

Manual calculations of ADAGRAD:

Step 1: $[x, y], \eta = 0.1, \text{epochs} = 1, m = 1, c = -1, \epsilon = 10^{-8}$

$$g_m = 0, g_c = 0$$

Step 2: iter = 1

Step 3: Sample = 1

$$\text{Step 4: } g_m = -[y_i - mx_i - c]x_i$$

$$\Rightarrow -[3.4 - (1 \times 0.2) + 1] \times 0.2$$

$$= -[3.4 - 0.2 + 1] \times 0.2 = [4.2] \times 0.2 \Rightarrow 0.84$$

$$g_c = -[4.2]$$

$$\text{Step 5: } G_m = G_m + (g_m)^2 = 0 + (0.84)^2 = 0.7056$$

$$G_c = G_c + (g_c)^2 = 0 + (4.2)^2 = 17.64$$

$$\text{Step 6: } \Delta m = \frac{-0.1}{\sqrt{0.7056 + 10^{-8}}} \times (-0.84) = 0.09999$$

$$\Delta c = \frac{-0.1}{\sqrt{17.64 + 10^{-8}}} \times (-4.2) = 0.09999$$

$$\text{Step 7: } m = m + \Delta m = (1 + 0.09999) = 1.09999$$

$$c = c + \Delta c = (-1 + 0.09999) = -0.90001$$

$$\text{Step 8: } \text{Sample} = \text{sample} + 1 = 1 + 1 = 2$$

Step 9: $2 > 2 = \text{false}$

go to step 4

Step 10: $q_m = -[y_i - m x_i - c] x_i$

$$= -[3.8 - (1 \times 1.9999) + 0.001] \times 0.4$$

$$= (-1.8011) \times 0.4 \Rightarrow -0.72044$$

$$q_c = -1.8011$$

$$\text{Step 11: } G_m = G_m + (q_m)^2 = 0.7056 + 0.5190 = 1.2246$$

$$G_c = G_c + (q_c)^2 = 17.64 + 3.2439 = 20.8839$$

$$\text{Step 12: } \Delta m = \frac{-0.1}{\sqrt{1.2246 + 10^8}} \times (-0.72044) = 0.065702$$

$$\Delta c = \frac{-0.1}{\sqrt{20.8839 + 10^8}} \times (-1.8011) = 0.03941$$

Step 13:

$$m = 1.9999 + 0.065702 = 2.0650$$

$$c = -0.001 + 0.3941 = 0.3931$$

Step 14: $\text{Sample} = \text{Sample} + 1 = 2 + 1 = 3 > 2$ true

Go to step 15

Step 15: $\text{iter} = \text{iter} + 1 = 1 + 1 = 2$

Step 16: $\text{iter} > \text{epochs} = 2 > 2 = \text{false}$
Go to step 17

Step 17: $\text{sample} = 1$

Step 18: $g_m = -[3.4 - (2.0650 \times 0.2) - 0.3931] \times 0.2$

$$g_m = -[2.5939] \times 0.2 = -0.5187$$

$$g_c = -2.5939$$

Step 19: $G_m = g_m + (g_m)^2 = 1.2246 + 0.2690 = 1.4936$

$$G_c = (g_c) + (g_c)^2 = 20.8839 + 6.7283 = 27.6122$$

Step 20: $\Delta m = \frac{-0.1}{\sqrt{1.4936 + 10^{-8}}} \times (-0.5187) = 0.01789$

$$\Delta c = \frac{-0.1}{\sqrt{27.6122 + 10^{-8}}} \times (-2.5939) = 0.04936$$

Step 21: $m = m + \Delta m = 2.0650 + 0.01789 = 2.08289$

$$c = c + \Delta c = 0.3931 + 0.04936 = 0.44246$$

Step 22: ~~Sample~~ $\text{Sample} = \text{sample} + 1 = 1 + 1 = 2 > 2$ false

Go to step 23.

$$\begin{aligned}\text{Step 23: } \hat{f}_m &= -[3.8 - (2.08289 \times 0.4) - 0.44246] \times 0.4 \\ &= -[2.5243] \times 0.4 = -1.00972\end{aligned}$$

$$q_c = -2.5243$$

$$\begin{aligned}\text{Step 24: } G_m &= G_m + (q_m)^2 = 1.4936 + (-1.00972)^2 \\ &= 2.5131\end{aligned}$$

$$\begin{aligned}G_c &= G_c + (q_c)^2 = 27.6129 + (-2.5243)^2 \\ &= 33.9842\end{aligned}$$

$$\text{Step 25: } \Delta m = \frac{-0.1}{\sqrt{(2.5131 \times 10^{-8})}} \times (-1.00972)$$

$$= 0.06369$$

$$\Delta c = \frac{-0.1}{\sqrt{33.9842 \times 10^{-8}}} \times (-2.5243)$$

$$= 0.0433$$

$$\text{Step 26: } m = m + \Delta m = 2.08289 + 0.06369 = 2.14658$$

$$C = C + \Delta C = 0.44246 + 0.0433 = 0.48576$$

$$\text{Step 27: } \text{Sample} = \text{Sample} + 1 = 2 + 1 = 3 \text{ no. of samples}$$

Go to next step.

$$\text{Step 28: } \text{iter} = \text{iter} + 1 = 2 + 1 = 3 > \text{epochs}$$

Go to next step.

Step 29: Print (m, c)

Step 30: Calculate mean square error.

$$= \frac{1}{2 \times 2} \sum (y_i - \hat{y}_i)^2 = \frac{1}{4} \left[(3.4 - (2.14658 \times 0.2) - 0.4876)^2 \right. \\ \left. + (3.8 - (2.14658 \times 0.4) - 0.48576)^2 \right]$$

$$MSE = \underline{\underline{3.05121}}$$