Erreta

Q1

(c) ... size of the randomly chosen group. Let E[Y] = μ and ...

Partial solution to HW 6

Q1

Checkout out chapter 5 Q5 solution in Janko Graver's book.

Q2

(Panchenko Exercise 1.5.2) There could be many, my solution use shifted normalized Poisson Set discrete random variable X. We define the pmf f_X to be

$$f_X(x) \begin{cases} 1 - e^{-10}, & \text{if } x = 0; \\ e^{-10} e^{\lambda} \frac{\lambda^{(x-1)}}{(x-1)!}, & \text{if } x \in \mathbb{N}. \end{cases}$$

Then, $P(X > 0) = e^{-10}$ is easy to check. Now we have to set the correct λ. By some calculation you can see that

$$\begin{aligned} E[X] &= \sum_{x=1}^{\infty} ((x-1)+1) e^{-10} e^{\lambda} \frac{\lambda^{(x-1)}}{(x-1)!} \\ &= e^{-10} \underbrace{\sum_{x=0}^{\infty} x e^{\lambda} \frac{\lambda^{x}}{(x)!}}_{\text{expectation of Poisson}(\lambda)} + e^{-10} \\ &= e^{-10}(\lambda + 1) \end{aligned}$$

Hence, set $\lambda = e^{20} - 1$ we will have $E[X] = e^{10}$.

$\square$

Q3

This is the same as the exercise in RC4 exercise for the indicator method. The number of pairs of animals alive, N, is not an easy random variable to compute. Therefore, we may consider "when will N increase?". This is easier, N increase 1 if a pair of animals is alive. Hence, the indicator we consider is $1_{A_i}, i = 1, \ldots, n$, where A_i means the event that the ith pair of animal is alive.

Then, we have $E[1_{A_i}] = P(A_i) = \binom{2(n-1)}{m} / \binom{2n}{m}$

Consequently, by linearity of expectation,
$E[N] = \sum_{i=1}^{n} E[1_{A_i}] = nP(A_1) = \frac{(2n-m)(2n-m-1)}{2(2n-1)}$ pairs.

□

Q4

Q5

Q6

Q7

Q8

Q9

Q10

Q11