

Assignment -11 :

Let consider a sample dataset have one input (x_i^a) and one output (y_i^a), and number of samples 4.

Develop a simple linear regression model using Nesterov Accelerated Gradient (NAG) optimizer.

Sample(i)	x_i^a	y_i^a
1	0.2	3.4
2	0.4	3.8
3	0.6	4.2
4	0.8	4.6

→ Do manual calculations for 2 iterations with first two samples.

Step 1: $[x, y]$, $m=1$, $c=-1$, $\eta=0.1$, epochs = 2, $\gamma=0.9$,
 $v_m = v_c = 0$, $ns=2$

Step 2: iter = 1

Step 3: sample = 1

$$\begin{aligned}
 \text{Step 4: } g_m &= \frac{\partial E}{\partial m} = - (y_i - (m + \gamma v_m) x_i - (c + \gamma v_c)) x_i \\
 &= - (3.4 - (1 + (0.9)(0)) 0.2 - (-1 + (0.9)(0))) (0.2) \\
 &= - (3.2 + 1) 0.2 \\
 g_m &= -0.84
 \end{aligned}$$

$$g_c = \frac{\partial E}{\partial c} = -(y_i - (m + \gamma v_m) x_i - (c + \gamma v_c))$$

$$= -(3.4 - (1 + (0.9)(0)) 0.2 - (-1 + (0.9)(0)))$$

$$g_c = -4.2$$

Step 5: $v_m = \gamma v_m - \eta g_m$

$$= (0.9)(0) - (-0.1)(-0.84)$$

$$v_m = -0.084$$

$$v_c = \gamma v_c - \eta g_c$$

$$= (0.9)(0) - (-0.1)(-4.2)$$

$$v_c = -0.42$$

Step 6: $m = m + v_m = 1 + (-0.084) = 0.916$

$$c = c + v_c = -1 - 0.42 = -1.42$$

Step 7: $\text{sample} = \text{sample} + 1 = 1 + 1 = 2$

Step 8: if (sample > ns)
 $2 > 2 \rightarrow \text{false}$
 goto step 4

Step 4: $g_m = \frac{\partial E}{\partial m} = -(3.8 - (0.916)(0.4) - (-1.42 + (0.9)(-0.084)) 0.4$

$$- (-1.42 + (0.9)(-0.42))) 0.4$$

$$= -(3.463 + 1.798) 0.4$$

$$g_m = -2.104$$

$$g_c = \frac{\partial E}{\partial c} = - (3.8 - (0.916 + (0.9)(-0.084)(0.4) - (-1.42 + (0.9)(-0.42)))$$

$$= - (2.914 + 1.798)$$

$$g_c = -4.712$$

Step 5: $v_m = \gamma v_m - \eta g_m$

$$= (0.9)(-0.084) - (-0.1)(-2.104)$$

$$v_m = -0.286$$

$$v_c = \gamma v_c - \eta g_c$$

$$= (0.9)(-0.42) - (-0.1)(-4.712)$$

$$v_c = -0.849$$

Step 6: $m = m + v_m = 0.916 - 0.286 = 0.63$

$$c = c + v_c = -1.42 - 0.849 = -2.269$$

Step 7: $\text{sample} = \text{sample} + 1 = 2 + 1 = 3$

Step 8: if (sample > ns)

$$3 > 2 \rightarrow \text{True}$$

goto next step

Step 9: $\text{iter} = \text{iter} + 1 = 1 + 1 = 2$

Step 10: if (iter > epochs)

$$2 > 2 \text{ false}$$

goto step 3

Step 3: $\text{sample} = 1$

$$\text{Step 4: } g_m = \frac{\partial E}{\partial m} = - \left(3.4 - (0.63 + (0.9)(-0.286))(0.2) \right. \\ \left. - (-2.269 + (0.9)(-0.849)) \right) 0.2$$

$$= - (3.325 - (-3.0331)) 0.2$$

$$g_m = -1.271$$

$$g_c = \frac{\partial E}{\partial c} = - \left(3.4 - (0.63 + (0.9)(-0.286))(0.2) \right. \\ \left. - (-2.269 + (0.9)(-0.849)) \right)$$

$$g_c = -6.3581$$

$$\text{Step 5: } v_m = \delta v_m - \eta g_m \\ = (0.9)(-0.286) - (-0.1)(-1.271)$$

$$v_m = -0.384$$

$$v_c = \delta v_c - \eta g_c \\ = (0.9)(-0.849) - (-0.1)(-6.3581)$$

$$v_c = -1.399$$

$$\text{Step 6: } m = m + v_m = 0.63 - 0.384 = 0.246$$

$$c = c + v_c = -2.269 - 1.399 = -3.668$$

$$\text{Step 7: } \text{sample} = \text{sample} + 1 = 1 + 1 = 2$$

if (sample > ns)

~~a~~ > 2 ~~True~~ False

goto step 4

Step 4:

$$g_m = \frac{\partial E}{\partial m} = - \left[3.8 - (0.246 + (0.9)(-0.384))(0.4) - (-3.668 + (0.9)(-1.399)) \right] (0.4)$$

$$= - \left[3.839 - (-4.927) \right] (0.4)$$

$$g_m = -3.506$$

$$g_c = \frac{\partial E}{\partial c} = - \left[3.8 - (0.246 + (0.9)(-0.384))(0.4) - (-3.668 + (0.9)(-1.399)) \right] (0.4)$$

$$g_c = -8.766$$

$$\text{Step 5: } V_m = \eta V_m - \eta g_m = (0.9)(-0.384) - (-0.1)(-3.506)$$

$$V_m = -0.696$$

$$V_c = \eta V_c - \eta g_c = (0.9)(-1.399) - (-0.1)(-8.766)$$

$$V_c = -2.1357$$

$$\text{Step 6: } m = m + V_m = 0.246 + (-0.696)$$

$$m = -0.45$$

$$c = c + V_c = -3.668 + (-2.1357)$$

$$c = -5.803$$

$$\text{Step 7: } \text{sample} = \text{sample} + 1$$

$$= 2 + 1$$

$$= 3$$

Step 8: if (sample > ns)
 $3 > 2 \rightarrow \text{True}$
goto next step

Step 9: iter = iter + 1
 $= 2 + 1 = 3$

Step 10: if (iter > epochs)
 $3 > 2 \rightarrow \text{True}$
goto next step

Step 11 :- print m, c values.

$$m = -0.45$$

$$c = \underline{\underline{-5.803}}$$