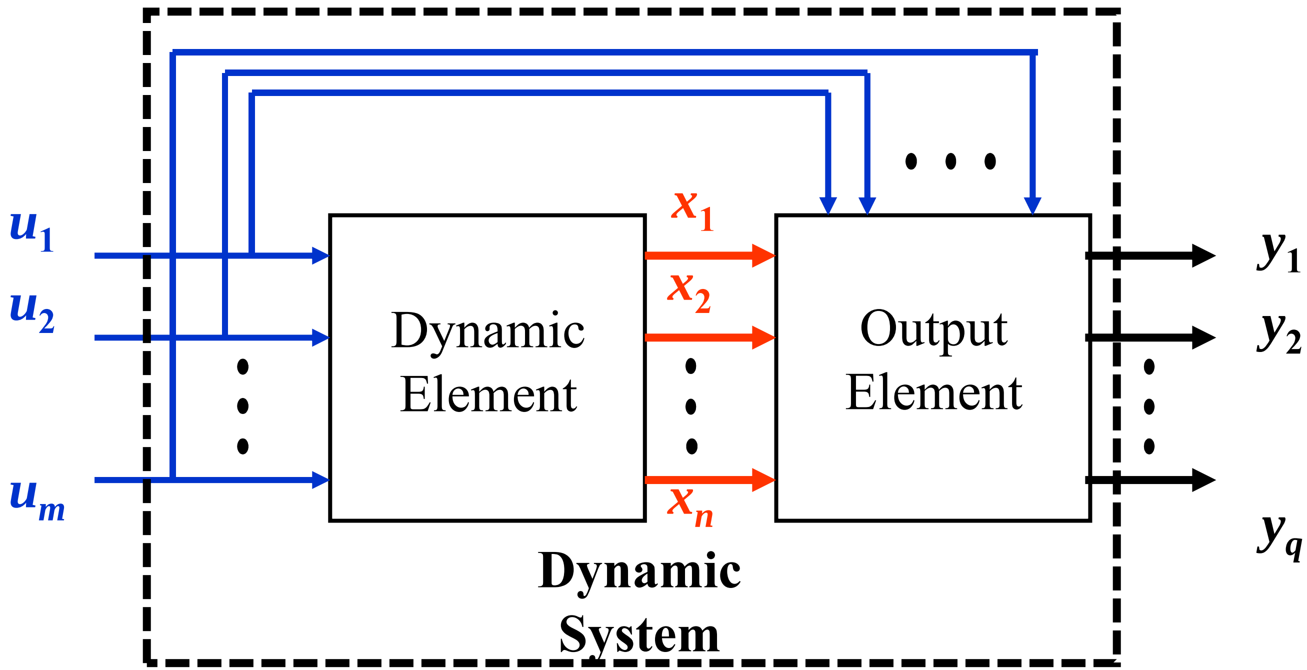
状态变量：动态系统的状态变量是确定动态系统状态的最小一组变量；

State Vector: The vector containing all n state variables.

状态向量 :若完全描述一个给定的系统的行为需要n个状态变量，那么这n个状态变量可以看成是向量$\mathbf{x}$的n个分量，这个向量就称为状态向量;

2.4.1 State Differential Equations

Consider the MIMO system that contains n state variables, the number of input signals and output signals are m and q respectively.



2.4.1 State Differential Equations

Consider the MIMO system that contains ***n*** state variables, the number of input signals and output signals are ***m*** and ***q*** respectively.

$$\begin{aligned}
\dot{x}_1(t) &= f_1(x_1, x_2, \cdots, x_n; u_1, u_2, \cdots, u_m; t) \\
\dot{x}_2(t) &= f_2(x_1, x_2, \cdots, x_n; u_1, u_2, \cdots, u_m; t) \\
&\vdots \\
\dot{x}_n(t) &= f_n(x_1, x_2, \cdots, x_n; u_1, u_2, \cdots, u_m; t)
\end{aligned}$$

$$\begin{aligned}
y_1(t) &= g_1(x_1, x_2, \cdots, x_n; u_1, u_2, \cdots, u_m; t) \\
y_2(t) &= g_2(x_1, x_2, \cdots, x_n; u_1, u_2, \cdots, u_m; t) \\
&\vdots \\
y_q(t) &= g_q(x_1, x_2, \cdots, x_n; u_1, u_2, \cdots, u_m; t)
\end{aligned}$$

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$$

$$\mathbf{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_q(t) \end{bmatrix}$$

$$\mathbf{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix}$$

2.4.1 State Differential Equations

Consider the MIMO system that contains *n* state variables, the number of input signals and output signals are *m* and *q* respectively.

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{f}[\mathbf{x}, \mathbf{u}, t] \\ \mathbf{y}(t) &= \mathbf{g}(\mathbf{x}, \mathbf{u}, t) \end{aligned} \qquad (2.23)$$

$$\begin{aligned} \dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + b_{11}u_1 + b_{12}u_2 + \cdots + b_{1m}u_m \\ \dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + b_{21}u_1 + b_{22}u_2 + \cdots + b_{2m}u_m \\ &\vdots \\ \dot{x}_n &= a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n + b_{n1}u_1 + b_{n2}u_2 + \cdots + b_{nm}u_m \end{aligned}$$

2.4.1 State Differential Equations

Consider the MIMO system that contains n state variables, the number of input signals and output signals are m and q respectively.

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{f}[\mathbf{x},\mathbf{u},t] \\ \mathbf{y}(t) &= \mathbf{g}(\mathbf{x},\mathbf{u},t)\end{aligned} \qquad (2.23)$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \vdots\\ \dot{x}_n\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12} & \cdots & a_{1n}\\ a_{21} & a_{22} & \cdots & a_{2n}\\ \vdots & \vdots & & \vdots\\ a_{n1} & a_{n2} & \cdots & a_{nn}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{bmatrix}+\begin{bmatrix}b_{11} & \cdots & b_{1m}\\ \vdots & & \vdots\\ b_{n1} & \cdots & b_{nm}\end{bmatrix}\begin{bmatrix}u_1\\ \vdots\\ u_m\end{bmatrix}$$

2.4.1 State Differential Equations

The state variable models of LTI systems:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad \textit{State Differential Equation}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \quad \textit{Output Equation} \tag{2.24}$$

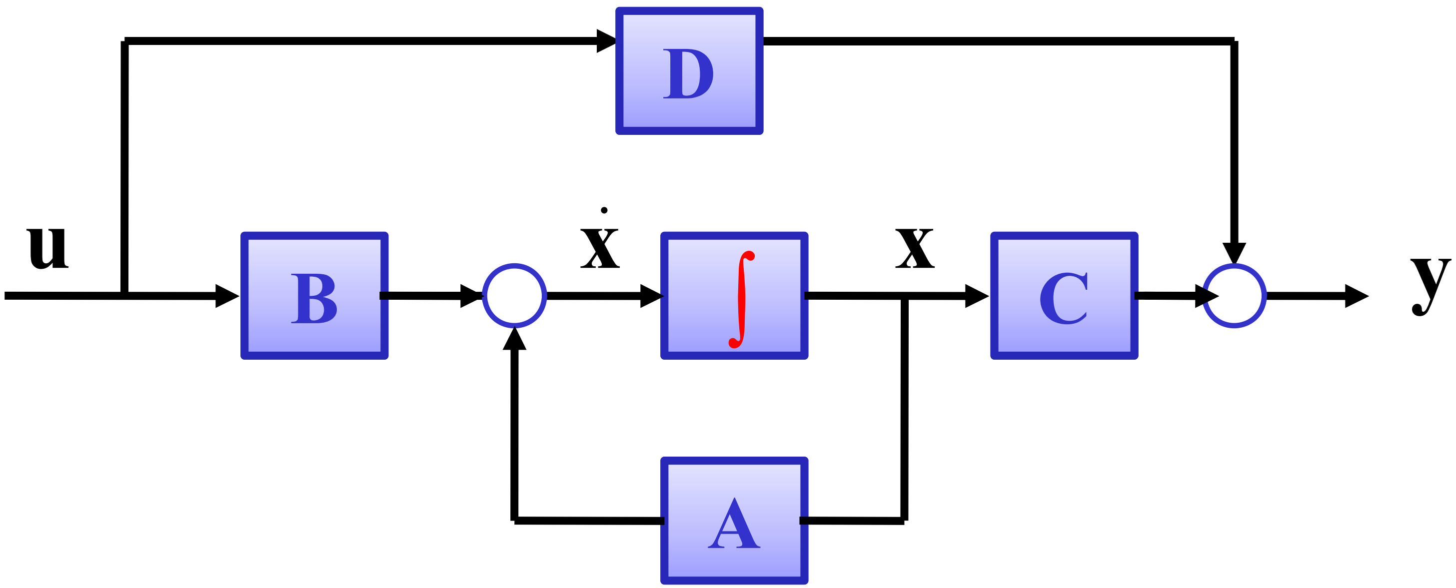
where $t \geq 0, x_0 = x(0)$ (Initial condition)

$\mathbf{x} \in R^n$,	*State Vector*	(状态向量)
$\mathbf{u} \in R^m$,	*Input Vector*	(输入向量)
$\mathbf{y} \in R^q$	*Output Vector*	(输出向量)
$\mathbf{A} \in R^{n\times n}$,	*System Matrix*	（系统矩阵）
$\mathbf{B} \in R^{n\times m}$,	*Input Matrix*	(输入矩阵)
$\mathbf{C} \in R^{q\times n}$,	*Output Matrix*	(输出矩阵)
$\mathbf{D} \in R^{q\times m}$,	*Feedforword Matrix*	(前馈矩阵)

2.4.1 State Differential Equations

The state variable models of LTI systems:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad \textit{State Differential Equation}$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \quad \textit{Output Equation} \qquad (2.24)$$



状态变量描述时LTI系统的结构图

2.4.1 State Differential Equations

选择状态变量必须要满足：

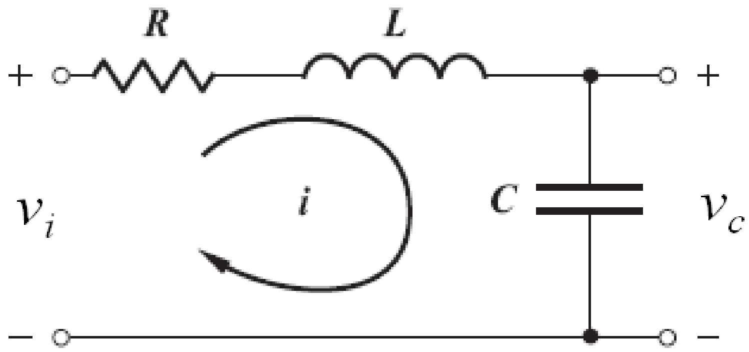
- 选择最小的状态变量为状态向量的元素。这些最少的状态变量可以充分地完全描述系统的状态；
- 状态向量的元素必须是线性无关的；

通过对状态微分方程的分析可以发现：

- 状态空间表达式是系统的一种完全描述，其核心是状态方程；
- 系统的状态空间表达式不是唯一的；

2.4.1 State Differential Equations

<E2.4> Determine the state variable model for the circuit. The input variable is v_i the output variable is v_c.



Solution:

Select v_c (the voltage of the capacitor) and i_L (the current of the inductance) as the state variables, we have

$$i_L = C\dot{v}_C, \qquad v_L = L\dot{i}_L$$

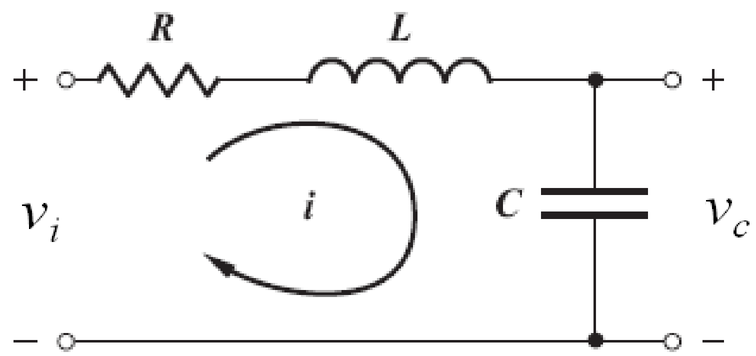
$$v_R + v_L + v_C = v_i$$

$$Ri_L + L\dot{i}_L + v_C = v_i$$

2.4.1 State Differential Equations

$$\dot{v}_C = \frac{1}{C} i_L$$

$$\dot{i}_L = -\frac{1}{L} v_C - \frac{R}{L} i_L + \frac{1}{L} v_i$$



Let $\begin{cases} x_1 = v_C \\ x_2 = i_L \end{cases}$, Output $\mathbf{y} = y = v_C(t)$ Input $\mathbf{u} = u = v_i(t)$

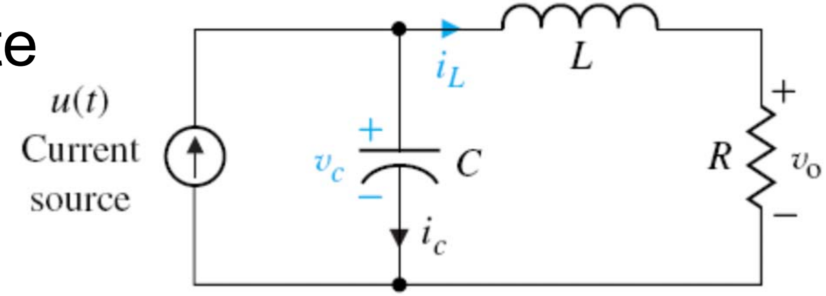
State Variable Model

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

2.4.1 State Differential Equations

<E2.5> Determine the state variable model for the circuit. The input variable is u_i the output variable is v_o.



