自动控制原理

Automatic Control Theory

西南交通大学
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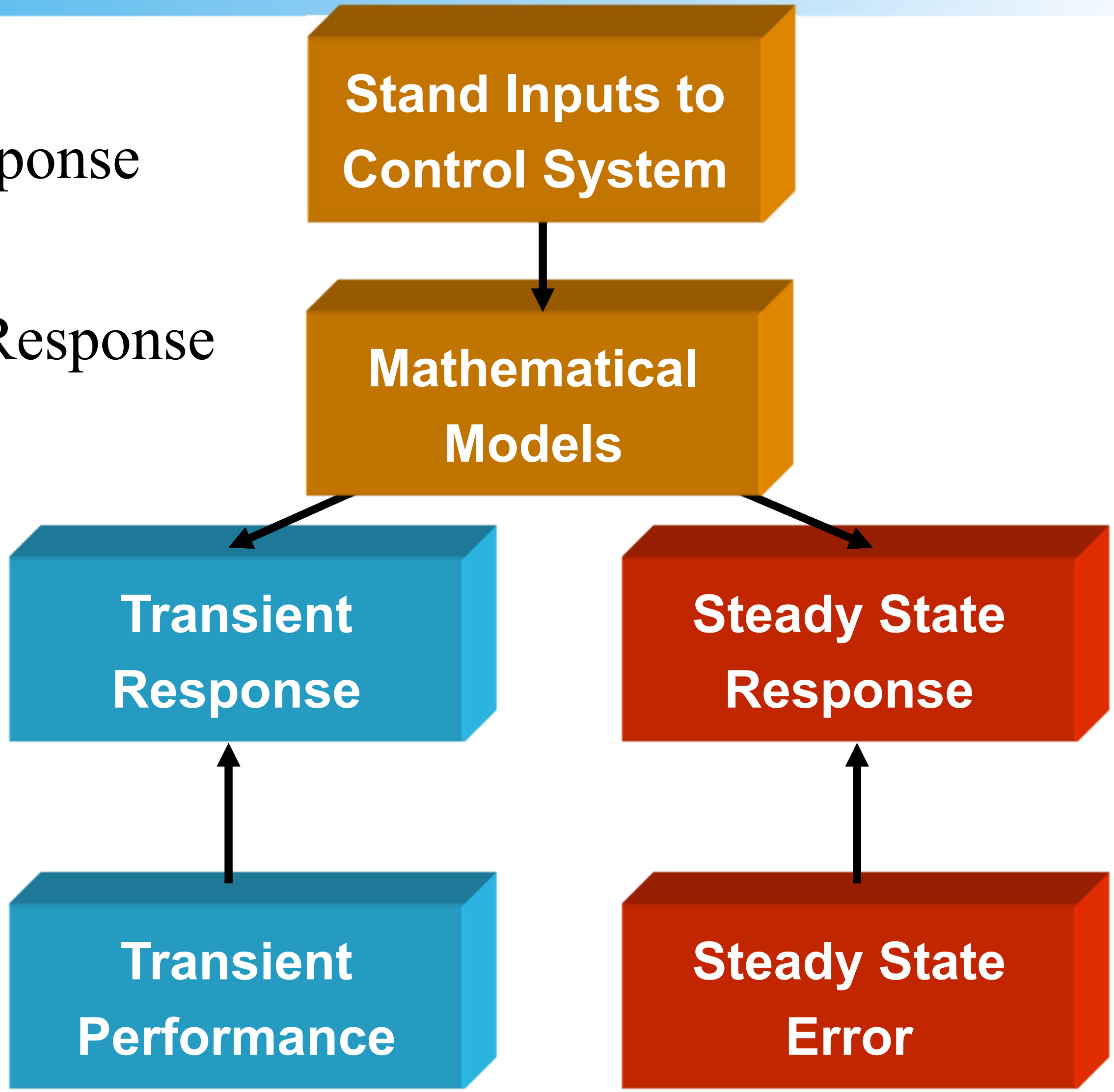
Chapter3 控制系统的运动分析

3.1 Introduction

System response :

- Transient Response
  动态响应
- Steady State Response
  稳态响应

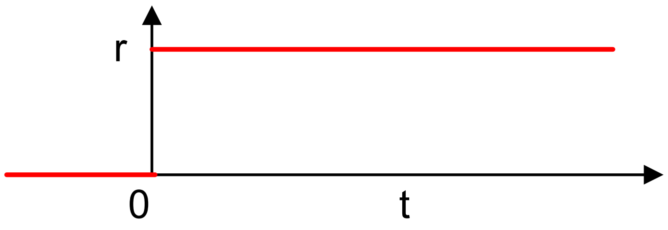


3.1 Introduction

- ***Standard Inputs to Control Systems 典型输入信号***

1) *Step Function 阶跃信号*

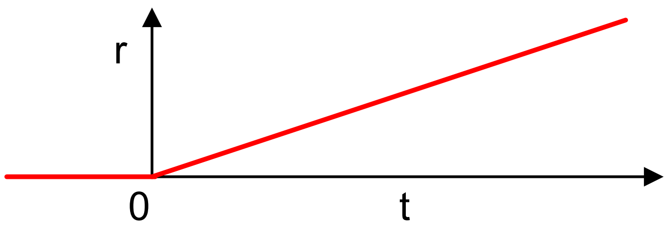
$$r(t)=\begin{cases}0, t<0\\ A, t>0\end{cases}, R(s)=A/s$$



Also represented as 1(t) or U(t)

2) *Ramp Function 斜坡信号*

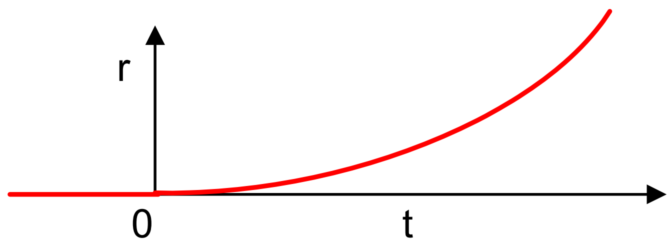
$$r(t)=\begin{cases}0, t<0\\ At, t\geq 0\end{cases}, R(s)=\frac{A}{s^2}$$



3.1 Introduction

3) Parabolic Function 抛物线信号

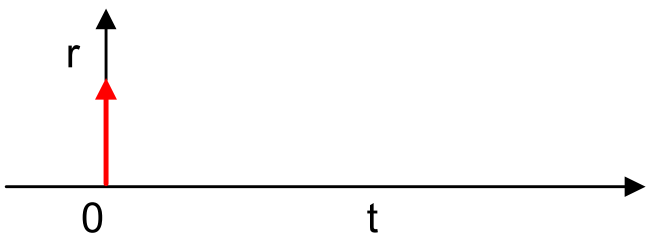
$$r(t)=\begin{cases}0, t<0\\ \frac{1}{2}At^2, t\geq 0\end{cases}, R(s)=\frac{A}{s^3}$$



when A=1， Unit step 单位阶跃、Unit ramp 单位斜坡、Unit parabolic 单位抛物线 function (input)

4) Impulse Function 脉冲信号

$$r(t)=\begin{cases}\lim\limits_{t_0\to 0}\frac{A}{t_0}, 0<t<t_0\\ 0, other\end{cases}, R(s)=A$$



3.1 Introduction

Unit Impulse
$$\delta(t-t_0)=\begin{cases}\infty, t=t_0\\0, t\neq t_0\end{cases}$$

Characteristic
$$\int_{-\infty}^{\infty}\delta(t)dt=1\rightarrow\int_{0-}^{0+}\delta(t)dt=1$$

sampling
$$\int_{-\infty}^{\infty}f(t)\delta(t)dt=f(0)\quad \text{or}$$
$$\int_{-\infty}^{\infty}f(t)\delta(t-t_0)dt=f(t_0)$$

单位脉冲函数作为典型输入信号，用于考察系统的脉冲响应，分析系统的固有性质：

Input: $r(t)=\delta(t), R(s)=1$

Output: $Y(s)=G(s)R(s)=G(s), y(t)=g(t)$

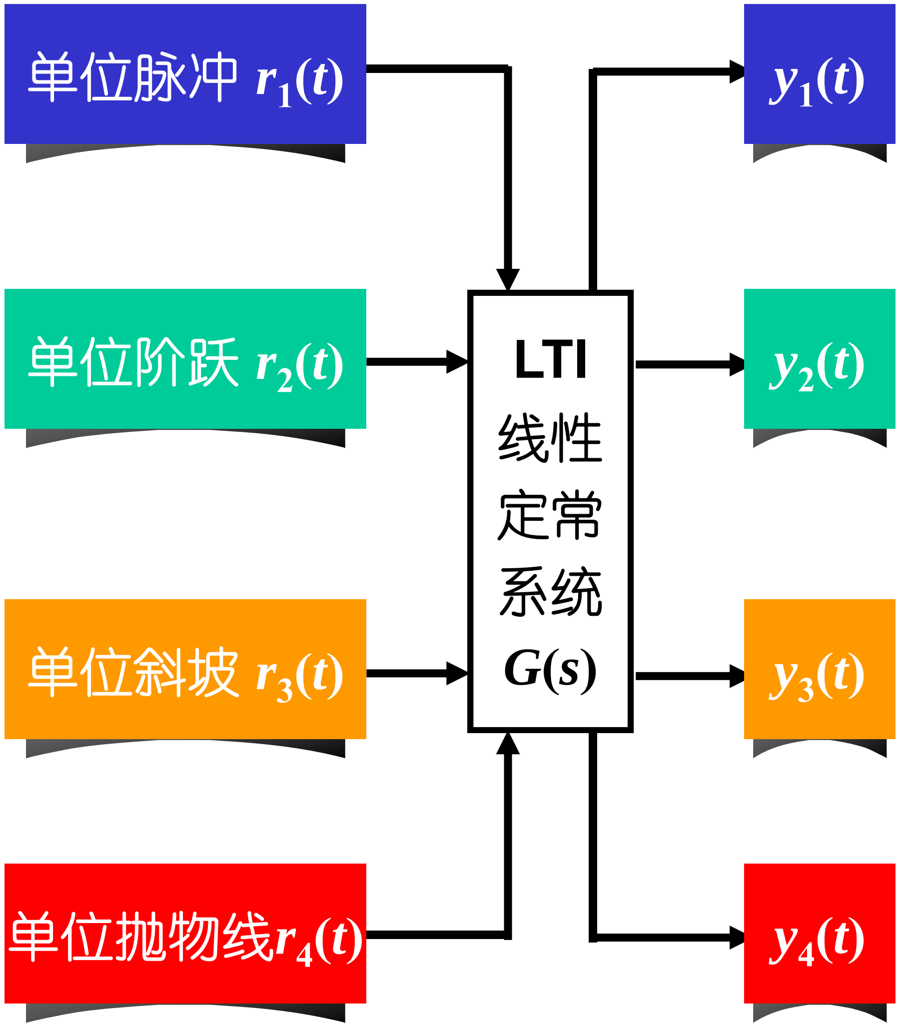
3.1 Introduction

- Relationship among 4 functions

$$r_1(t) = \frac{d}{dt} r_2(t)$$

$$r_2(t) = \frac{d}{dt} r_3(t)$$

$$r_3(t) = \frac{d}{dt} r_4(t)$$



$$y_1(t) = \frac{d}{dt} y_2(t)$$

$$y_2(t) = \frac{d}{dt} y_3(t)$$

$$y_3(t) = \frac{d}{dt} y_4(t)$$

3.1 Introduction

Characteristic of LTI System 线性定常系统的一个特性：

系统对输入信号导数的响应，等于系统对该输入信号响应的导数；或者，系统对输入信号积分的响应，等于系统对该输入信号响应的积分，积分常数由零初始条件确定。

5) Sinusoidal Function 正弦信号

$$r(t) = A\sin(\omega t + \varphi),$$

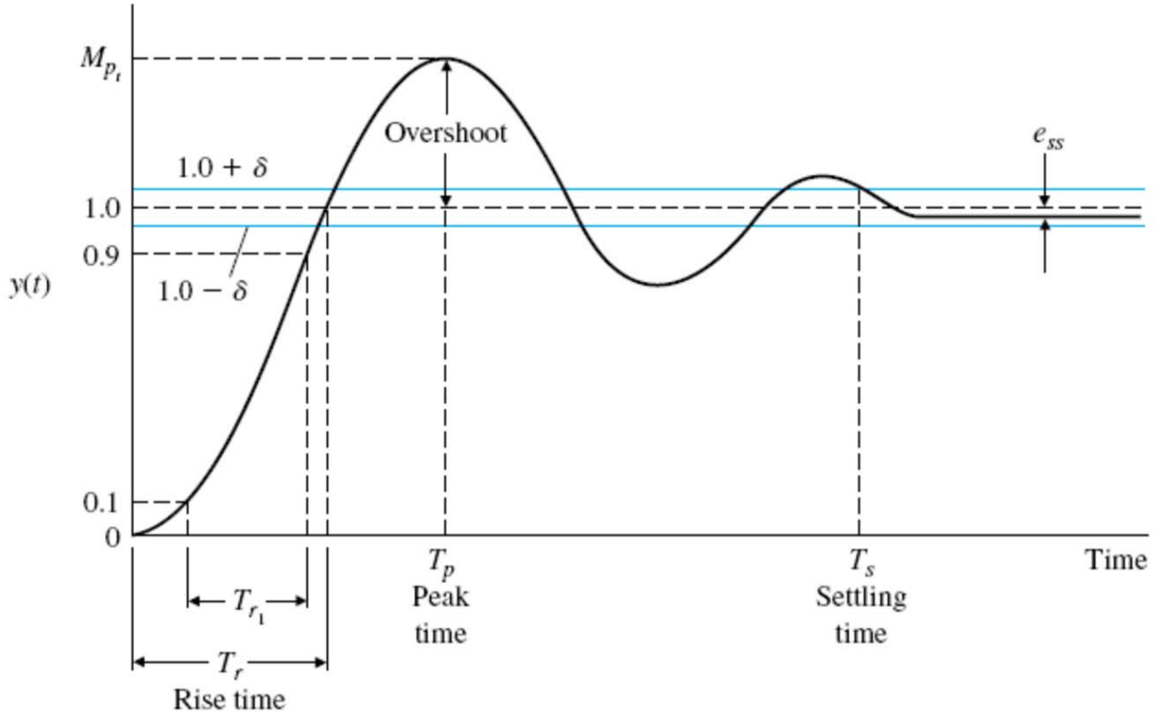
$$R(s) = Ae^{\frac{\varphi}{\omega}s}\frac{\omega}{s^2+\omega^2},$$

返回

3.2 Performance of control system

3.2.1 Step response and performance specifications

单位阶跃响应与性能指标



Transient response of a control system

3.2.1Step response and performance specifications

- ***Transient response performance specifications***
  暂态性能指标

  a) **Delay Time (延迟时间)**T_d: The delay time, measures the time to 50% of the magnitude of the input. 系统响应从0上升到稳态值的50%所需要的时间

  b) **Rise Time(上升时间)**T_r:

  1) The 0-100% rise time, measures the time to 100% of the magnitude of the input. (Under-damped systems) 系统响应从0上升到稳态值所需时间(有振荡系统)
  2) The 10%-90% rise time, measures the time from 10% to 90% of the magnitude of the input. (Over-damped systems)系统响应从稳态值的10%上升到90%所需时间(无振荡系统)

3.2.1Step response and performance specifications

c) ***Peak time (峰值时间)*** T_p: The time for a system to respond to a step input and rise to a peak response.

