Magnetic Mirror Effect in Magnetron Plasma: Modeling of Plasma Parameters

1 Lorentz Force

The equations of motion for a charged particle under the influence of Electric and Magnetic fields is described by the Lorentz Force in the S.I. units as

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \tag{1}$$

along with the expression for the velocity

$$\frac{d\mathbf{x}}{dt} = \mathbf{v} \tag{2}$$

These equations are discretized to obtain

$$\frac{\mathbf{v}_{k+1} - \mathbf{v}_k}{\Delta t} = \frac{q}{m}\left[\mathbf{E}_k + \frac{(\mathbf{v}_{k+1} + \mathbf{v}_k)}{2} \times \mathbf{B}_k\right] \tag{3}$$

from the Lorentz Force equation (1), and

$$\frac{\mathbf{x}_{k+1} - \mathbf{x}_k}{\Delta t} = \mathbf{v}_{k+1} \tag{4}$$

from the expression for velocity in equation (2).

2 Boris Algorithm

The Boris Algorithm splits equation (2) into three equations.

$$\frac{\mathbf{v}^- - \mathbf{v}_k}{(\Delta t/2)} = \frac{q}{m}\mathbf{E}_k \qquad or \qquad \frac{\mathbf{v}^- - \mathbf{v}_k}{\Delta t} = \frac{1}{2}\frac{q}{m}\mathbf{E}_k \tag{5}$$

which is often called the first half of the electric pulse.

$$\frac{\mathbf{v}^+ - \mathbf{v}^-}{\Delta t} = \frac{q}{m}\left(\frac{\mathbf{v}^+ + \mathbf{v}^-}{2}\right)\mathbf{B}_k \tag{6}$$

which is often called rotation by the magnetic field.

$$\frac{\mathbf{v}_{k+1} - \mathbf{v}^+}{(\Delta t/2)} = \frac{q}{m}\mathbf{E}_k \qquad or \qquad \frac{\mathbf{v}_{k+1} - \mathbf{v}^+}{\Delta t} = \frac{1}{2}\frac{q}{m}\mathbf{E}_k \tag{7}$$

which is often called the second half of the electric pulse.

Adding equations (5), (6) and (7) gives

$$\frac{\mathbf{v}_{k+1} - \mathbf{v}_k}{\Delta t} = \frac{q}{m}\left[\mathbf{E}_k + \frac{(\mathbf{v}^+ + \mathbf{v}^-)}{2} \times \mathbf{B}_k\right]$$

which is almost the discretized Lorentz equation (2) except that $(\mathbf{v}^+ + \mathbf{v}^-)$ is substituted for $\left(\mathbf{v}^{k+1} + \mathbf{v}^k\right)$. However, subtracting equation (5) from equation (7) gives $(\mathbf{v}^+ + \mathbf{v}^-) = \left(\mathbf{v}^{k+1} + \mathbf{v}^k\right)$, giving the discretized Lorentz equation (2). This means that the Boris algorithm is equivalent to the discretized Lorentz equation.

The equations (5), (6) and (7) can be written slightly different as

$$\begin{aligned} \mathbf{v}^- &= \mathbf{v}_k + q'\mathbf{E}_k \\ \mathbf{v}^+ &= \mathbf{v}^- + 2q'\left(\mathbf{v}^- \times \mathbf{B}_k\right) \\ \mathbf{v}_{k+1} &= \mathbf{v}^+ + q'\mathbf{E}_k \\ \mathbf{x}_{k+1} &= \mathbf{x}_k + \Delta t \;\; \mathbf{v}_{k+1} \end{aligned} \tag{8}$$

alongwith equation (4) where $q' = \frac{q}{m}\frac{\Delta t}{2}$. These equations are used to update the velocity of the particle under the influence of the Lorentz force under the Boris update strategy.

References

[1] Qin, H., Zhang, S., Xiao, J., & Tang, W. M. (April, 2013). *Why is Boris algorithm so good?*. Princeton Plasma Physics Laboratory, PPPL-4872.