Solution:

Select v_C (the voltage of the capacitor) and i_L (the current of the inductance) as the state variables, we have

$$i_C = C\dot{v}_C, \qquad v_L = L\dot{i}_L$$

$$KCL: i_C = C\dot{v}_C = u_i - i_L$$

$$KVL: L\dot{i}_L = v_C - Ri_L$$

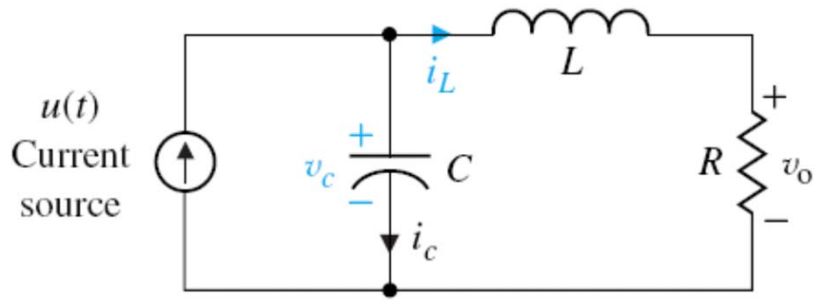
$$\text{Output: } v_o = Ri_L$$

2.4.1 State Differential Equations

Let $x_1 = v_C, x_2 = i_L$

$$\dot{x}_1 = -\frac{1}{C}x_2 + \frac{1}{C}u_i$$

$$\dot{x}_2 = +\frac{1}{L}x_1 - \frac{R}{L}x_2$$



State Variable Model

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & -\frac{1}{C}\\ \frac{1}{L} & -\frac{R}{L}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}\frac{1}{C}\\ 0\end{bmatrix}u_i$$

$$y = \begin{bmatrix}0 & R\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

2.4.2 Time response of SV models

For LTI system

The state differential equation is $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$

To get $\mathbf{x}(t)$

$$\dot{\mathbf{x}}(t) - \mathbf{A}\mathbf{x}(t) = \mathbf{B}\mathbf{u}(t)$$

Both side multiply with $e^{-\mathbf{A}t}$

$$e^{-\mathbf{A}t}[\dot{\mathbf{x}}(t) - \mathbf{A}\mathbf{x}(t)] = e^{-\mathbf{A}t}\mathbf{B}\mathbf{u}(t)$$

$$\frac{d[e^{-\mathbf{A}t}\mathbf{x}(t)]}{dt} = e^{-\mathbf{A}t}\mathbf{B}\mathbf{u}(t)$$

The integral from zero to infinity

$$\int_0^t \frac{d[e^{-\mathbf{A}\tau}\mathbf{x}(\tau)]}{d\tau} d\tau = \int_0^t e^{-\mathbf{A}\tau}\mathbf{B}\mathbf{u}(\tau) d\tau$$

2.4.2 Time response of SV models

$$e^{-\mathbf{A}\tau}\mathbf{x}(\tau)\Big|_0^t = \int_0^t e^{-\mathbf{A}\tau}\mathbf{B}\mathbf{u}(\tau)d\tau$$

$$e^{-\mathbf{A}t}\mathbf{x}(t) = \mathbf{x}(0) + \int_0^t e^{-\mathbf{A}\tau}\mathbf{B}\mathbf{u}(\tau)d\tau$$

两边左乘$e^{\mathbf{A}t}$

$$\Rightarrow \quad \mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}\mathbf{u}(\tau)d\tau \qquad (2.25)$$

Thus $\mathbf{x}(t) = \exp(\mathbf{A}t)\mathbf{x}(0) + \int_0^t \exp[\mathbf{A}(t-\tau)]\mathbf{B}\mathbf{u}(\tau)d\tau$

The **matrix exponential function**: $e^{\mathbf{A}t}$

$$e^{\mathbf{A}t} = \mathbf{I}_n + \mathbf{A}t + \frac{\mathbf{A}^2}{2!}t^2 + \cdots = \sum_{k=1}^{\infty}\frac{\mathbf{A}^k}{k!}t^k \qquad (2.26)$$

2.4.2 Time response of SV models

$$\mathbf{\Phi}(t) = e^{\mathbf{A}t}$$

State Transition Matrix

状态转移矩阵

$$\mathbf{x}(t) = \mathbf{\Phi}(t)\mathbf{x}(0) + \int_0^t \mathbf{\Phi}(t-\tau)\mathbf{B}\mathbf{u}(t)d\tau$$

How to get State Transition Matrix

- According to the definition

$$e^{\mathbf{A}t} = \mathbf{I}_n + \mathbf{A}t + \frac{\mathbf{A}^2}{2!}t^2 + \cdots = \sum_{k=1}^{\infty}\frac{\mathbf{A}^k}{k!}t^k$$

- Complex Number Domain

$$e^{\mathbf{A}t} = \mathscr{L}^{-1}[(s\mathbf{I}-\mathbf{A})^{-1}] \qquad (2.27)$$

返回

2.5 The transform between I/O models and SV models

2.5.1 Transform state variable models into transfer function models 由状态空间模型转换为传递函数(阵)

nth-order Linear Time Invariant System SV model

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$$

Initial conditions are set to zero. Taking Laplace transform.

We have:

$$s\mathbf{X}(s) = \mathbf{A}\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s) \quad (2.28)$$

$$\mathbf{Y}(s) = \mathbf{C}\mathbf{X}(s) + \mathbf{D}\mathbf{U}(s) \quad (2.29)$$

Taking matrix operation of Eq.(2.28), we have ：

$$(s\mathbf{I} - \mathbf{A})\mathbf{X}(s) = \mathbf{B}\mathbf{U}(s) \quad (2.30)$$

$$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\mathbf{U}(s) \quad (2.31)$$

2.5 The transform between I/O models and SV models

Substituting Eq. (2.31) into Eq. (2.29), we obtain :

$$\mathbf{Y}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\mathbf{U}(s) + \mathbf{D}\mathbf{U}(s)$$

$$= [\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}]\mathbf{U}(s)$$

$$\boxed{\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}}$$

Transfer Function Matrix

传递函数矩阵

For a SISO system, $U(s)$,$Y(s)$ are single input and single output respectively, we have the transfer function

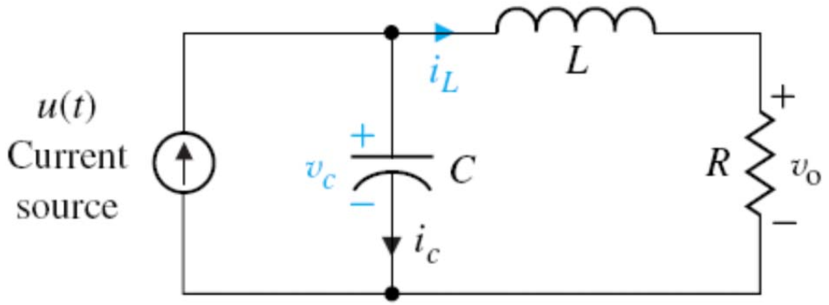
对于SISO系统, $U(s)$,$Y(s)$是标量，则传递函数为 :

$$G(s) = \frac{Y(s)}{U(s)} = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + D \qquad (2.32)$$

2.5 The transform between I/O models and SV models

<E2.6> Determine the transfer function for the RLC circuit as described by the differential equations.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & -\dfrac{1}{C}\\ \dfrac{1}{L} & -\dfrac{R}{L}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}\dfrac{1}{C}\\ 0\end{bmatrix}u$$



$$y=\begin{bmatrix}0 & R\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

We have

$$[s\mathbf{I}-\mathbf{A}]=\begin{bmatrix}s & \dfrac{1}{C}\\ -\dfrac{1}{L} & (s+\dfrac{R}{L})\end{bmatrix}$$

2.5 The transform between I/O models and SV models

Therefore, we obtain

$$[s\mathbf{I}-\mathbf{A}]^{-1}=\frac{adj(s\mathbf{I}-\mathbf{A})}{\det(s\mathbf{I}-\mathbf{A})}=\frac{1}{s^2+\frac{R}{L}s+\frac{1}{LC}}\begin{bmatrix}(s+\frac{R}{L}) & -\frac{1}{C}\\ \frac{1}{L} & s\end{bmatrix}$$

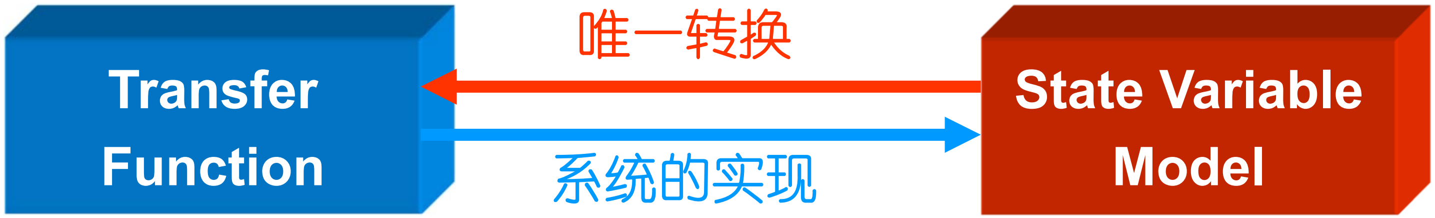
The Transfer Function is

$$G(s)=\mathbf{C}[s\mathbf{I}-\mathbf{A}]^{-1}\mathbf{B}=\begin{bmatrix}0 & R\end{bmatrix}\begin{bmatrix}\frac{(s+\frac{R}{L})}{\Delta(s)} & \frac{-\frac{1}{C}}{\Delta(s)}\\ \frac{1}{L\Delta(s)} & \frac{s}{\Delta(s)}\end{bmatrix}\begin{bmatrix}\frac{1}{C}\\ 0\end{bmatrix}$$

$$=\frac{R/LC}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

2.5 The transform between I/O models and SV models

2.5.2 *Transform transfer function models into state variable models* *由传递函数转换为状态空间模型*



由状态空间模型可以唯一的转换为一个传递函数(阵)；
由传递函数转换为状态空间模型，称为系统的实现问题。
由于状态量选择的多样性，对于一个传递函数(阵)，系统地实现是不唯一的。这里给出一种“**能控规范型 Controllable Canonical Form**”的实现形式。

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2.5 The transform between I/O models and SV models

- **The numerator of transfer function is constant**

  当传递函数的分子为常数项时

Transfer function of *n-th* order system:

$$G(s)=\frac{Y(s)}{U(s)}=\frac{1}{s^n+a_{n-1}s^{n-1}+\cdots+a_1s+a_0} \qquad (2.33)$$

Initial conditions are set to zero. Taking the inverse Laplace transform, we have:

在零初始条件下，运用拉普拉斯反变换得到

$$\frac{d^n y(t)}{dt^n}+a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}}+\cdots+a_1\frac{dy(t)}{dt}+a_0y(t)=u(t) \qquad (2.34)$$

2.5 The transform between I/O models and SV models

$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_1 \frac{dy(t)}{dt} + a_0 y(t) = u(t) \qquad (2.34)$$

Selecting State Variable

$$\begin{cases} x_1 = y(t) \\ x_2 = \dfrac{dy(t)}{dt} \\ x_3 = \dfrac{d^2 y(t)}{dt^2} \\ \vdots \\ x_n = \dfrac{d^{n-1} y(t)}{dt^{n-1}} \end{cases}$$

We have

$$\begin{cases} \dot{x}_1 = \dfrac{dy(t)}{dt} = x_2 \\ \dot{x}_2 = \dfrac{d^2 y(t)}{dt^2} = x_3 \\ \vdots \\ \dot{x}_{n-1} = \dfrac{d^{n-1} y(t)}{dt^{n-1}} = x_n \\ \dot{x}_n = \dfrac{d^n y(t)}{dt^n} = -a_0 x_1 - a_1 x_2 - \cdots - a_{n-1} x_n + u(t) \end{cases}$$

2.5 The transform between I/O models and SV models

The State Variable Model

$$
\begin{cases}
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u & (2.35) \\
y = [1 \quad 0 \quad \cdots \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} & (2.36)
\end{cases}
$$

能控规范型Controllable Canonical Form

2.5 The transform between I/O models and SV models

- ***The numerator of transfer function is polynomial***

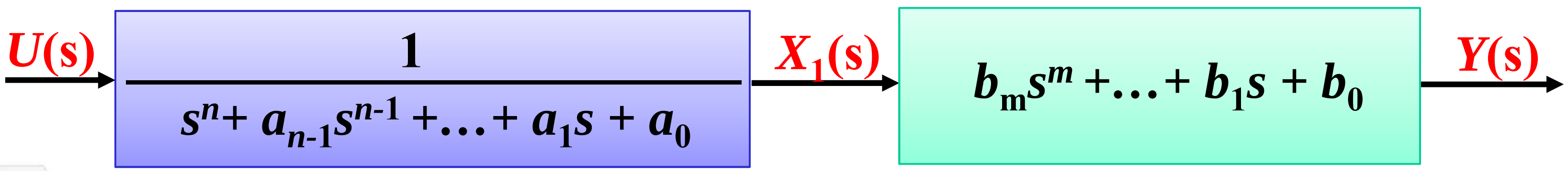
  当传递函数的分子为多项式，$m<n$

Transfer function of *n-th* order system :

$$G(s)=\frac{Y(s)}{U(s)}=\frac{b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0} \tag{2.37}$$

$$=\frac{1}{s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0}\left[b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0\right]$$

$$\frac{X_1(s)}{U(s)}=\frac{1}{s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0} \tag{2.38}$$



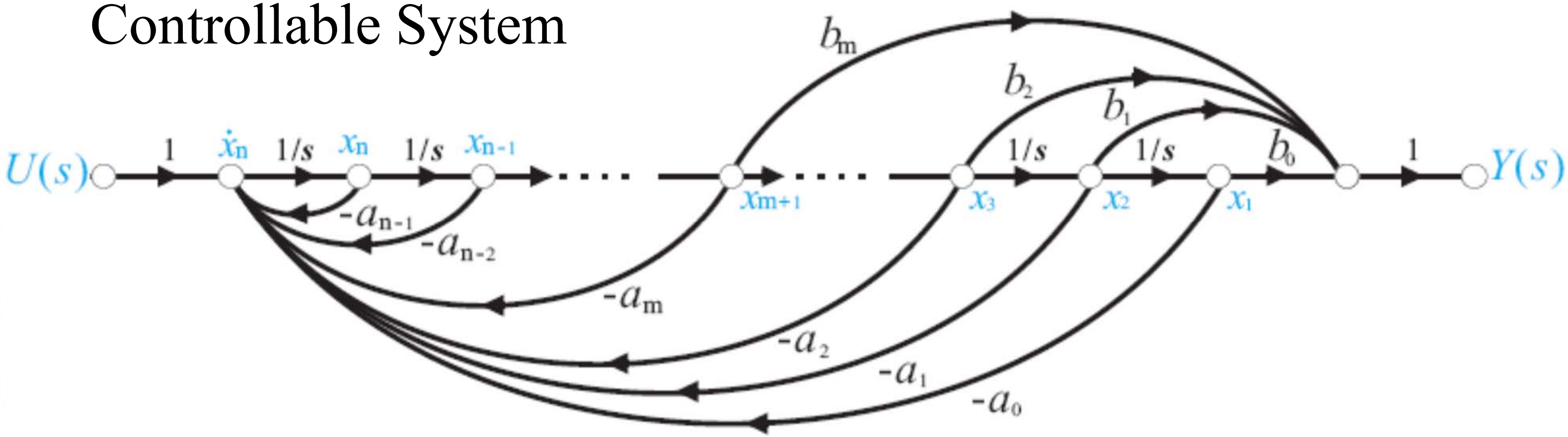
2.5 The transform between I/O models and SV models

therefore $Y(s) = X_1(s)\left[b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0\right]$ (2.39)

Taking the inverse Laplace transform, we have:

$$y(t) = b_m \frac{d^m x_1}{dt^m} + b_{m-1}\frac{d^{m-1}x_1}{dt^{m-1}} + \cdots + b_1\frac{dx_1}{dt} + b_0 x_1 \quad (2.40)$$
$$= b_0 x_1 + b_1 x_2 + \cdots + b_{m-1}x_m + b_m x_{m+1}$$

Controllable System



2.5 The transform between I/O models and SV models

State Differential Equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u \qquad (2.35)$$