系统响应达到最大峰值所需要的时间.

d) ***Percent Overshoot (最大)超调量*** σ: The amount by which the system output response proceeds beyond the desired response.

系统响应超出稳态值的最大偏离量(常以百分比表示)

$$\boxed{\sigma\% \overset{def}{=} \frac{y(T_p) - y(\infty)}{y(\infty)} \times 100\%} \qquad (3.1)$$

3.2.1Step response and performance specifications

e) ***Settling time*(调节时间)**T_s: The time required for the system output to settle within a certain percentage of the input amplitude.

系统响应与稳态值之差达到误差±△所需要的最小时间；

$$|y(t)-y(\infty)|\le y(\infty)\Delta,\quad t\ge T_s$$

返回

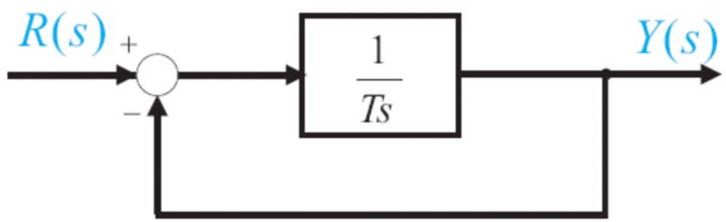
3.2.2 Transient response of 1st order system

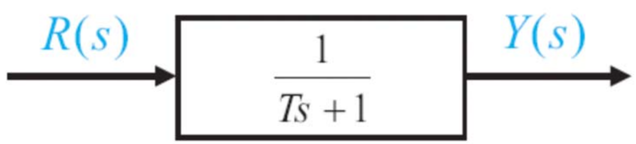
First order inertia system (一阶惯性系统)

- Closed-loop transfer function :

$$G(s)=\frac{Y(s)}{R(s)}=\frac{1}{Ts+1}$$

- Block diagram :





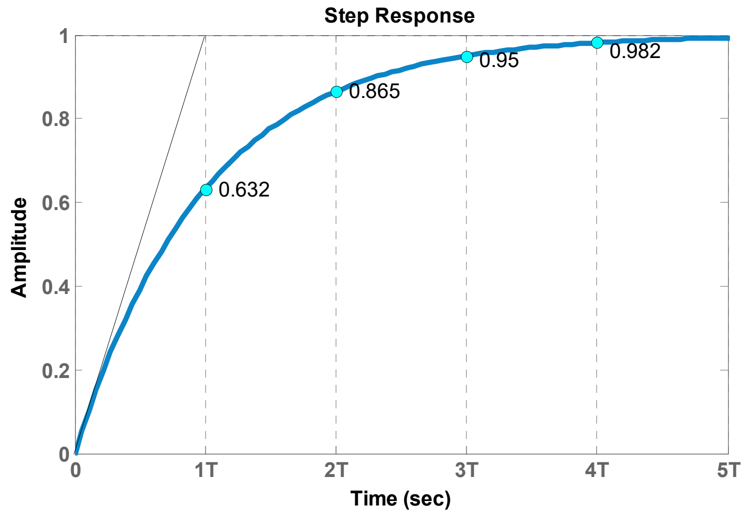
3.2.2 Transient response of 1st order system

- Unit step input response

$$r(t)=1(t), R(s)=1/s$$

$$Y(s)=G(s)R(s)=\frac{1}{Ts+1}\frac{1}{s}$$

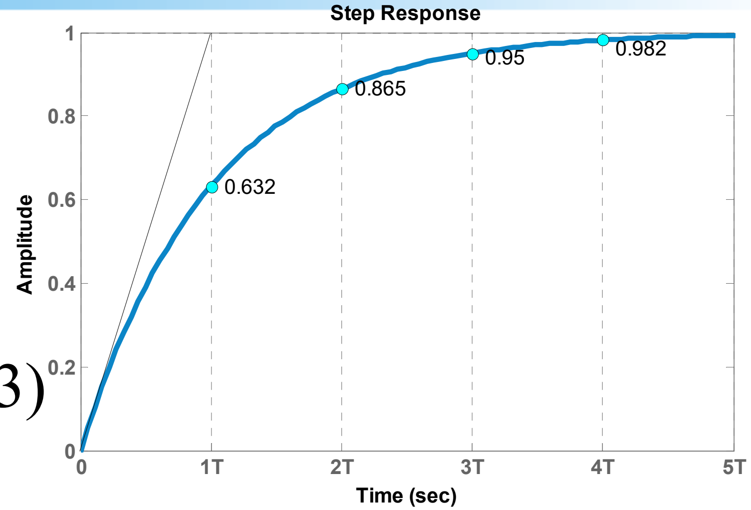
$$=\frac{1}{s}-\frac{T}{Ts+1}=\frac{1}{s}-\frac{1}{s+(1/T)}$$



$$\boxed{y(t)=1-e^{-\frac{t}{T}}, t\geq 0} \qquad (3.2)$$

3.2.2 Transient response of 1st order system

- Performance specifications



★ **Delay Time** T_d:

$$y(t)\big|_{t=T_d} = 1 - e^{-T_d/T} = 0.5$$

$$\boxed{T_d = -T\ln(0.5) = 0.69T} \quad (3.3)$$

★ **Raise Time** T_r:

$$y(t_{0.1}) = 0.1 = 1 - e^{-t_{0.1}/T} \Rightarrow t_{0.1} = -T\ln 0.9 = 0.105T$$

$$y(t_{0.9}) = 0.9 = 1 - e^{-t_{0.9}/T} \Rightarrow t_{0.9} = -T\ln 0.1 = 2.303T$$

$$\boxed{T_r = t_{0.9} - t_{0.1} = 2.20T} \quad (3.4)$$

★ **Settling Time** T_s:

$$\boxed{T_s(5\%) = 3T \qquad T_s(2\%) = 4T}$$

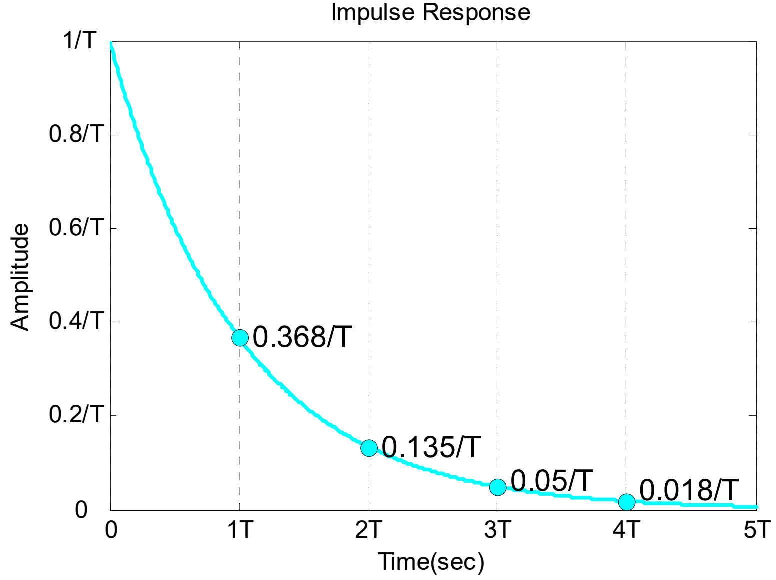
3.2.2 Transient response of 1st order system

- Unit Pulse input response

$$r(t)=\delta(t), R(s)=1$$

$$Y(s)=G(s)R(s)$$

$$=\frac{1}{Ts+1}1=\frac{1/T}{s+1/T}$$



$$\boxed{y(t)=\frac{1}{T}e^{-\frac{t}{T}}, t\geq 0} \qquad (3.5)$$

3.2.2 Transient response of 1st order system

- Unit ramp input response

$$r(t)=t1(t), R(s)=1/s^2$$

$$Y(s)=G(s)R(s)$$

$$=\frac{1}{Ts+1}\frac{1}{s^2}=\frac{1}{s^2}-\frac{T}{s}+\frac{T^2}{Ts+1}$$

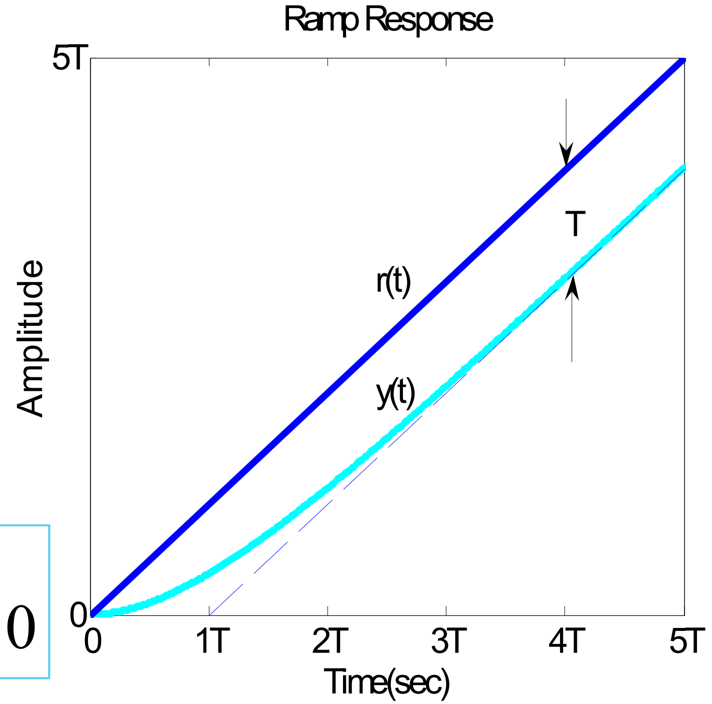
$$\boxed{y(t)=t-T+Te^{-\frac{t}{T}}=t-T(1-e^{-\frac{t}{T}}), t\geq 0}$$

$$\boxed{e(t)=r(t)-y(t)=T(1-e^{-\frac{t}{T}})}$$

(3.6)

(3.7)

$$e(\infty)=T$$



返回

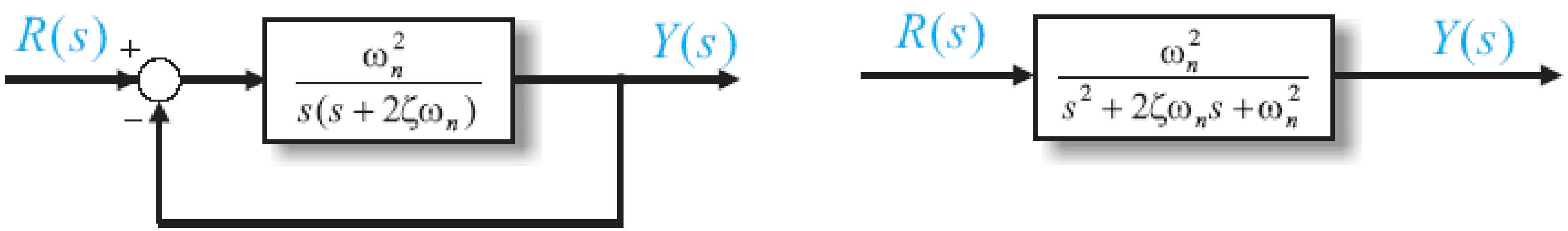
3.2.3Transient response of 2nd order system

Second order system(二阶典型(无零点)系统)

- Closed-loop transfer Function：

$$T(s)=\frac{Y(s)}{R(s)}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \qquad (3.8)$$

- Block diagram：



3.2.3Transient response of 2nd order system

- Characteristic Equation:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \qquad (3.9)$$

- Characteristic Roots:

$$-p_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} \qquad (3.10)$$

According the value of ζ, discuss as following:

$\zeta < 0$, Real($-p_{1,2}$) > 0	System is unstable	系统不稳定
$\zeta = 0$	Undamped system	无阻尼系统
$0 < \zeta < 1$	Under-damped system	欠阻尼系统
$\zeta = 1$	Critically damped system	临界阻尼系统
$\zeta > 1$	Over-damped system	过阻尼系统

3.2.3Transient response of 2nd order system

- ***Unit-step response of 2nd order system***

- Under-damped($0<\zeta<1$)

$$-p_{1,2}=-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}=\sigma \pm j\omega_d \qquad (3.11)$$

