

Gradient descent for multivariable problems

$$f(x, y) = x^2 + y^2 + 10$$

Step 1: let  $x = -1$ ,  $y = +1$ ,  $\eta = 0.1$ , epoch = ~~100~~ 2

Step 2:  $itr = 1$

Step 3:  $\frac{\partial f}{\partial x} = 2x = -2$

$$\frac{\partial f}{\partial y} = 2y = 2$$

Step 4:  $\Delta x = -\eta \frac{\partial f}{\partial x} = -0.1(-2) = 0.2$

$$\Delta y = -\eta \frac{\partial f}{\partial y} = -0.1(2) = -0.2$$

Step 5:  $x = x + \Delta x \Rightarrow -1 + 0.2 = -0.8$

$y = y + \Delta y \Rightarrow 1 - 0.2 = 0.8$

Step 6:  $itr = itr + 1 \Rightarrow 1 + 1 = 2$

Step 7: if ( $itr > epoch$ )

if ( $2 > 2$ )

false so go to step ③

Step 3:  $\frac{\partial f}{\partial x} = 2x = 2(-0.8) = -1.6$

$$\frac{\partial f}{\partial y} = 2y = 2(0.8) = 1.6$$

Step 4:  $\Delta x = -\eta \frac{\partial f}{\partial x} = -0.1(-1.6) = 0.16$

$$\Delta y = -\eta \frac{\partial f}{\partial y} = -0.1(1.6) = -0.16$$

Step 5:  $x = x + \Delta x = -0.8 + 0.16 = -0.64$

$$y = y + \Delta y = 0.8 - 0.16 = 0.64$$

Step 6:  $itr = itr + 1 \Rightarrow 2 + 1 = 3$

Step 7: if  $(3 > 2)$   
true so

$$x = -0.64, y = 0.64$$

$$f(x, y) = x^2 + y^2 + 10$$

$$= (-0.64)^2 + (0.64)^2 + 10$$

$$\therefore f(x, y) = 10.8192$$

Calculations for SGD

$x$	$y$
0.2	3.4
0.4	2.8

Step 1:  $(x, y)$ ,  $m=1$ ,  $C=-1$ ,  $\eta=0.1$  epochs = 2

Step 2:  $itr=1$

Step 3: Sample = 1

$$\text{Step 4: } \frac{\partial E}{\partial m} = -[3.4 - 1(0.2) + 1](0.2)$$

$$\frac{\partial E}{\partial m} = -0.84$$

$$\frac{\partial E}{\partial C} = -(3.4 - 1(0.2) + 1)$$

$$\frac{\partial E}{\partial C} = -4.2$$

$$\text{Step 5: } \Delta m = -\eta \frac{\partial E}{\partial m} = -0.1(-0.84)$$

$$\Delta m = 0.084$$

$$\Delta C = -\eta \frac{\partial E}{\partial C} = -0.1(-4.2)$$

$$\Delta C = 0.42$$

$$\text{Step 6: } m = m + \Delta m = 1 + 0.084$$

$$m = 1.084$$

$$C = C + \Delta C = -1 + 0.42$$

$$C = -0.58$$

$$\text{Step 7: } \text{sample} = 1 + 1 = 2$$

$$\text{Step 8: } \text{if } (\text{Sample} > n_s)$$

$$(2 > 2)$$

false go to step ④



$$\text{Step 4: } \frac{\partial E}{\partial m} = -[3.8 - (1.084)(0.4) + 0.58](0.4)$$

$$\frac{\partial E}{\partial m} = ~~-0.9436~~ -1.578$$

$$\frac{\partial E}{\partial c} = -[3.8 - (1.084)(0.4) + 0.58]$$

$$\frac{\partial E}{\partial c} = ~~-0.9436~~ -3.9464$$

$$\text{Step 5: } \Delta m = -\eta \frac{\partial E}{\partial m} = -0.1(-~~0.9436~~^{1.578})$$

$$\Delta m = ~~0.09436~~ 0.1578$$

$$\Delta c = -\eta \frac{\partial E}{\partial c} = -0.1(-3.9464)$$

$$\Delta c = ~~0.39464~~ 0.39464$$

$$\text{Step 6: } m = m + \Delta m = 1.084 + ~~0.09436~~ 0.1578$$

$$m = ~~1.084~~ 1.2418$$

$$c = c + \Delta c = -0.58 + 0.39464$$

$$c = ~~-0.34~~ -0.18536$$

$$\text{Step 7: } \text{Sample} = 2 + 1 = 3$$

$$\text{Step 8: } \text{if (sample} > n)$$

$$3 > 2$$

true so

$$\text{Step 9: } \text{itr} = \text{itr} + 1 = 1 + 1 = 2$$

$$\text{Step 10: } \text{if (itr} > \text{epoch)}$$

$$2 > 2$$

false

$$\text{Step 11: } \text{Sample} = 1$$

$$\text{Step 12: } \frac{\partial E}{\partial m} = -[\overset{3.4}{~~3.8~~} - 1.2418(3.4) + 0.18536](0.2)$$