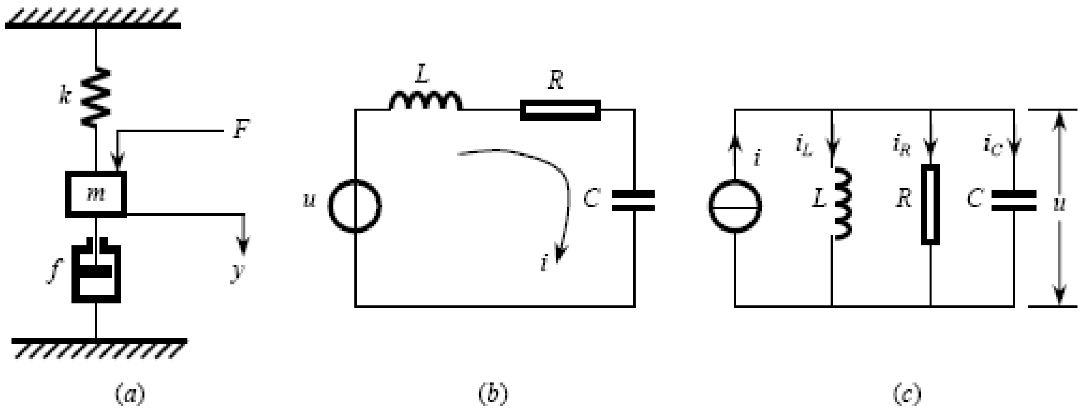
Output Equation

$$y = \begin{bmatrix} b_0 & b_1 & \cdots & b_m & 0 \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_{m+1} \\ \vdots \\ x_n \end{bmatrix} \qquad (2.41)$$

返回

2.6 Examples of mathematic models of systems

2.6.1 Mechanical and Electrical System



- Mechanical System

$$m\ddot{y} = \sum F = F - F_1 - F_2$$

$$F_1 = f\,\dot{y},$$ f—Friction Constant 阻尼系数

$$F_2 = ky,$$ k—Spring Constant 弹簧刚度系数系数

$$m\ddot{y} + f\,\dot{y} + ky = F \qquad (2.42)$$

2.6.1 Mechanical and Electrical System

- RLC series circuit (电荷q为变量)

$$i=\dot{q}$$

$$L\ddot{q}+R\dot{q}+\frac{1}{C}q=u \qquad (2.43)$$

- RLC parallel circuit（磁链 ψ 为变量）

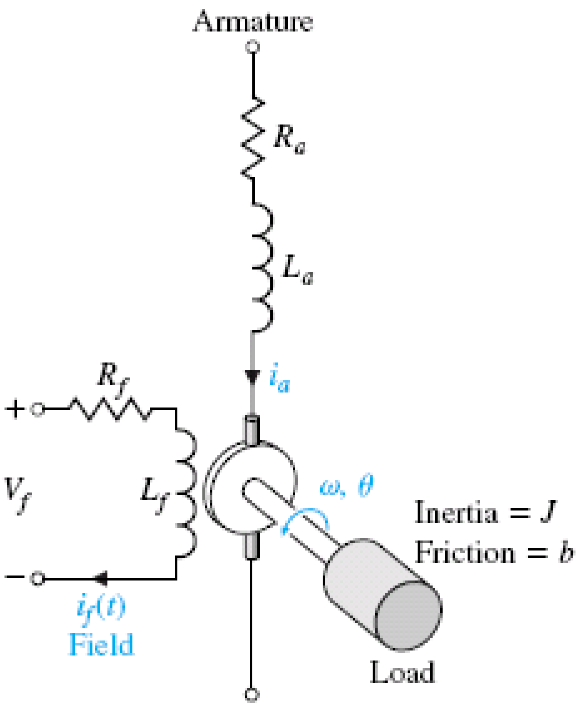
$$u=\dot{\psi}$$

$$C\ddot{\psi}+\frac{1}{R}\dot{\psi}+\frac{1}{L}\psi=i \qquad (2.44)$$

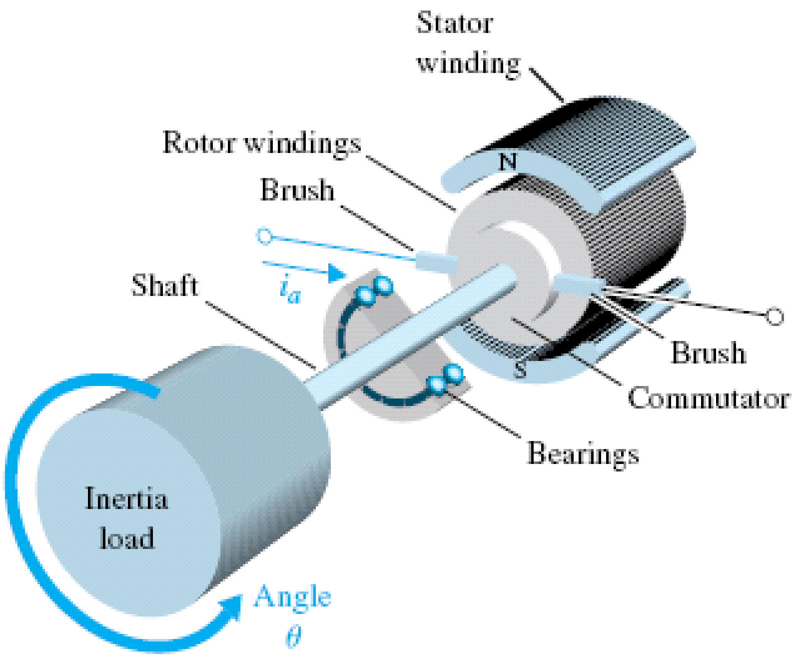
Equ. (2.42-2.44) Same Mathematical Models

2.6.2 DC Motor Feedback Control System

1. State variable model of dc motor



(a)

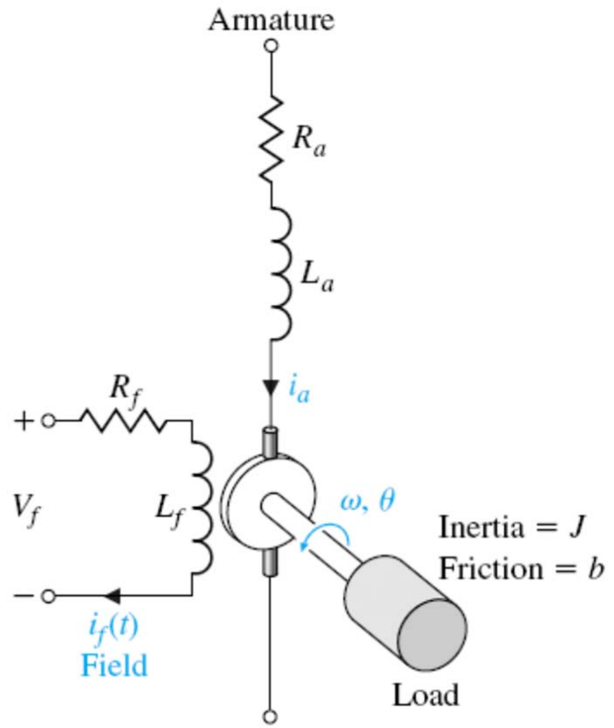


(b)

2.6.2 DC Motor Feedback Control System

- 电枢回路方程

$$L_a \frac{di_a}{dt} + R_a i_a + e_m = u_a \qquad (2.45)$$



L_a电枢回路电感(H)

R_a电枢回路电阻(Ω)

$e_m = C'_e \Phi \omega = C_e \omega$电动机反电势

C_e为电动机反电势系数(V/(rad/s))

2.6.2 DC Motor Feedback Control System

- 电动机运动方程

$$J\frac{d\omega}{dt}+f\omega+M_L=M \qquad (2.46)$$

J 和f 分别为折算到电动机轴上的转动惯量(kg.m^2)和粘性摩擦系数(N.m.s/rad)

M_L为折算置电动机轴上的 负载转矩$(\mathrm{N}\cdot\mathrm{m})$

$M=C_m i_a$为电动机电磁转矩$(\mathrm{N}\cdot\mathrm{m})$

C_m为电动机转矩系数$(\mathrm{N}\cdot\mathrm{m}/\mathrm{A})$

2.6.2 DC Motor Feedback Control System

(2.45)和(2.46)式可以写成

$$\frac{di_a}{dt}=-\frac{R_a}{L_a}i_a-\frac{C_e}{L_a}\omega+\frac{1}{L_a}u_a \tag{2.47}$$

$$\frac{d\omega}{dt}=\frac{C_m}{J}i_a-\frac{f}{J}\omega-\frac{1}{J}M_L \tag{2.48}$$

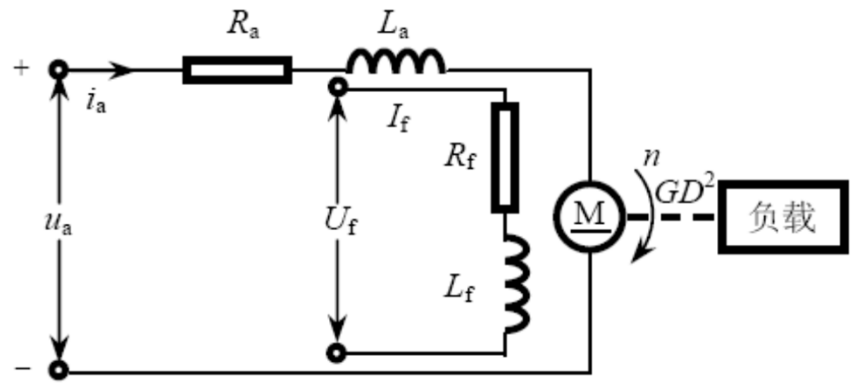
令 $x_1=i_a;x_2=\omega$,可得到系统状态微分方程

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-R_a/L_a & -C_e/L_a\\ C_m/J & -f/J\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1/L_a & 0\\ 0 & -1/J\end{bmatrix}\begin{bmatrix}u_a\\ M_L\end{bmatrix} \tag{2.49}$$

2.6.2 DC Motor Feedback Control System

2. Transfer function of dc motor

- 以转速作为输出
  - 电枢回路环节



由电枢回路方程

$$L_a \frac{di_a}{dt} + R_a i_a + K_e n = u_a \tag{2.49}$$

其中n为转速(r/min)，K_e为反电势系数

令$T_a = L_a / R_a$(电枢回路时间常数)

传递函数

$$\frac{I_a(s)}{U_a(s) - K_e N(s)} = \frac{1/R_a}{1+T_a s} \tag{2.50}$$

2.6.2 DC Motor Feedback Control System

- 转动环节

电动机功率较大时其粘性摩擦系数可以忽略不计，则运动方程为:

$$J_G \frac{dn}{dt} + M_L = M \qquad (2.51)$$

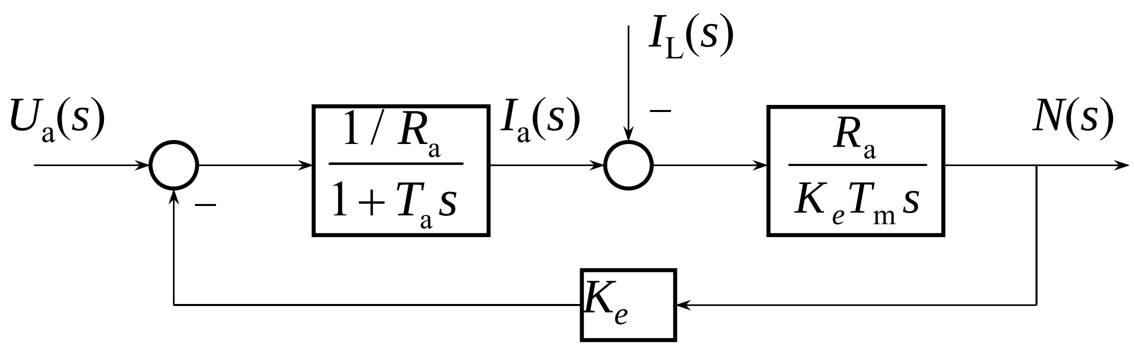
其中 $J_G = \dfrac{GD^2}{375}, M = K_m i_a, M_L = K_m i_L$

令 $T_m = \dfrac{GD^2}{375} \dfrac{R_a}{K_e K_m}$ (机电时间常数)

得到传递函数 $$\frac{N(s)}{I_a(s) - I_L(s)} = \frac{K_m}{J_G s} = \frac{R_a}{K_e T_m s} \qquad (2.52)$$

2.6.2 DC Motor Feedback Control System

可以得到枢控电动机结构图



枢控电动机结构图

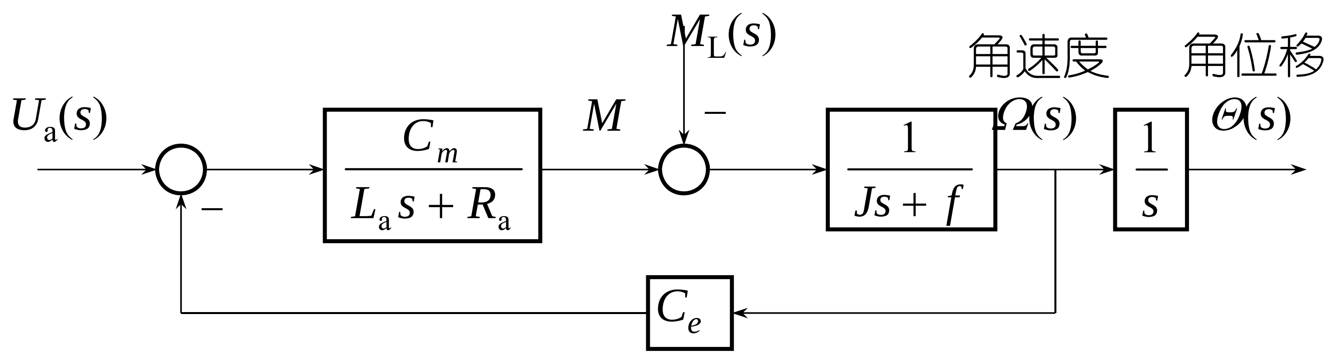
2.6.2 DC Motor Feedback Control System

- 以角速度或角位移作为输出量(小功率伺服系统)

由于电动机功率较小，要考虑粘性摩擦

由(2.46)式,转动环节的输入输出方程为：

$$(Js + f)\Omega(s) = M(s) - M_L(s) \tag{2.53}$$



枢控电动机框图模型

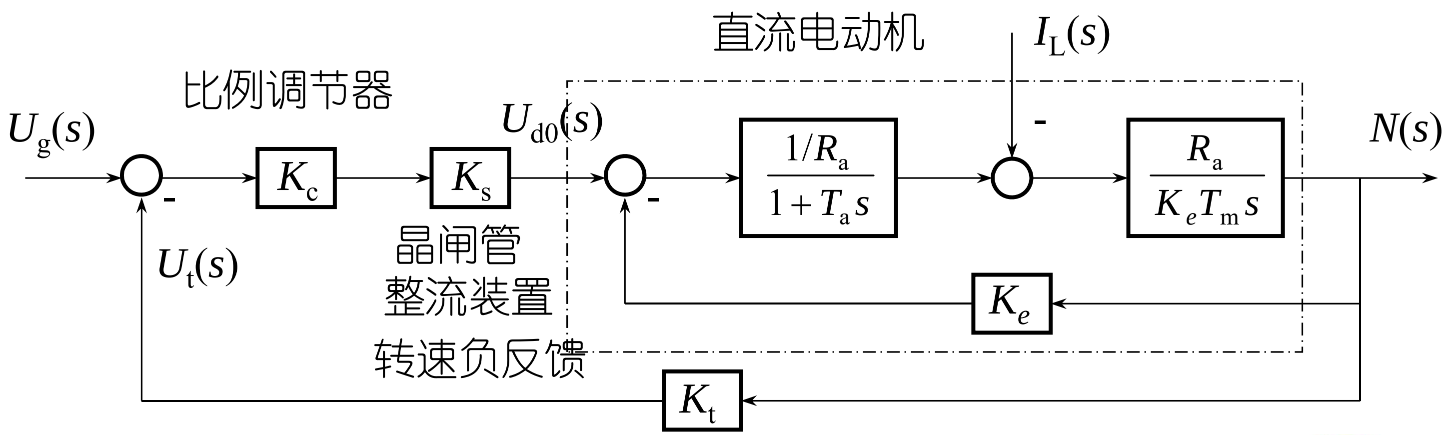
2.6.2 DC Motor Feedback Control System

3. 枢控电动机直流调速系统结构框图

- 以测速发电机作为转速负反馈环节，有

$$u_t = K_t n$$

- 采用比例调节器时，系统的结构框图



返回

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Summary

- In this chapter, we have been concerned with quantitative mathematical models of control systems.
- Introduced the I/O model which includes differential equations, transfer function and transfer function in frequency domain.
- Introduced the graphic models, mainly talked about the block diagram with it's transformation and reduction.
- Introduced the state variable models.
- Introduced the transform between I/O models and state variable models.

