Algorithms I

Tutorial 8

November 4, 2016

Problem 1

Suppose that the graph $G = (V, E)$ is represented as an adjacency matrix. Give a simple implementation of Prim's algorithm for this case that runs in $O(V^2)$ time.

Problem 2

Give an example of a directed graph with negative-weight edges for which Dijkstra's algorithm produces incorrect answers.

Problem 3

Professor Gaedel has written a program he claims implements Dijsktra's algorithm. The program produces the distance and the parent for each vertex v in the graph. Assume your are given the graph and the output of the professor's program, i.e., the distance and the parent for each vertex v. Design an $O(V + E)$ algorithm to determine whether the distance and the parent attributes match those of some shortest-path tree. You may assume that all edge weights are non-negative

Problem 4

Let $G = (V, E)$ be a weighted graph and T be its shortest-path tree from source s. Assume all weights in G are increased by the same amount, i.e. $\forall e \in E, w'_e = w_e + c$. Is tree T still the shortest-path tree (from source s) of the modified graph? If yes, prove the statement. Otherwise, give a counter example.

Problem 5

Let $G = (V, E)$ be a weighted undirected graph. Let $s, t \in V$ and $s \neq t$. Design an $O(E \log V)$ algorithm to find all vertices v such that v lies on at least one of the shortest paths between s and t.

Problem 6

How can you improve the time complexity of Dijkstra's algorithm to $O(V^2)$ if the graph given is dense?

Problem 7

How can we use the Floyd-Warshall algorithm to detect the presence of a negative weighted cycle?

Problem 8

Describe an algorithm to find the length of the shortest cycle in a graph.

Problem 9

CLRS 25.2-5 (Alternative definition of predecessor matrix π)