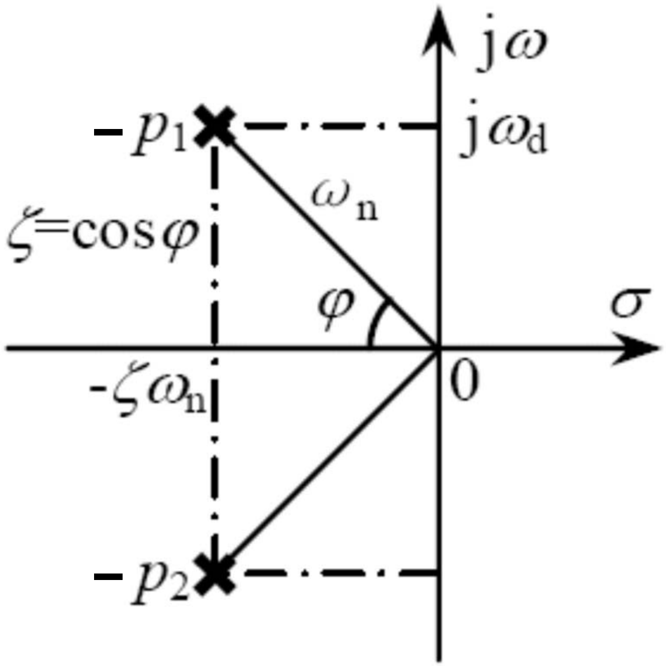
ζ Damping ratio
阻尼比

ω_n Undamped natural frequency
无阻尼自然振荡(角)频率

ω_d Damped natural frequency
阻尼自然振荡(角)频率

σ Damping constant
阻尼系数或衰减系数

φ Damped angle
阻尼角



3.2.3Transient response of 2nd order system

$$Y(s)=\frac{\omega_n^2}{(s^2+2\zeta\omega_n s+\omega_n^2)}\frac{1}{s}=\frac{1}{s}-\frac{s+2\zeta\omega_n}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$=\frac{1}{s}-\frac{s+\zeta\omega_n+\dfrac{\zeta}{\sqrt{1-\zeta^2}}\omega_n\sqrt{1-\zeta^2}}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)} \quad (3.12)$$

$$\mathcal{L}^{-1}[Y(s)]=y(t)=1-e^{-\zeta\omega_n t}\left(\cos\omega_n\sqrt{1-\zeta^2}t+\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_n\sqrt{1-\zeta^2}t\right)$$

$$=1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left(\omega_n\sqrt{1-\zeta^2}t+\varphi\right),t\geq 0 \quad (3.13)$$

$$\varphi=\arctan\frac{\sqrt{1-\zeta^2}}{\zeta}=\arccos\zeta \quad (3.14)$$

3.2.3Transient response of 2nd order system

Transient response of unit step input for a second order system is oscillatory as $0 < \zeta < 1$.

$y(\infty)=1$, Steady State Error is zero　无稳态误差；

3.2.3Transient response of 2nd order system

- Undamped ($\zeta = 0$)

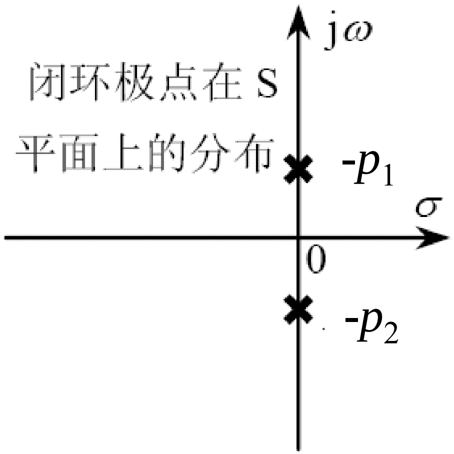
由(3.12), (3.13)令$\zeta=0$得到无阻尼时的阶跃响应

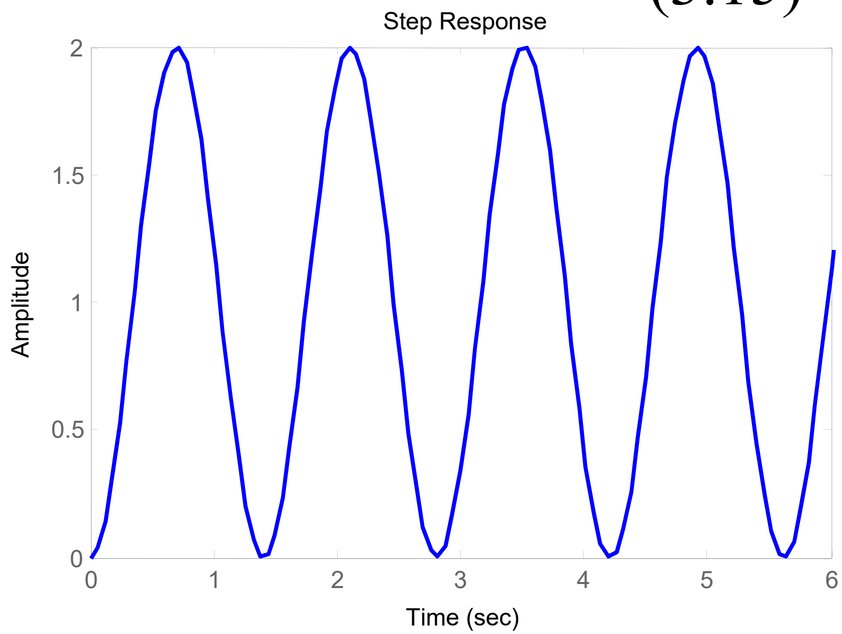
$$Y(s)=\frac{\omega_n^2}{(s^2+\omega_n^2)}\frac{1}{s}=\frac{1}{s}-\frac{s}{s^2+\omega_n^2}$$

$$y(t)=1-\cos\omega_n t,\quad t\geq 0 \qquad (3.15)$$

(3.15)是一个无衰减的振荡;

闭环极点在 S 平面上的分布





3.2.3Transient response of 2nd order system

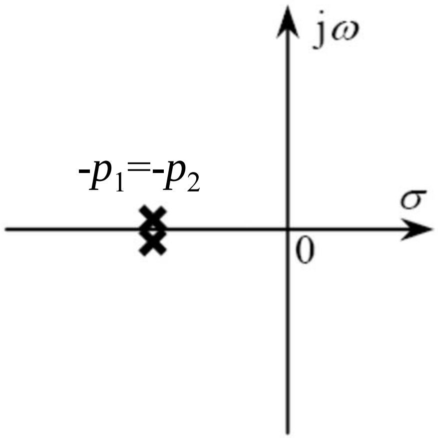
- Critically damped ($\zeta = 1$)

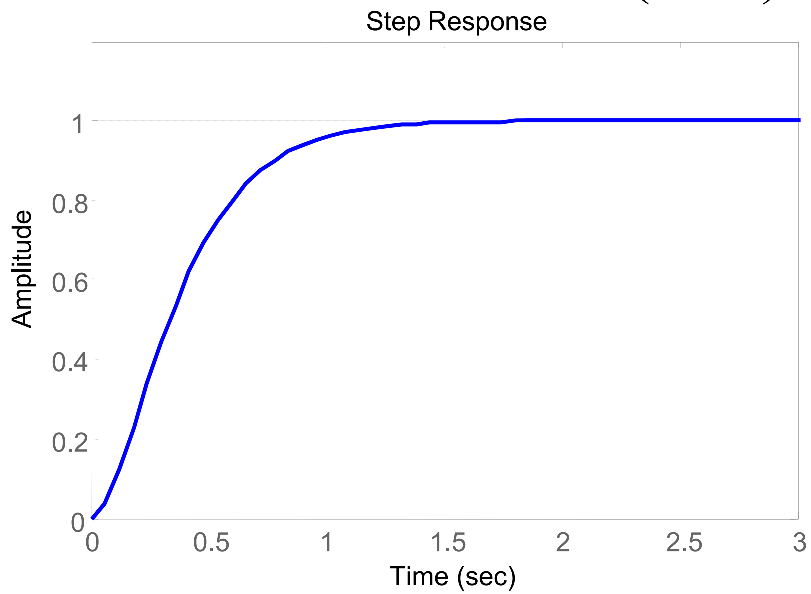
$$-p_{1,2} = \sigma \pm j\omega_d = -\omega_n$$

$$Y(s) = \frac{\omega_n^{\ 2}}{(s+\omega_n)^2}\frac{1}{s} = \frac{1}{s} - \frac{1}{s+\omega_n} - \frac{\omega_n}{(s+\omega_n)^2}$$

$$y(t) = 1 - e^{-\omega_n t}(1+\omega_n t), \quad t \ge 0 \qquad (3.16)$$

(3.16)是无振荡的上升曲线;





3.2.3Transient response of 2nd order system

- Over damped ($\zeta > 1$)

$$-p_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} = -\omega_n(\zeta \mp \sqrt{\zeta^2 - 1})$$

$$Y(s) = \frac{\omega_n{}^2}{(s+p_1)(s+p_2)}\frac{1}{s} = \frac{1}{s} + \frac{\omega_n}{2\sqrt{\zeta^2 - 1}}\left(\frac{-1/p_1}{s+p_1} + \frac{1/p_2}{s+p_2}\right)$$

$$y(t) = 1 + \frac{\omega_n}{2\sqrt{\zeta^2 - 1}}\left(\frac{-1}{p_1}e^{-p_1 t} + \frac{1}{p_2}e^{-p_2 t}\right), \quad t \geq 0 \qquad (3.17)$$

若令 $T_1 = \dfrac{1}{|-p_1|} = \dfrac{1}{p_1} = \dfrac{1}{\omega_n(\zeta - \sqrt{\zeta^2 - 1})}$

$$T_2 = \frac{1}{|-p_2|} = \frac{1}{p_2} = \frac{1}{\omega_n(\zeta + \sqrt{\zeta^2 - 1})}$$

为过阻尼二阶规范系统 的两个时间常数，可得

3.2.3Transient response of 2nd order system

$$y(t)=1-\frac{\omega_n}{2\sqrt{\zeta^2-1}}\left(T_1e^{-\frac{t}{T_1}}-T_2e^{-\frac{t}{T_2}}\right)$$

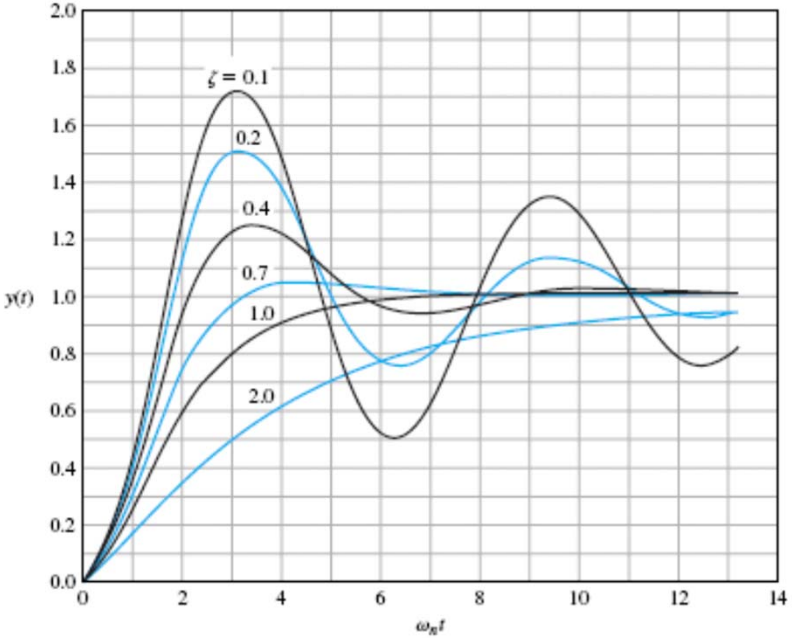
$$=1-\frac{1}{2\sqrt{\zeta^2-1}}\left(\frac{1}{\zeta-\sqrt{\zeta^2-1}}e^{-\frac{t}{T_1}}-\frac{1}{\zeta+\sqrt{\zeta^2-1}}e^{-\frac{t}{T_2}}\right),t\geq 0 \quad (3.18)$$

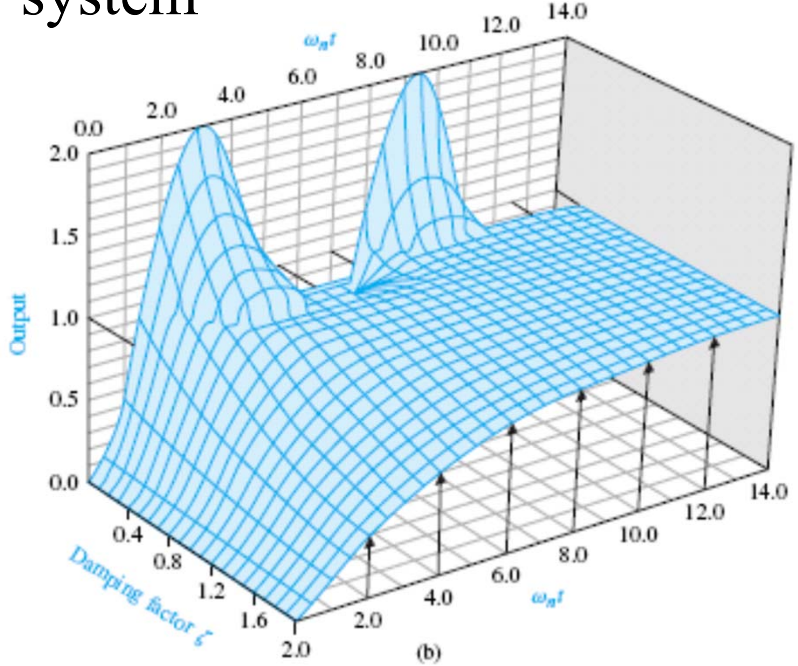
$$\frac{T_1}{T_2}=\frac{|-p_2|}{|-p_1|}=\frac{\zeta+\sqrt{\zeta^2-1}}{\zeta-\sqrt{\zeta^2-1}}=\left(\zeta+\sqrt{\zeta^2-1}\right)^2$$

当$\zeta>>1,T_1>>T_2,e^{-\frac{t}{T_2}}$项的衰减比$e^{-\frac{t}{T_1}}$项快得多($e^{-\frac{t}{T_1}}$项的系数也较大)对于系统暂态响应$e^{-\frac{t}{T_2}}$项在后期的影响很小，因此当$\zeta>>1,T_1>>T_2,(|-\mathrm{p}_2|>>|-\mathrm{p}_1|)$,系统暂态响应近似于一阶系统

