

$$G(s)H(s) = \frac{1}{(1 + 0.5s)(1 + 2s)}$$

$$G(j\omega)H(j\omega) = \frac{1}{(1 + \frac{j\omega}{2})(1 + \frac{j\omega}{0.5})}$$

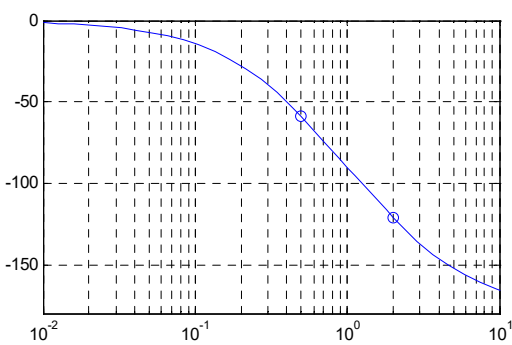
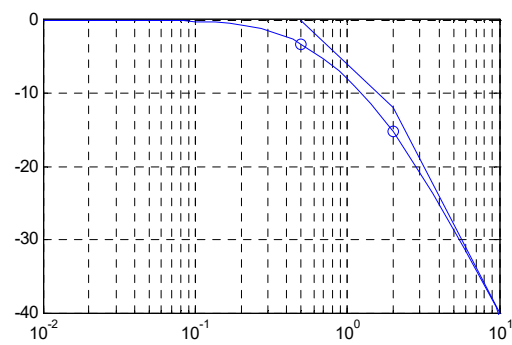
$$\omega_1 = 0.5 \text{ rad/sec} \quad L_{0.5} = -20 \log \sqrt{1 + \left(\frac{\omega}{0.5}\right)^2}$$

$$\omega_2 = 2 \text{ rad/sec} \quad L_2 = -20 \log \sqrt{1 + \left(\frac{\omega}{2}\right)^2}$$

	0.5	2
L0.5	-3.0103	-12.3045
L2	-0.2633	-3.0103
dB	-3.2736	-15.3148

$$\varphi(\omega) = -tg^{-1}\left(\frac{\omega}{0.5}\right) - tg^{-1}\left(\frac{\omega}{2}\right)$$

0.1	0.2	0.5	1	2	5
-14.17	-27.51	-59.04	-90	-120.96	-152.49



$$G(s)H(s) = \frac{(1 + 0.5s)}{s^2}$$

$$G(j\omega)H(j\omega) = \frac{(1 + \frac{j\omega}{2})}{j\omega^2}$$

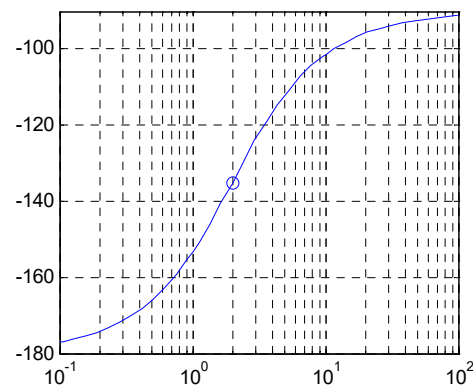
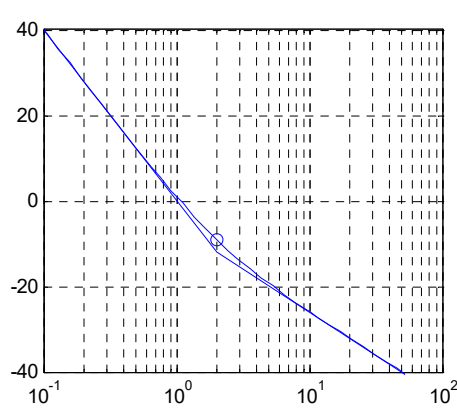
$$\omega_1 = 2 \text{ rad/sec} \quad L_2 = -20 \log \sqrt{1 + \left(\frac{\omega}{2}\right)^2}$$

$$L_I = -20 \log \omega \quad L_I = -20 \log \omega$$

	2
L2	3.0103
LI	-6.02
LI	-6.02
dB	9.03

$$\varphi(\omega) = tg^{-1}\left(\frac{\omega}{2}\right) - 90 - 90$$

2
-135



$$G(s)H(s) = \frac{s+10}{s^2+6s+10}$$

$$G(j\omega)H(j\omega) = \frac{(1+\frac{j\omega}{10})}{\left(1-\left(\frac{\omega}{\sqrt{10}}\right)^2\right) + \frac{j6\omega}{10}}$$

