AB-024 : INTRODUCTORY GEAR EQUATIONS

Introduction

An Introduction to Gear Equations

There are a couple of ways to get the exact speed you need on a motor, although we'd like to think that we provide a fairly comprehensive range of gearmotors. If you can't find an off the shelf solution, we're always happy to help discuss custom variations or encoder / PID solutions with you - just drop our engineers a line.

For quickly experimenting or prototyping, you may wish to build your own cheap gear chain. Here we'll take a look at a couple of basic calculations to get you started and which should provide an easy

Relationship Between Two Gears

When two gears are connected (we say 'in mesh') the speed relationship is very simple and is dependant only on the number of teeth in each gear. That is, the ratio of teeth is equal to the inverse ratio of speed. It is easier to understand when written as an equation:
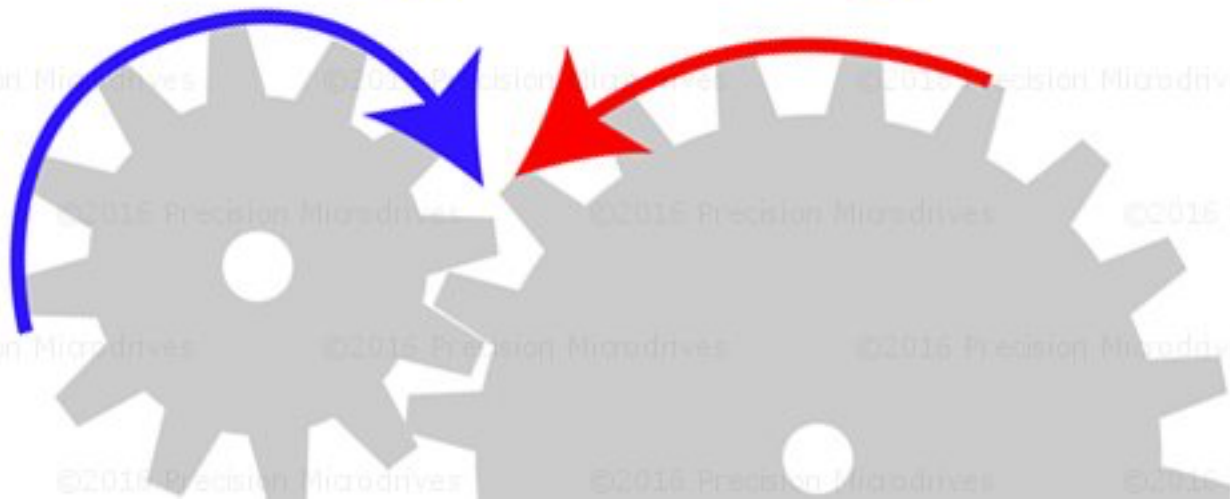
$$\frac{Teeth_A}{Teeth_B} = \frac{RPM_B}{RPM_A}$$

Or you may find it easier to remember that the product of the number of teeth and speed equal between the gears, i.e. the number of teeth times the speed of gear A will equal the number of teeth times the speed of gear B:

$$RPM_A \times Teeth_A = RPM_B \times Teeth_B$$

Whichever way is easier for you to remember, we can calculate any one variable assuming we have the other three. Often, we may already have a motor and desired output speed - using the first equation we can find the required ratio of teeth on the gears.

Two gears in mesh

For example, our 108-106 has a no load speed of 18,000 RPM at 3V. Let's say we wanted to reduce this to under 5,000 RPM:

$$\frac{Teeth_A}{Teeth_B} = \frac{RPM_B}{RPM_A}$$

$$\frac{Teeth_A}{Teeth_B} = \frac{5,000 RPM}{18,000 RPM}$$

$$\frac{Teeth_A}{Teeth_B} = 0.277778$$

OR

$$\frac{Teeth_B}{Teeth_A} = 3.6$$

This means our second gear needs to have at least 3.6 times as many teeth as our first gear. If gear A has 10 teeth, gear B must have at least 36. You may have to compromise when it comes to gear selection, not every value of teeth is available and you may find even numbers more common than odd.