3.2.3Transient response of 2nd order system

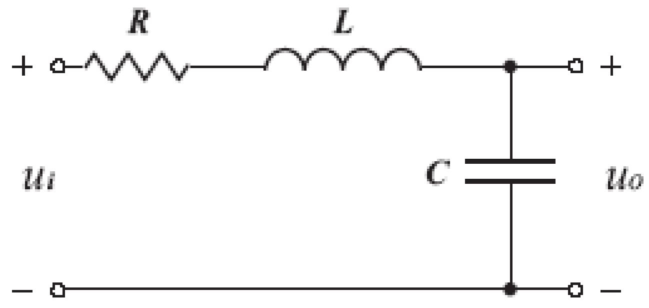
Transient response of second order system





ζ	$\zeta = 0$ 无阻尼	$0 < \zeta < 1$ 欠阻尼	$\zeta = 1$ 临界阻尼	$\zeta > 1$过阻尼
响应	无衰减振荡	衰减振荡	无振荡	无振荡

3.2.3Transient response of 2nd order system



<E 3.1>: RLC circuit

Transfer function:

$$G(s)=\frac{Y(s)}{R(s)}=\frac{1}{LCs^2+RCs+1}=\frac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

$$\omega_n=\frac{1}{\sqrt{LC}} \qquad 2\zeta\omega_n=\frac{R}{L} \qquad \zeta=\frac{R}{2\omega_n L}=\frac{1}{2}\sqrt{LC}\frac{R}{L}=\frac{1}{2}\sqrt{\frac{R}{\frac{1}{Cs}}\frac{R}{sL}}$$

ζ大–R较大(R为耗能元件) Ls,$1/Cs$ 较小\(L,C储能元件)

R较大,能耗较大(如上串联电路中)磁能和场能相互转换过程中在R上耗能较多，使得振荡衰减较快，甚至不能产生振荡。

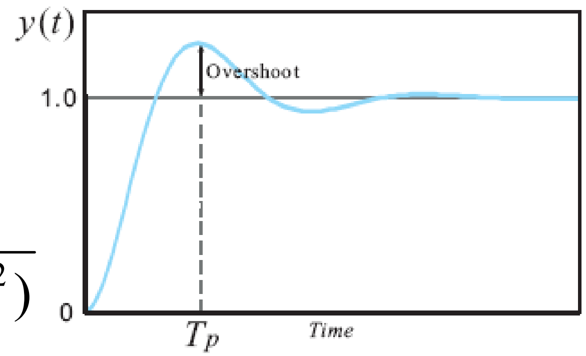
3.2.3Transient response of 2nd order system

- **Transient response performance of a under-damped second order system**

1) Peak Time T_p: 响应曲线第一次达到峰值的时间

$$\left.\frac{dy(t)}{dt}\right|_{t=T_p}=0 \qquad sY(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$=\frac{\omega_n}{\sqrt{1-\zeta^2}}\frac{\omega_n\sqrt{1-\zeta^2}}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)}$$



$$\frac{dy(t)}{dt}=\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_d t=0$$

$$\sin\omega_d t=0\Rightarrow\omega_d t=n\pi\Rightarrow t=n\pi/\omega_d,(n=0,1,2,\cdots)$$

第一次到达峰值, 取$n=1$

$$\boxed{T_p=\frac{\pi}{\omega_d}=\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}} \qquad (3.19)$$

3.2.3Transient response of 2nd order system

2) Percent Overshoot P.O. $\sigma\%$:

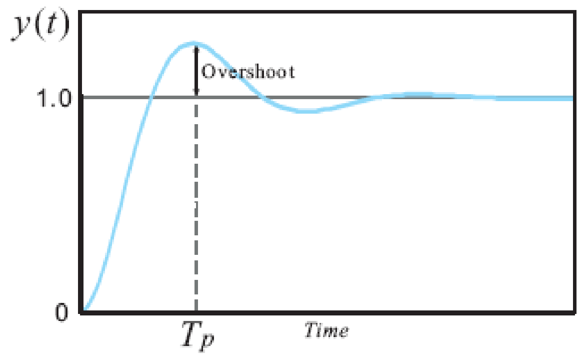
$$t = T_p = \pi / \omega_d$$

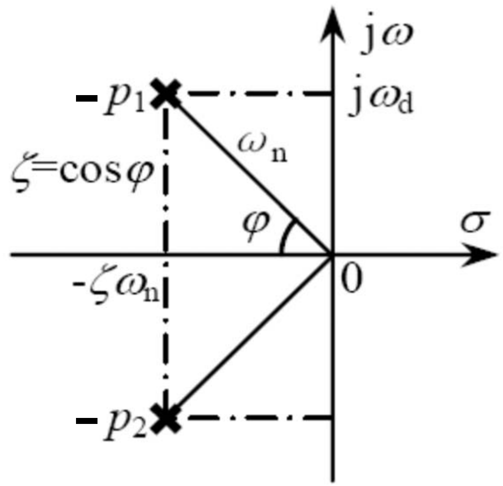
$$\sigma\% = \left[y(T_p) - 1 \right] \times 100\%$$

$$= -\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n T_p} \sin(\omega_d T_p + \varphi)$$

$$= -\frac{1}{\sqrt{1-\zeta^2}} \exp(-\zeta\omega_n \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}) \sin(\pi + \varphi)$$

since $\sin(\pi + \varphi) = -\sin\varphi = -\sqrt{1-\zeta^2}$



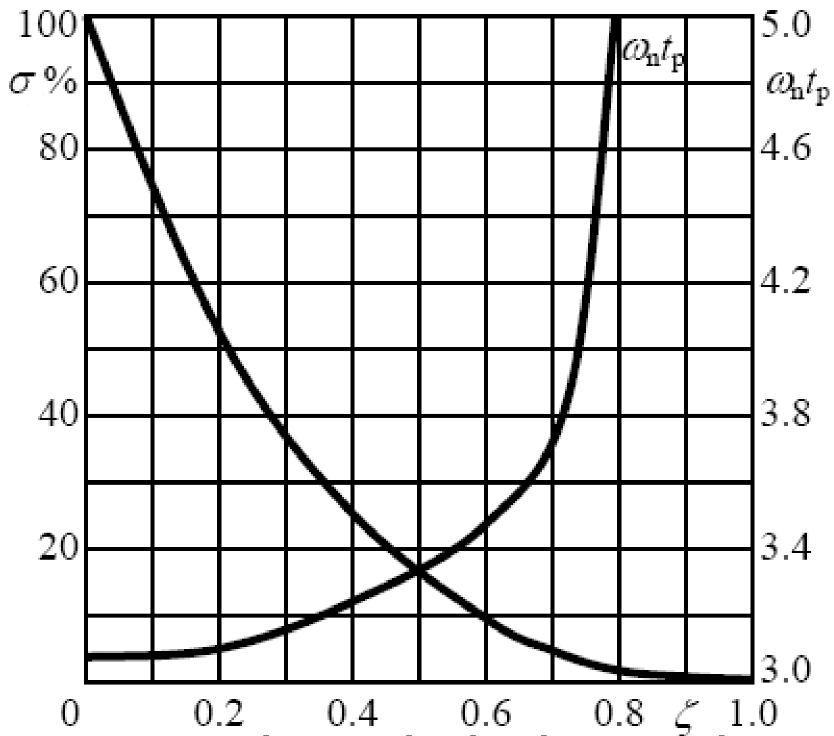


$$\boxed{\sigma\% = \exp(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}) \times 100\%} \qquad (3.20)$$

3.2.3Transient response of 2nd order system

The **percent overshoot** versus the **damping ratio** and the **normalized peak time** versus the **damping ratio** is shown below.

二阶规范系统的**超调量**和**峰值时间**与阻尼比的关系如下图所示

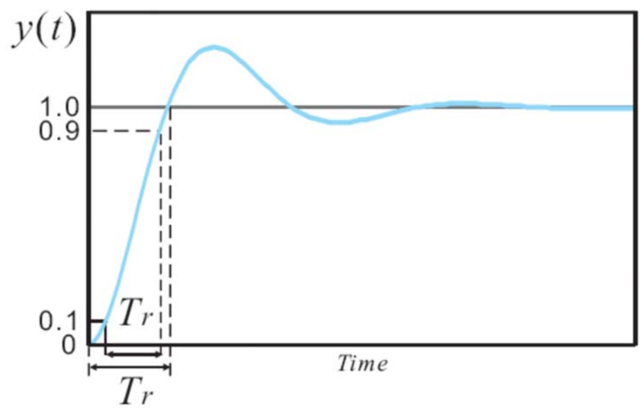


Note: Percent overshoot is independent of ω_n

超调量σ% 只是 ζ的函数,与 ω_n无关

3.2.3Transient response of 2nd order system

3) Rise Time上升时间 T_r:



采用 "0→100%"的上升时间定义

$$y(T_r) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n T_r} \sin(\omega_d T_r + \varphi)$$

$$= 1$$

Let $\sin(\omega_d T_r + \varphi) = 0 \Rightarrow \omega_d T_r + \varphi = \pi$

$$\boxed{T_r = \frac{\pi - \varphi}{\omega_d} = \frac{\pi - \varphi}{\omega_n \sqrt{1-\zeta^2}}} \qquad (3.21)$$

3.2.3Transient response of 2nd order system

4) Settling Time T_s:

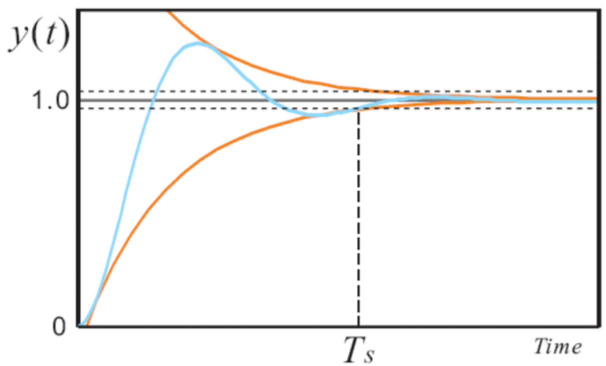
$$t \ge T_s : \left| y(t) - y(\infty) \right| \le y(\infty)\delta$$

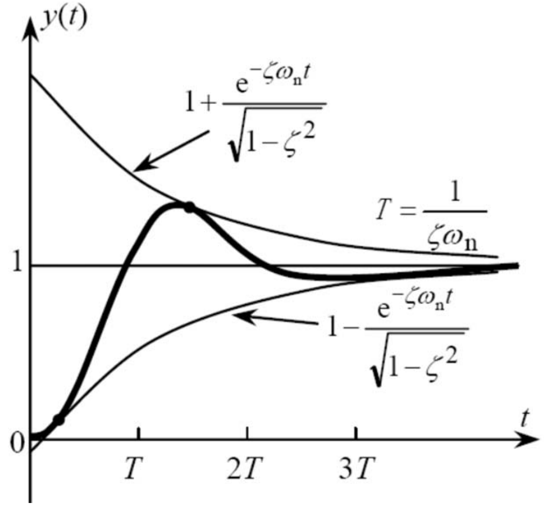
响应曲线的包络线： $1 \pm \dfrac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}$

$$\left| \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \varphi) \right| \le \delta$$

为了便于计算,近似取

$$\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n T_s} \approx \delta$$





$$\boxed{T_s = \frac{1}{\zeta\omega_n} \ln \frac{1}{\delta\sqrt{1-\zeta^2}} = \frac{1}{\zeta\omega_n}\left[-\ln\delta - \frac{1}{2}\ln(1-\zeta^2) \right]} \qquad (3.22)$$

3.2.3Transient response of 2nd order system

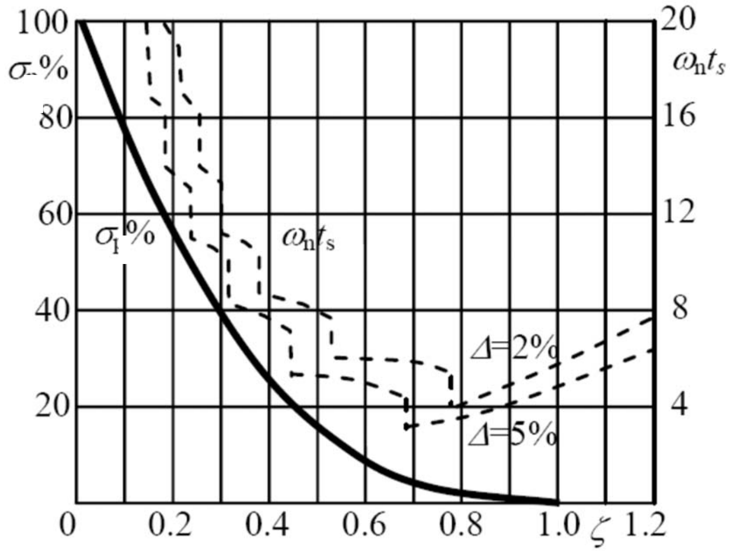
$$T_s(2\%)=\frac{1}{\zeta\omega_n}\left[4-\frac{1}{2}\ln(1-\zeta^2)\right] \tag{3.23}$$

$$T_s(5\%)=\frac{1}{\zeta\omega_n}\left[3-\frac{1}{2}\ln(1-\zeta^2)\right] \tag{3.24}$$

对于0<ζ<0.9,近似取

$$T_s(2\%)=\frac{4}{\zeta\omega_n} \tag{3.25}$$

$$T_s(5\%)=\frac{3}{\zeta\omega_n} \tag{3.26}$$



T_s的精确曲线实际上是不连续的,由T_s的定义,可知造成T_s为不连续的曲线,如图所示.