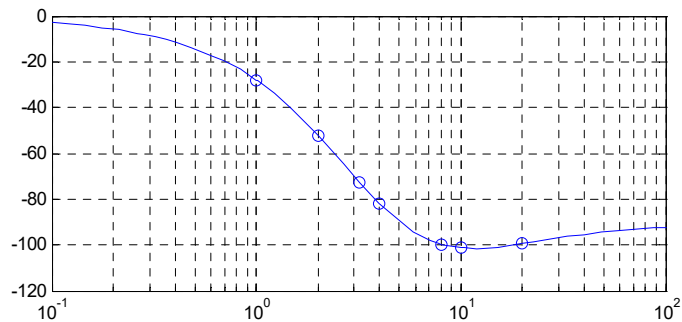
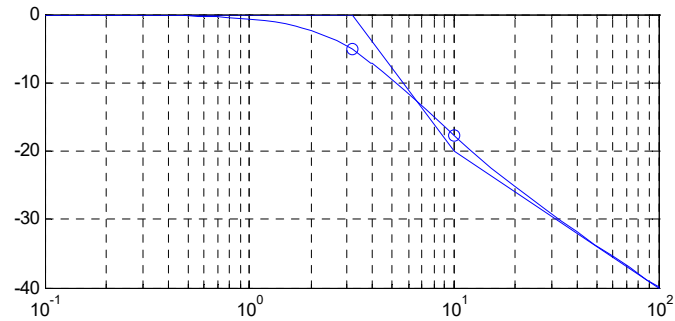
$$\omega_1 = 10 \text{ rad/sec} \quad L_{10} = 20 \log \sqrt{1+\left(\frac{\omega}{10}\right)^2}$$

$$\omega_2 = \sqrt{10} \text{ rad/sec} \quad \zeta = \frac{3}{\sqrt{10}}$$

$$L_2 = -20 \log \sqrt{\left[1-\left(\frac{\omega}{\sqrt{10}}\right)^2\right]^2 + \left(\frac{6\omega}{10}\right)^2}$$

	2	3.1623	4	10
L _{3.1623}	-2.5527	-5.563	-7.8675	-20.682
L ₁₀	0.1703	0.4139	0.6446	3.0103
dB	-2.3824	-5.1491	-7.2229	-17.672

$\omega < \sqrt{10}$	$\varphi(\omega) = tg^{-1}0.1\omega - tg^{-1}\frac{6\omega/10}{1-\omega^2/10}$							
ω	0.2	0.5	1	2	3	3.1623		
$\varphi(\omega)$	-5.72	-14.24	-27.98	-52.13	-70.12	-72.45		
$\omega > \sqrt{10}$	$\varphi(\omega) = tg^{-1}0.1\omega - 180^\circ + tg^{-1}\frac{6\omega/10}{1-\omega^2/10}$							
ω	4	5	10	20	30	50	100	
$\varphi(\omega)$	-82.23	-90	-101.3	-99.46	-97	-94.44	-92.27	



$$G(s)H(s) = \frac{30(s+8)}{s(s+2)(s+4)}$$

$$G(j\omega)H(j\omega) = \frac{30(j\frac{\omega}{8}+1)}{j\omega(j\frac{\omega}{2}+1)(j\frac{\omega}{4}+1)}$$

$$\omega_1 = 2 \text{ rad/sec} \quad L_2 = -20 \log \sqrt{1 + \left(\frac{\omega}{2}\right)^2}$$

$$\omega_2 = 4 \text{ rad/sec} \quad L_4 = -20 \log \sqrt{1 + \left(\frac{\omega}{4}\right)^2}$$