返回

Summary

- Read Chapter 4 page 183-218

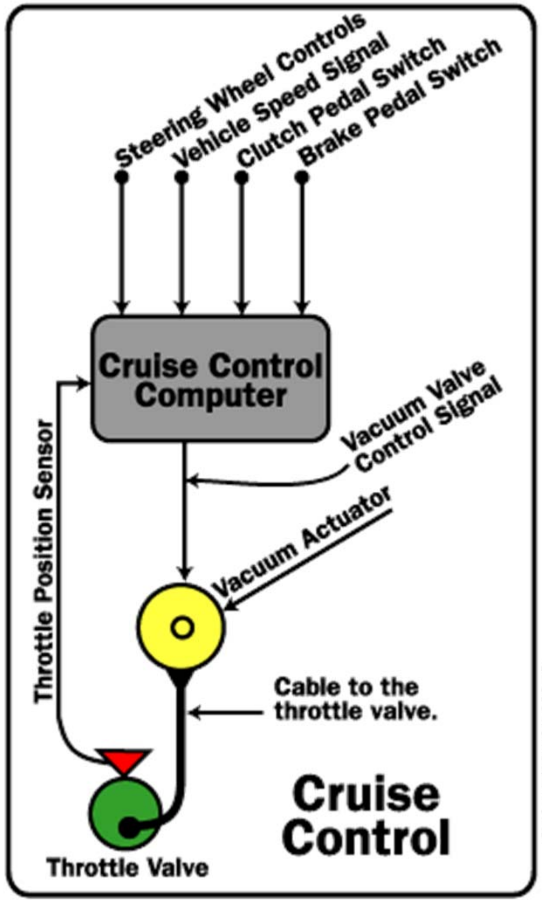
返回

Cruise Control System Design

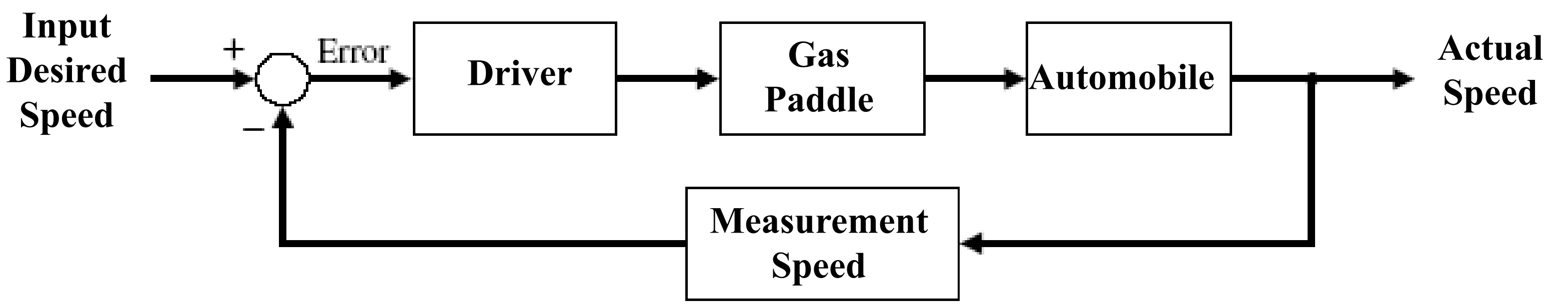
Design Task： Design a cruise control system;

Cruise control is the term used to describe a control system that regulates the speed of an automobile. Cruise control was commercially introduced in 1958 as an option on the Chrysler Imperial.

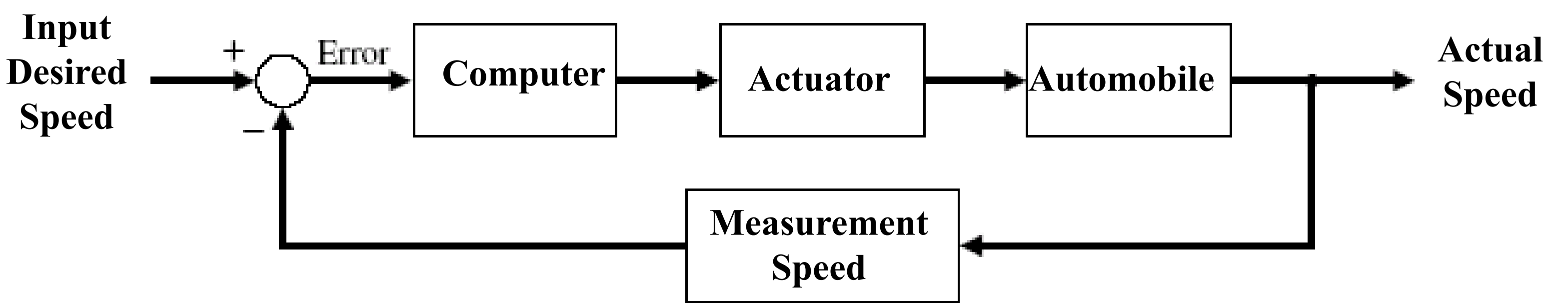
The basic operation of a cruise controller is to sense the speed of the vehicle, compare this speed to a desired reference, and then accelerate or decelerate the car as required.



Cruise Control System Design



Manual Cruise Control Block Diagram



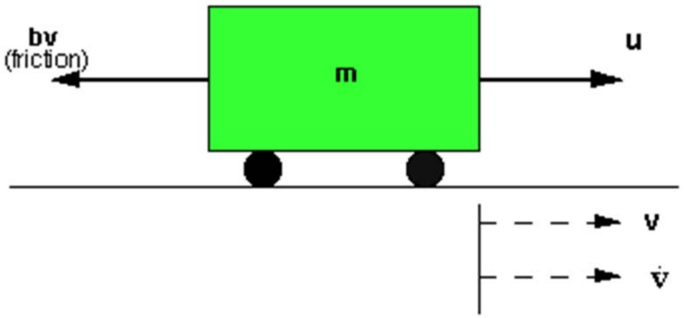
Automatic Cruise Control Block Diagram

Cruise Control System Design

Component of automobile：

Chassis Wheel Drive (Engine & Transmission)

1.Physical setup and mathematical equations：



Assume

Mass m=1000kg,

Friction Constant b=50N.sec/m

Speed v m/sec

Goal: Reach 10m/s less than 5Sec.

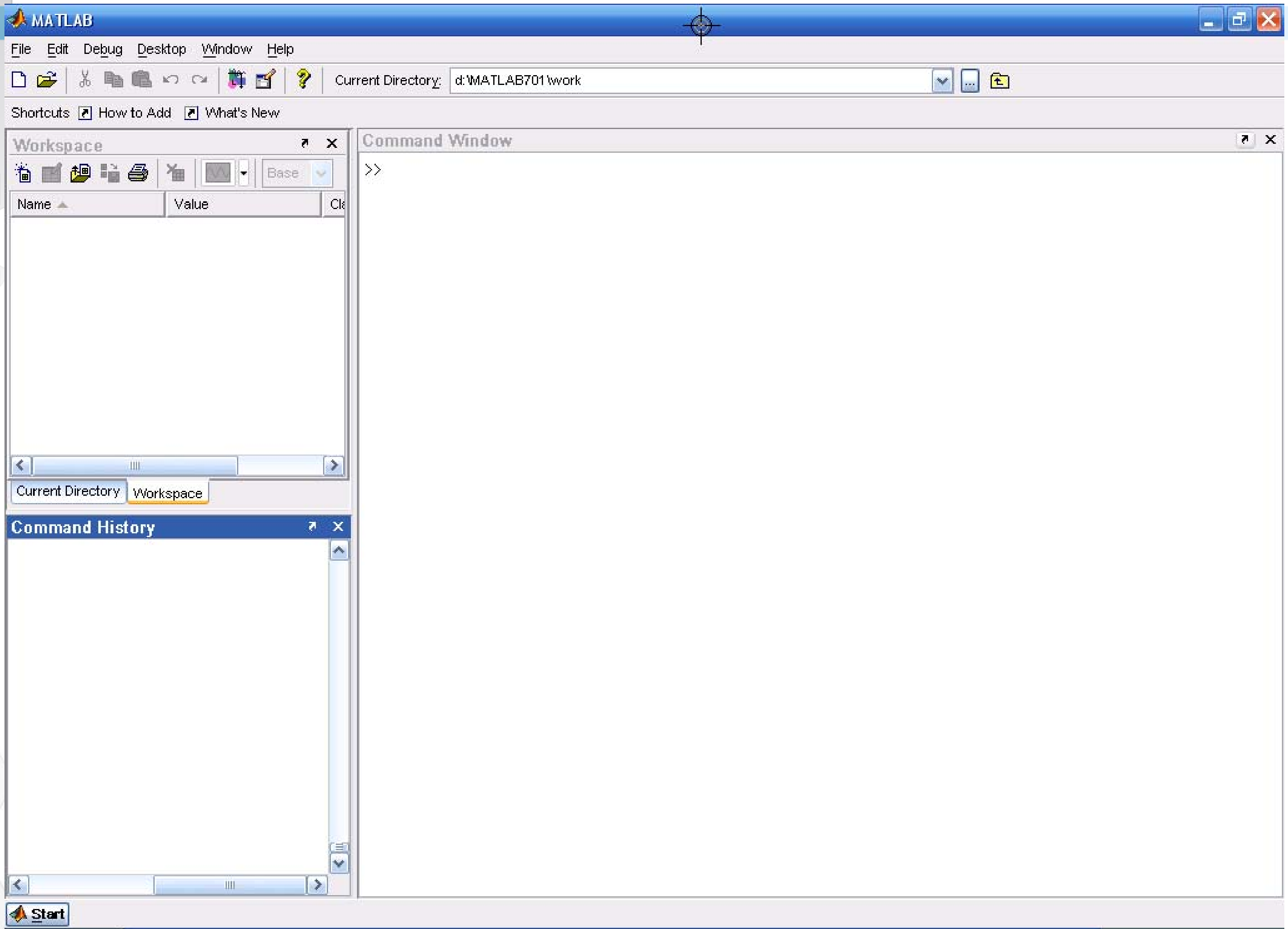
Keep the speed at 30m/s

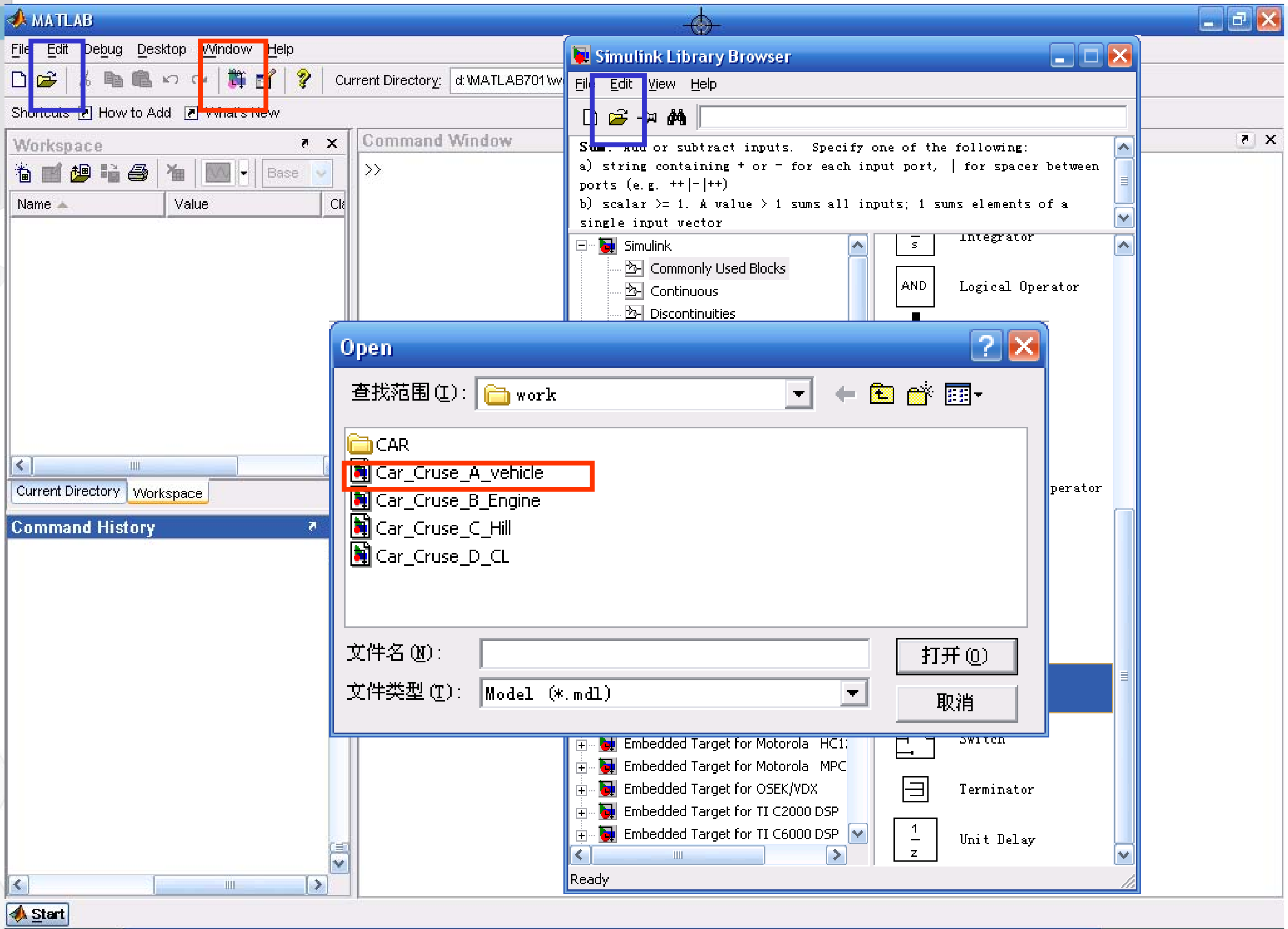
$$m\frac{dv}{dt} = -bv + F_{engine} + F_{hill}$$

$$\dot{x}_v = \left[-b/m\right] x_v + \left[1/m \quad 1/m\right] u_v$$

$$y_v = x_v$$

MATLAB 7.0


MATLAB
File Edit Debug Desktop Window Help
Current Directory: d:\MATLAB701\work
Shortcuts How to Add What's New
Workspace
Name Value
Current Directory Workspace
Command History
Command Window
>>
Start



