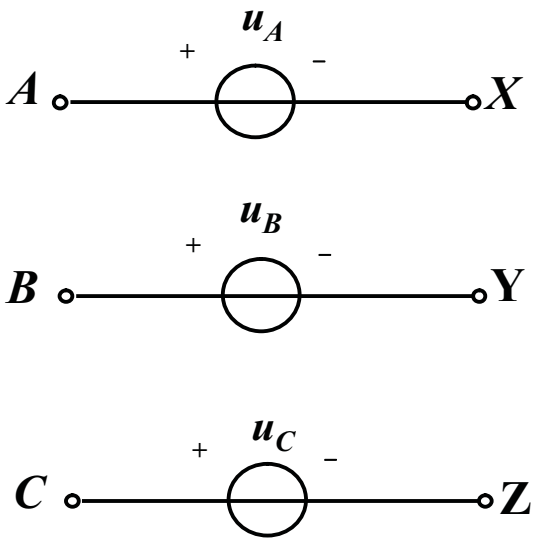
第八章　三相电路

§8-1　三相交流电路

一、三相电源



A、B、C ——绕组的始端（首端），电源的正极性端

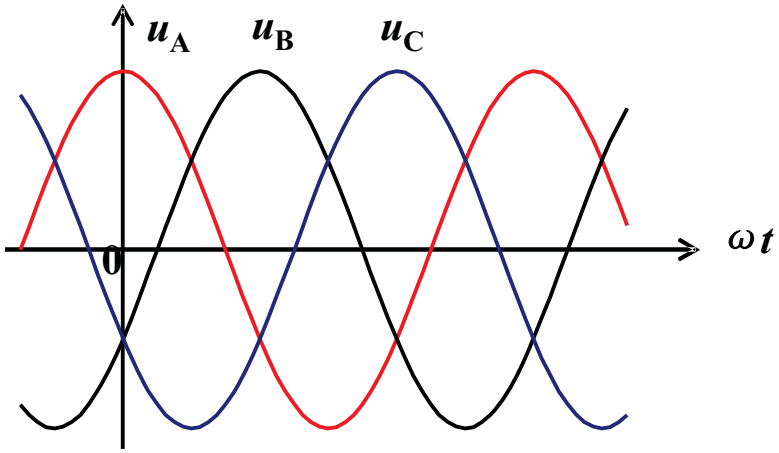
X、Y、Z ——绕组的终端（末端），电源的负极性端

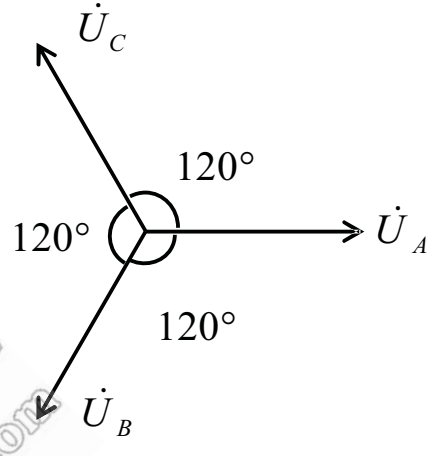
三相对称电源：A相、B相、C相

$$u_A = U_m \cos \omega t$$

$$u_B = U_m \cos(\omega t - 120°)$$

$$u_C = U_m \cos(\omega t + 120°)$$





相量形式：

$$\dot{U}_A = U\angle 0°$$
$$\dot{U}_B = U\angle -120°$$
$$\dot{U}_C = U\angle 120°$$

今后均指正序（顺序）。

正序（顺序）：A、B、C相序为顺时针

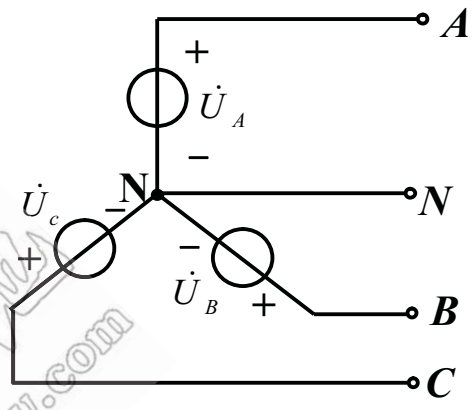
负序（逆序）：A、B、C相序为逆时针

$$\dot{U}_A + \dot{U}_B + \dot{U}_C = 0$$

$$u_A + u_B + u_C = 0$$

二、三相电源的联接

1、星形联接（Y）



公共点（*N*）：中点、零点

中线：中点引出的线（地线）

端线：始端引出的线（火线）

相电压：端线与中线间的电压。 $\dot{U}_{AN}$、$\dot{U}_{BN}$、$\dot{U}_{CN}$

简写为：$\dot{U}_A$、$\dot{U}_B$、$\dot{U}_C$

线电压：端线与端线间的电压。如：$\dot{U}_{AB}$、$\dot{U}_{BC}$、$\dot{U}_{CA}$

设 $\dot{U}_A = U_P \angle 0°$，$\dot{U}_B = U_P \angle -120°$，$\dot{U}_C = U_P \angle 120°$

则：$\dot{U}_{AB} = \dot{U}_A - \dot{U}_B = \sqrt{3} U_P \angle 30°$

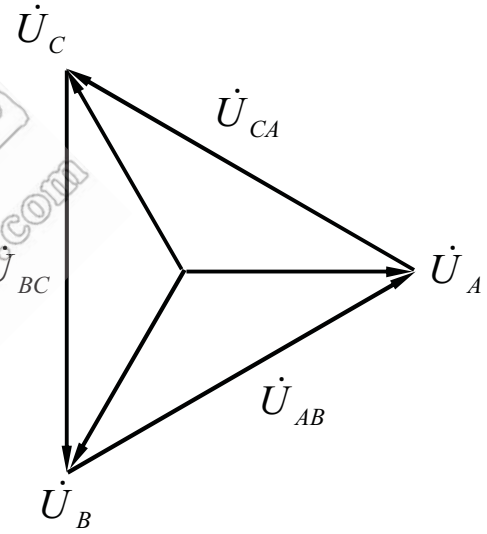
$= U_l \angle 30° = \sqrt{3}\, \dot{U}_A \angle 30°$

