

Algorithms – I
Tutorial 4
August 25th, 2017

1. In Free city of Braavos, they have a very strange monetary system. They have golden dragons (golden coins). Each of them has an integer number written on it. A coin n can also be exchanged in Iron bank into three kind of coins having values $n/2$, $n/3$ and $n/4$. But these numbers are all rounded down (the bank have to make profit). You can also sell golden dragons for American dollars. The exchange rate is 1:1. But you can not buy golden dragons. You have one gold coin of value $n > 0$. What is the maximum amount of American dollars you can get for it?

2. Harry doesn't like boredom. That's why whenever he gets bored, he comes up with games. One long winter evening he came up with a game and decided to play it. In this game, there are N integers on a board. In one move, he can pick any integer ' k ' from the board and then all the integers with value ' $k+1$ ' and ' $k-1$ ' gets destroyed. This move yields him ' k ' points. Harry is a perfectionist, so he decided to get as many points as possible. Could you help him out?

3. Arya and Sansa are playing a two player game. There is a pile having n stones. Players take turns removing stones from the piles, such that each move involves removing atmost k ($k > 1$) stones from the pile. The winner of the game is the player who removes the last stone from play. In other words, if there are no stones available at the start of a player's turn, they have lost the game. Arya starts first. The goal is to find whether Arya can guarantee her win if both players play optimally. Here, optimally means if a player can ensure his/her win, he/she will. Your algorithm should run in $O(nk)$ time.

4. Prove or disprove the following:



- a) The minimum value of any max-heap must be present in a leaf.
- b) The second minimum value of any max-heap must be present in a leaf.

5. To convert an array of size n to a heap, we discussed the makeHeap process. Suppose that instead of calling makeHeap on an array of size n , we insert the i -th element in a heap of size $i-1$ for $i = 1, 2, \dots, n-1$. Compare the performance of the heap-making procedure of Part (a) with the makeHeap procedure.

6. A d -ary heap is like a binary heap, but nonleaf nodes have d children instead of 2 children.

(a) How would you represent a d -ary heap in an array?

(b) What is the height of a d -ary heap of n elements in terms of n and d ?

(c) Give an efficient implementation of EXTRACT-MAX in a d -ary max-heap. Analyze its running time in terms of n and d .

7. Propose an algorithm for printing ' k ' largest elements in an array of ' n ' elements in $O(n + k \log n)$ time. Can you now make it more efficient by doing it in $O(n + k \log k)$ time?