po-hw-1

September 21, 2018

1 Homework 1: Thermodynamics of Seawater

For this assignment, you may use the following equation of state:

$$\rho \simeq \rho_0 \left[1 - \frac{gz}{c_0^2} - \alpha_0(1 - \gamma_B z)(\Theta - \Theta_0) - \frac{\gamma_C}{2}(\Theta - \Theta_0)^2 + \beta_0(S_A - S_{A0}) \right]$$

This equation, and its approximate parameters, are given in the following paper:
https://agupubs.onlinelibrary.wiley.com/doi/epdf/10.1002/2015GL065525

parameter	value
ρ_0	1027 kg m^{-3}
g	9.81 m s^{-2}
c	1500 m s^{-1}
α_0	1.67 $\times 10^{-4}$ K^{-1}
γ_B	1.12 $\times 10^{-4}$ K^{-1} m^{-1}
β	0.78$\times 10^{-3}$ $(g/kg)^{-1}$
Θ_0	10 deg. C
S_0	35 g/kg

1.1 1) Derive the following quantities in analytical form (10 points)

- thermal expansion coefficient
- haline contraction coefficient

Comment on the nonlinearities, or lack thereof, in your results. Give a physical interpretation for the parameters γ_B and γ_C. What would happen if you set them to zero?

(To make life easier, wherever the expression $1/\rho$ appears, you may approximate it as $1/\rho_0$, in keeping with the smallness of the density anomalies.)

1.2 2) Volume and Mass (10 points)

The Mediterranean Sea has a surface area of approx. 2,500,000 km^2 and an average depth of 1500 m.

1. What is its volume?

2. Assume the density increases linearly in depth from 1027 to 1035 kg m^{-3}. What is its total mass?
3. Let's say that the annual mean rate of evaporation minus precipitation out of the sea is 10 cm / year (net evaporation). How long will it take to evaporate away the entire basin?

1.3 3) Mixing Ratios - Two Sources (20 points)

This problem uses the fact that Θ and S_A are conserved to learn about the composition of water found in the ocean interior.

Imagine you have two "source" water masses: a polar one ($\Theta_1 = 2°$ C, $S_1 = 32$ g/kg) and a tropical one ($\Theta_1 = 20°$ C, $S_1 = 35$ g/kg). These two water masses mix in the interior to form thermocline water.

- What is the Θ and S of the thermocline water assuming that each source water mass makes a 50/50 contribution?
- What if the polar water only contributes 10% and the tropical water contributes 90%?
- Find the general rule for the thermocline water composition given the mixing ratio R. ($R = 1$ means 100% polar water; $R = 0$ means 100% thermocline water)
- Plot the possible values that the thermocline water can take from $R = 0$ to $R = 1$ in a Θ-S diagram. Why are the sources often called "end members"?

1.4 4) Mixing Ratios - Three Sources (20 points)

The assumption that there were only two source end members is highly simplistic. What if we have three source end members?

End member 1: Θ_1, S_1, contributes fraction R_1 to the interior water

End member 2: Θ_2, S_2, contributes fraction R_2 to the interior water

End member 3: Θ_3, S_3, contributes fraction R_3 to the interior water

Given the observed thermocline temperature and salinity Θ and S, find an expression for R_1 and R_2 and R_3.

Some hints: - note that $R_1 + R_2 + R_3 = 1$, so we only need to solve for two out of three fractions - try to formulate this as a linear algebra problem - there are some circumstances under which this can't be solved: what are they?

1.5 5) Cabbeling of Antarctic Intermediate Water (20 points)

Along the sloping isopycnals of the southern ocean, cool, fresh surface water mixes with warmer, saltier deeper water. Consider two water masses:

Surface Origin: $S_A = 34.2$ g/kg, $\Theta = 1°$ C

Deep Origin: $S_A = 34.4$ g/kg, $\Theta = 2.65°$ C

1. The two water masses both make their way to 500 m depth. Calculate the density of each before mixing.
2. Now let the two water masses mix with a 50/50 mixing ratio. What is the resulting Θ and S of the mixed water mass? How does it compare to the average Θ and S before mixing?
3. What is the resulting density after mixing? How does it compare to the average density before mixing?

This is cabbeling.

1.6   6) Thermobaric Effect on NADW and ABW

Typical water mass properties of North Atlantic Deep Water (NADW) are $S_A = 34.9$ g/kg, $\Theta = 1°$ C.

Typical water mass properties of Antarctic Bottom Water (NADW) are $S_A = 34.6$ g/kg, $\Theta = -1.5°$ C.

1. Calculate the potential density of these water masses with respect to 4000 m depth. Which is denser in the deep ocean?
2. Calculate the potential density of these water masses with respect to 0 m depth. Which is denser at the surface?
3. Repeat the calculation taking $\gamma_B = 0$.
4. Comment on the role of the thermobaric effect in the global ocean stratification.