$\dot{U}_{BC} = \dot{U}_B - \dot{U}_C = U_l \angle -90°$

$= \sqrt{3}\, \dot{U}_B \angle 30°$

$\dot{U}_{CA} = \dot{U}_C - \dot{U}_A = U_l \angle 150°$

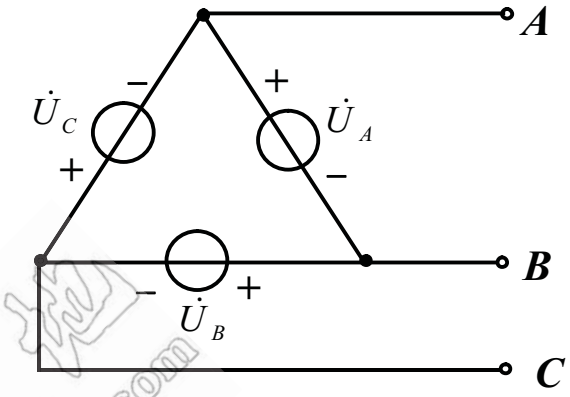
$= \sqrt{3}\, \dot{U}_C \angle 30°$



$\therefore \quad U_{线} = \sqrt{3}\, U_{相}$

线电压 $\dot{U}_{AB}$、$\dot{U}_{BC}$、$\dot{U}_{CA}$ 对称。

2、三角形联接（Δ）

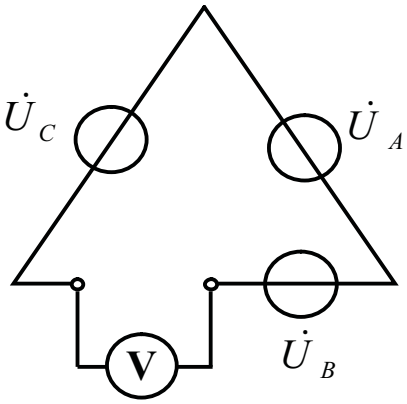


$$\dot{U}_{AB}=\dot{U}_A$$
$$\dot{U}_{BC}=\dot{U}_B$$
$$\dot{U}_{CA}=\dot{U}_C$$

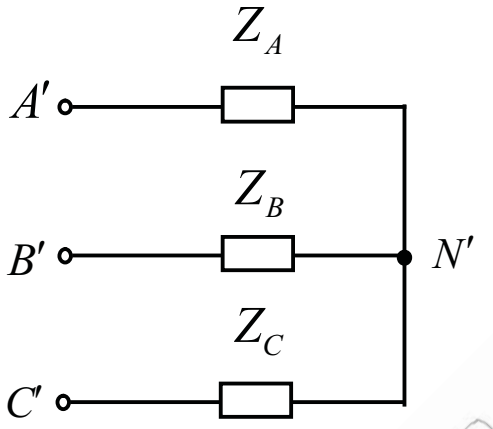
$$\therefore \quad U_l=U_P$$

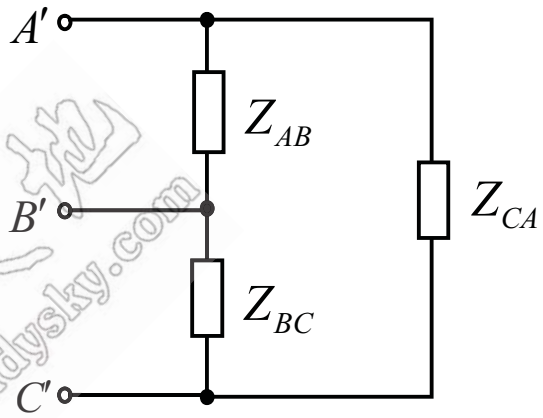
如一相接反，如*A*相

$$-\dot{U}_A+\dot{U}_B+\dot{U}_C=-2\dot{U}_A$$



三、 三相负载的联接





