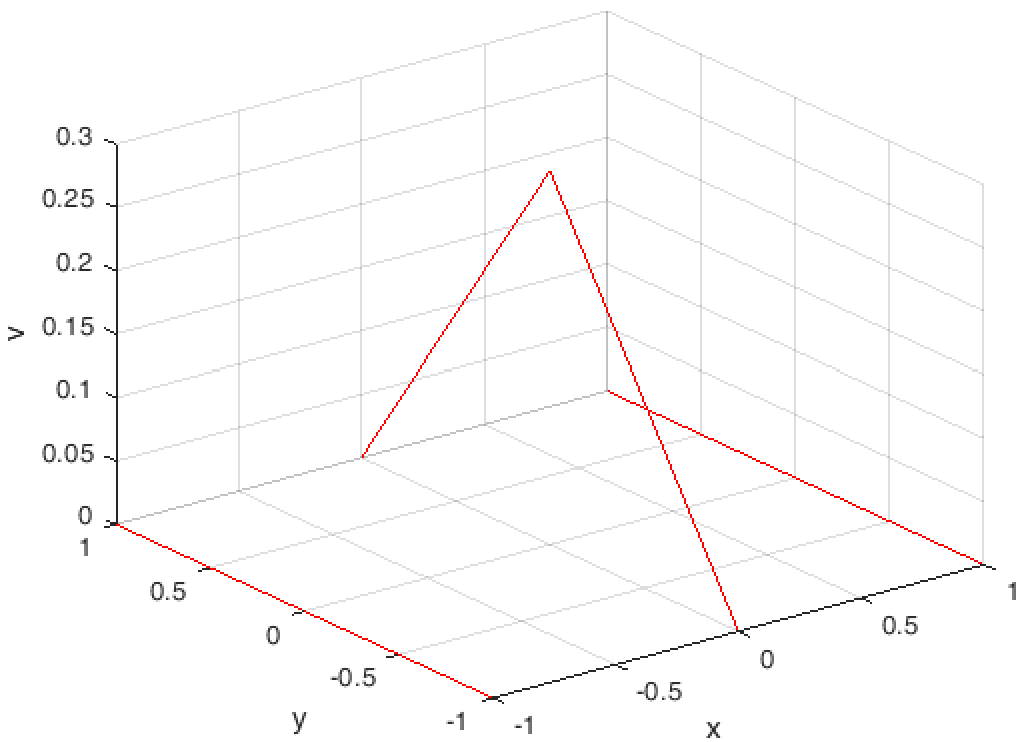
```
%{
    Using the Rayleigh-Ritz method
    ------------------------------

   Solve Poisson's equation in a square -1<=x<=1, -1<=y<=1, subject
   to the homogeneous boundary conditions V(x,+-1)=0=V(+-1,y).

   Solve:

    Due to the symmetry of the problem, we choose the basis function
    as
        Umn=(1-x^2)(1-y^2)(x^(2m)y^(2n)+x^(2n)y^(2m)), m,n=0,1,2,...

    Hence,
        ? = (1-x^2)(1-y^2)[a1+a2(x^2+y^2)+a3x^2y^2+a4(x^4+y^4)+...]
Case 1: When m=n=0, we obtain the first approximation (N=1) as
                        ?=a1u1
    where u1=(1-x^2)(1-y^2).

        A11={Lu1,u1}=-256/45,
        B1={g,u1}=-16?0/9
    Hence,
        -256a1/45=-16?0/9 ==> a1=5/16?0
    and
        ? = 5?0(1-x^2)(1-y^2)/16

Case 2: When m = n = 1, we obtain the second-order approximation (N =
 2) as
        ? = a1 u1 + a2 u2

    where u1 = (1 ? x2)(1 ? y2), u2 = (1 ? x2)(1 ? y2)(x^2 + y^2). A11
 and B1
    are the same as in case 1.
        A12 = A21 ={Lu1, u2} = -1024/525,
        A22 = {Lu2, u2} = -11.264/4725,
        B2 = {g, u2} = -32/45?o
    Hence,

    ? = (1-x^2)(1-y^2)(0.2922+0.0592(x^2+y^2))?o

Case 3: When m = n = 2, we obtain the third-order approximation (N=3)
 as
        ? = a1 u1 + a2 u2 + a3 u3

    where u1 = (1 ? x2)(1 ? y2),
          u2 = (1 ? x2)(1 ? y2)(x^2 + y^2),
          u3 = (1 ? x2)(1 ? y2)(x^2 + y^2 + x^2y^2).

        A11={Lu1,u1}=-256/45,
        B1={g,u1}=-16?0/9,

        A12 = A21 ={Lu1, u2} = -1024/525,
```

```
        A22 = {Lu2, u2} = -11.264/4725,
        B2 = {g, u2} = -32/45?o

        A13 = A31 ={Lu1, u3} = ?,
        A33 = {Lu3, u3} = ?,
        B3 = {g, u3} = ?
%}
close all ; clear all ; clc ;
format short;
syms x y;

rho = 1;
N=3;
B = zeros(N,1);
A = zeros(N,N);

u(1,:) = (1-x.^2).*(1-y.^2);
u(2,:) = (1-x.^2).*(1-y.^2).*(x.^2+y.^2);
u(3,:) = (1-x.^2).*(1-y.^2).*(x.^2+y.^2 + x.^2.*y.^2);

for i=1:N
    firstInt = int(-u(i)*rho,'x',0,1);
    secondInt = int(firstInt,'y',0,1);
    B(i) = secondInt;
    for j=i:N
        dfx = diff(u(i),x,length(u(i)));
        dfxx = diff(dfx,x,length(u(i)));

        dfy = diff(u(i),y,length(u(i)));
        dfyy = diff(dfy,y,length(u(i)));
        lu = (dfxx+dfyy);

        lu_u = lu.*u(j);

        firstInt = int(lu_u,'x',0,1);
        secondInt = int(firstInt,'y',0,1);
        A(i,j) = secondInt;
        A(j,i) = secondInt;
    end
end

a = A\B;


V(x,y) = u(1).*((a(1)+a(2).*(x.^2+y.^2))+a(3).*(x.^2.*y.^2));

[X,Y] = meshgrid(-1:1);
plot3(X,Y,V(X,Y),'r-');
xlabel('x');
ylabel('y');
zlabel('v');
grid;

fprintf('\t\tx\t\ty\t\tV(x,y)\n');
```

```
datasave = zeros(10,3);
iter=1;
for i=-1:0.5:1
    datasave(iter,:) = [i 0 V(i,0)];
    iter=iter+1;
end
for i=-1:0.5:1
    datasave(iter,:) = [0 i V(0,i)];
    iter=iter+1;
end
disp(datasave)
```

```
  x   y   V(x,y)
   -1.0000         0         0
   -0.5000         0    0.2058
         0         0    0.2949
    0.5000         0    0.2058
    1.0000         0         0
         0   -1.0000         0
         0   -0.5000    0.2058
         0         0    0.2949
         0    0.5000    0.2058
         0    1.0000         0
```



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