Dispatch and Scheduling for EV Charging Using Logic-Based Benders Decomposition

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***Abstract*—Coordinated charging is an effective approach for public charging stations to alleviate the conflict between massive electric vehicles (EVs) and limited charging facilities. This letter proposes a novel approach for the facility coordinator to make an immediate decision on the dispatch and scheduling of newly arriving EVs. The model is presented as a combination of mixed integer programming (MIP) and constraint programming (CP) using logic-based Benders decomposition, where CP is used to generate combinatorial cuts at the incumbent node of the MIP branch-and-cut algorithm. A lazy constraint callback approach is implemented to accelerate the solving by integrating the MIP and CP within a single search tree. Computational experiments demonstrate that the hybrid MIP-CP approach is over 100 times faster than the traditional method.**

***Index Terms*—Electric vehicle, charging scheduling, constraint programming, logic-based Benders decomposition.**

I. Introduction

With the rising adoption of electric vehicles (EVs), it is necessary for the public parking infrastructure with limited charging facilities to implement coordinated charging strategies, i.e. shifting the charging sessions of EVs [1]. There has been a growing number of papers that propose dispatch and scheduling mechanism for EV charging [2], [3]. The EV management systems in these papers are devoted to admitting as many EVs as possible while ensuring their charging needs before the departure time. The main issue of the management systems in [1]–[3] is that the dispatch and scheduling of EV charging are coupled, and the problem is modeled as a mixed integer programming (MIP) with a large number of variables. Especially for the scheduling problem, the MIP model needs to assign a binary variable to the state-of-charge of each EV at each time slot, which brings tremendous burden to the real-time application of the EV management system.

To solve the problem, this letter proposes a new approach to simplify the model of charging scheduling using constraint programming (CP). Since CP is built on interval variables, the size of CP model is independent of the number of time slots, which makes CP a state-of-the-art tool in solving scheduling problems. Note that although the EV charging scheduling is simplified, the EV dispatch problem is still a MIP. Therefore, the hybrid MIP-CP model is decoupled into two stages, and a logic-based Benders decomposition (LBBD) algorithm [4] is applied to solve the model. To further accelerate the solving, a lazy constraint callback (LCC) technology [5] is introduced to integrate the LBBD in a single Branch and Check (B&C) search tree.

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II. Model Formulation

This letter assumes that the charging management system manages multiple parking facilities. Each facility is equipped with single-output-multiple-cables (SOMC) chargers [3]. The MIP-based dispatch model optimizes the charging location as well as the charging rate of all newly arriving EVs, while the CP-based scheduling model checks whether these dispatched EVs can be scheduled.

A. MIP-Based Dispatch Model

Let I_p and I_a be the set of parked and arriving EVs, and $I = I_p \cup I_a$ be the set of all the EVs that submit the request of charging. Let J be the set of public parking facilities that managed by the EV management system, and K be the set of charging rates that the system can provide. Let binary variable $x_{ijk} = 1$ if EV i is assigned to charge at parking facility j using charging rate k and 0 otherwise. The charging demand of each EV is represented by D_i. The coefficient of economic losses caused by rejecting EV i to charge is denoted by c_i^r, while the additional cost of assigning EV i to parking facility j is represented by c_{ij}^a. Note that c_{ij}^a should be set to zero if parking facility j is the original destination of EV i.

The MIP-based dispatch model is stated as follows.

$$\min \quad \sum_{i\in I_a} c_i^r (1 - \sum_j \sum_k x_{ijk}) + \sum_{i\in I_a} \sum_j c_{ij}^a \sum_k x_{ijk} \tag{1}$$

$$s.t. \quad \sum_j \sum_k x_{ijk} \le 1 \quad \forall i \in I_a \tag{2}$$

$$x_{ijk} \in \Omega \quad \forall i \in I, j \in J, k \in K \tag{3}$$

The objective in (1) minimizes the total capital loss of the management system. Constraint (2) means that each EV can only be charged at one station using one type of charging rate if its charging request is accepted. Expression (3) is a generic constraint to force certain variables to be 0 or 1 according to specific limits. For example, x_{ijk} should remain the same for all parked EVs, or EV i is unwilling to park at j, or EV i can only be charged at charging rate k, etc.

B. CP-Based Scheduling Model

In this part, the scheduling of EV charging is formulated as a CP model. First, a new class of variables, namely interval variables, is introduced as a basic building block of the EV charging scheduling problem. The interval variable represents an interval of time during which a charging session happens and whose position in time is an unknown of the scheduling problem. An interval variable is characterized by a start value, an end value and a size. As to the charging scheduling model in this letter, let y_i be the interval variable of EV i whose domain can be defined as $\{[a_i, d_i)|a_i, d_i \in \mathbb{Z}, a_i \leq d_i\}$, where a_i and d_i are the arrival and departure time. The time horizon is divided into N_T equal-length time slots, so that the size of y_i can be denoted by $e_i - s_i$, where s_i and e_i are two integer variables representing the index of time slot when EV i starts and ends its charging. To improve readability, a diagram for the interval variable is shown in Fig. 1(a).