负载对称：$Z_A = Z_B = Z_C$

$$Z_{AB} = Z_{BC} = Z_{CA}$$

对称三相电路：电源、负载均对称。否则为不对称三相电路。

第八章　三相电路

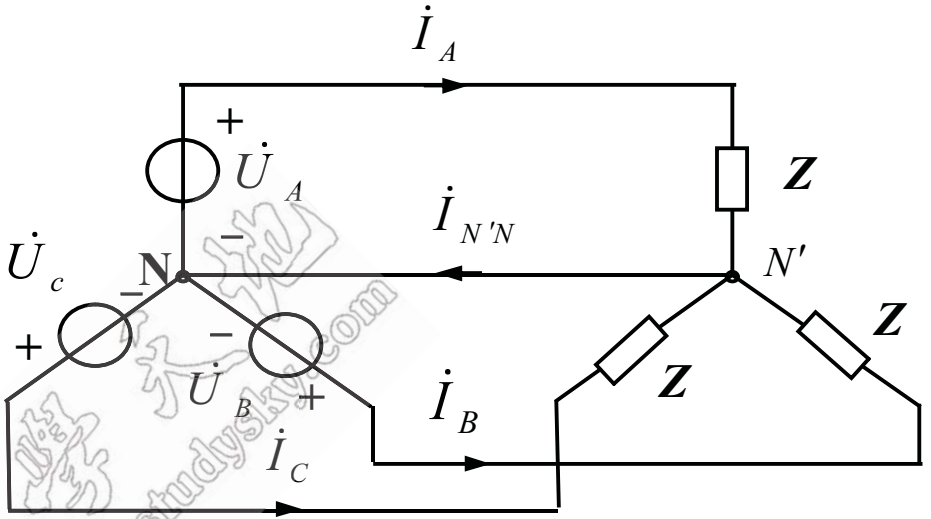
§8-2　对称三相电路的计算

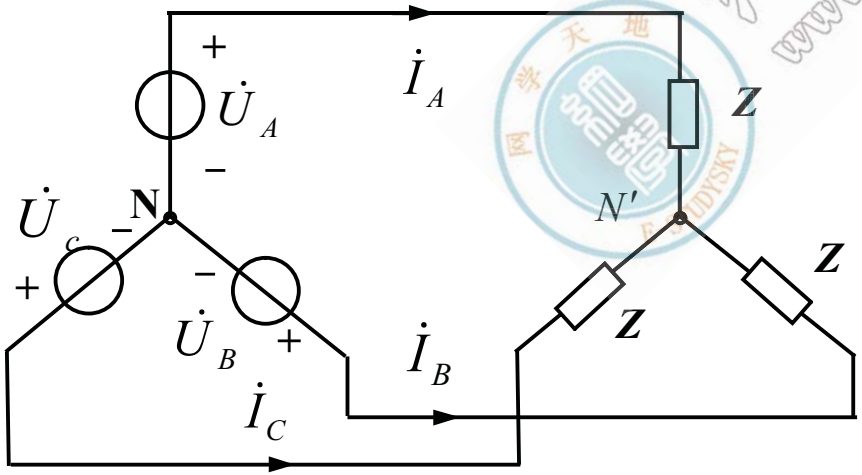
一、负载Y接

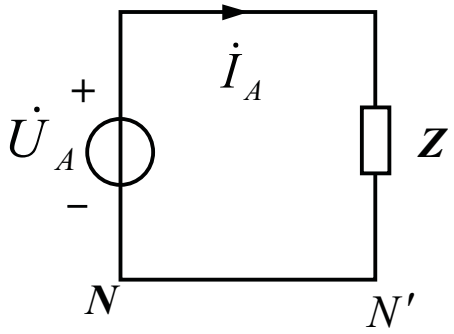
Y_0-Y_0

$$\dot{I}_A=\frac{\dot{U}_A}{Z},\quad \dot{I}_B=\frac{\dot{U}_B}{Z},\quad \dot{I}_C=\frac{\dot{U}_C}{Z}$$

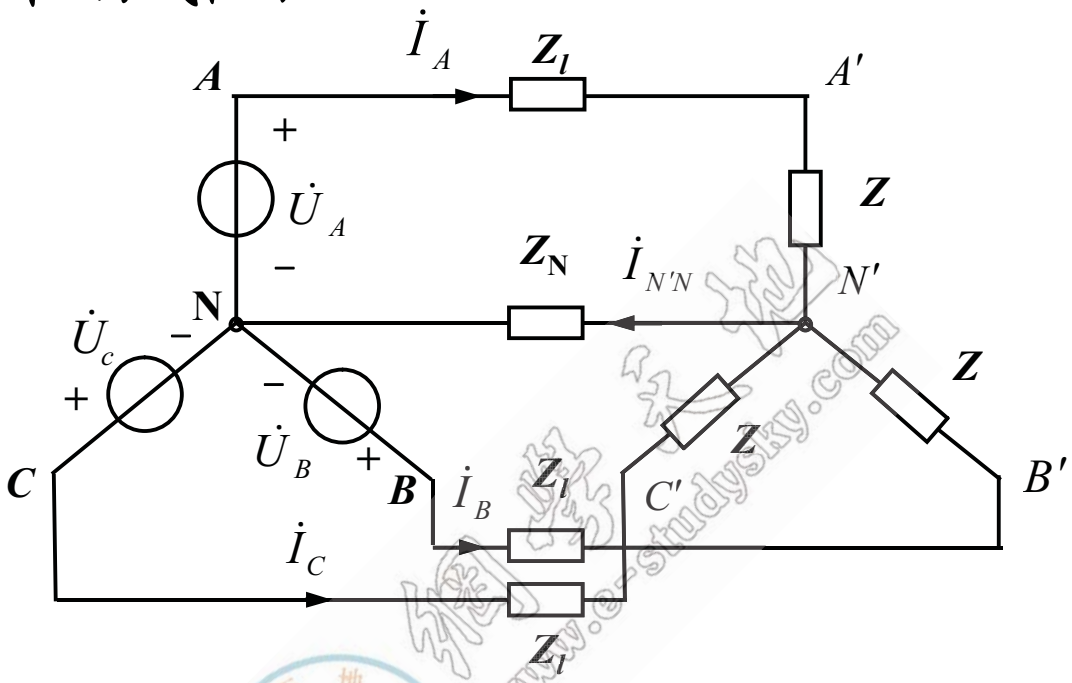
$$\dot{I}_{N'N}=\dot{I}_A+\dot{I}_B+\dot{I}_C=\frac{\dot{U}_A+\dot{U}_B+\dot{U}_C}{Z}=0$$







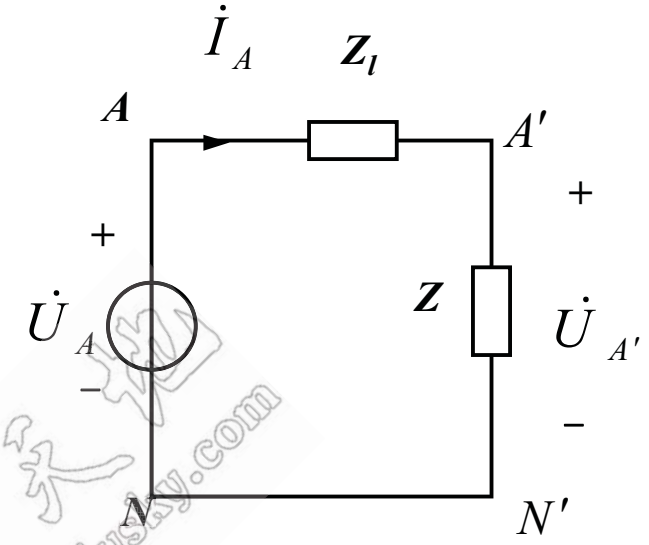
有中线阻抗和端线阻抗



$$(\frac{1}{Z+Z_l}+\frac{1}{Z+Z_l}+\frac{1}{Z+Z_l}+\frac{1}{Z_N})\dot{U}_{N'N}=\frac{\dot{U}_A+\dot{U}_B+\dot{U}_C}{Z+Z_l}$$