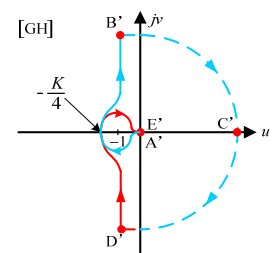
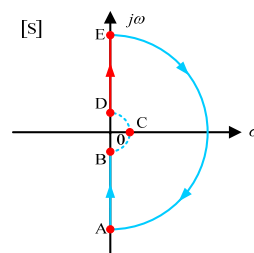
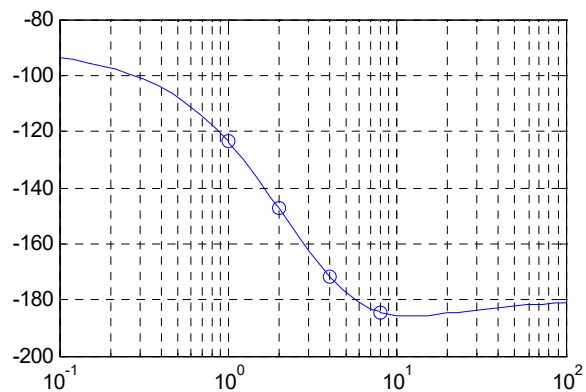
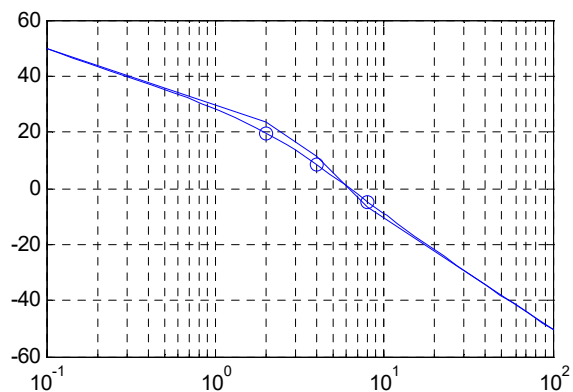
$$\omega_3 = 8 \text{ rad/sec} \quad L_8 = 20 \log \sqrt{1 + \left(\frac{\omega}{8}\right)^2}$$

$$\text{积分环节} \quad L_1 = 20 \log \omega$$

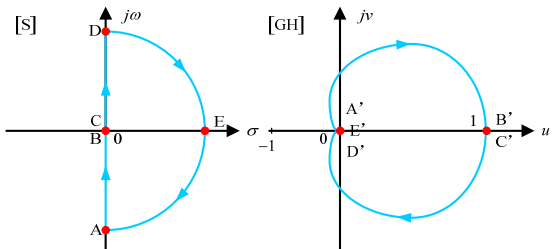
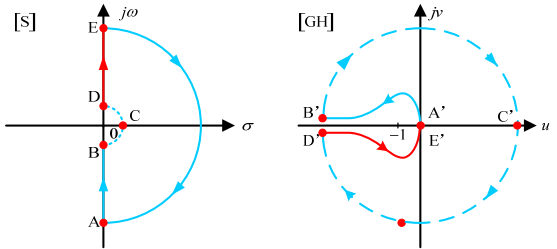
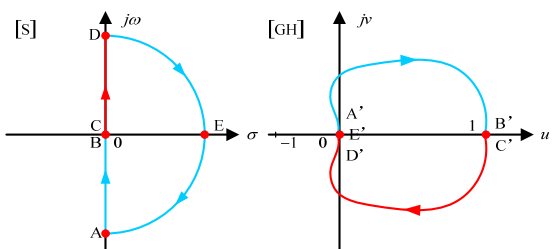
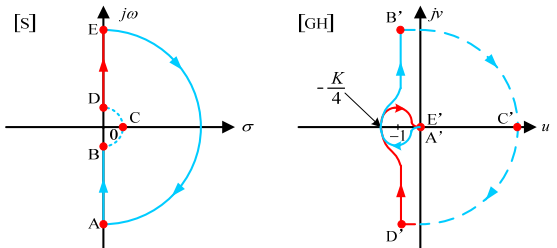
	2	4	8
L2	-3.01	-6.99	-12.3
L4	-0.97	-3.01	-6.99
L8	0.26	0.97	3.01
L1	-6.02	-12.04	-18.06
	-9.74	-21.07	-34.34
LK	29.54	29.54	29.54
	19.80	8.47	-4.8

$$\varphi(\omega) = \text{tg}^{-1}\left(\frac{\omega}{8}\right) - \text{tg}^{-1}\left(\frac{\omega}{4}\right) - \text{tg}^{-1}\left(\frac{\omega}{2}\right) - 90^\circ$$