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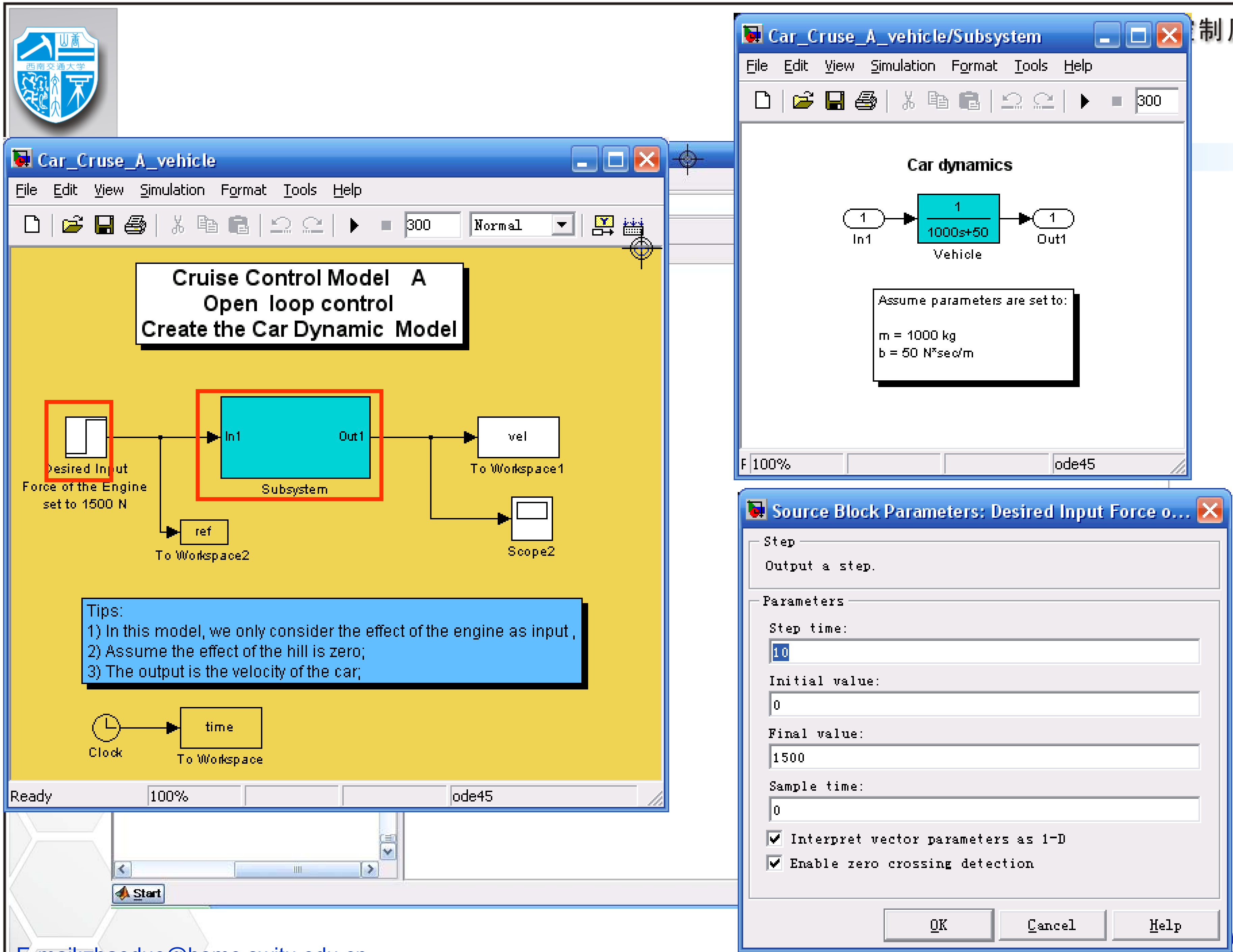

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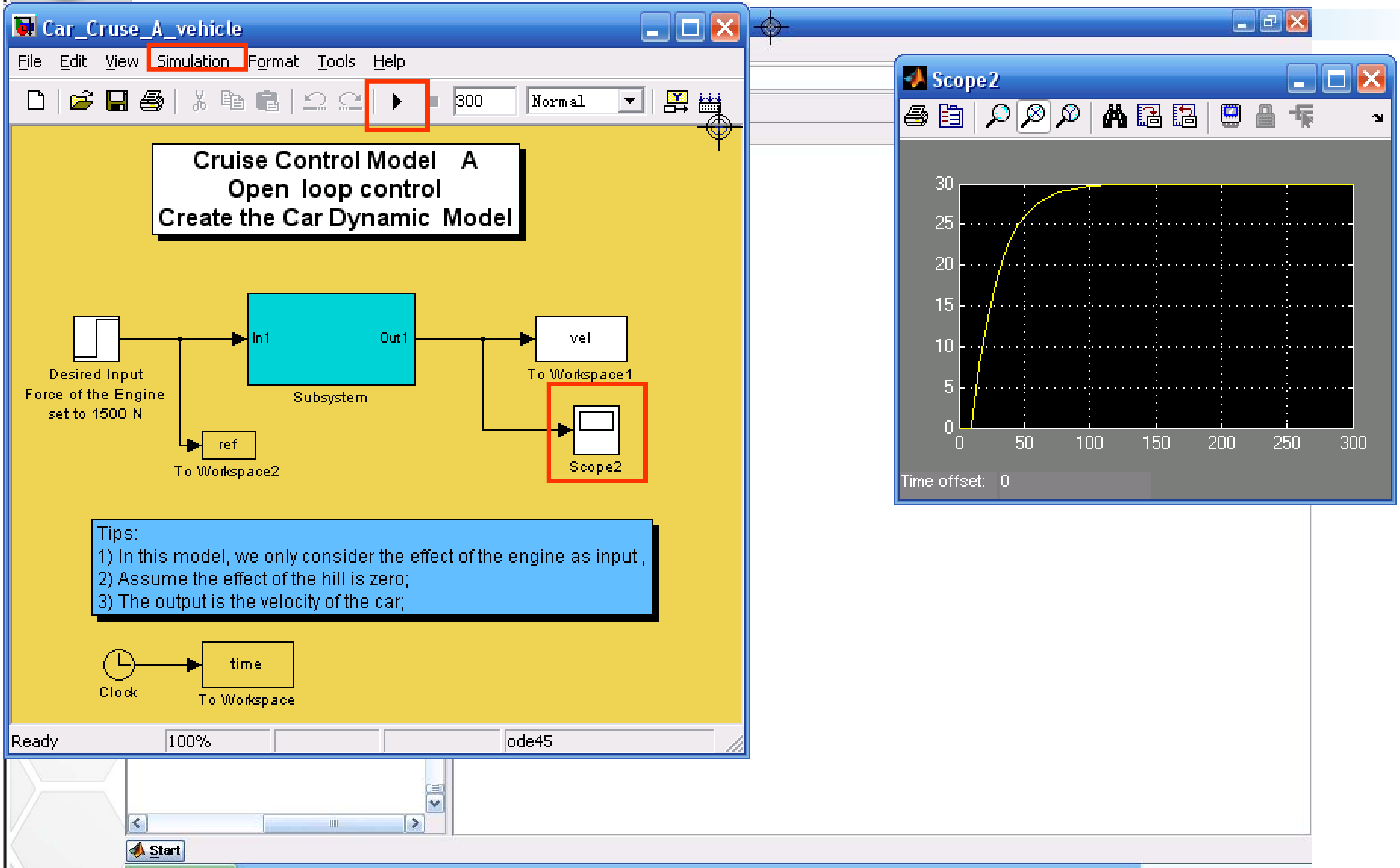

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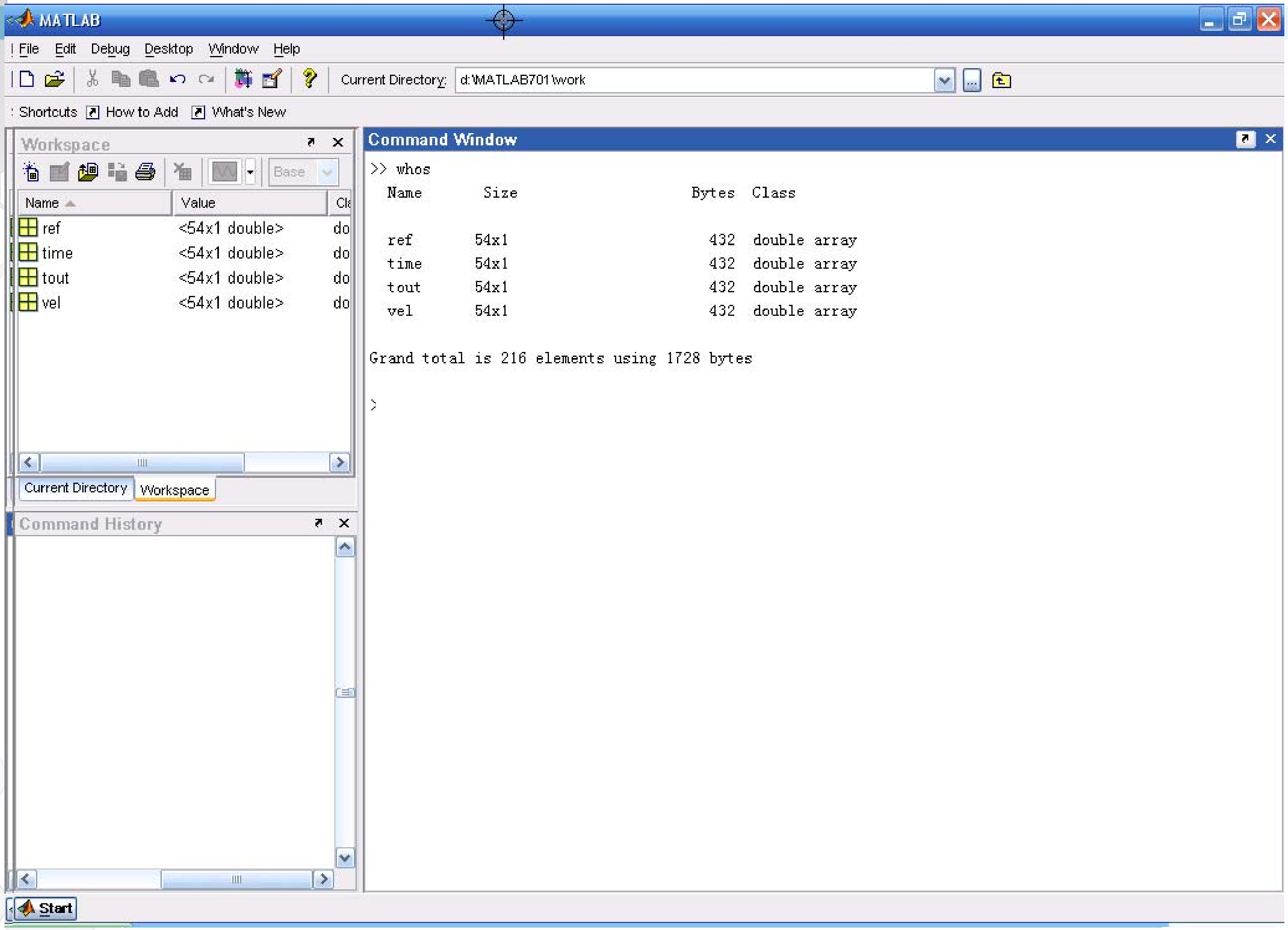

MATLAB
File Edit Debug Desktop Window Help
Current Directory: d:\MATLAB701\work
Shortcuts How to Add What's New
Workspace
Base
Name Value Cl
ref <54x1 double> do
time <54x1 double> do
tout <54x1 double> do
vel <54x1 double> do
Current Directory Workspace
Command History
Command Window
>> whos
Name Size Bytes Class
ref 54x1 432 double array
time 54x1 432 double array
tout 54x1 432 double array
vel 54x1 432 double array
Grand total is 216 elements using 1728 bytes
>
Start

