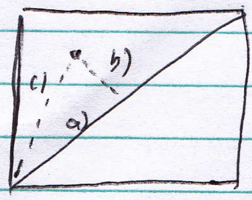
Mitic (2015) Improved GoF, p. 87, Herleitung

Tags: Euclidean Distance, length of vector
Euclidean norm
Pythagoras rule $a^2+b^2=c^2$ in triangle



b) $(x-x')^2 + (y-y')^2 = H = (x-y')^2 + (y-y')^2$

a) $x'^2 + y'^2 = 2y'^2$

c) $x^2 + y^2$

a) $= x^2 + y^2 - \left[(x-y')^2 + (y-y')^2 \right]$

$\Leftrightarrow x^2 + y^2 - \left[x^2 - 2xy' + y'^2 + y^2 - 2yy' + y'^2 \right]$

$\Leftrightarrow \cancel{x^2} + \cancel{y^2} - \cancel{x^2} + 2xy' - y'^2 - \cancel{y^2} + 2yy' - y'^2$

a) $= 2y'^2 = 2xy' - 2y'^2 + 2yy'$

$4y'^2 = 2xy' + 2yy' \quad |:y'$

$\frac{4y'\cdot\cancel{y'}}{\cancel{y'}} = 2x + 2y = 4y' \quad |:4$

$\frac{2x+2y}{4} = \frac{x+y}{2} = y'$

b) $= H = x^2 + y^2 - 2y'^2$

$= x^2 + y^2 - 2\cdot\left(\frac{x+y}{2}\right)^2$

$\Leftrightarrow x^2 + y^2 - 2\cdot\frac{(x^2+2xy+y^2)}{4}$

$\Leftrightarrow x^2 + y^2 - \frac{(x+y)^2}{2} = H^2 \quad |\cdot 2$

$2(x^2+y^2) - (x+y)^2 = 2H^2$

$\Leftrightarrow 2x^2 + 2y^2 - x^2 - 2xy - y^2 = 2H^2$

$\Leftrightarrow \underbrace{x^2 + y^2 - 2xy}_{\text{2. binomische Formel}} = (x-y)^2 = 2H^2 \quad |:2$

$\Leftrightarrow \frac{(x-y)^2}{2} = H^2 \quad |\sqrt{\ } \quad \Big| \quad \frac{|x-y|}{\sqrt{2}} = H$