解得： $\dot{U}_{N'N}=0$

单相电路求解

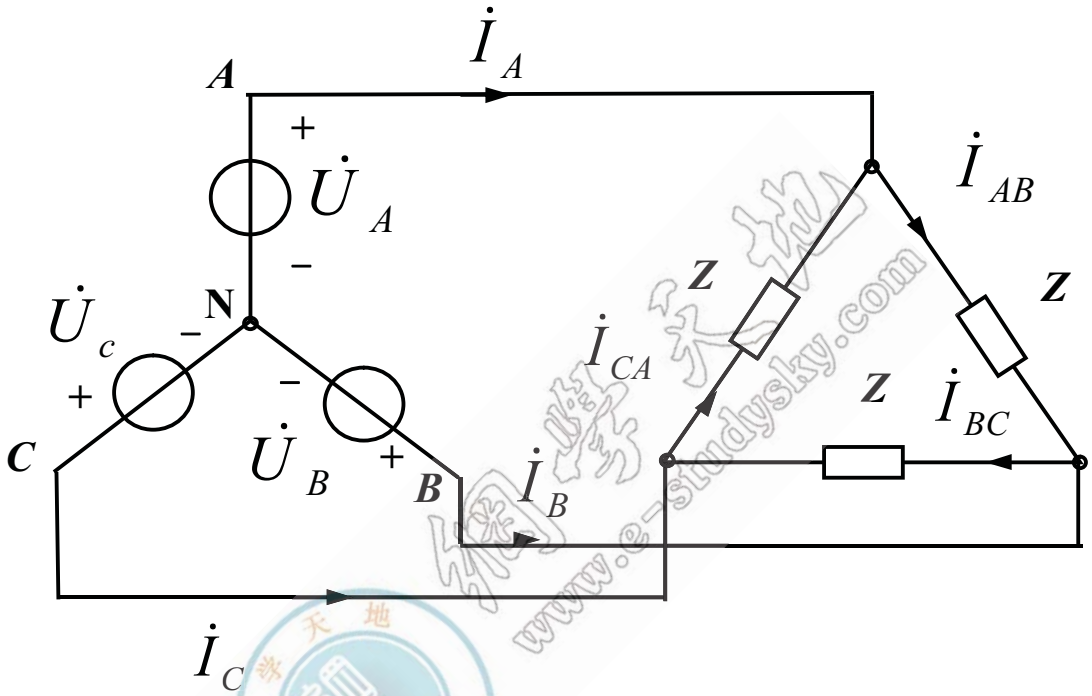


$$\dot{I}_A = \frac{\dot{U}_A}{Z_l + Z}, \quad \dot{U}_{A'} = \frac{Z}{Z_l + Z}\dot{U}_A$$

$$\dot{I}_B = \dot{I}_A \angle -120° \qquad \dot{I}_C = \dot{I}_A \angle 120°$$

$$\dot{U}_{B'} = \dot{U}_{A'} \angle -120° \qquad \dot{U}_{C'} = \dot{U}_{A'} \angle 120°$$

二、负载为Δ联接：Y—Δ



相电流 $\dot{I}_{AB}$ $\dot{I}_{BC}$ $\dot{I}_{CA}$

线电流 $\dot{I}_{A}$ $\dot{I}_{B}$ $\dot{I}_{C}$

$$\dot{I}_{AB}=\frac{\dot{U}_{AB}}{Z}=\frac{U_l\angle 0^\circ}{|Z|\angle\theta}=I_P\angle -\theta$$

$$\dot{I}_{BC}=\frac{\dot{U}_{BC}}{Z}=\frac{U_l\angle -120^\circ}{|Z|\angle\theta}=I_P\angle -120^\circ-\theta=\dot{I}_{AB}\angle -120^\circ$$

$$\dot{I}_{CA}=\frac{\dot{U}_{CA}}{Z}=I_P\angle 120^\circ-\theta=\dot{I}_{AB}\angle 120^\circ$$

∴ 相电流对称

线电流： $\dot{I}_A = \dot{I}_{AB} - \dot{I}_{CA} = I_P \angle -\theta - I_P \angle 120° - \theta$

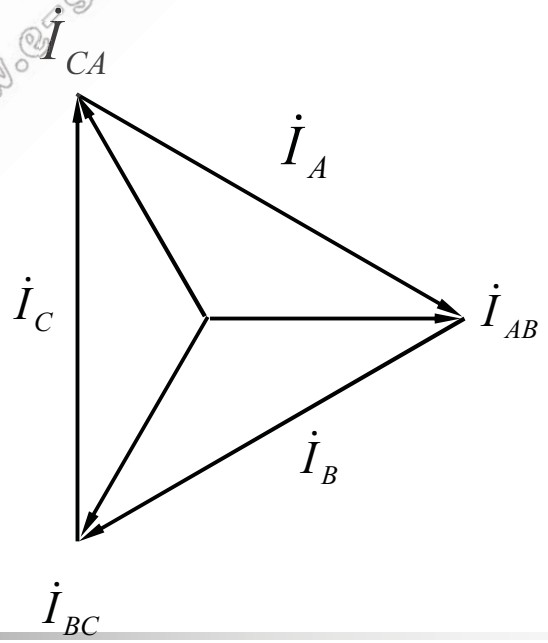
$= \sqrt{3}\, I_P \angle -\theta - 30° = \sqrt{3}\, \dot{I}_{AB} \angle -30°$

$\dot{I}_B = \dot{I}_{BC} - \dot{I}_{AB} = \sqrt{3}\, I_P \angle -\theta - 150° = \sqrt{3}\, \dot{I}_{BC} \angle -30°$

$\dot{I}_C = \dot{I}_{CA} - \dot{I}_{BC} = \sqrt{3}\, I_P \angle -\theta + 90° = \sqrt{3}\, \dot{I}_{CA} \angle -30°$

