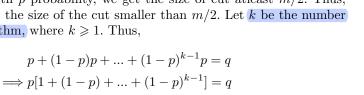
Tutorial Problem 5

Question- Number of runs of the basic algorithm of randomized max cut to get a fixed value of the probability that the expected cut size is at least |E|/2. **Answer-** Simple randomized algorithm outputs a cut of size at least |E|/2 with probability at least $\frac{1}{|E|+2}$.

Now, let q be the fixed probability that we intend to achieve. We define $p \ge \frac{1}{m+2}$, where |E| = m. With p probability, we get the size of cut at least m/2. Thus, with (1-p), we get the size of the cut smaller than m/2. Let k be the number of run by the algorithm, where $k \ge 1$. Thus,



$$\Rightarrow p\{1 + (1-p) + \dots + (1-p)\} = q$$

$$\Rightarrow p\frac{1 - (1-p)^k}{1 - (1-p)} = q$$

$$\Rightarrow 1 - (1-p)^k = q$$

$$\Rightarrow k = \frac{\log(1-q)}{\log(1-p)}$$

$$\Rightarrow k \geqslant \frac{\log(1-q)}{\log(1-\frac{1}{m+2})}$$

$$\Rightarrow k \geqslant \frac{\log(1-q)}{\log(\frac{m+1}{m+2})}$$

$$\Rightarrow k = \left[\frac{\log(1-q)}{\log(\frac{m+1}{m+2})}\right]$$

For example, let $q = \frac{1}{2}$ and = 1022. The required value of k is given bellow.

$$k = \left\lceil \frac{\log(1 - \frac{1}{2})}{\log(\frac{1022 + 1}{1022 + 2})} \right\rceil$$

$$\implies k = \left\lceil \frac{\log(\frac{1}{2})}{\log(\frac{1023}{1024})} \right\rceil$$

$$\implies k = \left\lceil 709.4 \right\rceil$$

$$\implies k = 710$$