A quick delve into a handful of our office supply returns the following sizes:

- 60 teeth

- 42 teeth

- 30 teeth

- 16 teeth
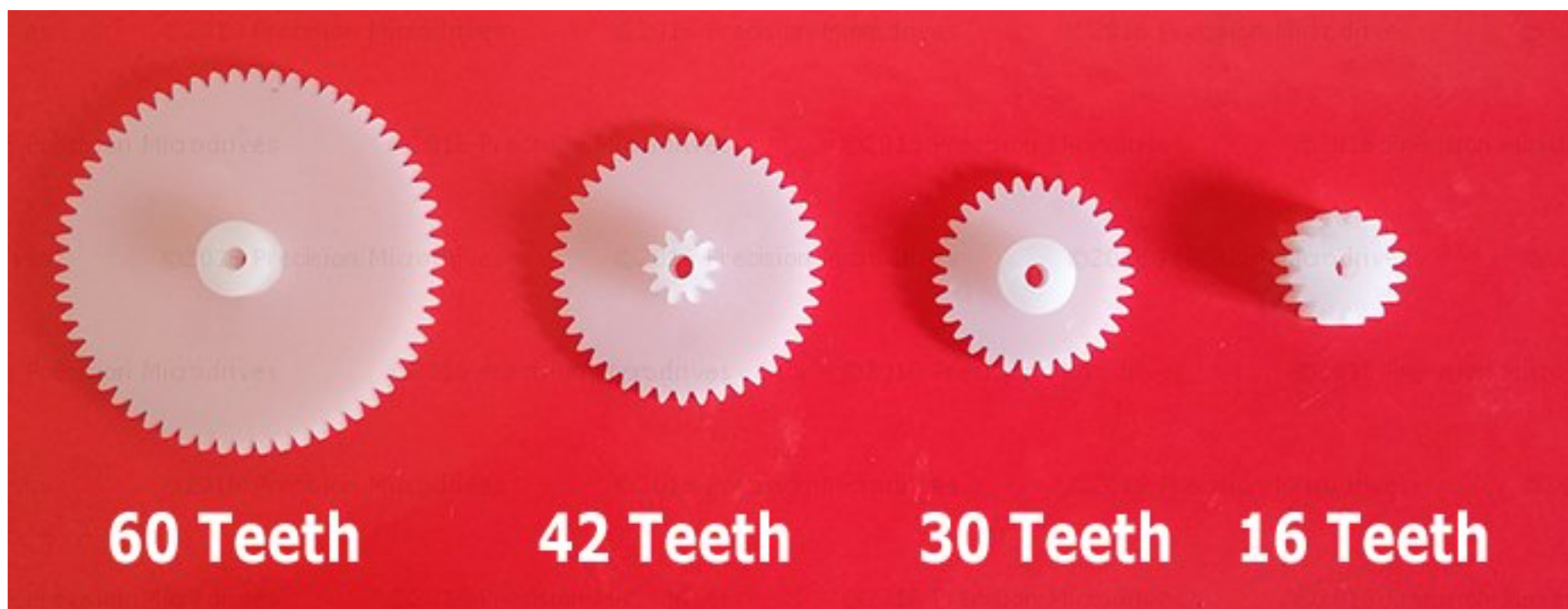
Gears are available in a range of diameters / teeth

Using the 16 tooth size as gear A and the 60 tooth size as gear B gives us a ratio of 3.75 between the gears. This equates to 4,800 RPM when used with the 108-106. Note that due to the nature of the way the gears mesh, they will turn in opposite directions.

Gear A
16 Teeth

Gear B
60 Teeth

$$RPM_B = RPM_A \times (16 / 60) = RPM_A / 3.75$$

A gear of 60 teeth attached to a 16 teeth gear reduces the speed by 3.75

If we needed to reduce the output speed even further, say close to 1,000 RPM, we would need to increase the gearing ratio - but how do we overcome size restrictions? Most applications are limited in space, and the more teeth you require the greater the diameter of the gear.