	$\varphi(\omega) = \text{tg}^{-1}\left(\frac{\omega}{8}\right) - \text{tg}^{-1}\left(\frac{\omega}{4}\right) - \text{tg}^{-1}\left(\frac{\omega}{2}\right) - 90^\circ$									
ω	0.2	0.5	1	2	5	8	10	20	50	100
$\varphi(\omega)$	-97.1409	-107.58	-123.48	-147.53	-177.53	-184.4	-185.55	-184.78	-182.23	-181.14



习题 2

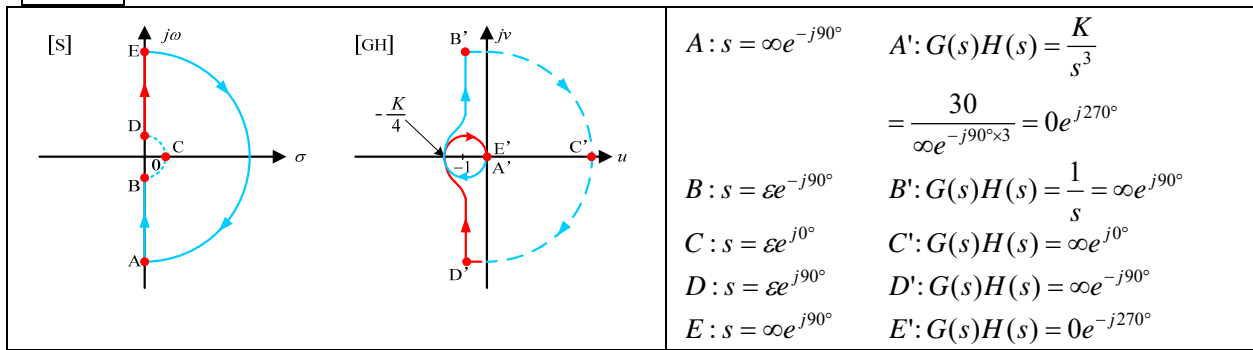
 <p>P=0 N=0 Z=N+P=0 系统稳定</p>	$A: s = \infty e^{-j90^\circ} \quad A': G(s)H(s) = \frac{K}{s^2}$ $= \frac{K}{\infty e^{-j90^\circ \times 2}} = 0 e^{j180^\circ}$ $B: s = 0 e^{-j90^\circ} \quad B': G(s)H(s) = e^{j0^\circ}$ $C: s = 0 e^{j90^\circ} \quad C': G(s)H(s) = e^{j0^\circ}$ $D: s = \infty e^{j90^\circ} \quad D': G(s)H(s) = 0 e^{-j180^\circ}$ $E: s = \infty e^{j0^\circ} \quad E': G(s)H(s) = 0 e^{j0^\circ}$
 <p>P=0 N=0 Z=N+P=0 系统稳定</p>	$A: s = \infty e^{-j90^\circ} \quad A': G(s)H(s) = \frac{0.5}{s}$ $= \frac{0.5}{\infty e^{-j90^\circ}} = 0 e^{j90^\circ}$ $B: s = \varepsilon e^{-j90^\circ} \quad B': G(s)H(s) = \frac{1}{s^2} = \infty e^{j180^\circ}$ $C: s = \varepsilon e^{j0^\circ} \quad C': G(s)H(s) = \infty e^{j0^\circ}$ $D: s = \varepsilon e^{j90^\circ} \quad D': G(s)H(s) = \infty e^{-j180^\circ}$ $E: s = \infty e^{j90^\circ} \quad E': G(s)H(s) = 0 e^{-j90^\circ}$
 <p>P=0 N=0 Z=N+P=0 系统稳定</p>	$A: s = \infty e^{-j90^\circ} \quad A': G(s)H(s) = \frac{1}{s}$ $= \frac{1}{\infty e^{-j90^\circ}} = 0 e^{j90^\circ}$ $B: s = 0 e^{-j90^\circ} \quad B': G(s)H(s) = 1$ $C: s = 0 e^{j90^\circ} \quad C': G(s)H(s) = 1$ $D: s = \infty e^{j90^\circ} \quad D': G(s)H(s) = 0 e^{-j90^\circ}$ $E: s = \infty e^{j0^\circ} \quad E': G(s)H(s) = 0 e^{j0^\circ}$
 <p>P=0 N=2 Z=N+P=2 系统不稳定</p>	$A: s = \infty e^{-j90^\circ} \quad A': G(s)H(s) = \frac{30}{s^2}$ $= \frac{30}{\infty e^{-j2 \times 90^\circ}} = 0 e^{j180^\circ}$ $B: s = \varepsilon e^{-j90^\circ} \quad B': G(s)H(s) = \frac{1}{s} = \infty e^{j90^\circ}$ $C: s = \varepsilon e^{j0^\circ} \quad C': G(s)H(s) = \infty e^{j0^\circ}$ $D: s = \varepsilon e^{j90^\circ} \quad D': G(s)H(s) = \infty e^{-j90^\circ}$ $E: s = \infty e^{j90^\circ} \quad E': G(s)H(s) = 0 e^{-j180^\circ}$