3.2.3Transient response of 2nd order system

5) Delay Time T_d:

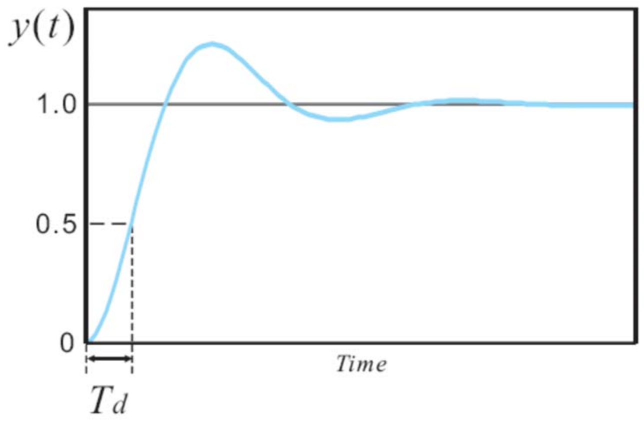
$$t = T_d, y(T_d) = 0.5$$

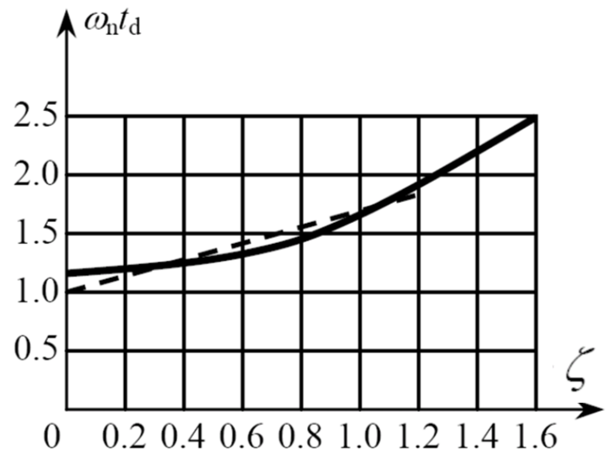
$$\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n T_d} \sin(\omega_d T_d + \varphi) = 0.5$$

T_d的求解由隐函数给出

$$\omega_n T_d = \frac{1}{\zeta} \ln \frac{2\sin(\omega_d T_d + \varphi)}{\sqrt{1-\zeta^2}} \tag{3.27}$$

其中曲线如图所示





3.2.3Transient response of 2nd order system

6) 振荡次数 *N*:

阻尼振荡周期:

$$\tau_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$$

由公式(3.25)或(3.26)可以给出振荡次数N 的近似计算公式:

$$\boxed{N = \frac{T_s}{\tau_d} = \frac{(3\sim 4)\sqrt{1-\zeta^2}}{2\pi\zeta}} \qquad (3.28)$$

3.2.3Transient response of 2nd order system

- 注：兼顾超调量和调节时间，控制系统常选择

  $\zeta = 0.4\sim0.8$，相应的 $\sigma\% = 25.4\%\sim1.5\%$

实际控制系统常选取工作在欠阻尼状态，只有当不允许出现超调或对象本身惯性很大时，才采用接近临界阻尼的过阻尼状态。

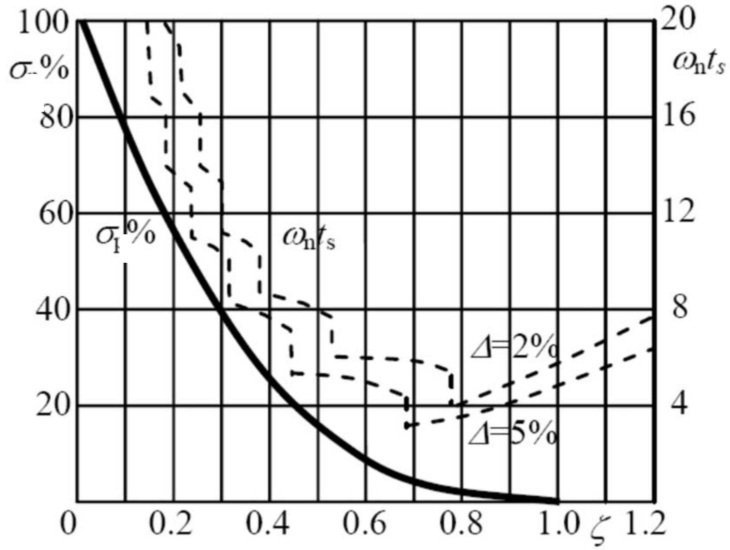
3.2.3Transient response of 2nd order system

二阶工程最佳参数

某些控制系统采用所谓“二阶工程最佳参数”作为控制系统工程设计的依据，即选择参数使

$$\zeta = 1/\sqrt{2} = 0.707, \text{相应的} \sigma\% = e^{-\pi} \times 100\% = 4.3\%$$

由 $\sigma\%$ 和 $\omega_n T_s$ 与 ζ 的关系曲线可见，此时控制系统较好地兼顾了暂态响应的平稳性与快速性。



返回

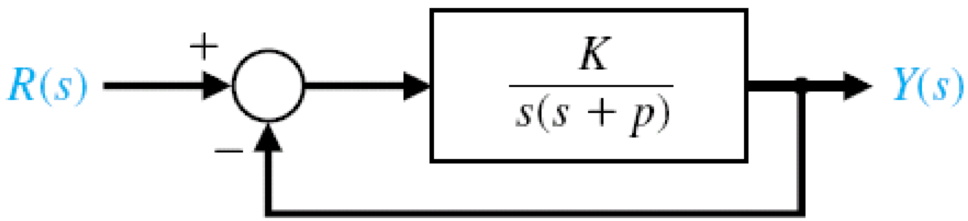
3.2.3Transient response of 2nd order system

<E 3.1> Parameter selection

A single-loop feedback control system is shown below. We desire to select the gain K and the parameter p so that the time-domain specifications will be satisfied.

a. The transient response to a step should be as fast as is attainable while retaining an overshoot of less than 5% (P.O.<5%).

b. The settling time to within 2% of the final value should be less than 4 seconds (T_s<4 (δ%=2%)).



$$P.O. = \sigma\% = \exp(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}})\times 100\% < 5\%$$

$$e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} < 0.05 \Rightarrow -\frac{\zeta\pi}{\sqrt{1-\zeta^2}} < \ln(0.05) \Rightarrow \frac{\zeta^2\pi^2}{1-\zeta^2} > 8.97$$

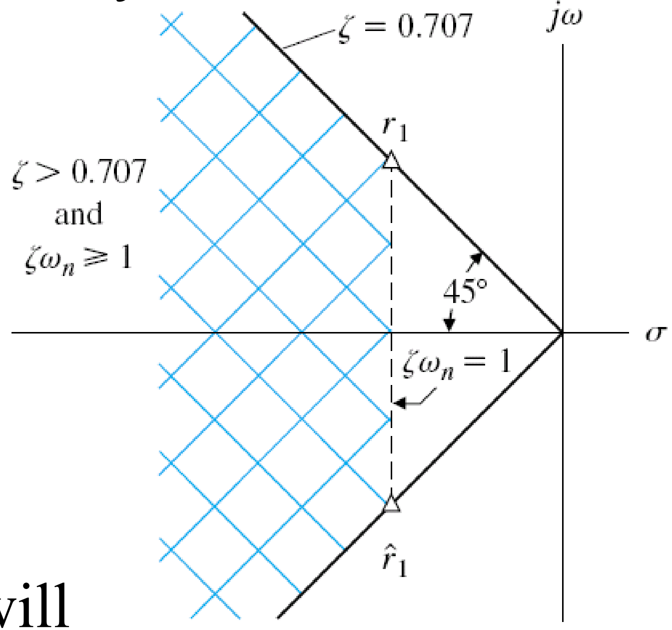
$$\zeta^2(\pi^2 + 8.97) > 8.97$$

$$\zeta > 0.69$$

$$\varphi = \cos^{-1}\zeta \approx 46.36°$$

$$T_s = \frac{4}{\zeta\omega_n} \le 4$$

$$\zeta\omega_n \ge 1$$



The region is shown in the figure will satisfy both time-domain requirements is shown cross-hatched on the *s*-plane.

When the closed-loop roots are chosen as

$$r_{1,2} = -1 \pm j1$$

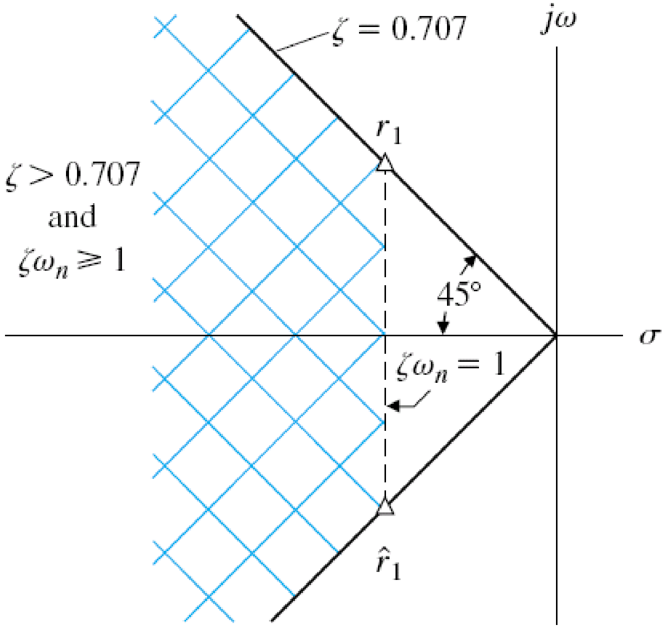
$$\text{So} \begin{cases} \zeta\omega_n = 1 \\ \zeta = 1/\sqrt{2} \end{cases} \Rightarrow \begin{cases} \omega_n = \sqrt{2} \\ \zeta = 1/\sqrt{2} \end{cases}$$

$$T(s) = \frac{G(s)}{1+G(s)}$$

$$= \frac{\omega_n^{\ 2}}{s^2 + 2\zeta\omega_n s + \omega_n^{\ 2}} = \frac{K}{s^2 + ps + K}$$

$$p = 2$$

$$K = 2$$



3.2.4 Effects of a 3rd zero on the 2nd system response

For a second order system, insert a 3rd zero, the closed-loop transfer function of the system:

$$T(s)=\frac{Y_Z(s)}{R(s)}=\frac{\omega_n{}^2(\tau s+1)}{s^2+2\zeta\omega_n s+\omega_n{}^2} \qquad (3.29)$$

$$=\frac{\omega_n{}^2(s+z)}{z(s^2+2\zeta\omega_n s+\omega_n{}^2)}$$

where, $-z=-\dfrac{1}{\tau}$ is the zero

3.2.4 Effects of a 3rd zero on the 2nd system response

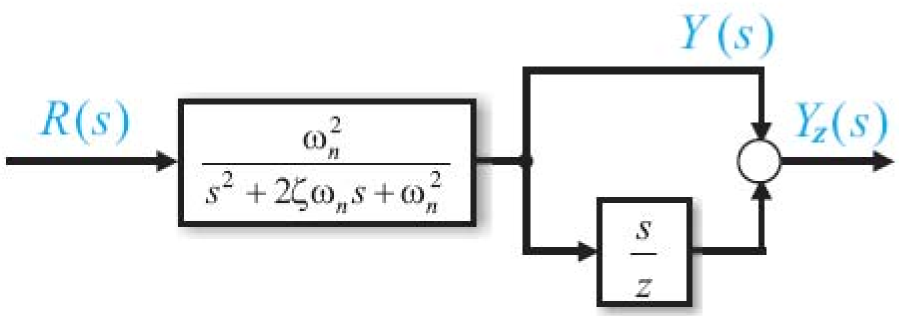
- Unit step response of the 2nd order system with a zero

$$r(t)=1(t), R(s)=1/s; \quad 0<\zeta<1$$

$$\Phi_Z(s)=\frac{\omega_n{}^2}{s^2+2\zeta\omega_n s+\omega_n{}^2}+\frac{s}{z}\frac{\omega_n{}^2}{s^2+2\zeta\omega_n s+\omega_n{}^2}$$

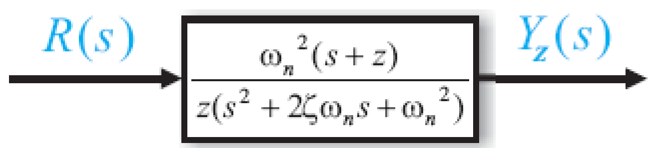
The block diagram

$$Y_Z(s)=Y(s)+\frac{s}{z}Y(s)$$



With zero initial condition

$$y_Z(t)=y(t)+\frac{1}{z}\dot{y}(t)$$



3.2.4 Effects of a 3rd zero on the 2nd system response

$$\frac{1}{z}\dot{y}(t)=\frac{e^{-\zeta\omega_n t}}{z\sqrt{1-\zeta^2}}\left(\zeta\omega_n\sin(\omega_d t+\varphi)-\omega_d\cos(\omega_d t+\varphi)\right)$$

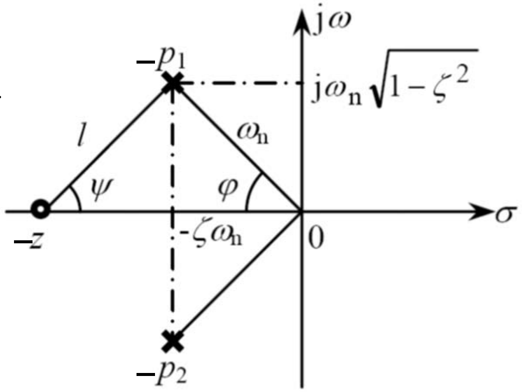
$$y_z(t)=y(t)+\frac{1}{z}\dot{y}(t)$$

$$=1-\frac{e^{-\zeta\omega_n t}}{z\sqrt{1-\zeta^2}}\left((z-\zeta\omega_n)\sin(\omega_d t+\varphi)-\omega_d\cos(\omega_d t+\varphi)\right)$$

$$=1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\frac{l}{z}\sin(\omega_d t+\varphi+\psi),t\geq 0 \qquad (3.30)$$

其中$l=\sqrt{(z-\zeta\omega_n)^2+\omega_d^{\ 2}}=\sqrt{z^2-2\zeta\omega_n z+\omega_n^{\ 2}}$

$$\psi=tg^{-1}\frac{\omega_n\sqrt{1-\zeta^2}}{z-\zeta\omega_n},\quad \varphi=tg^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$$

