CS21003 - Tutorial 11
November 10th, 2017

1. We are given a directed graph G(V, E) on which each edge (u,v) has an associated value r(u,v) , which is a real number in the range [0,1] that represents the reliability of a communication channel from vertex *u* to vertex *v*. We interpret r(u,v) as the probability that the channel from *u* to *v* will not fail, and we assume that these probabilities are independent. Give an efficient algorithm to find the most reliable path between two given vertices.

2. Prove or Disprove: Dijkstra's algorithm always produce shortest path on a directed graph with integer-valued edge weights.

3. Suppose that you are given a graph G = (V, E) and a its minimum spanning tree T. Suppose that we delete from G, one of the edges (u, v) ∈ T and let G' denote this new graph.

a) Is G' guaranteed to have a minimum spanning tree?
b) Assuming that G' has a minimum spanning tree T' . TRUE or FALSE: the number of edges in T' is no greater than the number of edges in T? Explain your answer in one sentence.
c) Assuming that G' has a minimum spanning tree T' , describe an algorithm for finding T'. What is the complexity of your algorithm?

4. Let G be an undirected connected weighted graph. Suppose the graph has at least one cycle (choose one). For that chosen cycle, let edge e be an edge that has strictly greater cost than all other edges in the cycle. (Such an edge might not exist, e.g. there might be two edges that have the same greatest cost). Show that e does not belong to any MST of G.
[Hint: proof by contradiction]

5. Consider a "reversed" Kruskal's algorithm for computing a MST. Initialize T to be the set of all edges in the graph. Now, consider edges from largest to smallest cost. For each edge, delete it from T if that edge belongs to a cycle in T. (Never mind how to implement this. Just note that union-find does not allow deletions, so an inefficient implementation of this reversed Kruskal is not obvious). Assuming all the edge costs are distinct, does this new algorithm correctly compute a MST?

6. Professor Gaedel has written a program he claims implements Dijsktra's algorithm. The program produces the distance and the parent for each vertex v in the graph. Assume your are given the graph and the output of the professor's program, i.e., the distance and the parent for each vertex V. Design an O(V+E) algorithm to determine whether the distance and the parent attributes match those of some shortest-path tree. You may assume that all edge weights are non-negative.

7. Consider a mixed unweighted graph G = (V, E) with both directed and undirected edges. Assume that initially there are no cycles in G which use only directed edges. Give an algorithm to assign direction to each of the undirected edges so that the completely directed graph so obtained has no cycles. Analyze the asymptotic complexity of the algorithm.
Hints: Use topological ordering of edges, O(V+E)