Homework 2

Deadline: April 24

1. If we have $f : \mathbb{R}^{n\times p} \to \mathbb{R}$, then $\frac{\mathrm{d}f}{\mathrm{d}\mathbf{X}} = \frac{\partial f}{\partial x_{ij}}$. Assume $f = \mathrm{tr}(\mathbf{AX})$. Prove that $\frac{\mathrm{d}f}{\mathrm{d}\mathbf{X}} = \mathbf{A}^T$.

2. Let $\mathbf{X}$ be an $n \times p$ matrix. Solve the following optimization problem: $\min \phi(\mathbf{Z}, \mathbf{V}|\mathbf{X}) = \|\mathbf{X} - \mathbf{Z}\mathbf{V}^T\|_F^2$, s.t. $\mathbf{V}^T\mathbf{V} = \mathbf{I}_q$ and $\mathbf{Z}^T\mathbf{1}_n = \mathbf{0}$, where $\mathbf{V}$ is a $p \times q$ matrix and $\mathbf{Z}$ is an $n \times q$ matrix.

3. Assume a mapping from the latent space into the data space is $\mathbf{x} = \mathbf{W}\mathbf{z} + \boldsymbol{\mu} + \sigma\boldsymbol{\epsilon}$, where $\mathbf{x} \in \mathbb{R}^p$, $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_p)$, $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_q)$, $\mathbf{z} \perp\!\!\!\perp \boldsymbol{\epsilon}$, and $\mathbf{W}^T\mathbf{W} = \mathbf{I}_q$. Please use EM algorithm to solve parameters: $\mathbf{W}$, $\boldsymbol{\mu}$, σ.

4. Please give the formula of Probabilistic Kernel PCA, and solve it.