$$G(s) = \frac{30 * (s+8)}{s(s+2)(s+4)} \Rightarrow G(j\omega) = \frac{30 * (j\omega+8)}{j\omega(j\omega+2)(j\omega+4)}$$

$$\text{Im}(G(j\omega)) = \frac{-30(2\omega^3 + 64\omega)}{\omega^6 + 20\omega^4 + 36\omega^2} = 0 \Rightarrow \omega^2 = 32$$

$$\text{Re}(G(j\omega)) = \frac{-30\omega^2(40 + \omega^2)}{\omega^6 + 20\omega^4 + 36\omega^2} \Big|_{\omega^2=32} = -1.25$$

习题 3



与实轴的交点:

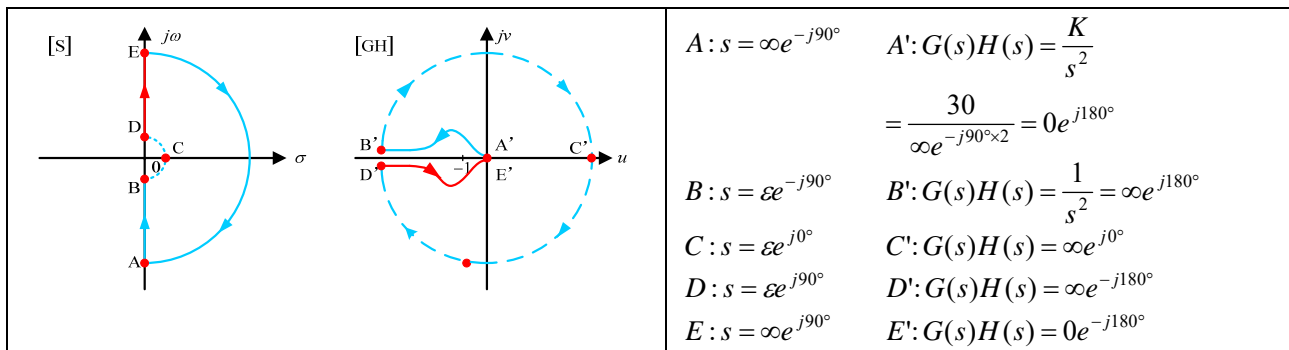
$$G(j\omega)H(j\omega) = \frac{K}{j\omega(-\omega^2 + j\omega + 4)} = \frac{K}{-\omega^2 + j\omega(4 - \omega^2)} = \frac{K[-\omega^2 - j\omega(4 - \omega^2)]}{\omega^4 - \omega^2(4 - \omega^2)^2}$$

$$\text{Im}[G(j\omega)H(j\omega)] = 0 \Rightarrow 4 - \omega^2 = 0 \Rightarrow \omega = \pm 2$$

$$\text{Re}[G(j\omega)H(j\omega)] = \frac{-K\omega^2}{\omega^4 - \omega^2(4 - \omega^2)^2} \Big|_{\omega=2} = -\frac{K}{4}$$