Let T_{ik} be the number of time slots needed for charging if EV i is assigned to charge at rate k, then the size of y_i must satisfy the following constraint.

$$size(y_i) = e_i - s_i = \sum_k T_{ik} x_{ijk} \quad \forall i \in I, j \in J \tag{4}$$

As can be observed, the state-of-charge for each EV i at any time can be determined with only one interval variable, which makes CP model independent of the number of time slots.

Since our CP-based scheduling model is built on interval variables rather than algebraic variables, the modeling method of CP constraints is converted from the addition of algebraic variables to the cumulation of interval variables. In this letter, a particular cumulative function, $pulse(u, v, h)$, is introduced to define a pulse function F such that $F(t) = h$ if $t \in [u, v)$, $F(t) = 0$ otherwise. A diagram for F is given in Fig. 1(b).

Then, for each parking facility j, a CP model is presented to verify whether there exists at least one feasible scheduling solution for all the EVs assigned to parking facility j. Firstly, the constraints of the utilization of chargers is given as:

$$\sum_{i \in I} pulse(s_i, e_i, x_{ijk}) \leq N_{jk} \quad \forall j \in J, k \in K \tag{5}$$

where the left side of (5) is the total number of operational chargers with a charging rate of k at parking facility j, and N_{jk} is this installation number of the charger.

For each parking facility, the total charging power can be described in (6) as a cumulative function.

$$C_j = \sum_{i \in I} pulse(s_i, e_i, \sum_{k \in K} p_k x_{ijk}) \quad \forall j \in J \tag{6}$$

Moreover, the total charging power C_j above cannot exceed its upper limit P_{jt} in any time slot t. Note that C_j is a CP expression, while P_{jt} is an algebraic function, which means that the constraint for the upper limit cannot be modeled by CP directly. To address the issue, a novel modeling method is proposed to use constant $\max_t(P_{jt})$ as a transition. Firstly, a cumulative function D_j is defined to represent the difference between P_{jt} and $\max_t(P_{jt})$ as follows.

$$D_j = \sum_{t \in T} pulse(t, t+1, \max_t(P_{jt}) - P_{jt}) \quad \forall j \in J \tag{7}$$

Then, by appending D_j to C_j, the upper limit of C_j can be changed correspondingly from P_{jt} to $\max_t(P_{jt})$. Therefore, the constraint for the upper limit of the total charging power can be formulated by CP as follows.

$$C_j + D_j \leq \max_t(P_{jt}) \quad \forall j \in J \tag{8}$$

To improve readability, a schematic diagram for the modeling approach above is depicted in Fig. 2.

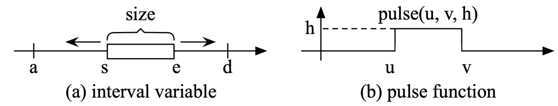

Fig. 1. Interval variable and pulse function.

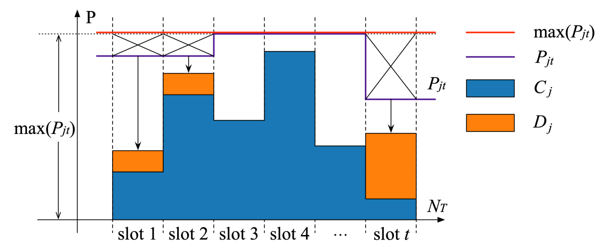

Fig. 2. A schematic diagram for the constraint of charging power.

III. Solution Methodology

Since CP is dual infeasible, the classical Benders decomposition algorithm is generalized to LBBD [4]. The MIP model is regarded as the master problem, and the CP model for each $j \in J$ is considered as the sub-problem. Binary variable x_{ijk} is chosen as the coupling variable between MIP and CP.

A. Cut Generation

The core idea behind the LBBD is similar to the classical Benders decomposition, i.e. working on the solution space of the master problem only and iteratively modifying the shape of the space based on the results of sub-problems.

Specifically, let $\hat{x}_{ijk}$ be the dispatch decision of EVs that determined in the master problem. If the CP model of parking facility j under the fixed $\hat{x}_{ijk}$ is infeasible, a logic based Benders cut will be generated in analogy with the combinatorial Benders cut as follows.

$$\sum_{k \in K} \sum_{i \in S_{jk}} (1 - x_{ijk}) \geq 1 \quad \forall j \in J \tag{9}$$

where S_{jk} is a set of EVs which are assigned to charge at parking facility j using charging rate k in the MIP model. In other words, $S_{jk} = \{i \mid \hat{x}_{ijk} = 1\}$. The cut in (9) prevents EVs from being given the same dispatch decision.