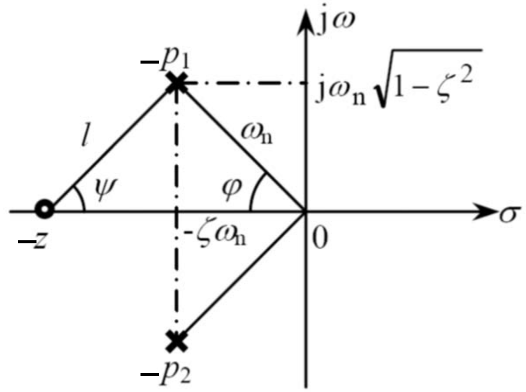


3.2.4 Effects of a 3rd zero on the 2nd system response

$$\frac{l}{z}=\frac{\sqrt{z^2-2\zeta\omega_n z+\omega_n^{\ 2}}}{z}=\sqrt{1-\frac{2\zeta\omega_n}{z}+\frac{\omega_n^{\ 2}}{z^2}}$$

$$=\frac{1}{\zeta}\sqrt{\zeta^2-2r\zeta^2+r^2}$$

其中$r=\dfrac{\zeta\omega_n}{z}$为复数极点实部与零点之比



$$\boxed{y(t)=1-\frac{\sqrt{\zeta^2-2r\zeta^2+r^2}}{\zeta\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_d t+\varphi+\psi),t\geq 0} \qquad (3.31)$$

3.2.4 Effects of a 3rd zero on the 2nd system response

- Transient response performance specifications

1) **Rise Time** T_{rz}:

$$T_{rz} = \frac{\pi - \varphi - \psi}{\omega_d} = T_r - \frac{\psi}{\omega_d} \qquad (3.32)$$

2) **Peak Time** T_p:

$$T_{pz} = \frac{\pi - \psi}{\omega_d} = T_p - \frac{\psi}{\omega_d} \qquad (3.33)$$

3) **Percent Overshoot** $\sigma\%$:

$$\sigma_z\% = \frac{1}{\zeta}\sqrt{\zeta^2 - 2r\zeta^2 + r^2}\, e^{-\zeta\omega_n T_{pz}} \times 100\%$$

$$= \frac{l}{z} e^{-\zeta\omega_n T_p} e^{\zeta\omega_n \frac{\psi}{\omega_d}} = \sigma\% \frac{l}{z} e^{\frac{\zeta\psi}{\sqrt{1-\zeta^2}}} \qquad (3.34)$$

3.2.4 Effects of a 3rd zero on the 2nd system response

4) Settling Time T_{sz}:

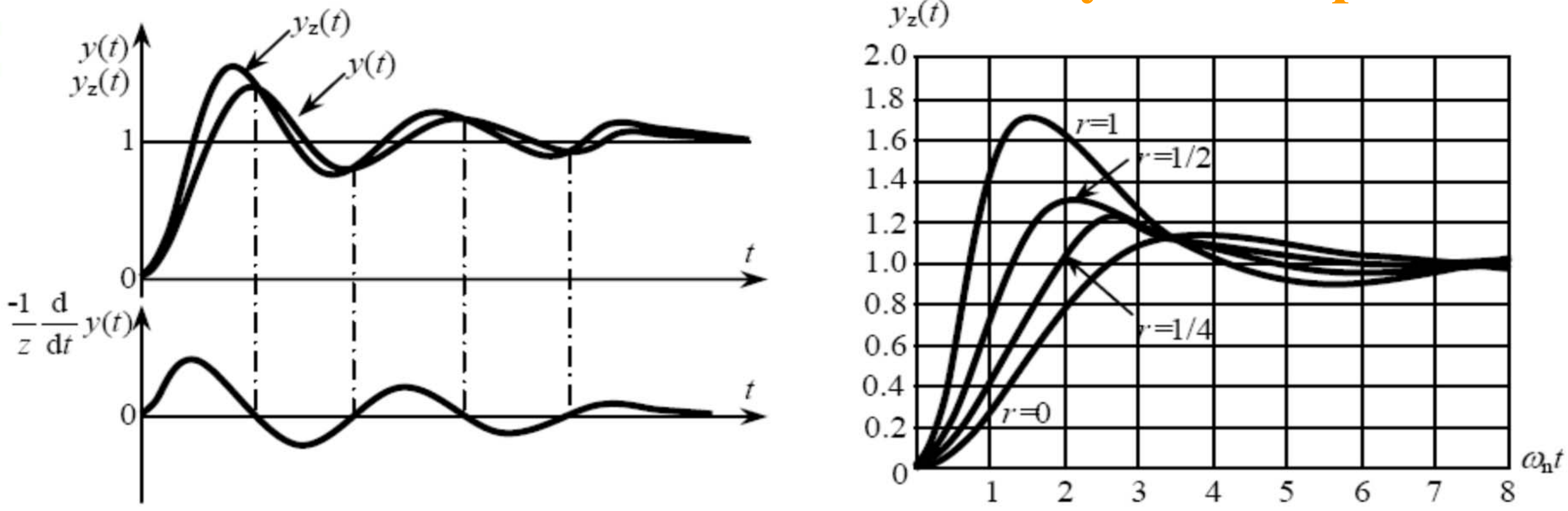
As the solution for T_s, we choose

$$\frac{l}{z}\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} = \delta$$

$$T_{sz} = \frac{1}{\zeta\omega_n}\left[-\ln\delta - \frac{1}{2}\ln(1-\zeta^2) + \ln\frac{l}{z}\right]$$

$$= T_s + \frac{1}{\zeta\omega_n}\ln\frac{l}{z} \qquad (3.35)$$

3.2.4 Effects of a 3rd zero on the 2nd system response



(a)闭环零点对系统暂态响应的影响 (b)单位阶跃响应曲线(ζ=0.5)

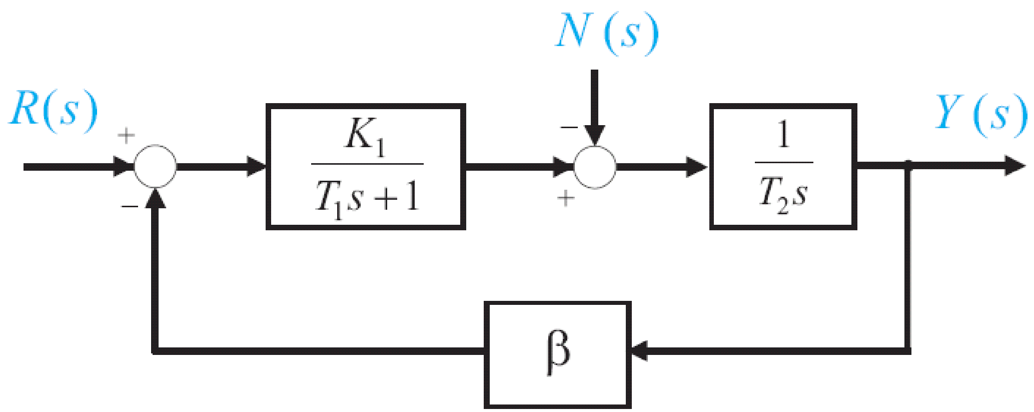
添加零点对原无零点规范二阶系统性能的影响：

- Peak time decrease 峰值时间提前；
- Percent overshooting increase 超调量增大(振荡加剧)；
- Settling time 调节时间

$$\frac{z}{\zeta\omega_n}=\frac{1}{r}$$ 越小，影响越大

3.2.4 Effects of a 3rd zero on the 2nd system response

<E 3.2>: The block diagram of a Dc motor control system



With input $r(t)$ when ($n(t) = 0$):

$$\frac{Y(s)}{R(s)} = \frac{K_1}{T_1 T_2 s^2 + T_2 s + \beta K_1}$$

无零点的二阶系统

With $n(t)$ (when $r(t)=0$):

$$\frac{Y(s)}{N(s)} = \frac{T_1 s + 1}{T_1 T_2 s^2 + T_2 s + \beta K_1}$$

有零点的二阶系统

返回

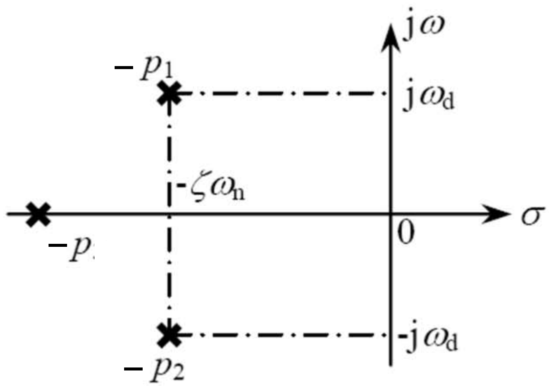
3.2.5 Transient response of 3rd order system

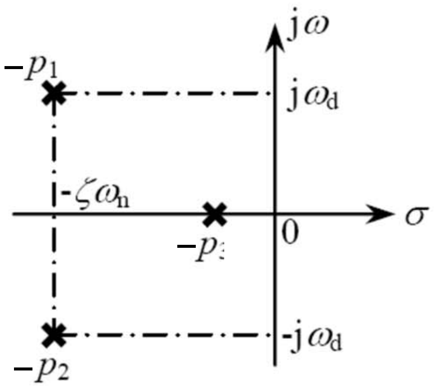
- Closed-loop transfer function of 3rd order system

$$\frac{Y(s)}{R(s)}=\frac{\omega_n^{\ 2}}{(Ts+1)(s^2+2\zeta\omega_n s+\omega_n^{\ 2})}$$

$$=\frac{\omega_n^{\ 2}p}{(s+p)\left[(s+\zeta\omega_n)^2+\omega_n^{\ 2}(1-\zeta^2)\right]} \qquad (3.36)$$

where $p = 1/T$





3.2.5 Transient response of 3rd order system

- Unit step input response of a 3rd order system

$$0<\zeta<1, r(t)=1(t), R(s)=1/s$$

when

$$Y(s)=\frac{\omega_n^{\ 2} p}{(s+p)(s^2+2\zeta\omega_n s+\omega_n^{\ 2})}\frac{1}{s}=\frac{A_0}{s}+\frac{A_1}{s+p}+\frac{A_2 s+A_3}{s^2+2\zeta\omega_n s+\omega_n^{\ 2}}$$

$$=\frac{A_0}{s}+\frac{A_1}{s+p}+\frac{A_2(s+\zeta\omega_n)-A_2\zeta\omega_n+A_3}{s^2+2\zeta\omega_n s+\omega_n^{\ 2}}$$

$$A_0=1 \quad A_1=\frac{-1}{\zeta^2\beta(\beta-2)+1} \quad A_2=\frac{-\zeta^2\beta(\beta-2)}{\zeta^2\beta(\beta-2)+1}$$

$$-A_2\zeta\omega_n+A_3=\frac{-\beta\zeta\omega_n[\zeta^2(\beta-2)+1]}{\zeta^2\beta(\beta-2)+1}=\frac{-\beta\zeta[\zeta^2(\beta-2)+1]\omega_n\sqrt{1-\zeta^2}}{\left[\zeta^2\beta(\beta-2)+1\right]\sqrt{1-\zeta^2}}$$

3.2.5 Transient response of 3rd order system

$$y(t)=1-\frac{e^{-pt}}{\zeta^2\beta(\beta-2)+1}-\frac{e^{-\zeta\omega_n t}}{\zeta^2\beta(\beta-2)+1}$$

$$\times\left[\zeta^2\beta(\beta-2)\cos\omega_d t+\frac{\beta\zeta[\zeta^2(\beta-2)+1]}{\sqrt{1-\zeta^2}}\sin\omega_d t\right]$$

$$=1-\frac{e^{-\beta\zeta\omega_n t}}{\zeta^2\beta(\beta-2)+1}-\frac{\beta\zeta e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}\sqrt{\zeta^2\beta(\beta-2)+1}}\sin(\omega_d t+\gamma), t\ge 0 \qquad (3.37)$$

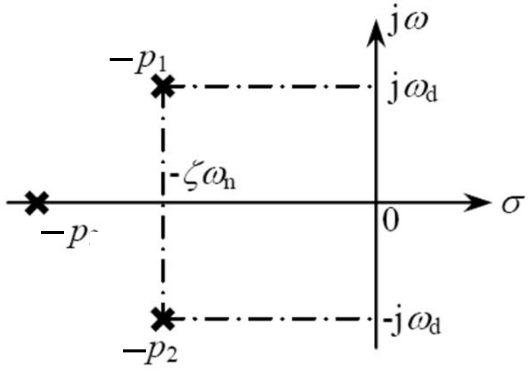
$$\gamma=tg^{-1}\frac{\zeta(\beta-2)\sqrt{1-\zeta^2}}{\zeta^2(\beta-2)+1}$$

$$\beta=\frac{p}{\zeta\omega_n},\left(\text{比较:有零点二阶系统,}\frac{1}{r}=\frac{z}{\zeta\omega_n}\right)$$

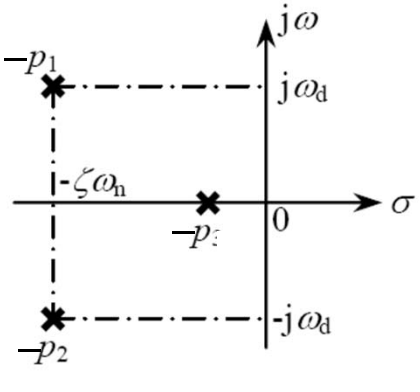
注: $\zeta^2\beta(\beta-2)+1=\zeta^2\beta^2-2\zeta^2\beta+1=\zeta^2(\beta-1)^2+(1-\zeta^2)>0$