$$\frac{\partial E}{\partial m} = 0.1273$$

$$\frac{\partial E}{\partial C} = - [3.4 - 1.2418(3.4) + 0.18536]$$

$$\frac{\partial E}{\partial C} = 0.6367$$

Step 5:  $\Delta m = -\eta \frac{\partial E}{\partial m} = -0.1(0.1273)$

$$\Delta m = -0.01273$$

$$\Delta C = -\eta \frac{\partial E}{\partial C} = -0.1(0.6367)$$

$$\Delta C = -0.06367$$

Step 6:  $m = m + \Delta m = 1.2418 + (-0.01273)$

$$m = 1.229$$

$$C = C + \Delta C = -0.18536 - 0.06367$$

$$C = -0.249$$

Step 7: Sample = 1 + 1 = 2

Step 8: if (Sample > ne)  
(2 > 2)

false

Step ④  $\frac{\partial E}{\partial m} = - [3.8 - (1.229)(0.4) + 0.249](0.4)$

$$\frac{\partial E}{\partial m} = -1.4229$$

$$\frac{\partial E}{\partial C} = - [3.8 - (1.229)(0.4) + 0.249]$$

$$\frac{\partial E}{\partial C} = -3.5574$$

Step ⑤:  $\Delta m = -\eta \frac{\partial E}{\partial m} = -0.1(-1.4229)$

$$\Delta m = 0.14229$$

$$\Delta C = -\eta \frac{\partial E}{\partial C} = -0.1(-3.5574)$$

$$\Delta C = 0.35574$$

Step ⑥:  $m = m + \Delta m = 1.229 + 0.14229$

$$m = 1.37129$$

$$C = C + \Delta C = -0.249 + 0.35574$$

$$C = 0.10674$$

Step 7:  $\text{sample} = 2 + 1 = 3$

Step 8: if ( $\text{Sample} > n_s$ )

$$3 > 2$$

true

Step 9:  $\text{itr} = 2 + 1 = 3$

step 10: if ( $\text{itr} > \text{epoch}$ )

$$3 > 2$$

Step 11:  $m = 1.37129$ ,  $C = 0.10674$

## Assignment - 5

### Calculations for MBGD

<u>x</u>	<u>y</u>	
0.2	3.4	} 1
0.4	3.8	
0.6	4.2	} 2
0.8	4.6	

1)  $(x, y)$ ,  $m=1$ ,  $c=-1$ ,  $\eta=0.1$  epochs = 2,  $b_s=2$

2)  $n_b = \frac{n_s}{b_s} = \frac{4}{2} = 2$

3)  $iter = 1$

4) Batch = 1

5)  $\frac{\partial E}{\partial m} = -\frac{1}{b_s} \sum_{i=1}^{b_s} (y_i - mx_i - c) x_i$

$$= -\frac{1}{2} \left[ (3.4 - (1)(0.2) - (-1)) 0.2 \right] + \left[ (3.8 - 0.4 + 1) 0.4 \right]$$
$$= -1.34$$

$$\frac{\partial E}{\partial c} = -\frac{1}{2} \left[ (3.4 - 0.2 + 1) + (3.8 - 0.4 + 1) \right]$$
$$= -4.3$$

6)  $\Delta m = -(0.1)(-1.34) = 0.134$   
 $\Delta c = -(0.1)(-4.3) = 0.43$

7)  $m = m + \Delta m = 1 + 0.134 = 1.134$

$$c = c + \Delta c = -1 + 0.43 = -0.57$$

8) Batch + 1

9) if  $(z > z)$   
false so



$$5) \frac{\partial E}{\partial m} = -\frac{1}{2} \left[ (4.2 - (1.1(0.6)) + 0.57) 0.6 + (4.6 - (1.134)(0.8) + 0.57) 0.8 \right]$$

$$= 2.932$$

$$\frac{\partial E}{\partial c} = -\frac{1}{2} \left[ (4.2 - (1.134)(0.6) + 0.57) + (4.6 - (1.134)(0.8) + 0.57) \right]$$