汽车速度控制系统设计实例

2. Mathematical model of engine：

$$\frac{d\tau}{dt} = -a\tau + K_i u_e$$

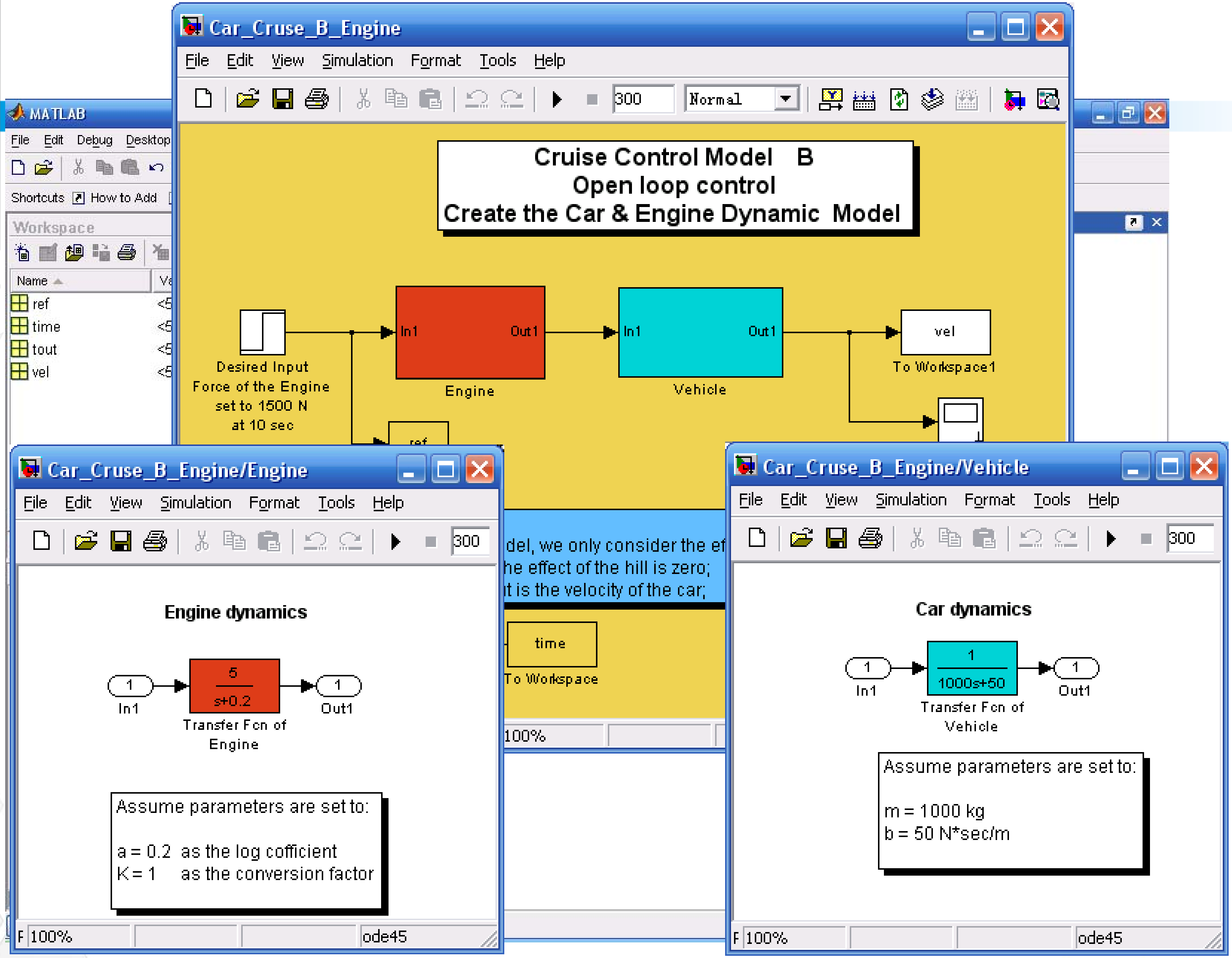
$$y_e = K_o\tau$$

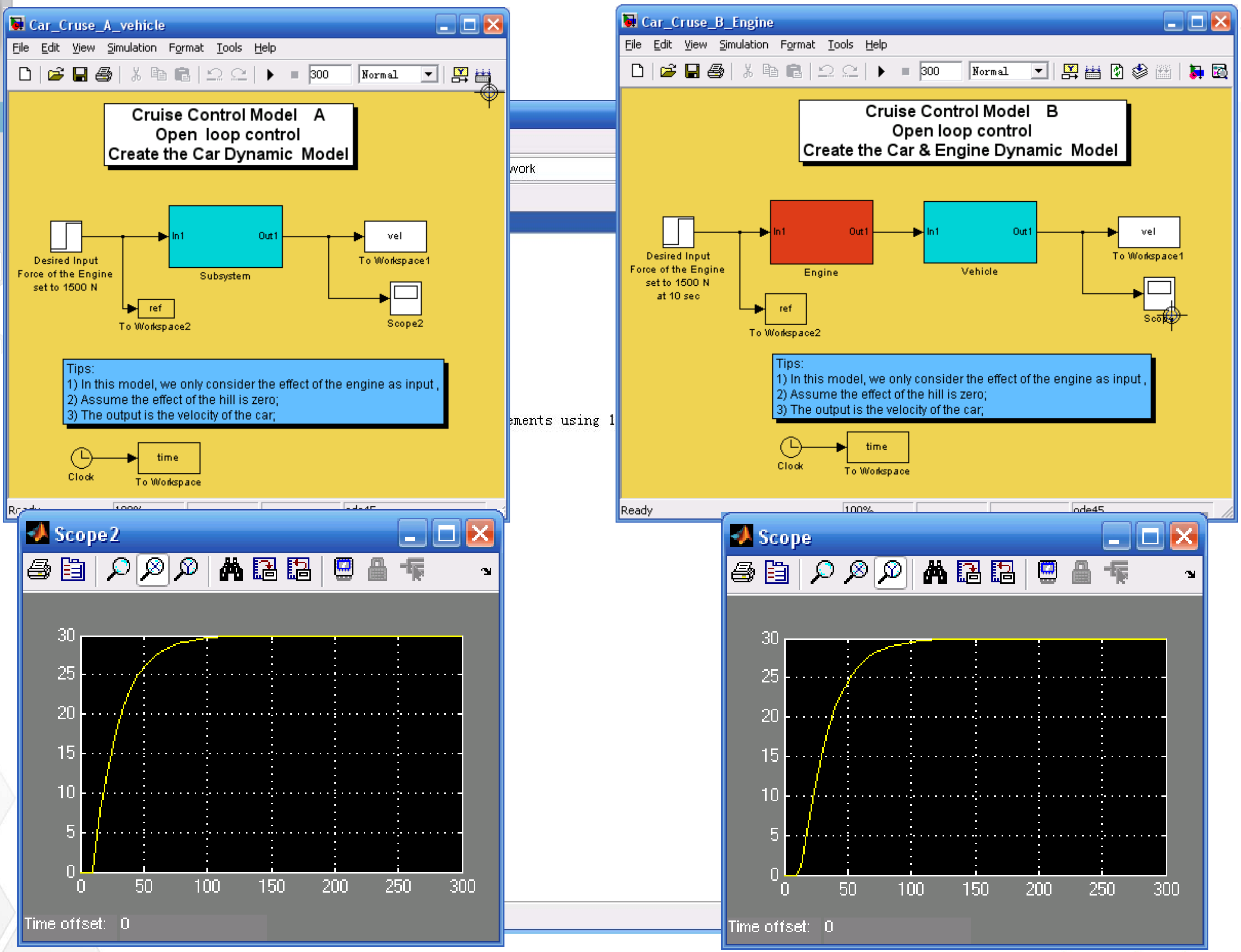
Assume a=0.2 Time constant,

K_o=5 引擎输出力矩与F_e之间的转换因子(如传动装置)

u_e 作为对引擎的输入,可选择合适的单位

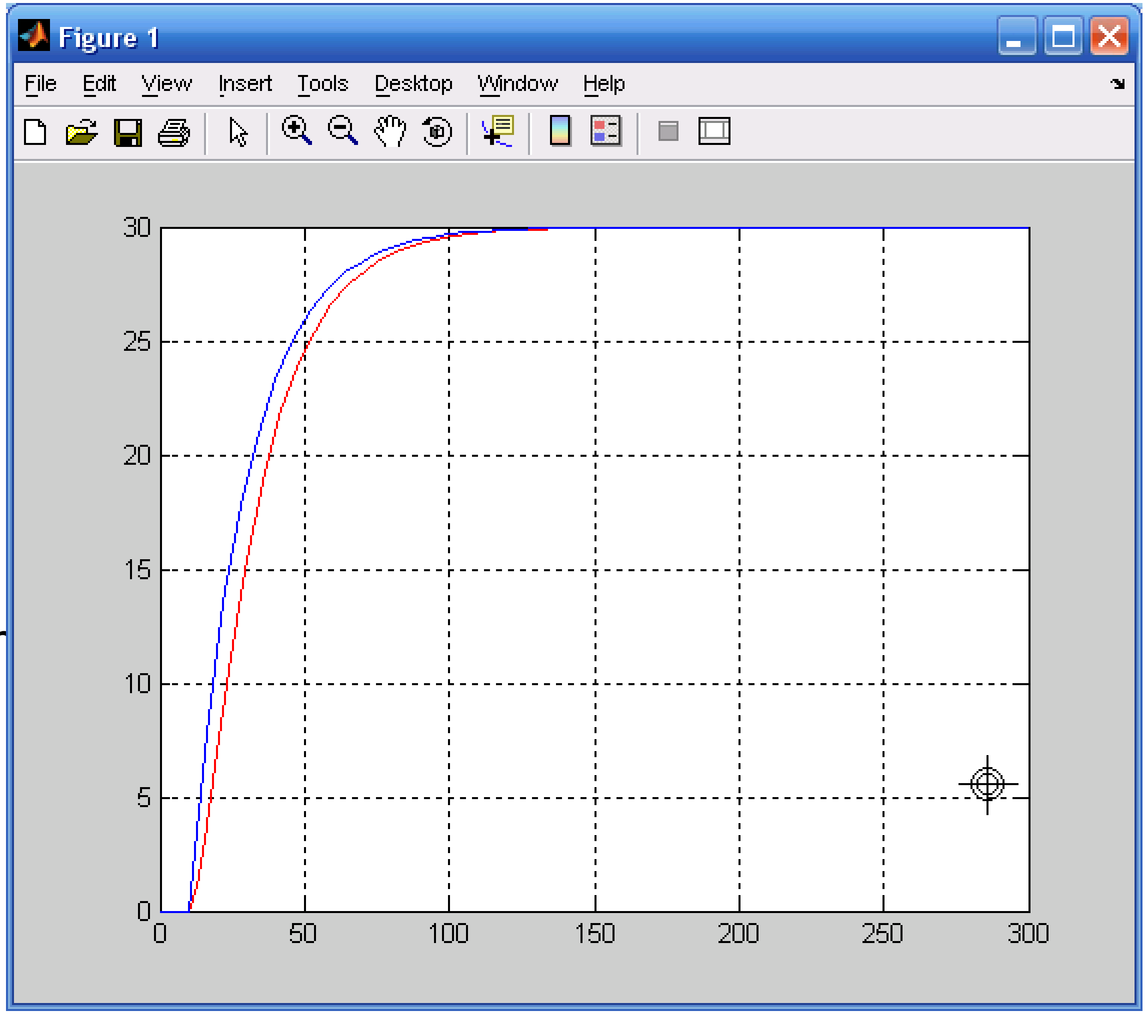
K_i=1引擎输入量与力矩之间的转换因子(如踏板角度等)


Car_Cruse_B_Engine
Cruise Control Model B
Open loop control
Create the Car & Engine Dynamic Model
Desired Input Force of the Engine set to 1500 N at 10 sec
Engine
Vehicle
vel
To Workspace1
time
To Workspace
del, we only consider the ef
he effect of the hill is zero;
t is the velocity of the car;
Car_Cruse_B_Engine/Engine
Engine dynamics
5
s+0.2
Transfer Fcn of Engine
In1
Out1
Assume parameters are set to:
a = 0.2 as the log cofficient
K = 1 as the conversion factor
Car_Cruse_B_Engine/Vehicle
Car dynamics
1
1000s+50
Transfer Fcn of Vehicle
Assume parameters are set to:
m = 1000 kg
b = 50 N*sec/m


Car_Cruse_A_vehicle
Cruise Control Model A
Open loop control
Create the Car Dynamic Model
Desired Input
Force of the Engine
set to 1500 N
Subsystem
To Workspace1
To Workspace2
Scope2
Tips:
1) In this model, we only consider the effect of the engine as input ,
2) Assume the effect of the hill is zero;
3) The output is the velocity of the car;
Clock
To Workspace
Car_Cruse_B_Engine
Cruise Control Model B
Open loop control
Create the Car & Engine Dynamic Model
Desired Input
Force of the Engine
set to 1500 N
at 10 sec
Engine
Vehicle
To Workspace1
To Workspace2
Tips:
1) In this model, we only consider the effect of the engine as input ,
2) Assume the effect of the hill is zero;
3) The output is the velocity of the car;
Clock
To Workspace
Scope2
Time offset: 0
Scope
Time offset: 0

```
>> whos
  Name         Size
  ref_a       54x1
  ref_b       56x1
  time_a       54x1
  time_b       56x1
  tout       56x1
  vel_a       54x1
  vel_b       56x1
Grand total is 386 elem

>> plot(time_a,vel_a)
>> grid
>> hold
Current plot held
>> plot(time_b,vel_b,'r')
```



汽车速度控制系统设计实例

汽车构成：底盘 车轮 驱动(引擎&传动系统)

