Partial solution to HW 4 and HW 5

HW4

Q6

The idea of disjoint union will be helpful to us. Given an set $A \in \mathbb{R}$, we may assume that $P(X \in A) \geq P(Y \in A)$. (Otherwise, replace X with Y in the following proof). The RHS can be written as

$$\begin{aligned}
\text{RHS} &= 1 - P(X = Y) \\
&= [P(X \in A) + P(X \in A^c)] - [P(X = Y, Y \in A) + P(X = Y, Y \in A^c)] \\
&= [P(X \in A) + P(X \in A^c)] - [P(X = Y, Y \in A) + P(X = Y, X \in A^c)] \\
&\geq P(X \in A) - P(Y \in A) + P(X \in A^c) - P(X \in A^c, X = Y) \\
&= LHS + P(X \in A^c, X \neq Y) \\
&\geq LHS
\end{aligned}$$

□

Another way of doing it, is to consider the LHS

$$\begin{aligned}
\text{LHS} &= P(X \in A) - P(Y \in A) \\
&= P(X \neq Y, X \in A) - P(X \neq Y, Y \in A) \qquad (1) \\
&\leq P(X \neq Y)
\end{aligned}$$

HW5

Q3

λ is the average 2. Therefore, the probability of scoring exactly 6 in one game is

$$p = e^{-\lambda} \frac{\lambda^6}{6!}$$

The problem ask for the probability that 'at least' one game scoring exactly 6 Field goals, which has the probability $1 - (1 - p)^{150}$

Q4

Check the note for recitation on Google drive.