$$= -4.17$$

$$6) \Delta m = 0.2932$$

$$\Delta c = 0.417$$

$$7) m = 1.13 + 0.293 = 1.42$$

$$c = -0.57 + 0.4 = -0.15$$

$$8) \text{Batch} + = 1$$

$$9) \text{if (batch} > n1)$$

$$3 > 2$$

$$10) \text{itr} + = 1$$

$$11) \text{if (2} > 2)$$

$$\text{false}$$

$$4) \text{Batch} = 1$$

$$\frac{\partial E}{\partial m} = -\frac{1}{2} \left[ (3.4 - (1.4)(0.2) - (0.5) 0.2 + (3.8 - (1.4)(0.4) + 0.15) 0.4 \right]$$

$$= -1.0029$$

$$\frac{\partial E}{\partial c} = -\frac{1}{2} \left[ (3.4 - (1.42)(0.2) + 0.1523) + (3.8 - (1.4)(0.4) + 0.15) \right]$$

$$= -3.3241$$



$$6) \Delta m = -0.1 (-1.0029) = 0.1002$$

$$\Delta C = -0.1 (-3.3241) = 0.3324$$

$$7) m + \Delta m = 1.42 + 0.1002 = 1.5$$

$$C + \Delta C = -0.19 + 0.3 = 0.19$$

$$8) \text{Batch } t = 1$$

$$9) \text{if } (2 > 2)$$

$$5) \frac{\partial E}{\partial m} = -\frac{1}{2} [(4.2 - (1.9)(0.6) - 0.17) 0.6 + (4.6 - (1.9)(0.8) - 0.17) 0.8]$$

$$= -2.21$$

$$\frac{\partial E}{\partial C} = -3.19$$

$$6) \Delta m = -0.1 \times -2.21 = 0.221$$

$$\Delta C = -0.1 \times -3.19 = 0.319$$

$$7) m + \Delta m = 1.5 + 0.22 = 1.7$$

$$C + \Delta C = 0.19 + 0.3 = 0.4$$

$$8) \text{Batch } t = 1$$

$$a) \text{if } (\text{Batch} > nb)$$

$$b) \text{if } t = 1$$

$$11) \text{if } (3 > 2)$$

$$12) m = 1.768$$

$$C = 0.494$$

## Assignment - 7

### Calculation for BGD

X	Y
0.2	3.4
0.4	3.8
0.6	4.2
0.8	4.6

1)  $(x, y)$ ,  $m=1$ ,  $c=-1$ ,  $\eta=0.1$ ,  $\text{epochs}=2$ ,  $n_s=2$

2)  $\text{its}=1$

3)  $\frac{\partial E}{\partial m} = -\frac{1}{n_s} \sum_{i=1}^{n_s} (y_i - mx_i - c) x_i$

$$= -\frac{1}{2} \left[ (3.4 - (-1)(0.2) + 1) 0.2 + (3.8 - (-1)(0.4) + 1) 0.4 \right]$$

$$= -1.34$$

$$\frac{\partial E}{\partial c} = -\frac{1}{2} \left[ (3.4 - 0.2 + 1) + (3.8 - 0.4 + 1) \right]$$

$$= -4.3$$

4)  $\Delta m = -\eta \frac{\partial E}{\partial m}$

$$= -0.1 \times -1.34 = 0.134$$

$$\Delta c = -\eta \frac{\partial E}{\partial c}$$

$$= -0.1(-4.3) = 0.43$$

5)  $m \leftarrow \Delta m$

$$= 1 + 0.134 = 1.134$$

$$c \leftarrow \Delta c$$

$$= -1 + 0.43 = -0.57$$

6)  $\text{its} \leftarrow \text{its} + 1$

7) if  $(z > 2)$

$$3) \frac{\partial F}{\partial m} = -\frac{1}{2} \left[ (3.4 - (1.134)(0.2) + 0.57)(0.2) + (3.8 - (1.134)(0.4) + 0.57)(0.4) \right]$$

$$= -1.157$$

$$\frac{\partial F}{\partial C} = -\frac{1}{2} \left[ 3.4 - (1.134)(0.2) + 0.57 + 3.8 - (1.134)(0.4) + 0.57 \right]$$

$$= -3.829$$

$$4) \Delta m = -0.1 \times 1.15 = 0.1157$$

$$\Delta C = -0.1 \times -3.8 = 0.3829$$

$$5) m + \Delta m = 1.134 + 0.1157 = 1.2497$$

$$C + \Delta C = -0.57 + 0.3829 = -0.187$$

$$6) i_{\text{for}} = 1$$

$$7) \text{ if } (i_{\text{for}} > \text{epoch})$$

$$3 > 2$$

$$8) m = 1.24$$

$$C = -0.187$$

Momentum optimizer

X	Y
0.2	3.4
0.4	3.8

step 1:  $[x, y]$ ,  $m=1$ ,  $C=-1$ ,  $\eta=0.1$  epochs = 2,  $\delta=0.9$

$$v_m = v_c = 0$$

Step 2:  $itr = 1$

Step 3: sample = 1

$$\begin{aligned} \text{step 4: } g_m &= \frac{\partial E}{\partial m} = -(y_i - mx_i - C)x_i \\ &= -(3.4 - 1(0.2) + 1)(0.2) \\ \frac{\partial E}{\partial m} &= -0.84 \end{aligned}$$