B. Branch and Check

After generating the Benders cuts, the next step is adding them to the master problem. In the classical Benders decomposition, each time a cut is added to the master problem, the solver has to solve it again. Although the new cuts may alter the structure of the search tree, it is highly probable that the solver will spend time revisiting candidate solutions that have already been eliminated earlier, which is time-consuming.

To accelerate the solving procedure, B&C is introduced to integrate the MIP and CP models into one single search tree. When branching the MIP master problem, every time a new incumbent solution $\hat{x}_{ijk}$ is found, the resulting sub-problems will be solved to obtain the cuts in (9). Then, these constraints will be enforced throughout the remainder of the search tree by using lazy constraint callback (LCC) technology [4], [5].

The solving procedure of the LBBD integrated with LCC technology is given as follows.

Algorithm 1: LBBD for EV charging

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build the MIP model (1)-(3);
while branching the MIP model do
    if an incumbent solution x̂_ijk is found then
        repeat
            solve the j-th CP model (4)-(8) under x̂_ijk;
            if model is infeasible then
                add (9) to the search tree by LCC;
            end
        until all parking facilities are checked;
    end
end
return the current {x_ijk, y_i} and terminate.
```

IV. Experimental Evaluation

The performance of the proposed approach is evaluated on a testing system which controls 3 dispersed parking facilities in urban area. Three types of charging rate, 50, 22 and 7 kW, are considered. The capacity of each parking facility is 1435, 688 and 832 kW. The length of time slot is set to 5 minutes, and the charging session for each EV is an integer multiple of 5 minutes. Detailed parameters of EVs, chargers and cost coefficient can be found in [6].

All algorithms are coded in Python and tested on a MacBook Pro. The MIP model is solved by Gurobi 9, while the CP model is solved by IBM CP Optimizer 12.10.

A. Case Study

In this section, it is assumed that the charging request for 450 EVs has been accepted by the management system. And at a certain time, another 50 EVs submitted their request to the system simultaneously.

Fig. 3 shows the dispatch and scheduling decision of the 50 arriving EVs, where the three sub-pictures correspond to the three charging stations respectively. For each EV, the whisker means the time interval whose caps on the ends are the time of arrival and departure. The box on the whisker denotes the charging session, and the charging rate can be identified by its color. As can be seen, all newly arriving EVs are dispatched and scheduled for charging successfully.

B. Computational Performance

To verify the effectiveness and superiority of the MIP-CP model and the solving strategy, the size and solving time of which are compared a traditional MIP model modified from [3] (solved by Gurobi directly) and a MIP-CP model (solved by LBBD without LCC). The results of 5 different cases are listed in Table I, where the two numbers in the first column are the number of parked and newly arriving EVs, the size of the models are measured by the number of variables.

As can be seen, the size of MIP-CP model is only 2% of the MIP model for the same case, and this is the reason why

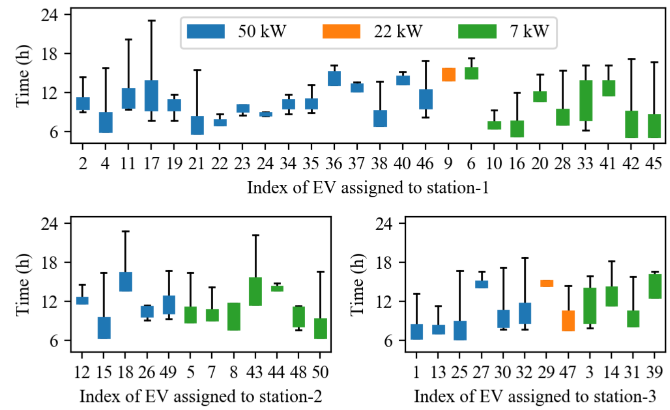

Fig. 3. The dispatch and scheduling of newly arriving EVs.

TABLE I
Comparison of Size and Time in Different Cases

Number	MIP		MIP-CP		MIP-CP with LCC	
	size	time (s)	size	time (s)	size	time (s)
(500,100)	80452	131.56	1880	4.17	1880	1.59
(600,100)	93396	230.73	2079	5.01	2079	1.84
(700,100)	108606	573.77	2234	4.82	2234	2.02
(800, 50)	115100	633.00	2318	4.98	2318	2.14
(850, 50)	122018	840.23	2431	6.65	2431	2.37

the CPU time for solving the MIP model is hundreds of times longer than the MIP-CP model. The comparison between the MIP-CP models with and without using LCC further shows that performing LBBD in a single B&C search tree is 2 to 3 times faster than the classical approach where the master and sub-problems must be solved iteratively. Moreover, as the size of model increases, the time for solving the MIP-CP model is about 2 seconds (if using LCC), while the time for solving the MIP model grows exponentially and reaches 14 minutes in the last case.

V. Conclusion

In this letter, a novel hybrid MIP-CP model is proposed to solve the dispatch and scheduling of EV charging based on LBBD algorithm. The high computational performance makes it a desirable tool for real-time application.

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