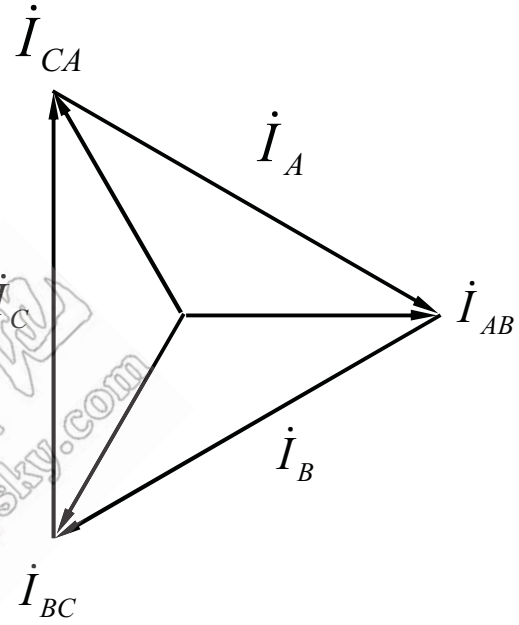
线电流对称

Δ： $I_l = \sqrt{3} I_P, \quad U_l = U_P$



一般　　$\Delta \to Y$

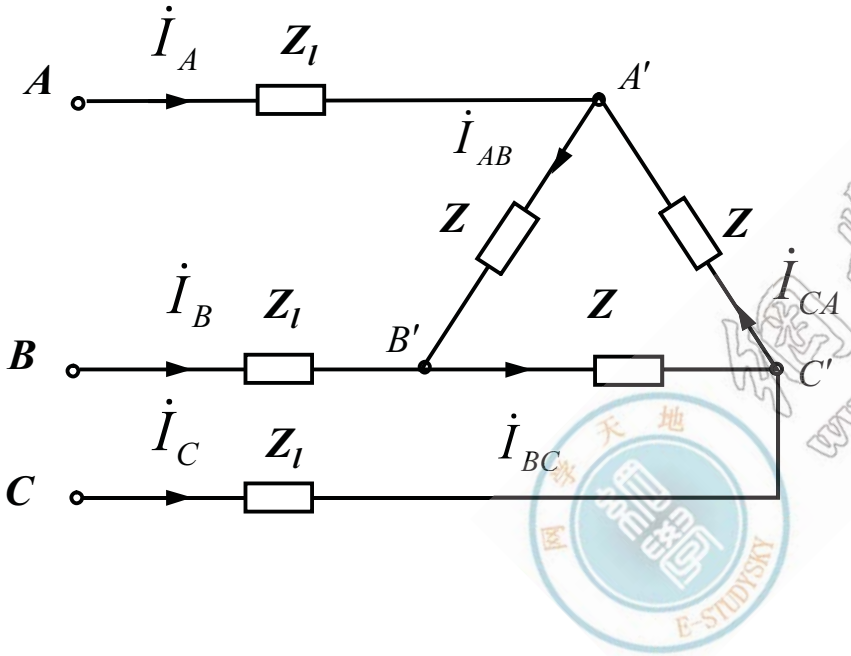
如 $\dot{I}_A$ 已知

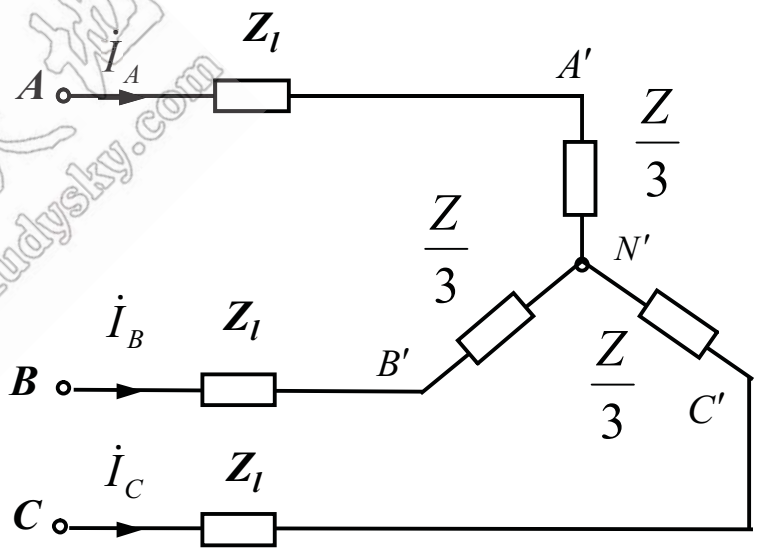


$$\dot{I}_{AB} = \frac{\dot{I}_A}{\sqrt{3}} \angle 30°$$

$$\dot{I}_{CA} = \frac{\dot{I}_A}{\sqrt{3}} \angle 150°$$

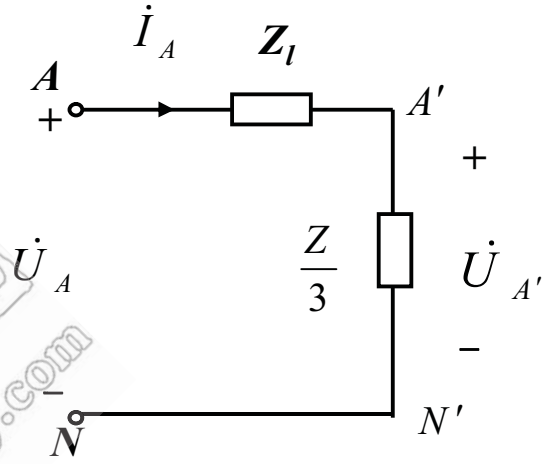
例8－1　三相对称电路。已知负载 $Z=300\angle 30°\Omega$，线路阻抗 $Z_l=10+j10\Omega$，电源侧线电压 $U_l=380V$。求各相、线电流及 $\dot{U}_{A'B'}$。





解： 负载 $\Delta \to Y$　　单相求解

设 $\dot{U}_A = \dfrac{U_l}{\sqrt{3}}\angle 0° \approx 220\angle 0°\ \ V$



则 $\dot{I}_A = \dfrac{\dot{U}_A}{Z_l + \frac{Z}{3}} = 1.93\angle -31.84°\ \ A$

$\dot{I}_B = 1.93\angle -31.84° - 120° = 1.93\angle -151.84°\ \ A$

$\dot{I}_C = 1.93\angle -31.84° + 120° = 1.93\angle 88.16°\ \ A$

对应的相电流：

$$\dot{I}_{AB} = \frac{\dot{I}_A}{\sqrt{3}} \angle 30° = 1.11 \angle -1.84° \ A$$

$$\dot{I}_{BC} = 1.11 \angle -121.84° \ A$$

$$\dot{I}_{CA} = 1.11 \angle 118.16° \ A$$

$$\dot{U}_{A'B'} = Z\dot{I}_{AB} = 333 \angle 28.16° V$$

另：

$$\dot{U}_{A'} = \frac{\frac{Z}{3}}{\frac{Z}{3} + Z_l} \dot{U}_A = 193.45 \angle -1.84° V$$

$$\dot{U}_{A'B'} = \sqrt{3}\dot{U}_{A'} \angle 30° = 335 \angle 28.16° \ V$$

第八章　三相电路

§8-3　三相电路的功率及其测量

一、有功功率（平均功率）P

Y接：$P = P_A + P_B + P_C$

$$= U_A I_A \cos\varphi_A + U_B I_B \cos\varphi_B + U_C I_C \cos\varphi_C$$

三相对称：$= 3U_P I_P \cos\varphi$

$$= \sqrt{3} U_l I_l \cos\varphi$$

$I_l = I_P$　　$U_l = \sqrt{3} U_P$

Δ接： $P = P_{AB} + P_{BC} + P_{CA}$

$$= U_{AB} I_{AB} \cos\varphi_{AB} + U_{BC} I_{BC} \cos\varphi_{BC} + U_{CA} I_{CA} \cos\varphi_{CA}$$