当 $K \geq 4$ 时, $P=0$, $N=2$, $Z=N+P=2$ 系统不稳定

当 $K < 4$ 时, $P=0$, $N=0$, $Z=N+P=0$ 系统稳定



显然, Nyquist 曲线不包围(-1,0)点, 因此当系统 $K > 0$ 时, 系统稳定

习题 4

(1) 如果 $P=0$, $N=2$, $Z=N+P=2$, 系统不稳定, 闭环在右半平面中有 2 个特征根

(2) 如果 $P=0$, $N=0$, $Z=N+P=0$, 系统 稳定, 闭环在右半平面中有 0 个特征根

习题 5

(1)

$$\varphi(\omega) = -90^\circ - \text{tg}^{-1}(\omega) - \text{tg}^{-1}(\omega/2) = 180^\circ$$

$$\Rightarrow \text{tg}^{-1}(\omega) + \text{tg}^{-1}(\omega/2) = -90^\circ \Rightarrow \text{tg}^{-1} \frac{\omega + \omega/2}{1 - \omega^2/2} = -90^\circ \Rightarrow 1 - \omega^2/2 = 0 \Rightarrow \omega = \pm\sqrt{2}$$

$$|g(j\omega)| = \frac{2}{|j\omega||j\omega + 1| \left| \frac{j\omega}{2} + 1 \right|} \Big|_{\omega=\pm\sqrt{2}} = \frac{2}{\sqrt{2}\sqrt{2+1} \sqrt{\frac{1}{2}+1}} = \frac{2}{3}$$

$$A(\omega) = -20 \log |g(j\omega)| = 3.5218 \text{ dB}$$

(2)

$$A(\omega) = -20 \log |g(j\omega)| = 16 \text{ dB} \Rightarrow -20 \log \frac{K/2}{|j\omega||j\omega+1| \left| \frac{j\omega}{2} + 1 \right|} = 16 \text{ dB}$$

$$\Rightarrow \log \frac{K/2}{|j\omega||j\omega+1| \left| \frac{j\omega}{2} + 1 \right|} = -0.8 \Rightarrow \frac{K/2}{|j\omega||j\omega+1| \left| \frac{j\omega}{2} + 1 \right|} = 10^{-0.8}$$