$$\begin{aligned} g_c &= \frac{\partial E}{\partial C} = -(3.4 - 1(0.2) + 1) \\ \frac{\partial E}{\partial C} &= -4.2 \end{aligned}$$

$$\begin{aligned} \text{Step 5: } v_m &= \delta v_m - \eta g_m \\ &= (0.9)(0) - (0.1)(-0.84) \end{aligned}$$

$$v_m = 0.084$$

$$\begin{aligned} v_c &= \delta v_c - \eta g_c \\ &= (0.9)(0) - (0.1)(-4.2) \end{aligned}$$

$$v_c = 0.42$$

$$\begin{aligned} \text{Step 6: } m &= m + v_m = 1 + 0.084 = 1.084 \\ C &= C + v_c = -1 + 0.42 = -0.58 \end{aligned}$$

Step 7: sample = 1 + 1 = 2

Step 8: If (sample  $\geq$  ns)  
2  $\geq$  2



Step 4:  $g_m = \frac{\partial E}{\partial m} = - [3.8 - (1.084)(0.4) + 0.58](0.4)$

$$g_m = -1.57856$$

$$g_c = \frac{\partial E}{\partial c} = - [3.8 - (1.084)(0.4) + 0.58]$$

$$g_c = \frac{\partial E}{\partial c} = ~~-1.57856~~ -3.9464$$

Step 5:  $v_m = \delta v_m - \eta g_m$

$$= (0.9)(0.084) - (0.1)(-1.57856)$$

$$v_m = 0.233456$$

$$v_c = \delta v_c - \eta g_c$$

$$= (0.9)(0.42) - (0.1)(-3.9464)$$

$$v_c = 0.77264$$

Step 6:  $m = m + v_m = 1.084 + 0.233456$

$$m = 1.3174$$

$$c = c + v_c = -0.58 + 0.77264$$

$$c = 0.19264$$

Step 7: Sample = 2 + 1 = 3

Step 8: if (3 > 2)

Step 9: i/r = i/r + 1 = 1 + 1 = 2

Step 10: if (2 > 2)

Step 3: Sample = 1

Step 4:  $g_m = \frac{\partial E}{\partial m} = - [3.4 - (1.3174)(0.2) - 0.19264]$

$$g_m = -0.5887$$

$$g_c = \frac{\partial F}{\partial c} = -[3.4 - (1.3174)(0.2) - 0.19264]$$

$$g_c = -2.9438$$

Step 5:

$$V_m = \delta V_m - \eta g_m$$

$$= (0.9)(0.23345) - (0.1)(-0.5887)$$

$$V_m = 0.26897$$

$$V_c = \delta V_c - \eta g_c$$

$$= (0.9)(0.77264) - (0.1)(-2.9438)$$

$$V_c = 0.9897$$

Step 6:  $m = m + V_m = 1.3174 + 0.26897$

$$m = 1.5863$$

$$c = c + V_c = 0.19264 + 0.9897$$

$$c = 1.18234$$

Step 7: Sample = 1 + 1 = 2

Step 8: If (Sample > 10)  
 $z > 2$

Step 4:  $g_m = \frac{\partial F}{\partial m} = -[3.8 - (1.586)(0.4) + 1.1823](0.4)$

$$g_m = -1.7391$$

$$g_c = \frac{\partial F}{\partial c} = -[3.8 - (1.586)(0.4) + 1.1823]$$

$$g_c = -4.3479$$

Step 5:  $V_m = \delta V_m - \eta g_m$

$$= (0.9)(0.26897) - (0.1)(-1.7391)$$

$$V_m = 0.41604$$

$$V_c = \delta V_c - \eta g_c$$

$$= (0.9)(0.9897) - (0.1)(-4.3479)$$

$$V_c = 1.3255$$

Step 6:  $m = m + V_m = 1.5863 + 0.41604$

$$m = 2.0023$$

$$C = C + V_c = 1.18234 + 1.3255$$

$$C = 2.507$$

Step 7: Sample =  $2 + 1 = 3$

Step 8: if ( $3 > 2$ )

Step 9: itr =  $2 + 1 = 3$

Step 10: if ( $3 > 2$ )

Step 11:  $m = 2.00, C = 2.507$