$\Rightarrow e^{-pt}$项的系数总是为负数

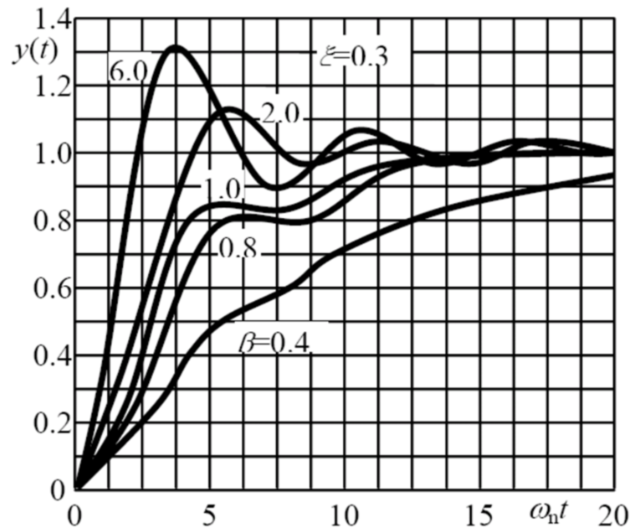
3.2.5 Transient response of 3rd order system



(a)β>1 时



(b)β<1 时



Discussion:

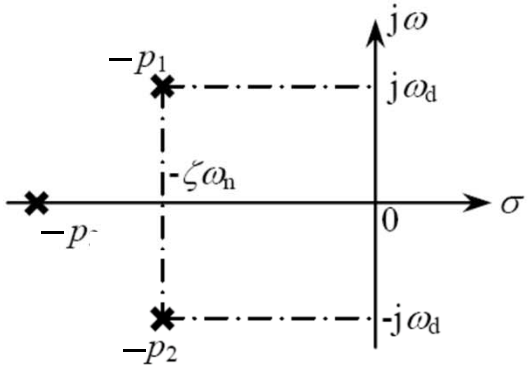
1) $y(t)$与$\zeta, \omega_n, \beta = \dfrac{p}{\zeta\omega_n}$有关

$\beta \to \infty$，相当于二阶系统；

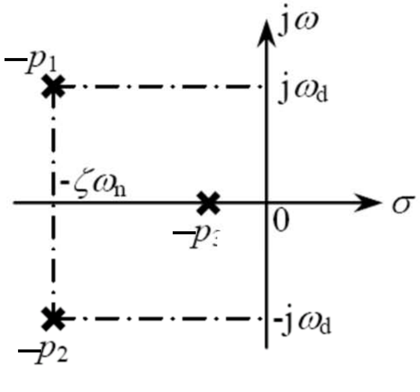
$\beta >> 1$，共轭复数极点为主导极点，响应主要呈现为二阶特性；

$\beta << 1$，实极点为主导极点，响应主要呈现为一阶特性；

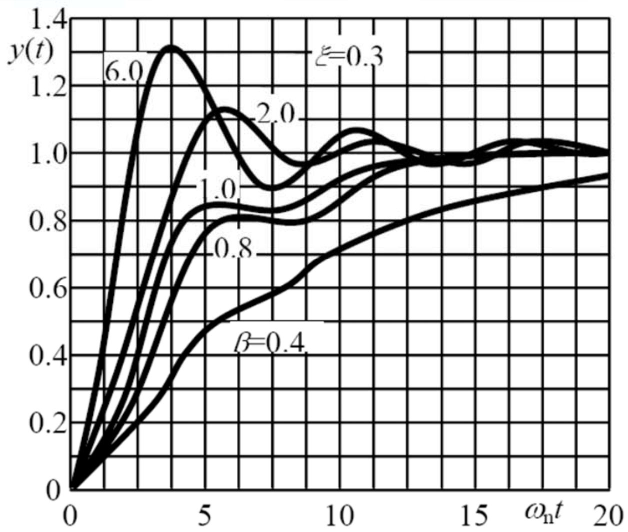
3.2.5 Transient response of 3rd order system



(a)β>1 时



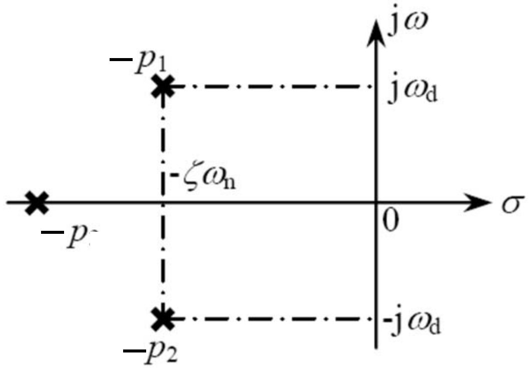
(b)β<1 时



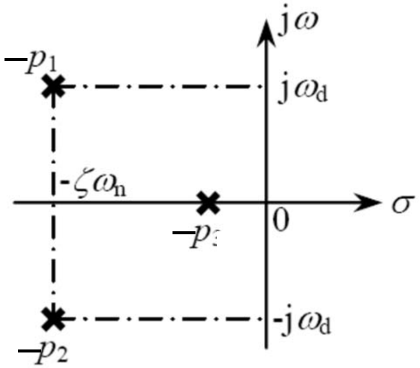
Discussion:

2) 当$\beta \geq 5$左右(或者$\beta \leq 1/5$左右)，可按照主导极点共轭复数极点(或按照主导极点实极点)估算暂态响应特性；

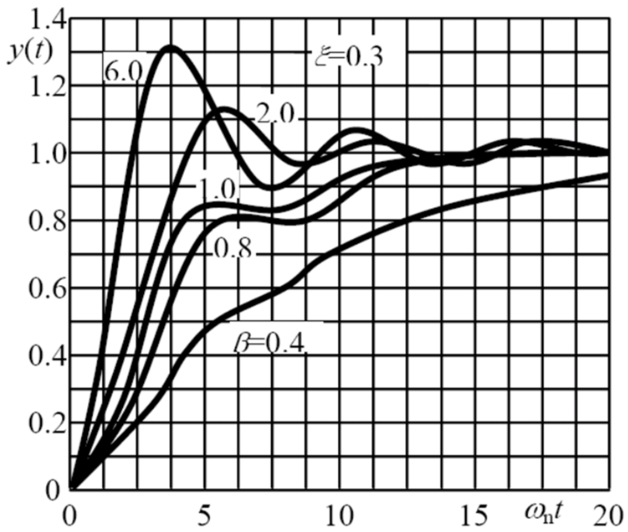
3.2.5 Transient response of 3rd order system



(a)β>1 时

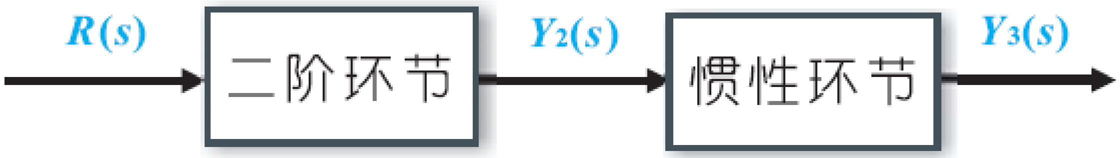


(b)β<1 时



Discussion:

3) 实极点的影响：振荡性 减弱，超调量减小，响 应速度变慢，相当于增加了系 统的惯性；



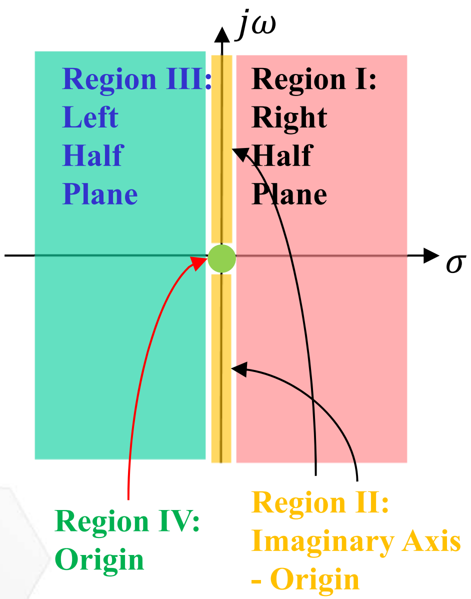
返回

3.3 Steady state error

Final Value Theory（终值定理）

Time domain **S domain**

$$f(t\to\infty)=\lim_{t\to\infty} f(t) \qquad \lim_{t\to\infty} f(t)=\lim_{s\to 0} sF(s) \qquad F(s)=\mathscr{L}(f(t))$$



If the system has Poles in the following Region:

Region I: **System response is unstable** **Not fit for this situation.** $\lim_{s\to 0} sF(s)=\lim_{s\to 0} s\frac{1}{s-5}=0$ ☒	**Region II:** **System response if oscillatory** **Not fit for this situation.** $\lim_{s\to 0} sF(s)=\lim_{s\to 0} s\frac{1}{s^2+5}=0$ ☒
Region III: **System response is stable** **Final value is always 0** $\lim_{s\to 0} sF(s)=\lim_{s\to 0} s\frac{1}{s+5}=0$ ☑	**Region IV:** **Output is the integral of the input signal (impulse r(t)=1).** $\lim_{s\to 0} sF(s)=\lim_{s\to 0} s\frac{1}{s}=1$ ☑

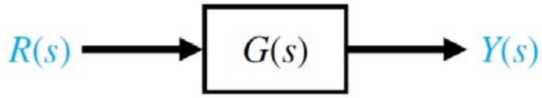
System Type: Number of poles at the origin.

Type I System; Type II System; Type III System and …

3.3 Steady state error

3.3.1 Steady State Error

Open loop system



$$e_o(t) = r(t) - y(t)$$

$$E_o(s) = R(s) - Y(s)$$

$$= (1 - G(s))R(s)$$

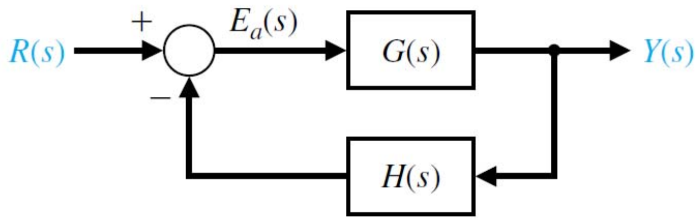
If input is unit step

$$e_o(\infty) = \lim_{s\to 0} sE_o(s)$$

$$= \lim_{s\to 0} s(1 - G(s))\frac{1}{s}$$

$$= 1 - G(0)$$

Closed loop system



$$e_c(t) = r(t) - y(t)$$

$$E_c(s) = R(s) - Y(s)$$

$$= (\frac{1}{1+G(s)})R(s), where H(s) = 1$$

If input is unit step

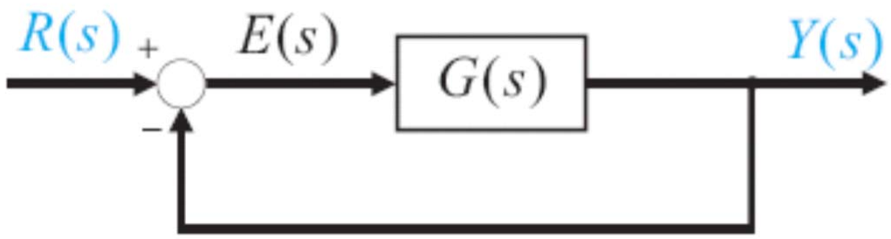
$$e_c(\infty) = \lim_{s\to 0} sE_c(s)$$

$$= \lim_{s\to 0} s(\frac{1}{1+G(s)})\frac{1}{s}$$

$$= \frac{1}{1+G(0)}$$

3.3 Steady state error

3.3.2 Steady state error of a unit feedback system



Error

$$e(t)=r(t)-y(t)$$

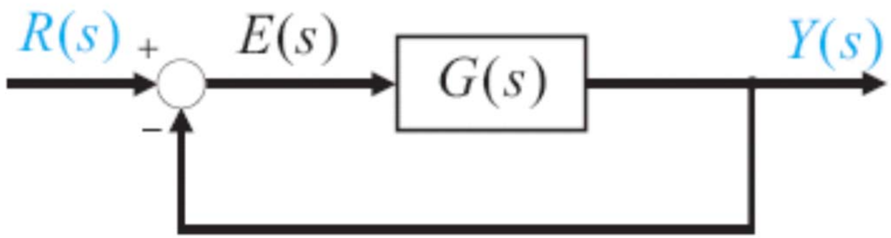
$$E(s)=R(s)-Y(s)$$

$$E(s)=R(s)-\frac{G(s)}{1+G(s)}R(s)$$

$$=\frac{1}{1+G(s)}R(s)=\Phi_e(s)R(s) \qquad (3.39)$$

$$\Phi_e(s)=\frac{1}{1+G(s)}=\frac{1}{1+G_k(s)}$$ 称为误差传递函数

3.3 Steady state error



Steady state error

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{sR(s)}{1+G(s)} \qquad (3.40)$$

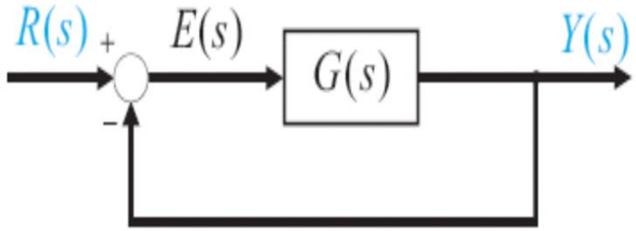
Determined by **loop transfer function** and **input** .

稳态误差由**开环传递函数**和**输入**决定

3.3 Steady state error

[E3.3] Try to get the **Steady State Error** for the unit feedback control system, where the transfer function is

$$G(s)=K(0.5s+1)/[s(s+1)(2s+1)]$$



and input is unit ramp signal.

