

Linear Algebra

1. Scalars

A scalar is a single number (0-dimensional quantity).

Operations:

- Addition: $a + b$
- Multiplication: $a * b$
- Division: a / b

Scalars are used as elements of vectors, matrices, and tensors.

2. Vectors

A vector is a **1D array** of numbers (magnitude and direction).

Basic Operations:

$a = [1, 2, 3, 4]$

$b = [1, 2, 3, 4]$

- Addition : $a + b$
- Subtraction : $a - b$
- Scalar Multiplication : $2 * a$
- Dot product : $a * b$

3. Matrices

A matrix is a **2D array** of numbers, arranged in rows and columns.

$A = \text{np.array}([[1, 2], [3, 4]])$

$B = \text{np.array}([[5, 6], [7, 8]])$

Basic Operations:

Matrix Addition: $A + B$

Matrix Multiplication: $A*B$ # `np.dot(A, B)`

Transpose: $A.T$

Determinant: #`np.linalg.det(A)`

Inverse: `np.linalg.inv(A)`

4. Tensors

A tensor is a **generalization of vectors and matrices to more dimensions**:

5. Gradients

The gradient is a **vector of partial derivatives** of a function with respect to its variables.

Numerical Gradient Example:

For a function $f(x)=x^2$

Linear Algebra in ML/AI

- **Scalars:** Learning rate, loss values.
- **Vectors:** Features of a single data point.
- **Matrices:** Dataset representation (rows = samples, columns = features).
- **Tensors:** Image datasets (e.g., $28 \times 28 \times 3$ RGB images).
- **Gradients:** Used in optimization (e.g., gradient descent).