The Variation of Radiation Pressure and Breathing of BLR

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1 Equation of motion

Gravitation:

$$F_{grav} = -\frac{GMm}{r^2} \tag{1}$$

Radiation pressure:

$$F_{rad} = \frac{GMm}{r^2} \cdot \frac{3l}{2\sigma_T N_{cl}} \tag{2}$$

Here, l is the ratio between actual luminosity L and Eddington luminosity $L = \frac{4\pi GMm_p c}{\sigma_T}$.
If we suppose that the density of the gas in clouds varies with radius as

$$\rho_{cl} = \rho_{cl0}(\frac{r}{r_0})^{-s} \tag{3}$$

then we know the column density varies as

$$N_{cl} = N_{cl0}(r/r_0)^{-\frac{2}{3}s} \tag{4}$$

Drag force:

$$F_{drag} = -6\pi\mu R_{cl} v \tag{5}$$

We suppose the viscosity μ varies as

$$\mu = \mu_0(\frac{r}{r_0})^{\nu} \tag{6}$$

The radius of clouds varies as

$$R_{cl} = R_{cl0}(\frac{r}{r_0})^{\frac{s}{3}} \tag{7}$$

To simplify our calculation, we must redefine our scales. we use the typical size of BLR r_0 as the scale of length and Kepler velocity $v_0 = \sqrt{\frac{GM}{r_0}}$ as the scale of velocity. Then the acceleration of the cloud can be written as

$$a_{grav} = -\frac{1}{r^2} \tag{8}$$

$$a_{rad} = \frac{\alpha f(t)}{r^{2-\frac{2}{3}s}}, \quad \alpha = \frac{3l_0}{2\sigma_T N_{cl0}} \tag{9}$$

(Here, the variation of light curve can be written as $l = l_0 * f(t)$)

$$a_{drag} = -\beta r^{\frac{s}{3}+\nu} v, \quad \beta = \frac{6\pi\mu_0 R_{cl0} r_0}{m v_0} \tag{10}$$

If we define the Reynolds number as

$$\mathrm{Re} = \frac{\rho u L}{\mu} \tag{11}$$

Here, u is the mean velocity of the cloud relative to the background gas and L is a characteristic length, ρ is the density of the background gas. After a few lines of algebra, we can get

$$\beta = \frac{\Lambda}{\mathrm{Re}_0}, \quad \Lambda = \frac{9}{2}\frac{r_0}{R_{cl0}}\frac{\rho_0}{\rho_{cl0}}\frac{u_0}{v_0} \tag{12}$$

As the drag force can not change the direction of angular momentum of the cloud, the motion of the cloud is constrained in a plane. We use polar coordinates to describe its motion.

$$\ddot{r} = \frac{L^2}{r^3} - \frac{1}{r^2} + \frac{\alpha f(t)}{r^{2-\frac{2}{3}s}} - \beta r^{\frac{s}{3}+\nu}\dot{r} \tag{13}$$

$$\dot{L} = -\beta r^{\frac{s}{3}+\nu} L \tag{14}$$

2 The perturbation theory

At first, we study the motion of clouds when the radiation pressure and drag force can be viewed as a perturbation,*i.e.*$\alpha \ll 1, \beta \ll 1$. The 0^{th} order approximation is just circular motion

$$r = 1, \quad L = 1 \tag{15}$$

To the linear order,

$$\delta\ddot{r} = \tag{16}$$