$$\Rightarrow K = 0.9509$$

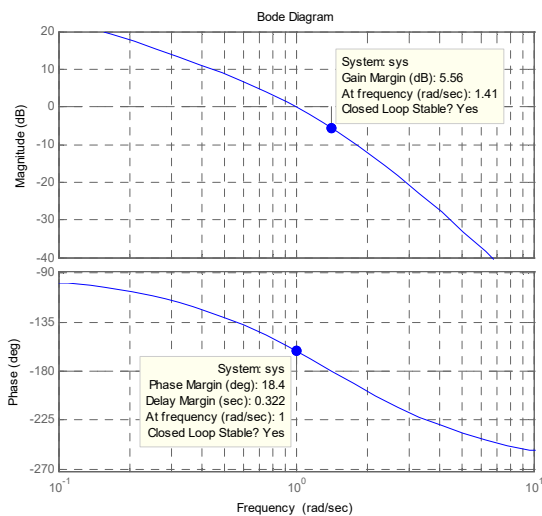
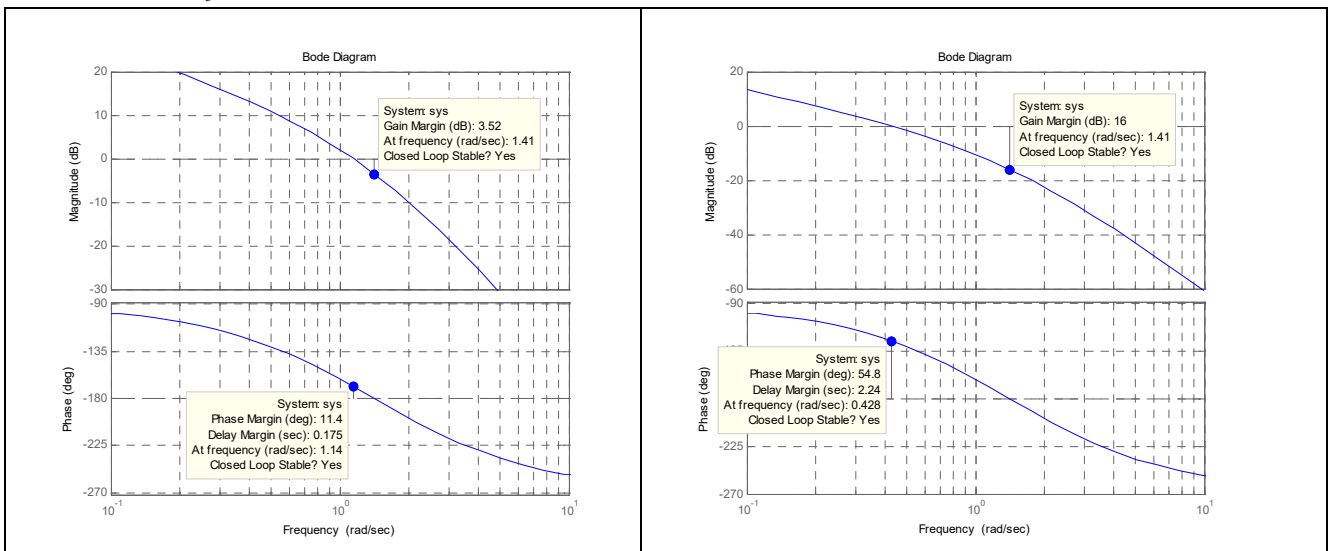
(3)

$$|g(j\omega_c)| = \frac{\sqrt{10}/2}{|j\omega_c||j\omega_c+1| \left| \frac{j\omega_c}{2} + 1 \right|} = 1 \Rightarrow \frac{\sqrt{10}}{\sqrt{\omega_c^2(1+\omega_c^2)(\omega_c^2+4)}} = 1 \Rightarrow \omega_c^6 + 5\omega_c^4 + 4\omega_c^2 = 10$$

$$\Rightarrow \omega_c^2 = 1 \Rightarrow \omega_c = 1 \text{ rad/sec}$$

$$\varphi(\omega_c) = -90^\circ - \tan^{-1}(\omega_c) - \tan^{-1}(\omega_c/2) \Big|_{\omega_c=1} = -161.57^\circ$$

$$\gamma = 180^\circ + \varphi(\omega_c) = 18.43^\circ$$



方法一：时域分析法得特征方程为

$$1 + \frac{50 \times \frac{30}{s^2(0.1s+1)}}{1 + \frac{120s}{s^2(0.1s+1)}} = 0 \Rightarrow 0.1s^3 + s^2 + 120s + 1500 = 0$$

$\because 120 < 1500 \times 0.1 \therefore$ 系统不稳定。

方法二：采用频域分析法计算。开环传递函数为

$$G_k(s) = \frac{1500(0.08s+1)}{s^2(0.1s+1)}$$

计算幅值穿越频率

$$L(\omega) = \begin{cases} 20 \lg \frac{1500}{\omega^2} & (\omega \leq 10) \\ 20 \lg \frac{1500}{\omega^2 \times 0.1\omega} & (10 \leq \omega \leq 12.5) \\ 20 \lg \frac{1500 \times 0.08\omega}{\omega^2 \times 0.1\omega} & (\omega \geq 12.5) \end{cases} \Rightarrow 20 \lg \frac{1500 \times 0.08\omega_c}{\omega_c^2 \times 0.1\omega_c} = 0 \Rightarrow \omega_c = 34.64$$

计算相角裕量

$$\gamma = 180^\circ - 2 \times 90^\circ - \lg^{-1} 0.1 \times 34.64 - \lg^{-1} 0.08 \times 34.64 = -144.1^\circ < 0$$

结论：系统不稳定。