Adding In Gears

Your first instinct may be to simply add a third gear, but as we will see that this has no effect on the output speed. Remember when two gears are in mesh:

$$RPM_A \times Teeth_A = RPM_B \times Teeth_B$$

So the relationship between our third and second gear is given as:

$$RPM_B \times Teeth_B = RPM_C \times Teeth_C$$

Therefore:

$$RPM_A \times Teeth_A = RPM_B \times Teeth_B = RPM_C \times Teeth_C$$

$$RPM_A \times Teeth_A = RPM_C \times Teeth_C$$

$$\frac{Teeth_A}{Teeth_C} = \frac{RPM_C}{RPM_A}$$

The relationship between the speed of the input and the speed of the output is only dependant upon the ratio of teeth between the input and the output gears. However, the addition of the second gear does ensure the input and output gears turn in the same direction.
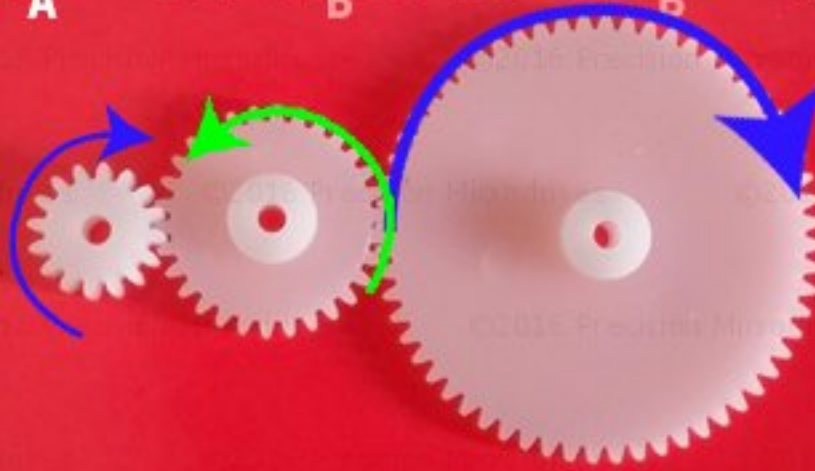
An intermediary gear will change the direction of rotation, but has no effect on speed

Compound Gears

This is overcome by using what is known as a compound gear, where a single gear has two different sets of teeth:

Compound gears have two sets of teeth

With the compound gear the speed is preserved between the gear's two sets of teeth. Using a compound gear, lets look at how our speed and teeth relationship is changed. To explain, we will treat the compound gear as two separate gears (B and C) that are forced to the same speed. We now have two separate meshes:

$$RPM_A \times Teeth_A = RPM_B \times Teeth_B$$

and

$$RPM_C \times Teeth_C = RPM_D \times Teeth_D$$

However the speeds of gear B and gear C are equal, we'll rename it RPM_{BC} to clarify:

$$RPM_A \times Teeth_A = RPM_{BC} \times Teeth_B$$

$$RPM_{BC} \times Teeth_C = RPM_D \times Teeth_D$$

We can rearrange each equation to give a value for RPM_{BC}:

$$RPM_{BC} = \frac{RPM_A \times Teeth_A}{Teeth_B}$$

$$RPM_{BC} = \frac{RPM_D \times Teeth_D}{Teeth_C}$$

$$\frac{RPM_D \times Teeth_D}{Teeth_C} = \frac{RPM_A \times Teeth_A}{Teeth_B}$$

Our output speed is therefore:

$$RPM_D = RPM_A \times \frac{Teeth_A \times Teeth_C}{Teeth_B \times Teeth_D}$$

Teeth A and C can be thought of as transmitting power, i.e. these are forcing teeth B and D to rotate. You can think of the output speed as the input speed times the product of transmitting teeth divided by the product of receiving teeth:

$$OutputSpeed = InputSpeed \times \frac{Product\,of\,Transmitting\,Teeth}{Product\,of\,Receiving\,Teeth}$$
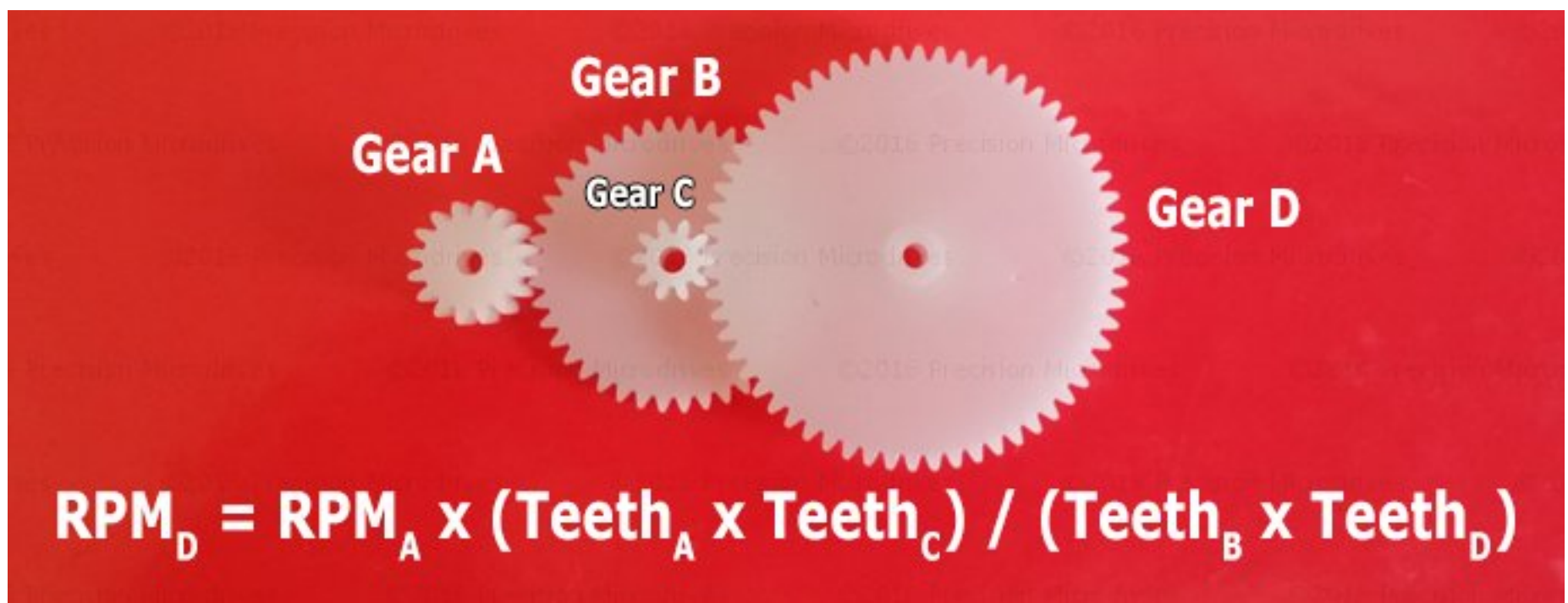
Compound gears enable further speed reduction

Back to our example, it just so happens the 42 tooth gear happens to be a compound gear with a 10

tooth secondary gear. We'll continue to use the smallest gear to drive and the biggest as the final stage to maximise our overall gearing. So our setup now looks as follows:

- Gear A: 16 teeth

- Gear B: 42 teeth (compound with Gear C)