三相对称： $= 3U_P I_P \cos\varphi$

$$= \sqrt{3} U_l I_l \cos\varphi$$

$$U_l = U_P \qquad I_l = \sqrt{3} I_P$$

对称三相电路： $P = 3U_P I_P \cos\varphi = \sqrt{3} U_l I_l \cos\varphi$

$\cos\varphi$ ——负载的功率因数

二、无功功率Q

对称三相：$Q = 3U_P I_P \sin\varphi = \sqrt{3}U_l I_l \sin\varphi$

三、视在功率S

对称三相：$S = 3U_P I_P = \sqrt{3}U_l I_l = \sqrt{P^2 + Q^2}$

四、瞬时功率p

$$p = p_A + p_B + p_C = u_A i_A + u_B i_B + u_C i_C$$

$$= \sqrt{2}U_P \cos\omega t \cdot \sqrt{2}I_P \cos(\omega t - \varphi)$$

$$+ \sqrt{2}U_P \cos(\omega t - 120^\circ) \cdot \sqrt{2}I_P \cos(\omega t - \varphi - 120^\circ)$$

$$+ \sqrt{2}U_P \cos(\omega t + 120^\circ) \cdot \sqrt{2}I_P \cos(\omega t - \varphi + 120^\circ)$$

$$= 3U_P I_P \cos\varphi + U_P I_P\left[\cos(2\omega t-\varphi)+\cos(2\omega t+120°-\varphi)+\cos(2\omega t-120°-\varphi)\right]$$

$$= 3U_P I_P \cos\varphi \quad = \sqrt{3}U_l I_l \cos\varphi$$

$$= P = 常数$$

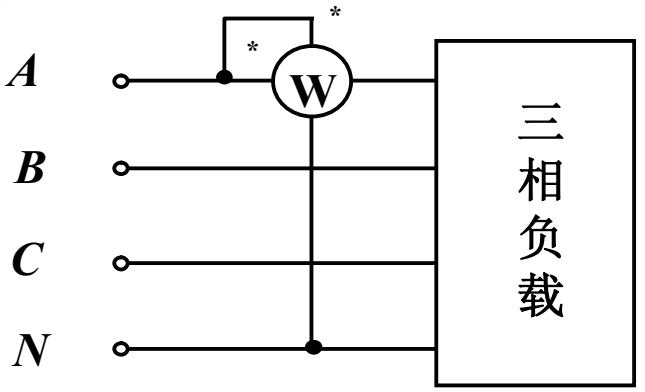
三相电动机

五、测量方法

1. 三相四线制

对称，测一相

$$P = 3\,\text{W}$$



不对称时，用三个表分别测量。

2. 三相三线制:

两表法

视为Y接 ，因为Δ→Y

$$p = p_A + p_B + p_C = u_A i_A + u_B i_B + u_C i_C$$

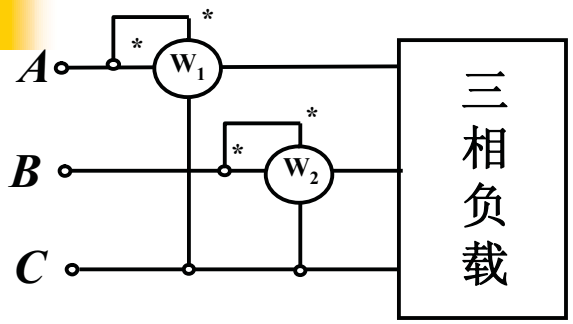
$$= (u_A - u_C) i_A + (u_B - u_C) i_B$$

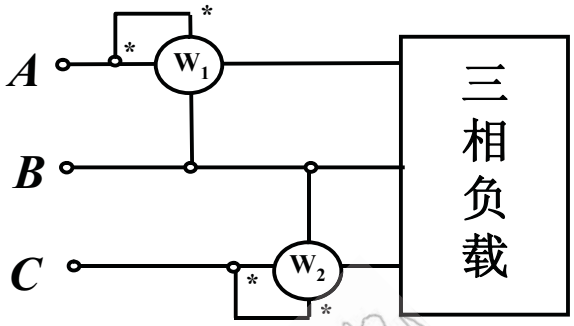
$$= u_{AC} i_A + u_{BC} i_B$$

注：$i_C = -(i_A + i_B)$

$$\therefore P = U_{AC} I_A \cos\varphi_1 + U_{BC} I_B \cos\varphi_2$$

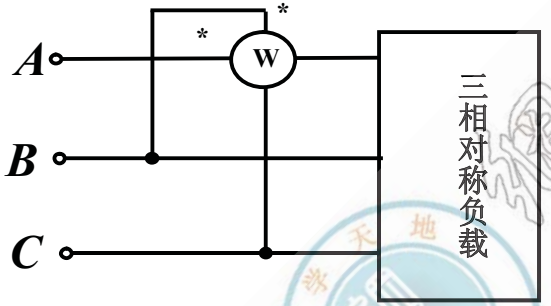
φ_1 ——$\dot{U}_{AC}$与$\dot{I}_A$的夹角　　φ_2 ——$\dot{U}_{BC}$与$\dot{I}_B$的夹角

