

Erreta

Q1

(c) ... size of the randomly chosen group. Let $E[Y] = \mu$ and ...

Partial solution to HW 6

Q1

Checkout out chapter 5 Q5 solution in Janko Graver's book.

Q2

(Panchenko Exercise 1.5.2) There could be many, my solution use shifted normalized Poisson Set discrete random variable X . We define the pmf f_X to be

$$f_X(x) \begin{cases} 1 - e^{-10}, & \text{if } x = 0; \\ e^{-10} e^{\lambda} \frac{\lambda^{(x-1)}}{(x-1)!}, & \text{if } x \in \mathbb{N}. \end{cases}$$

Then, $P(X > 0) = e^{-10}$ is easy to check. Now we have to set the correct λ . By some calculation you can see that

$$\begin{aligned} E[X] &= \sum_{x=1}^{\infty} ((x-1) + 1) e^{-10} e^{\lambda} \frac{\lambda^{(x-1)}}{(x-1)!} \\ &= e^{-10} \underbrace{\sum_{x=0}^{\infty} x e^{\lambda} \frac{\lambda^x}{(x)!}}_{\text{expectation of Poisson}(\lambda)} + e^{-10} \\ &= e^{-10}(\lambda + 1) \end{aligned}$$

Hence, set $\lambda = e^{10} - 1$ we will have $E[X] = e^{10}$.

□

Q3

This is the same as the exercise in RC4 exercise for the indicator method. The number of pairs of animals alive, N , is not an easy random variable to compute. Therefore, we may consider "when will N increase?". This is easier, N increase 1 if a pair of animals is alive. Hence, the indicator we consider is $1_{A_i}, i = 1, \dots, n$, where A_i means the event that the i th pair of animal is alive.

$$\text{Then, we have } E[1_{A_i}] = P(A_i) = \binom{2(n-1)}{m} / \binom{2n}{m}$$

$$\text{Consequently, by linearity of expectation,} \\ E[N] = \sum_{i=1}^n E[1_{A_i}] = nP(A_1) = \frac{(2n-m)(2n-m-1)}{2(2n-1)} \text{ pairs.}$$

Q4

Again, this is a problem using the indicator method.

Q5**Q6**

- (a) Calculate $E[X^2] = p$, and use $\text{Var}(X) = E[X^2] - E[X]^2$.
- (b) We did that in recitation class. If you forget, try to use the method introduced in handout Chapter 3 page 14 of from prof. Sheu.
- (c) Recall that $E[aX + b] = aE[X] + b$ due to the linearity of expectation. We have $\text{Var}(aX + b) = E[(aX + b - (aE[X] + b))^2] = E[a^2(X - E[X])^2] = a^2\text{Var}(X)$

Q7**Q8****Q9****Q10****Q11**