- Gear C: 10 teeth (compound with Gear B)
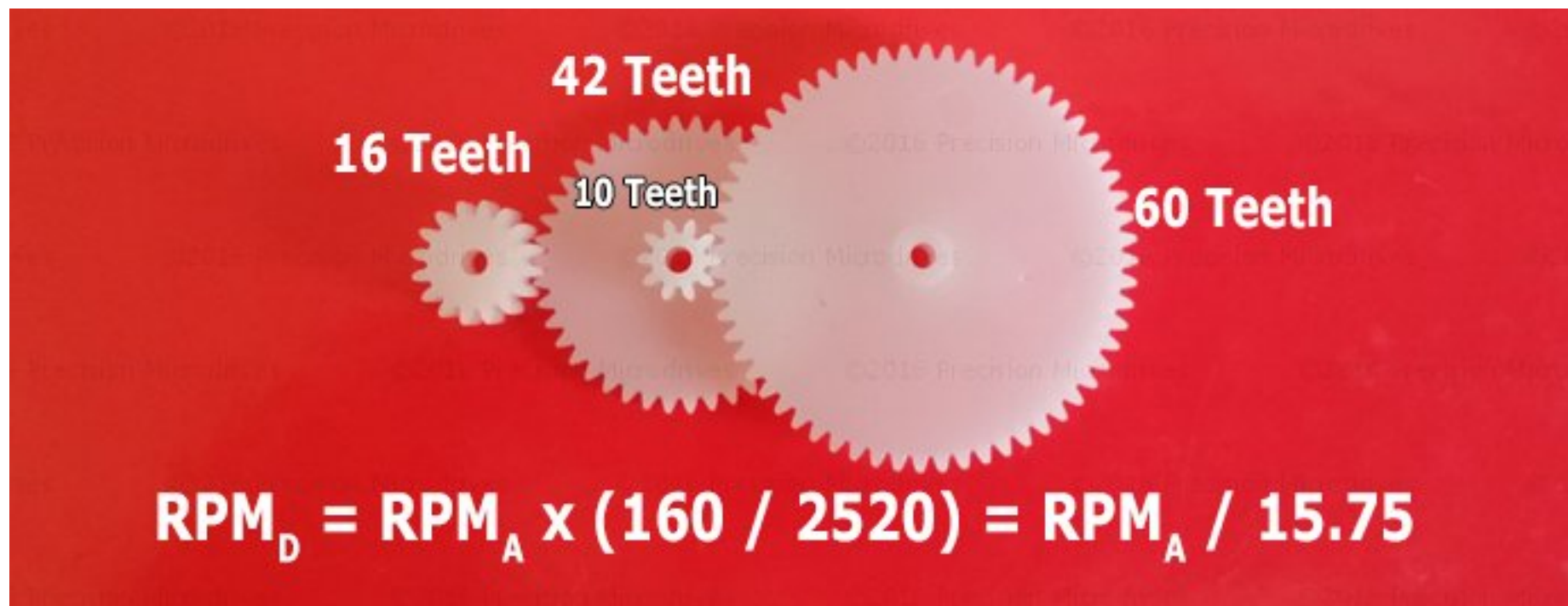
- Gear D: 60 teeth

Still using the 108-106 at 18,000 RPM, what would we expect the new output speed to be?

$$RPM_D = RPM_A \times \frac{Teeth_A \times Teeth_C}{Teeth_B \times Teeth_D}$$

$$RPM_D = 18,000 \times \frac{16 \times 10}{42 \times 60}$$

$$RPM_D = 1,143 RPM$$

Still a little above our target, but we've reduced the the output speed by ~94% - which is a significant reduction from only a few gears!

RPM$_D$ = RPM$_A$ x (160 / 2520) = RPM$_A$ / 15.75

Our 3 gear chain enabled a significant speed reduction

Once we understand how the speed is reduced through the gear chain, calculating the transfer of torque is simple: it is the inverse of the speed ratio.

As the output speed of the gear chain decreases, the system can deliver more torque. If you browse our product catalogue, you can see that for similar motors (for example our 212-4XX) as the Gear Ratio increases, the Rated Torque increases, and the Rated Speed decreases.

An important note is to remember that the speed depends on the level of required torque. So a motor's output speed is always at a specified level of torque. The Rated Speed is at the Rated Torque, and other values can be read from the Typical Performance characteristics graphs.

If the load is constant, a better way of thinking about it is not to consider the torque as an output of the motor but as an input to the system. With our known load at the output gear, we actually work backwards to find what load motor is seen by the motor. From there we can read the Typical Performance Characteristics and estimate the speed of the motor and other details such as the current draw.