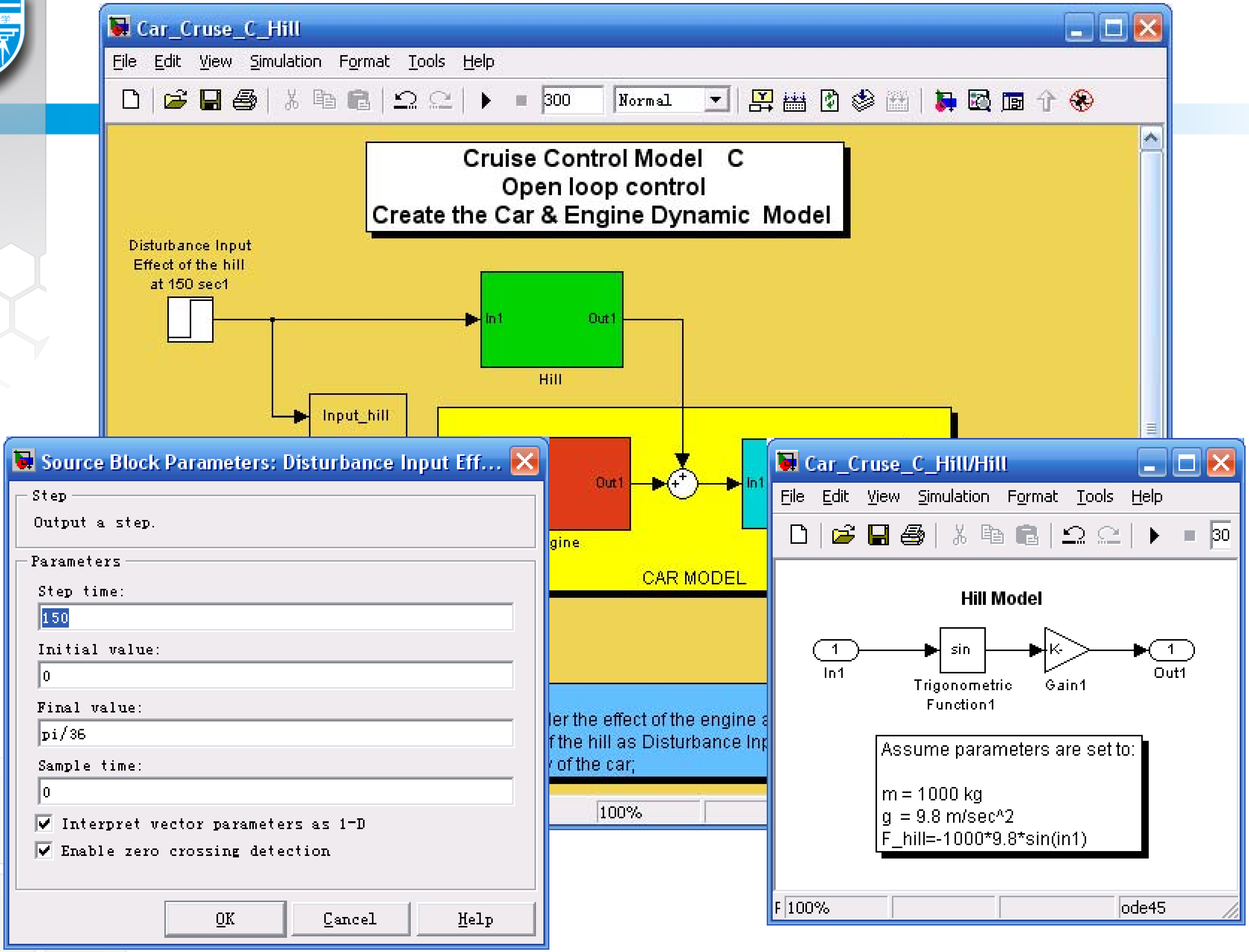
3. 考虑到路面坡度对汽车速度的影响：

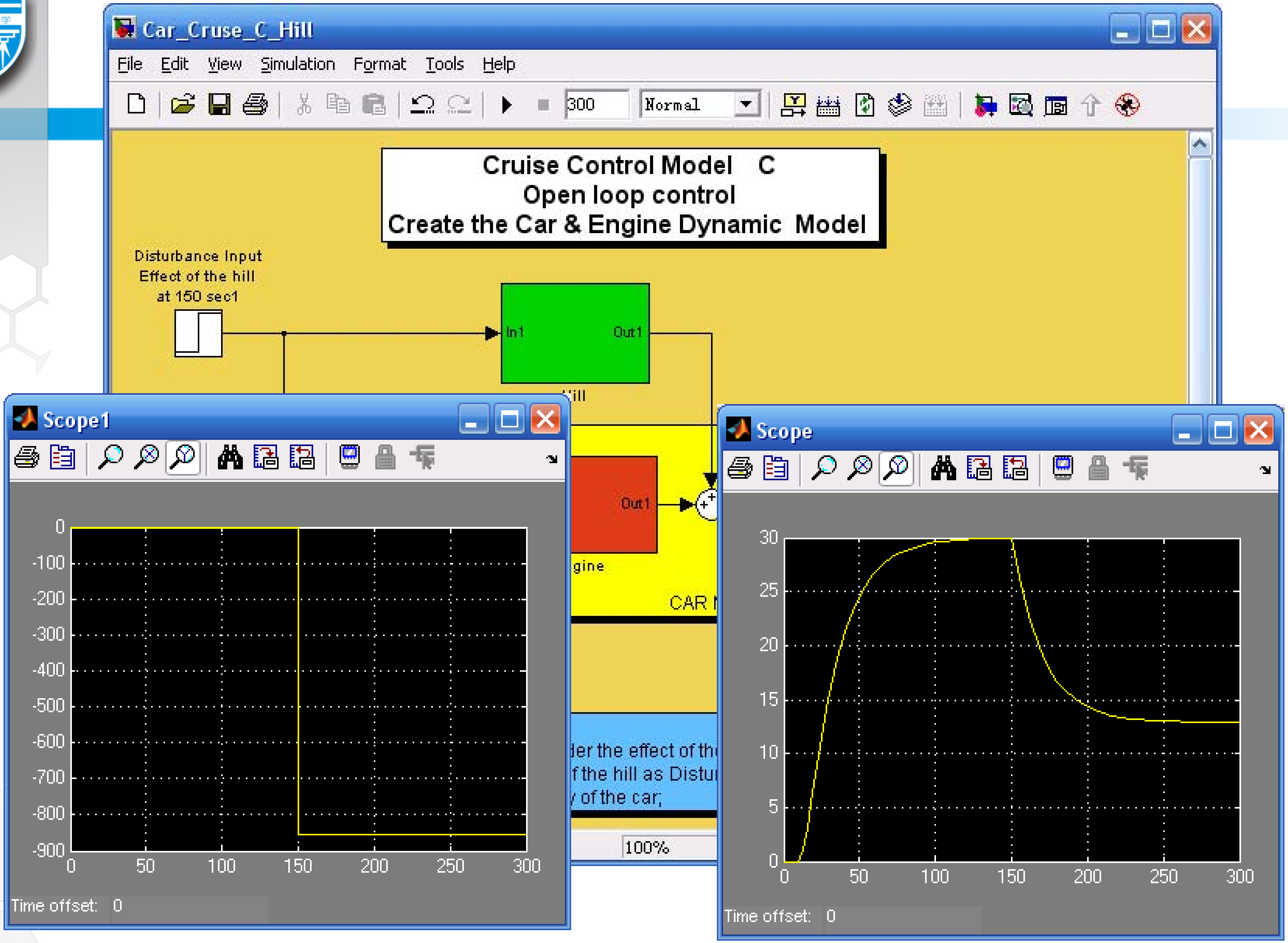
$$F_{hill} = -mg\sin(\theta)$$

g 重力加速度

θ 路面坡度

假设在路面有10度的上坡


Car_Cruse_C_Hill
File Edit View Simulation Format Tools Help
300
Normal
Cruise Control Model C
Open loop control
Create the Car & Engine Dynamic Model
Disturbance Input
Effect of the hill
at 150 sec1
In1
Out1
Hill
Input_hill
Out1
gine
In1
CAR MODEL
ler the effect of the engine a
f the hill as Disturbance Inp
of the car;
100%
Source Block Parameters: Disturbance Input Eff...
Step
Output a step.
Parameters
Step time:
150
Initial value:
0
Final value:
pi/36
Sample time:
0
Interpret vector parameters as 1-D
Enable zero crossing detection
OK
Cancel
Help
Car_Cruse_C_Hill/Hill
File Edit View Simulation Format Tools Help
30
Hill Model
1
In1
sin
Trigonometric
Function1
K-
Gain1
1
Out1
Assume parameters are set to:
m = 1000 kg
g = 9.8 m/sec^2
F_hill=-1000*9.8*sin(in1)
100%
ode45



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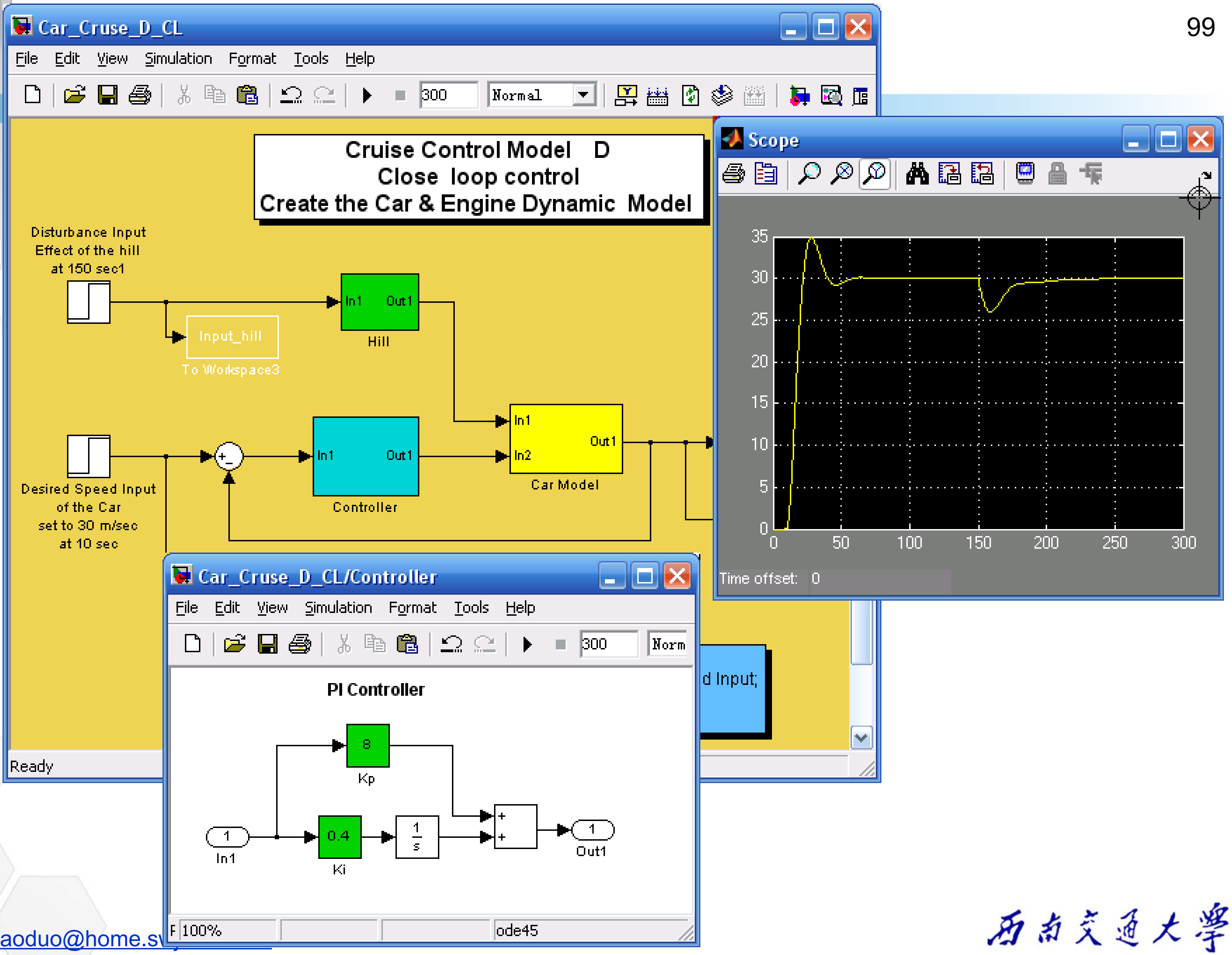
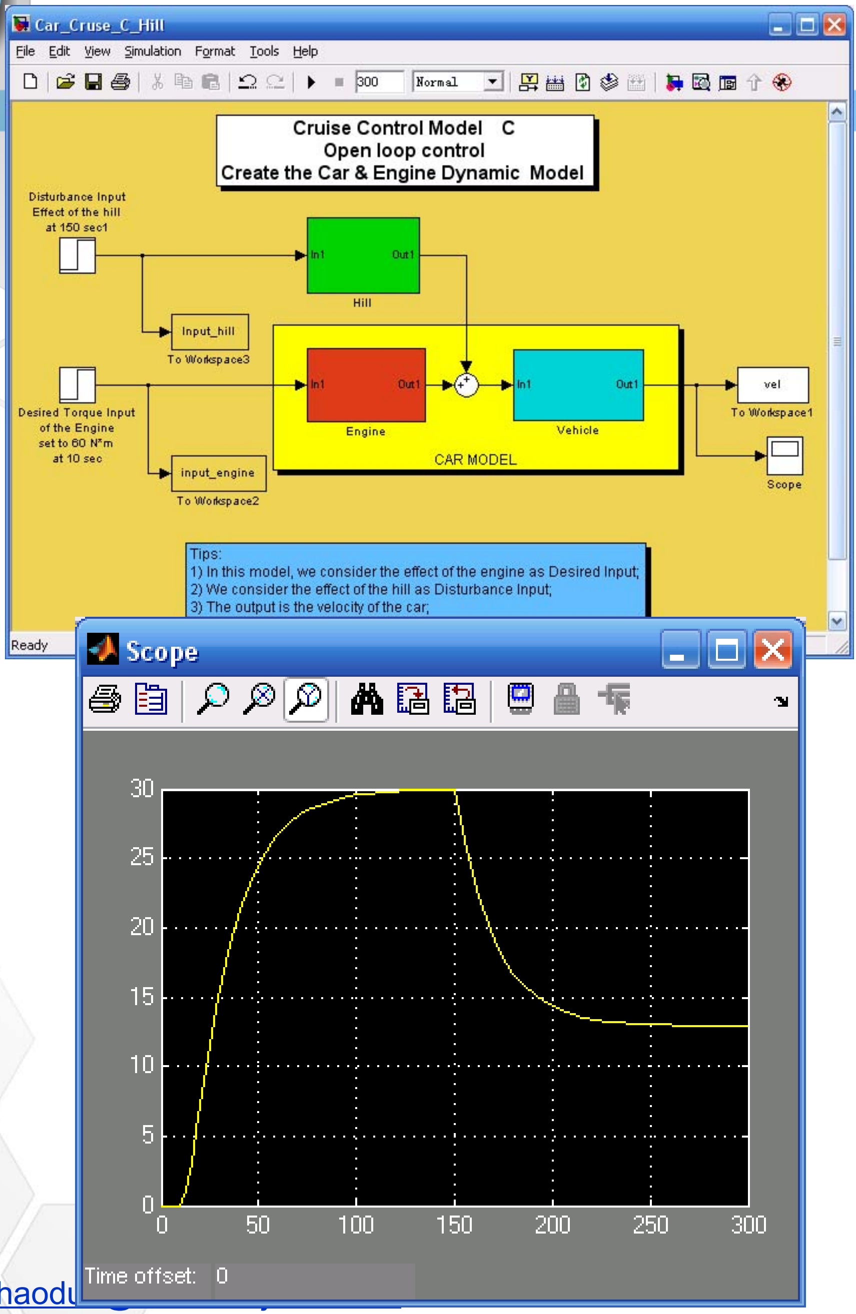
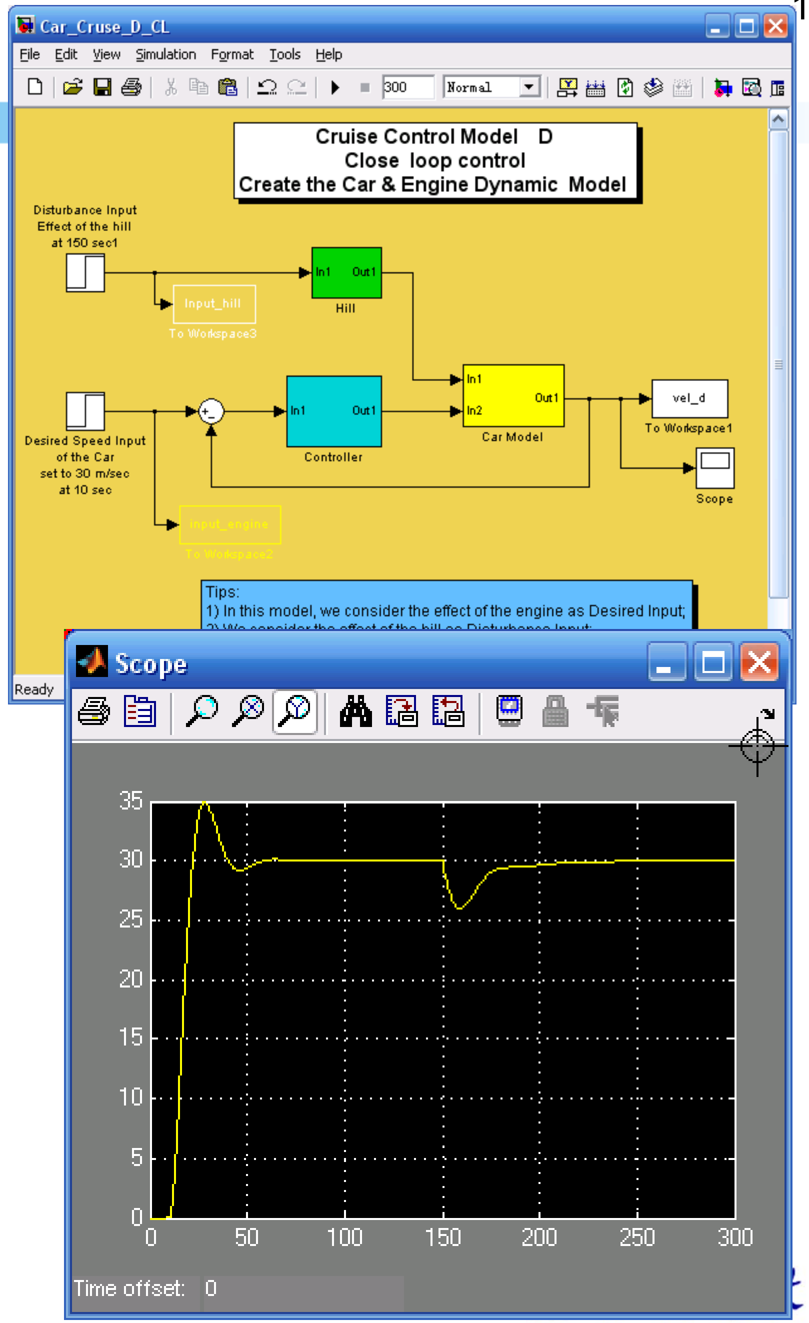

