Solution (Suggestion) to STATS 310 (MATH 230) Lecture notes by Sourave Chatterjee

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Abstract

It's the work accompanying Prof. Shue's Probability theory class in 2022 spring. We hope to cover exercises in Prof. Chatterjee's STATS 310 notes [Cha] from chapter 7 onward. Special thanks go to Pingbang Hu [Hu] who made the LaTeXtemplate.

Contents

7 Chapter7: Independece

This is a simple demo for you to take fancy notes in LaTeX!

7.1 Definition

We now see some common environment you'll need to complete your note.

Answer. 7.1.3.(AH) The 'only if' part is equivalent to definition 7.1.2, so showing the 'if' part is enough. We can start with index J with size 2, then use induction to generalize the fact to any finite size J.
$|J| = 2$. W.L.O.G, we set $J_2 = \{j_1, j_2\} \subset I$($J_2$ means J with size 2). Since cases with one of the event being $\Omega, \emptyset$ is trivial, what is left for showing independence of the events are $A_{j_1} \cap A_{j_2}^c$ and $A_{j_1}^c \cap A_{j_2}$.

$$\begin{aligned}
\mathbb{P}(A_{j_1} \cap A_{j_2}^c) &= \mathbb{P}(A_{j_1}) - \mathbb{P}(A_{j_1} \cap A_{j_2}) && \text{(Disjoint additivity)} \\
&= \mathbb{P}(A_{j_1}) - \mathbb{P}(A_{j_1})\mathbb{P}(A_{j_2}) && \text{(Given condition)} \\
&= \mathbb{P}(A_{j_1})(1 - \mathbb{P}(A_{j_2})) && \text{(Disjoint additivity)} \\
&= \mathbb{P}(A_{j_1})\mathbb{P}(A_{j_2}^c)
\end{aligned}$$

The process of $A^c_{j_1} \cap A_{j_2}$ is the same, and thus A_{j_1} and $A_{j_2}\}$ are independent. Now, suppose the statement is true for J with size up to $n-1$, we consider $J_n = J_{n-1} \cup \{j_n\}$. Again, we skip cases with $A_{j_n}, \Omega, \emptyset$,

$$\begin{aligned}
\mathbb{P}((\cap_{k=1}^{n-1} A_{j_k}) \cap A^c_{j_n}) &= [\prod_{k=1}^{n-1} \mathbb{P}(A_{j_k})](1 - \mathbb{P}(A^c_{j_n})) \\
&= [\prod_{k=1}^{n-1} \mathbb{P}(A_{j_k})]\mathbb{P}(A^c_{j_n})
\end{aligned}$$

By mathematical induction, the given statement is true for all finite $J \subset I$.

7.1.1 Internal Link

You should see all the common usages of internal links. Additionally, we can use citations as [**newton1726philosophiae**], which just link to the reference page!

Appendix

A Additional Proofs

A.1 Proof of ??

See https://en.wikipedia.org/wiki/Mass%E2%80%93energy_equivalence.

References

[Cha] Sourav Chatterjee. *STATS 310 (MATH 230) Lecture notes (ongoing,to be updated)*. URL: https://souravchatterjee.su.domains//stats310notes.pdf.

[Hu] Pingbang Hu. *Academic-Template*. URL: https://github.com/sleepymalc/Academic-Template.git.