Alternatively, if you want to calculate the maximum torque a motor can deliver with it's new gear chain, you would take the Stall Torque value from the motor's datasheet and use the relationship below to calculate the output torque. This is useful if you don't care about speed, you just want to make sure the the load is moved (or have a wide tolerance for your speed).

There are a few caveats with this, first at stall the speed is obviously 0 RPM - but using a 'close to stall' speed of 1 RPM avoids divisions / multiplications by zero. Secondly, sometimes stall torques are estimates - a motor doesn't always steadily decrease to 0 RPM so it would be wise to use a margin of error. Lastly, once a motor is running it can handle a greater torque load than when it's first starting to turn. That means the Stall Torque is likely to be a greater value than the maximum starting torque.

We'll consider each of these scenarios, but first let's review the relationship. For two gears in mesh, we're essentially replacing the Teeth with the Torque (pull the teeth out!):

$$RPM_A \times Teeth_A = RPM_B \times Teeth_B$$
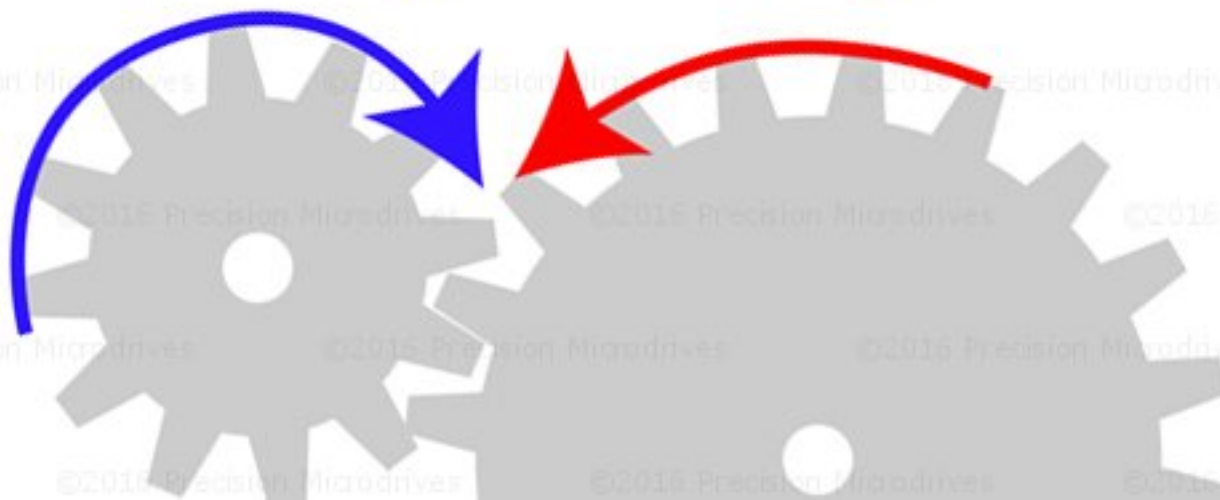
Torque ratio between two meshed gears
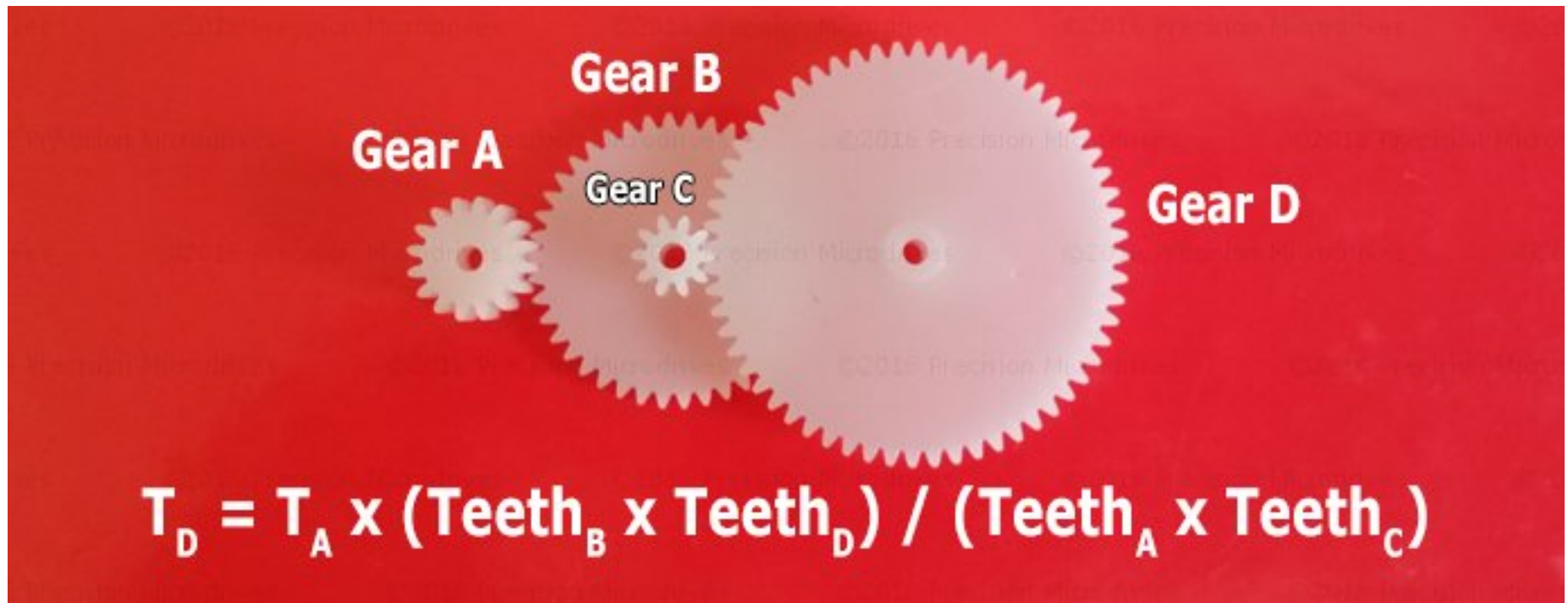
$$\frac{T_A}{T_B} = \frac{RPM_B}{RPM_A}$$