Car_Cruse_D_CL
File Edit View Simulation Format Tools Help
300
Normal
Cruise Control Model D
Close loop control
Create the Car & Engine Dynamic Model
Disturbance Input
Effect of the hill
at 150 sec1
Input_hill
To Workspace3
In1 Out1
Hill
In1
In2
Out1
Car Model
Desired Speed Input
of the Car
set to 30 m/sec
at 10 sec
In1 Out1
Controller
Ready
d Input;
Scope
35
30
25
20
15
10
5
0
0 50 100 150 200 250 300
Time offset: 0
Car_Cruse_D_CL/Controller
File Edit View Simulation Format Tools Help
300
Norm
PI Controller
8
Kp
1
In1
0.4
Ki
1/s
+
+
1
Out1
100%
ode45





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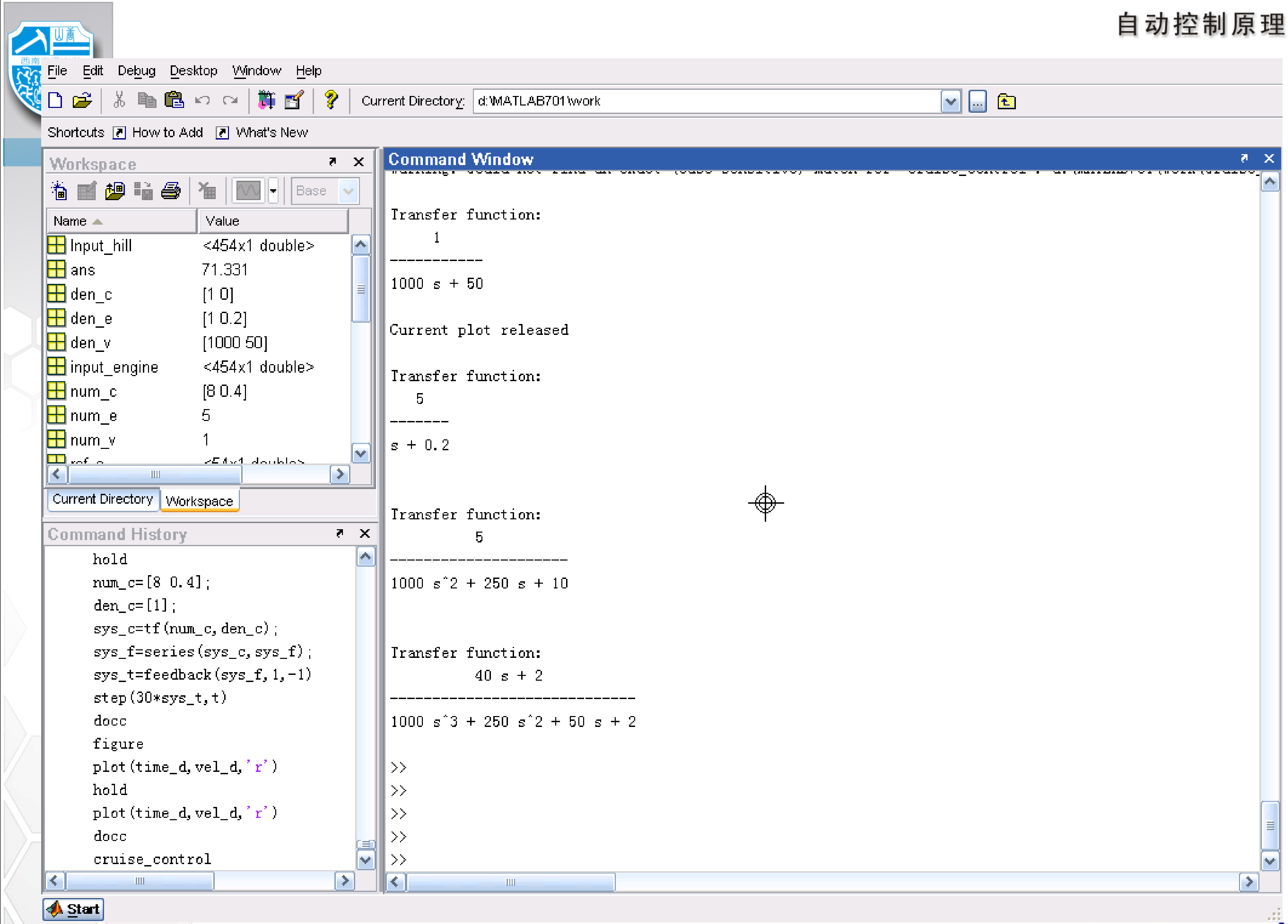
Editor - D:\MATLAB701\work\Cruise_control.m

```
t=[0:0.8:300];

num_v=[1];
den_v=[1000 50];
sys_v=tf(num_v,den_v)
step(1500*sys_v,t)
hold
num_e=[5];
den_e=[1 0.2];
sys_e=tf(num_e,den_e)

sys_f=series(sys_e,sys_v)

step(60*sys_f,t)

num_c=[8 0.4];
den_c=[1 0];
sys_c=tf(num_c,den_c);

sys_f=series(sys_c,sys_f);
sys_t=feedback(sys_f,1,-1)
step(30*sys_t,t)
```



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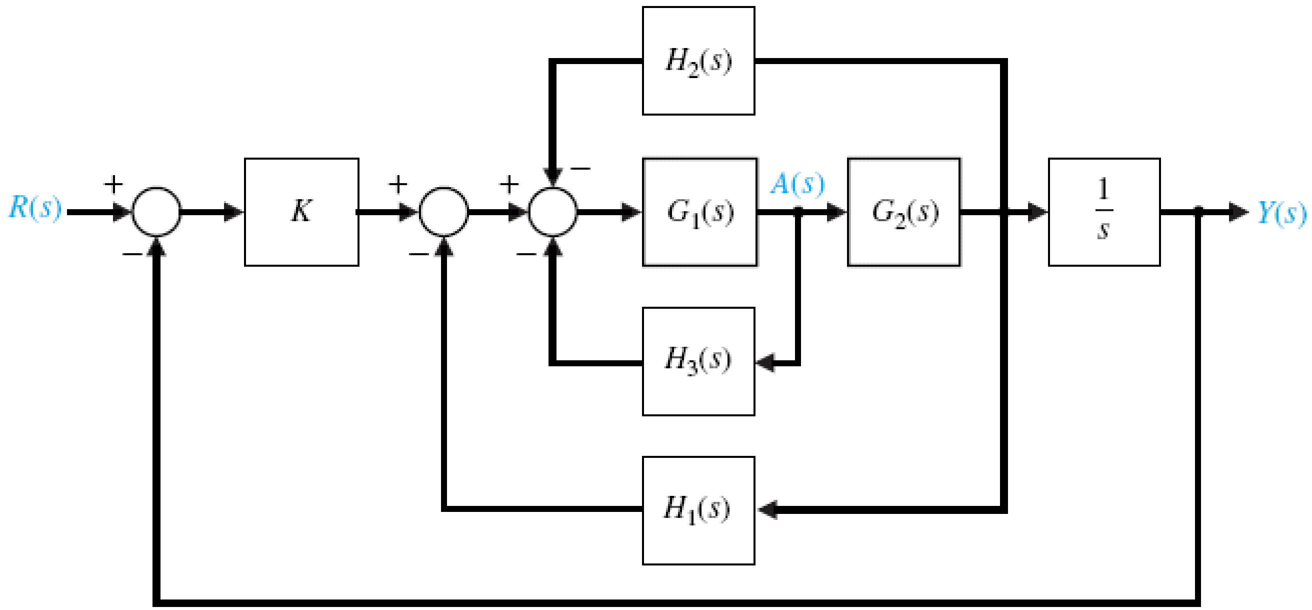
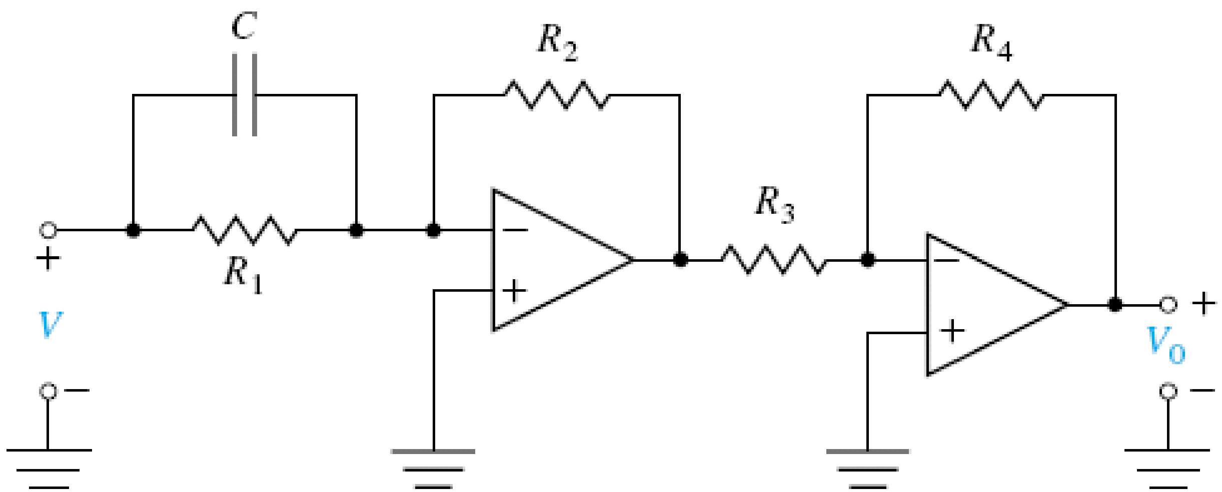
Homeworks

1. 系统方框图如图所示，计算传递函数 $T(s)=Y(s)/R(s)$



Homeworks

2. 假设如图所示运算放大器是理想的，各个参数取值为C=1μF,R_1=167kΩ,R_2=240kΩ,R_3=1kΩ, R_4=100kΩ，试计算运算放大器电路的传递函数$G(s)=V_o(s)/V(s)$



Homeworks

3. 假设以下两个系统的状态微分方程分别为：

(1) $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -14 & -7 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}u,$

$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{x}$

(2) $\dot{\mathbf{x}} = \begin{bmatrix} 1 & 1 & -1 \\ 4 & 3 & 0 \\ -2 & 1 & 10 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u,$

$y = \begin{bmatrix} 20 & 30 & 10 \end{bmatrix}\mathbf{x}$

试计算上述系统的传递函数$G_1(s)$， $G_2(s)$

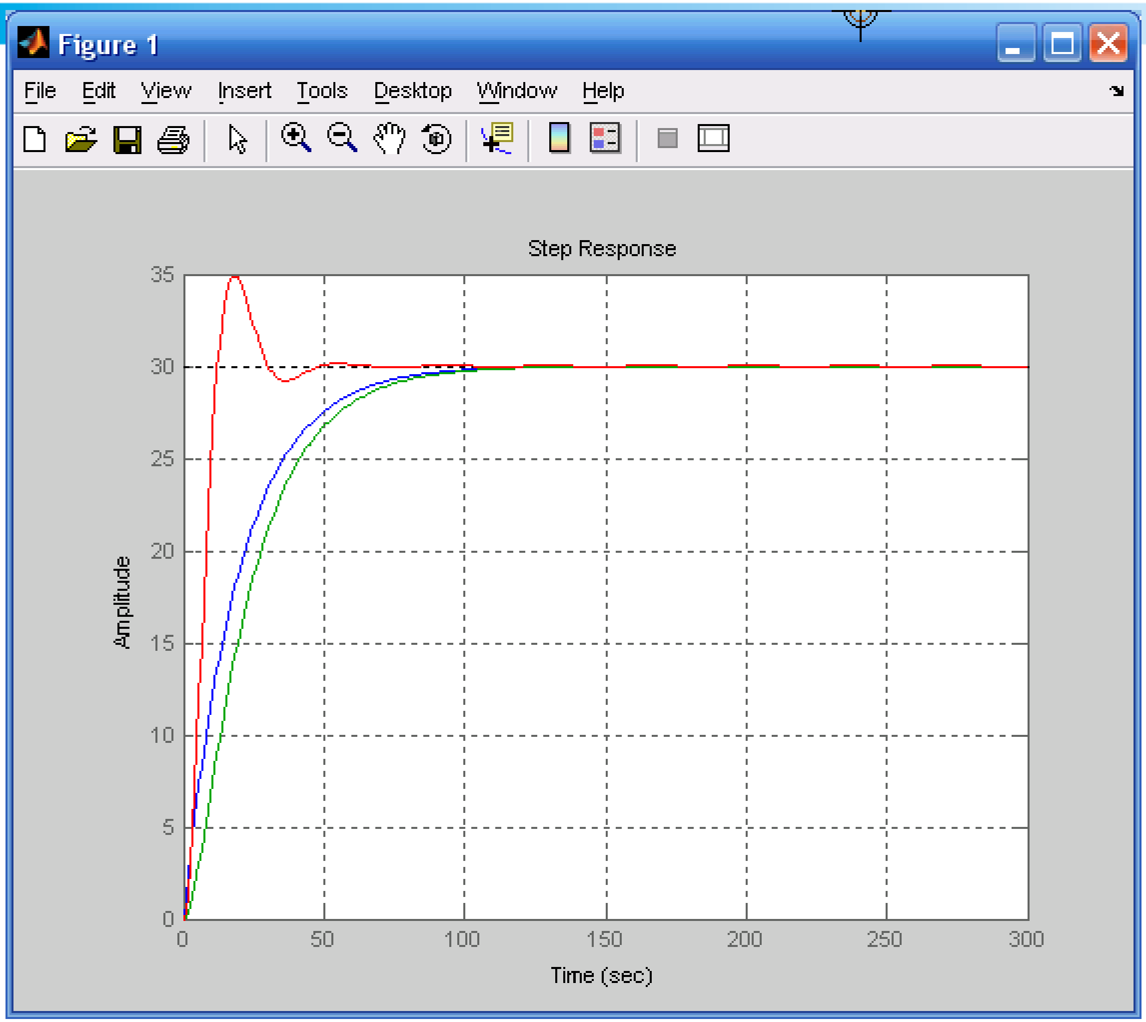
Homeworks

4. 假设以下两个系统的传递函数分别为：

$$(1)G(s)=\frac{8}{s^3+7s^2+14s+8}$$

$$(2)G(s)=\frac{8(s+5)}{s^3+12s^2+44s+48}$$

试写出上述系统的状态空间模型



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