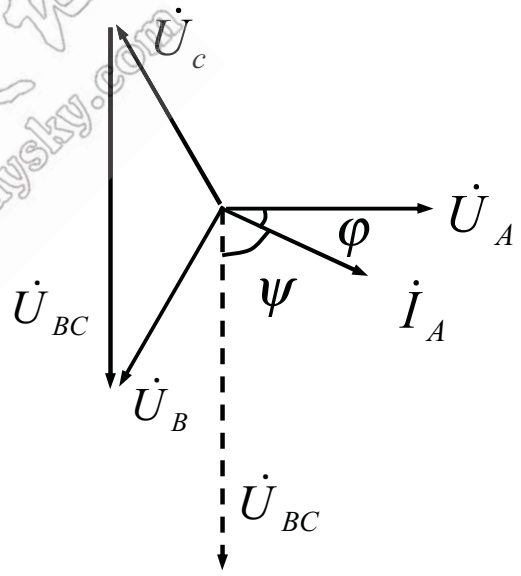




$$P = \textcircled{W}_1 + \textcircled{W}_2$$

3. 对称三相电路的无功测量：



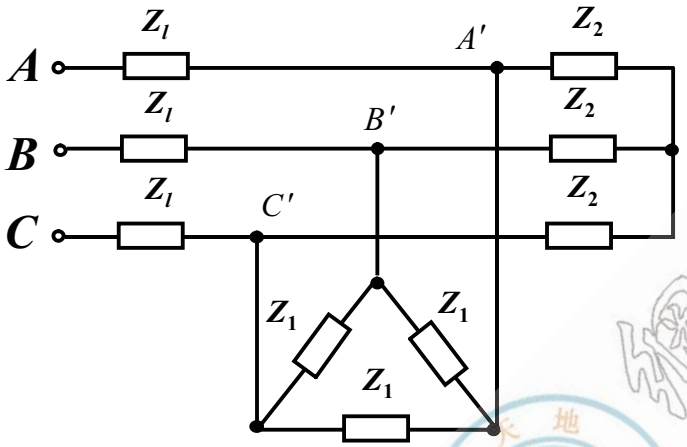


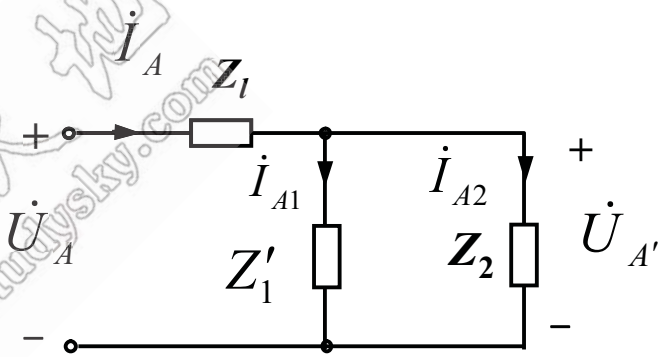
$$\textcircled{W} = U_{BC} I_A \cos\psi$$

$$= U_{BC} I_A \cos(90^\circ - \varphi) \quad = U_{BC} I_A \sin\varphi \quad = U_l I_l \sin\varphi$$

$$Q = \sqrt{3} U_l I_l \sin\varphi = \sqrt{3}\,\textcircled{W}$$

例8－2　　Z_1、Z_2为感性负载，Δ接的总功率为10kw，$\cos\varphi_1=0.8$；Y接的总功率7.5kw，$\cos\varphi_2=0.88$；线路阻抗 $Z_l=0.2+j0.3\Omega$ 电源对称，负载侧线电压$U_l=380V$，求电源侧线电压。





解：单相求解： $\Delta \rightarrow Y$　　$Z_1'=\dfrac{Z_1}{3}$

（1）求Z_1　　设Δ的相电流为I_{P1}

$$I_{P1}=\frac{P_1}{3U_{P1}\cos\varphi_1}=\frac{10^4}{3\times380\times0.8}=10.96A$$

$$|Z_1| = U_{P1}/I_{P1} = 34.67\Omega \qquad \varphi_1 = \cos^{-1}0.8 = 36.87°$$

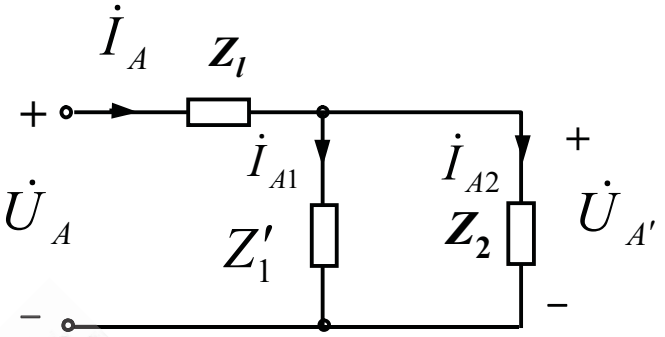
$$\therefore Z_1 = 34.67\underline{/36.87°}\ \Omega \qquad Z_1' = 11.56\underline{/36.87°}\ \Omega$$

（2）求Z_2：设Y接的相电流为I_{P2}

$$I_{P2} = \frac{P_2}{3U_{P2}\cos\varphi_2} = \frac{7500}{3\times\frac{380}{\sqrt{3}}\times 0.88} = 12.95A$$

$$|Z_2| = U_{P2}/I_{P2} = \frac{380}{\sqrt{3}}\Big/12.95 \approx 16.94\Omega \qquad \varphi_2 = \cos^{-1}0.88 = 28.36°$$

$$\therefore Z_2 = 16.94\underline{/28.36°}\ \Omega$$



（3） $U_{A'} = U_l / \sqrt{3} \approx 220V$

设 $\dot{U}_{A'} = 220/\underline{0°}\ V$

则 $\dot{I}_{A1} = \dot{U}_{A'} / Z_1' = 19.03/\underline{-36.87°}\ \text{A} = 15.22 - j11.42A$

$\dot{I}_{A2} = \dot{U}_{A'} / Z_2 = 12.98/\underline{-28.36°} = 11.42 - j6.16A$

$\dot{I}_A = \dot{I}_{A1} + \dot{I}_{A2} = 26.64 - j17.58 = 31.92/\underline{-33.42°}\ A$

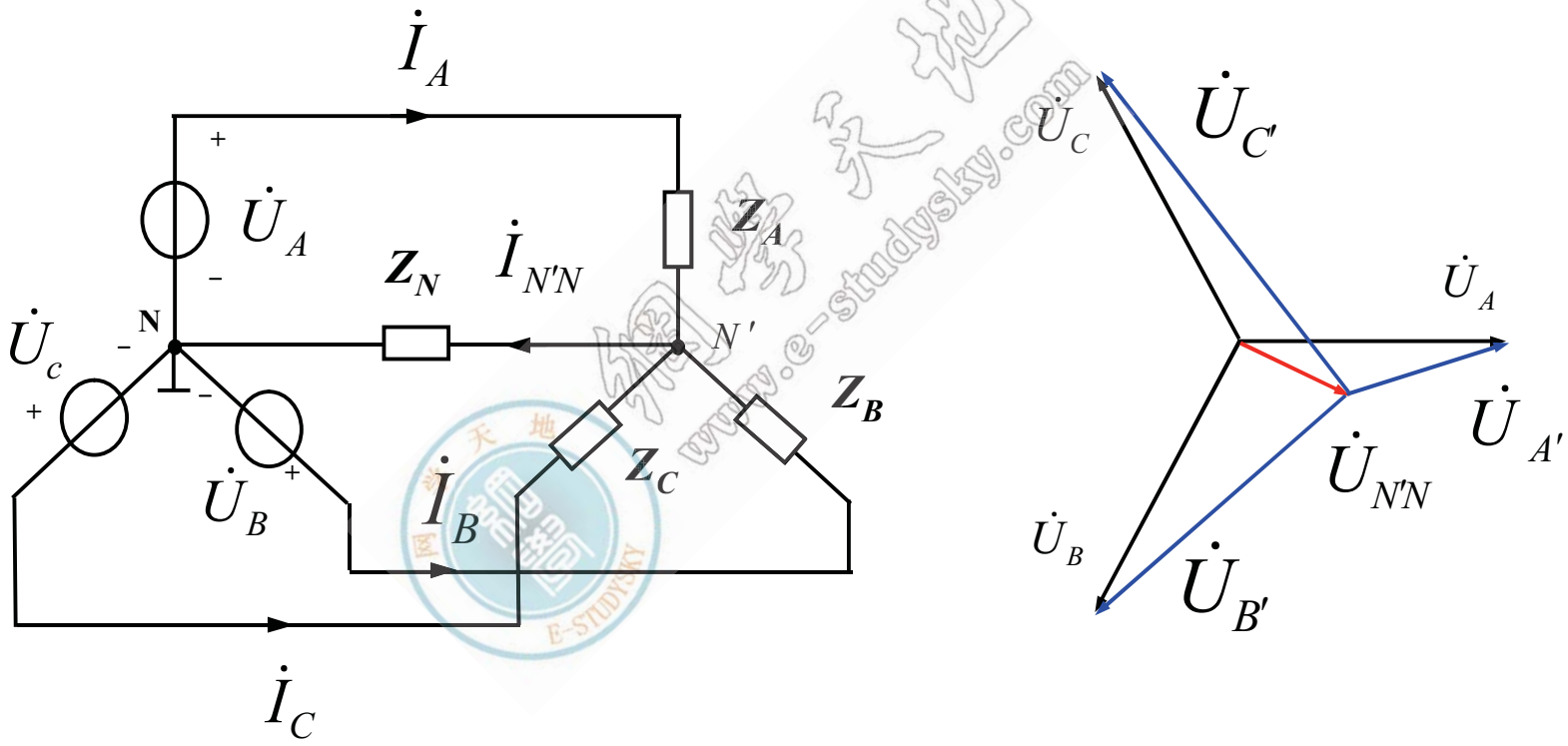
$\therefore \dot{U}_A = Z_l \dot{I}_A + \dot{U}_{A'} = 0.36/\underline{56.3°} \times 31.92/\underline{-33.42°} + 220$