Step1.Determin the stability of the system. The characteristic equation of the closed-loop system is

$$\Delta(s)=2s^3+3s^2+(1+0.5K)s+K$$

The Routh array is:

s^3	2	$(1+0.5K)$
s^2	3	K
s^1	$\dfrac{3-0.5K}{3}$	0
s^0	K	

Thus for a stable system, we require that: **$0<K<6$**

3.3 Steady state error

Step2.Determin $E(s)$:

$$E(s)=\Phi_e(s)R(s)=\frac{1}{1+G(s)}R(s)=\frac{1}{1+\dfrac{K(0.5s+1)}{s(s+1)(2s+1)}}\frac{1}{s^2}$$

$$=\frac{s(s+1)(2s+1)}{s(s+1)(2s+1)+K(0.5s+1)}\frac{1}{s^2}$$

Step3.To calculate the steady state error, we use final value theorem

$$e_{ss}=\lim_{s\to 0}sE(s)=\lim_{s\to 0}s\frac{s(s+1)(2s+1)}{s(s+1)(2s+1)+K(0.5s+1)}\frac{1}{s^2}=\frac{1}{K}$$

计算结果表明，稳态误差的大小，与系统的开环增益K有关。系统的开环增益越大，稳态误差越小。由此看出，**稳态精度与稳定性**对K的要求是矛盾的。

只有稳定的系统，才可以计算稳态误差

3.3 Steady state error

Loop transfer function (n^{th} order system)

$$G(s)=\frac{K\prod_{i=1}^{m}(T_i s+1)}{s^N\prod_{j=1}^{n-N}(\tau_j s+1)} \qquad (3.41)$$

***N**: The number of integration **N** reflect the **tracking ability** of the system.*

$N=0,1,\ldots$ Type zero system, Type one system respectively

N: 开环传递函数$G(s)$中**零极点**的重数，即串联的积分环节的个数，称为系统的类型(或无差阶数)

$N=0,1,2,\ldots$ 分别称为0型,1型,2型,…系统

3.3 Steady state error

以**静态误差系数**给出典型输入下的系统的**稳态误差**:

1) Step Input: $R(s)=\dfrac{A}{s}$

$$e_{ss}=\lim_{s\to 0} sE(s)=\lim_{s\to 0}\frac{s}{1+G(s)}\frac{A}{s}=\lim_{s\to 0}\frac{A}{1+G(s)} \qquad (3.42)$$

Position error constant

静态位置误差系数

$$K_p \overset{def}{=} \lim_{s\to 0} G(s)=\lim_{s\to 0}\frac{K}{s^N} \qquad (3.43)$$

Steady state error

$$e_{ss}=\frac{A}{1+K_p} \qquad (3.44)$$

Type zero system: $K_p=K \qquad e_{ss}=\dfrac{A}{1+K}$

Type one type two system: $K_p=\infty \qquad e_{ss}=0$

3.3 Steady state error

2) Ramp input: $R(s)=\dfrac{A}{s^2}$

$$e_{ss}=\lim_{s\to 0}sE(s)=\lim_{s\to 0}\frac{A}{s[1+G(s)]}=\lim_{s\to 0}\frac{A}{sG(s)} \qquad (3.45)$$

Velocity error constant
静态速度误差系数

$$K_v \overset{def}{=} \lim_{s\to 0}sG(s)=\lim_{s\to 0}\frac{K}{s^{N-1}} \qquad (3.46)$$

Steady state error

$$e_{ss}=\frac{A}{K_v} \qquad (3.47)$$

Type zero system: $K_v=0 \qquad e_{ss}=\infty$

Type one system: $K_v=K \qquad e_{ss}=\dfrac{A}{K}$

Type two system: $K_v=\infty \qquad e_{ss}=0$

3.3 Steady state error

3) Acceleration input: $R(s)=\dfrac{A}{s^3}$

$$e_{ss}=\lim_{s\to 0} sE(s)=\lim_{s\to 0}\frac{A}{s^2[1+G(s)]}=\lim_{s\to 0}\frac{A}{s^2G(s)} \tag{3.48}$$

Acceleration error constant
静态加速度误差系数

$$K_a \overset{def}{=} \lim_{s\to 0} s^2G(s)=\lim_{s\to 0}\frac{K}{s^{N-2}} \tag{3.49}$$

Steady state error

$$e_{ss}=\frac{A}{K_a} \tag{3.50}$$

Type zero, type one system: $K_a=0$ $\quad e_{ss}=\infty$

Type two system: $K_a=K$ $\quad e_{ss}=\dfrac{A}{K}$

3.3 Steady state error

- Summary of steady state error

系统的型(无差阶数)	误差系数			稳态误差		
	K_p	K_v	K_a	阶跃输入 $e_{ss}=\frac{A}{1+K_p}$	斜坡输入 $e_{ss}=\frac{A}{K_v}$	抛物线输入 $e_{ss}=\frac{A}{K_a}$
0 型	K	0	0	$A/(1+K)$	∞	∞
1 型	∞	K	0	0	A/K	∞
2 型	∞	∞	K	0	0	A/K

3.3 Steady state error

[E3.3]Try to get the **Steady State Error** for the unit feedback control system, where the transfer function of $G_1(s)=K_1+K_2/s$ and $G_2(s)=K/(\tau s+1)$.

(1) Consider the first situation when input is a step signal and the gain $K_2=0$.

$$\text{As } K_2=0 \text{ and } r(t)=u(t) \quad \Rightarrow G_1(s)=K_1 \text{ and } R(s)=\frac{A}{s}$$

$$G_o(s)=\frac{K_1K}{\tau s+1}$$

The system is stable. For $\triangle(s)=\tau s+1+K_1K$

$$e_{ss}=\frac{A}{1+K_p} \quad where K_p=K_1K$$

3.3 Steady state error

[E3.3] Try to get the **Steady State Error** for the unit feedback control system, where the transfer function of $G_1(s)=K_1+K_2/s$ and $G_2(s)=K/(\tau s+1)$.

(2) Consider the second situation when input is a step signal and the gain $K_2>0$.

$$\text{As } K_2 > 0 \text{ and } r(t) = u(t) \quad \Rightarrow G_1(s) = K_1 + K_2 / s \text{ and } R(s) = \frac{A}{s}$$

$$G_o(s) = \frac{(K_1 s + K_2)K}{s(\tau s + 1)}$$

The system is stable. For $\triangle(s)=\tau s^2+(1+K_1K)s+K_2K$

$$e_{ss} = \frac{A}{1+K_p} = 0, \quad where \quad K_p = \infty$$

3.3 Steady state error

[E3.3] Try to get the **Steady State Error** for the unit feedback control system, where the transfer function of $G_1(s)=K_1+K_2/s$ and $G_2(s)=K/(\tau s+1)$.

(3) Consider the third situation when input is a ramp signal and the gain $K_2>0$.

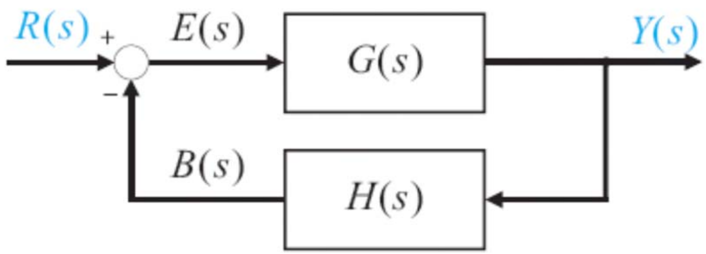
$$\text{As } K_2>0 \text{ and } r(t)=u(t) \quad \Rightarrow G_1(s)=K_1+K_2/s \text{ and } R(s)=\frac{A}{s^2}$$

$$G_o(s)=\frac{(K_1 s+K_2)K}{s(\tau s+1)} \qquad K_v=\lim_{s\to 0} sG_o(s)=K_2K$$

$$e_{ss}=\frac{A}{K_V}, \quad where \quad K_v=K_2K$$

3.3 Steady state error

3.3.3 Steady state error of nonunity feedback system



1） 折算到输入端（偏差）

$$e(t) = r(t) - b(t)$$

$$E(s) = R(s) - B(s) = R(s) - H(s)\frac{G(s)R(s)}{1+G(s)H(s)}$$

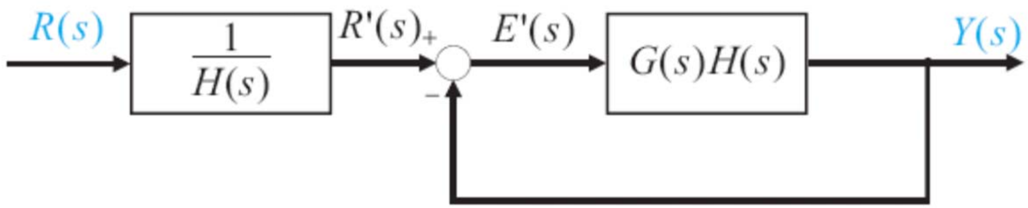
$$= \frac{1}{1+G(s)H(s)}R(s) = \Phi_e(s)R(s) \qquad (3.51)$$

$$\Phi_e(s) = \frac{1}{1+G(s)H(s)} = \frac{1}{1+G_k(s)}$$ 误差传递函数

$$G_k(s) = G(s)H(s)$$ 开环传递函数

3.3 Steady state error

2) 折算到输出端（误差）



$$R'(s) = \frac{1}{H(s)} R(s)$$

$$e'(t) = r'(t) - y(t)$$

$$E'(s) = R'(s) - Y(s) = R'(s) - \frac{G(s)H(s)}{1+G(s)H(s)} R'(s)$$

$$= \frac{1}{1+G(s)H(s)} R'(s) = \Phi_e(s) R'(s) = \frac{\Phi_e(s) R(s)}{H(s)} = \frac{E(s)}{H(s)} \qquad (3.52)$$

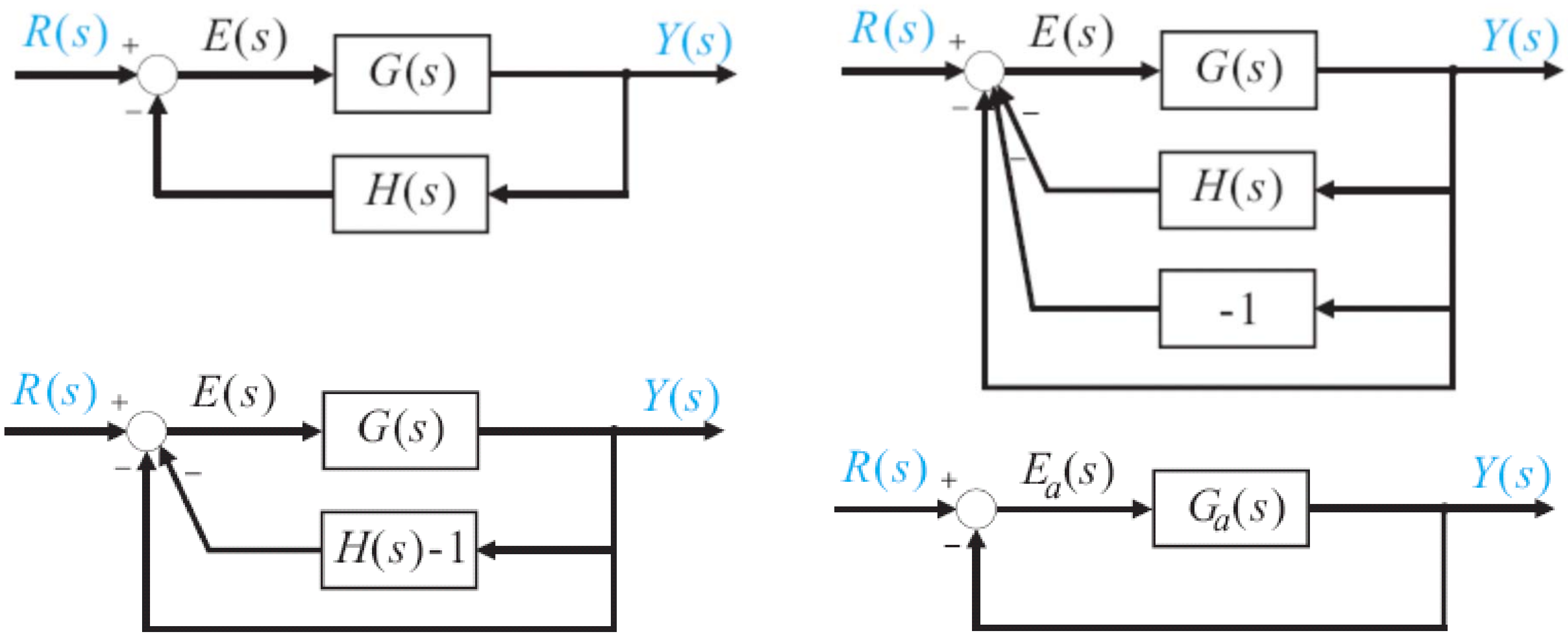
误差$e'(t)$的定义，物理意义明确；

偏差$e(t)$的定义，结构图中有对应的量,便于理论分析；

一般用$e(t)$进行误差分析；

3.3 Steady state error

非单位反馈系统可化为等效单位反馈系统讨论



$$G_a(s) = \frac{G(s)}{1 + G(s)H(s) - G(s)}$$

转换为讨论 $e_a(t) = r(t) - y(t)$

返回

Summary

In this chapter, we mainly focus on:

- Dynamic analysis of control system in time domain.

  控制系统时间域的运动分析；

- Consider the definition and measurement of the performance of a control system with step input.

  通过单位阶跃响应讨论控制系统暂态响应的性能指标；

- Performance specifications of second order system.

  二阶系统的暂态响应性能指标：

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \qquad \sigma\% = \exp\left(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)\times 100\%$$

$$T_s(2\%) = \frac{4}{\zeta\omega_n} \qquad T_s(5\%) = \frac{3}{\zeta\omega_n}$$

$$T_r = \frac{\pi-\varphi}{\omega_d} = \frac{\pi-\varphi}{\omega_n\sqrt{1-\zeta^2}} \qquad \varphi = tg^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$$

小结

Note: Percent overshoot is only dependent of ζ

$$\begin{cases}\zeta \to \sigma\% \\ \zeta,\omega_n \to T_s,T_r\end{cases},\ \left.\begin{cases}\sigma\% \to \zeta \\ T_s,T_r\end{cases}\right\} \to \omega_n$$

- Effects of a third pole and zero on the second-order system response.

  在二阶系统暂态响应分析的基础上增加零点和极点的影响

- Steady-state error analysis, steady-state error constant and steady-state error calculation.

  稳态误差分析，静态误差系数及稳态误差的计算；

返回

- 课后阅读第六章

“线性反馈控制系统的稳定性”