or

$$RPM_A \times T_A = RPM_B \times T_B$$

For a more complicated compound gear chain it will likely be easiest to first calculate the speed ratio using the teeth of the gears, then invert it for the torque ratio. Alternatively:

$$T_D = T_A \times \frac{Teeth_B \times TeethD}{Teeth_A \times Teeth_C}$$

Torque in a compound gear train

In this case, T_A would be the stall torque of the motor and T_D would be the maximum load on the output gear.

A Worked Example

Let's go through a worked example, say we have a known load of 2.5 mNm, the 108-106, and the gear chain above. We want to figure out how fast the load will move, we can calculate it in the following steps:

1. Calculate the gearing ratio

2. Calculate the torque seen by the motor

3. Read the Typical Performance Characteristics to see 108-106's speed at that torque

4. Use the gear ratio to determine the speed at the load

Using the same gear chain as above, we calculate the ratio for the torque:

Using the same gear chain as above, we calculate the ratio for the torque:

$$T_D = T_A \times \frac{Teeth_B \times Teeth_D}{Teeth_A \times Teeth_C}$$

$$T_D = T_A \times \frac{42 \times 60}{16 \times 10}$$

$$T_D = T_A \times 15.75$$

Now, we know the load T_D is 2.5 mNm, which means the torque seen by the motor is:

$$T_A = \frac{T_D}{15.75}$$

$$T_A = \frac{2.5}{15.75}$$

$$T_A = 0.159mNm$$

Reading the 108-106 datasheet, we would expect the motor to turn at (roughly) 14,000 RPM and draw 140 mA. With the motor's speed known, we can use the gear ratio to calculate the output speed:

$$RPM_D = \frac{RPM_A}{15.75}$$

$$RPM_D = \frac{14,000}{15.75}$$

$$RPM_D = 889RPM$$

A small caveat is all calculations we have ignored the friction and inertia present in the gear chain at rest (which would be very small for these plastic gears, but not for large metal gears).

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