$= 11.49/\underline{22.89°} + 220 = 230.59 + j4.47 = 230.63/\underline{1.11°}V$

∴ 电源侧线电压 $= \sqrt{3} \times 230.63 = 399.46V$

§8-4 不对称三相电路的计算

按一般正弦电路处理



如电源对称负载不对称的三相电路的求解：

结点法：设N为参考结点

$$\left(\frac{1}{Z_A}+\frac{1}{Z_B}+\frac{1}{Z_C}+\frac{1}{Z_N}\right)\dot{U}_{N'N}=\frac{\dot{U}_A}{Z_A}+\frac{\dot{U}_B}{Z_B}+\frac{\dot{U}_C}{Z_C}$$

$$\dot{U}_{N'N}\neq 0$$

中性点位移或称中性点漂移

$$\dot{I}_A=\frac{\dot{U}_A-\dot{U}_{N'N}}{Z_A}$$

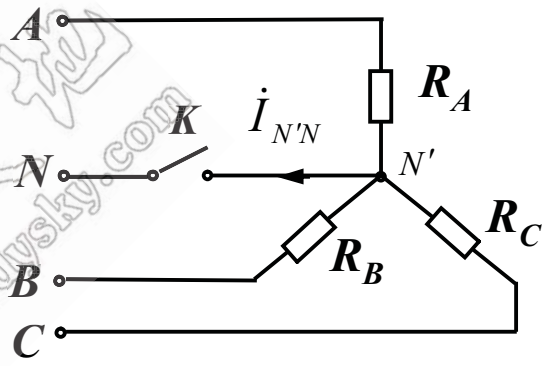
$$\dot{I}_B=\frac{\dot{U}_B-\dot{U}_{N'N}}{Z_B}\neq \dot{I}_A\angle -120^\circ$$

$$\dot{I}_C=\frac{\dot{U}_C-\dot{U}_{N'N}}{Z_C}\neq \dot{I}_A\angle 120^\circ$$

例8－3　电源对称，其相电压 $U_P = 220V$ ，负载是三个白炽灯，其额定工作电压为220V，A、B两相灯泡为100W，C相灯泡为25W。求

（1）　当开关K打开时，各相灯泡承受的电压以及它们实际承载的功率。

（2）　当开关K闭合时的中线电流 $\dot{I}_{N'N}$ 。



解：
$$R_A = R_B = \frac{U^2}{P_A} = \frac{220^2}{100} = 484\Omega$$

$$R_C = \frac{U^2}{P_C} = \frac{220^2}{25} = 1936\Omega$$

（1） 开关K打开时

设 $\dot{U}_{AN}=220\angle 0°V$

则 $\dot{U}_{BN}=220\angle -120°V,\quad \dot{U}_{CN}=220\angle 120°V$

结点法

$$\dot{U}_{N'N}=\frac{\dfrac{220}{484}+\dfrac{220\angle -120°}{484}+\dfrac{220\angle 120°}{1936}}{\dfrac{2}{484}+\dfrac{1}{1936}}=73.33\angle -60°V$$

$$\dot{U}_{AN'} = \dot{U}_{AN} - \dot{U}_{N'N} = 220 - 73.33\underline{/-60°}$$
$$= 183.33 + j63.51 = 193.97\underline{/19.11°}V$$

$$\dot{U}_{BN'} = \dot{U}_{BN} - \dot{U}_{N'N} = 220\underline{/-120°} - 73.33\underline{/-60°} = 193.97\underline{/139.11°}V$$

$$\dot{U}_{CN'} = \dot{U}_{CN} - \dot{U}_{N'N} = 220\underline{/120°} - 73.33\underline{/-60°} = 293.3\underline{/120°}V$$

$$P_A = P_B = \frac{193.97^2}{484} = 77.74W$$

$$P_C = \frac{293.3^2}{1936} = 44.434W$$

（2） 开关K闭合时

$$\dot{I}_A = \frac{\dot{U}_{AN}}{R_A} = \frac{220\underline{/0°}}{484} = 0.455\underline{/0°}\ A$$

$$\dot{I}_B = \frac{\dot{U}_{BN}}{R_B} = \frac{220\underline{/-120°}}{484} = 0.455\underline{/-120°}\ A$$

$$\dot{I}_C = \frac{\dot{U}_{CN}}{R_C} = \frac{220\underline{/120°}}{1936} = 0.114\underline{/120°}\ A$$

$$\dot{I}_{N'N} = \dot{I}_A + \dot{I}_B + \dot{I}_C = 0.34\underline{/-60°}\ A$$