

Image and video processing: From Mars to Hollywood with a stop at the hospital.

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Week #3

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1. Q: Consider that the maximal value of an image I_1 is M and its minimal is m (with M different than m). An intensity transform that maps the image I_1 onto I_2 such that the maximal value of I_2 is L and the minimal is 0 is:

- $(I_1 - m / M - m) \cdot L$
- $(I_1 / M - m) \cdot L$
- $(L / M - m) \cdot I_1$
- Such transform does not exist.

A: The correct answer is $(I_1 - m / M - m) \cdot L$. We see that if $I_1 = M$ then the fraction becomes 1 and the value is mapped to L . If $I_1 = m$, then the fraction is zero and so is the mapped value. All other values for I_1 will lead to fractions in the interval $[0, 1]$ and as such the values of I_2 are in the interval $[0, L]$.

2. Q: Why global discrete histogram equalization does not, in general, yield a flat (uniform) histogram?
- Actually, global discrete histogram equalization always yields flat histograms by definition.
 - Because images are in color.
 - Because in global histogram equalization, all pixels with the same value are mapped to the same value.
 - Because the histogram equalization mathematical derivation doesn't exist for discrete signals.

A: Since we cannot split pixels of the same value to be mapped to different ones, complete flat histograms are not achievable in general. All pixels having the same value in the original image are mapped to the same value after global equalization.

3. Q: Discrete histogram equalization is an invertible operation, meaning we can recover the original image from the equalized one by inverting the operation, since:

- Actually, histogram equalization is in general non-invertible.
- Images have unique histograms
- There is a unique histogram equalization formula per image.
- Pixels with different values are mapped to pixels with different values.

A: In global histogram equalization, it can happen that pixels with different values in the original image are mapped to the same value, and therefore the operation is not invertible.

4. Q: Given an image with only 2 pixels and 3 possible values for each one. Determine the number of possible different images and the number of possible different histograms

- 9 images and 9 histograms.
- 9 images and 6 histograms
- 6 images and 6 histograms.
- 6 images and 9 histograms.

A: Each pixel can have any of the 3 possible values, and therefore there are $3 \times 3 = 9$ possible images. On the other hand, since the order does not matter for the histograms, there are only 6 possible different histograms, e.g., considering the possible values as 0,1,2, the images 01 and 10 have the same histogram.

5. Q: Which integer x number minimizes $\sum_{i=1}^{99} |x - i|$

- 49
- 51
- 50.5
- 50

A: 50. Remember that the median minimizes such functional.

6. Q: Applying a 3×3 averaging filter to an image a large (infinity) number of times is

- Equivalent to replacing all the pixel values by 0.
- The same as applying it a single time.
- Equivalent to replacing all the pixel values by the average of the values in the original image.
- The same as applying a median filter.

A: As discussed in the video, this leads to a constant value. It is also equivalent to applying an averaging operation with an increasing kernel, until the kernel has “infinity” support and then we obtain the average of all pixel values. As an extra exercise, experiment this writing a simple program in your computer.

7. Q: Which integer x number minimizes $\sum_{i=3}^1 |x-i|^2$?

- 1
- 2
- 3
- 4

A: Remember the average minimizes this function.

8. Q: Consider a row of pixels with values 1, 1, 1, 1, 5, 1, 1, 1, 1. When we apply an average and a median filter of size 3, the output values of the 5th pixel starting from the left are

- 7/3 and 1 respectively
- 1 for both operations
- 5 and 1 respectively
- 9/3 and 1 respectively

A: 7/3 and 1 respectively. And also note how the median filter gets rid of this pointwise noise.

9. Q: Consider a row of pixels with values 1, 1, 1, 1, 5, 5, 5, 5, 5. When we apply an average and a median filter of size 3, the output values of the 5th pixel starting from the left are

- 11/3 and 5 respectively.
- 5 for both operations.
- 1 and 5 respectively.
- 7/3 and 1 respectively.

A: 11/3 and 5 respectively. And also note how the median filter preserves the edge but the average does not.

10. Q: Consider an image denoising operation T , and write $T(I)$ the application of T to the image I .

- If T is the non-local means algorithm, then $T(T(I))=T(I)$.
- If T is the non-local means algorithm, then $T_{\infty}(I)=\text{average}(I)$, where T_{∞} stands for applying T an infinite number of times and $\text{average}(I)$ is the pixel average of the image I .

- If T is the non-local means algorithm, then there is no image for which $T(I)=I$.
- None of the above statements is correct.

A: None of the above statements is correct. Note that the first option is not correct because even the neighborhood will be different after a single non-local means application. The second options is not correct, that is a property of the standard average filter but not necessarily of non-local means. The third option is not correct, e.g., the equality is valid for any constant image.