

Assignment - 9:

Let consider a sample dataset have one input (x_i^a) and one output (y_i^a), and a number of samples 4. Develop a simple linear regression model using momentum optimizer.

Sample(i)	x_i^a	y_i^a
1	0.2	3.4
2	0.4	3.8
3	0.6	4.2
4	0.8	4.6

→ Do manual calculations for 2 iterations with first two samples.

Step 1:- $[x, y]$, $m=1$, $c=-1$, $\eta=0.1$, epochs=2, $\gamma=0.9$,
 $v_m = v_c = 0$, $ns=2$

Step 2: iter=1

Step 3: sample=1

$$\begin{aligned} \text{Step 4: } g_m &= \frac{\partial E}{\partial m} = -(y_i - mx_i - c) x_i \\ &= -(3.4 - (1)(0.2) + 1)(0.2) \\ g_m &= -0.84 \end{aligned}$$

$$\begin{aligned} g_c &= \frac{\partial E}{\partial c} = -(y_i - mx_i - c) \\ &= -(3.4 - (1)(0.2) + 1) \\ g_c &= -4.2 \end{aligned}$$

$$\begin{aligned}\text{Step 5: } v_m &= \gamma v_m - \eta g_m \\ &= (0.9)(0) - (-0.1)(-0.84) \\ v_m &= -0.084\end{aligned}$$

$$\begin{aligned}v_c &= \gamma v_c - \eta g_c \\ &= (0.9)(0) - (-0.1)(-4.2) \\ v_c &= -0.42\end{aligned}$$

$$\begin{aligned}\text{Step 6: } m &= m + v_m = 1 + (-0.084) = 0.916 \\ c &= c + v_c = -1 - 0.42 = -1.42\end{aligned}$$

$$\text{Step 7: } \text{sample} = \text{sample} + 1 = 1 + 1 = 2$$

$$\begin{aligned}\text{Step 8: } &\text{if (sample} > \text{ns)} \\ &2 > 2 \rightarrow \text{false} \\ &\text{goto step 4.}\end{aligned}$$

$$\begin{aligned}\text{Step 4: } g_m &= \frac{\partial E}{\partial m} = - (3.8 - (0.916)(0.4) + 1.42)(0.4) \\ g_m &= -1.941\end{aligned}$$

$$\begin{aligned}g_c &= \frac{\partial E}{\partial c} = - (3.8 - (0.916)(0.4) + 1.42) \\ g_c &= -4.853\end{aligned}$$

$$\begin{aligned}\text{Step 5: } v_m &= \gamma v_m - \eta g_m \\ &= (0.9)(-0.084) - (-0.1)(-1.941) \\ v_m &= -0.269\end{aligned}$$

$$\begin{aligned}v_c &= \gamma v_c - \eta g_c \\ &= (0.9)(-0.42) - (-0.1)(-4.853) \\ v_c &= -0.863\end{aligned}$$

$$\text{Step 6: } m = m + v_m = 0.916 + (-0.269)$$

$$m = 0.647$$

$$c = c + v_c = -1.42 - 0.863 = -2.283$$

$$\text{Step 7: } \text{sample} = \text{sample} + 1 = 2 + 1 = 3$$

$$\text{Step 8: } \text{if (sample} > \text{ns)}$$

$$3 > 2 \text{ True}$$

\Rightarrow goto next step

$$\text{Step 9: } \text{iter} = \text{iter} + 1 = 1 + 1 = 2$$

$$\text{Step 10: } \text{if (iter} > \text{epochs)}$$

$$2 > 2 \rightarrow \text{false}$$

goto step 3

$$\text{Step 3: } \text{sample} = 1$$

$$\text{Step 4: } g_m = \frac{\partial E}{\partial m} = - (3.4 - (0.647)(0.2) + 2.283)(0.2)$$

$$g_m = -1.110$$

$$g_c = \frac{\partial E}{\partial c} = - (3.4 - (0.647)(0.2) + 2.283)$$

$$g_c = -5.553$$

$$\text{Step 5: } v_m = \eta v_m - \eta g_m$$

$$= (0.9)(-0.269) - (-0.1)(-1.110)$$

$$v_m = -0.353$$

$$v_c = \eta v_c - \eta g_c$$

$$v_c = (0.9)(-0.863) - (-0.1)(-5.553)$$

$$v_c = -1.332$$

$$\text{Step 6: } m = m + v_m = 0.6463 + (-0.353)$$

$$m = 0.293$$

$$c = c + v_c = -2.283 - 1.332$$

$$c = -3.615$$

$$\text{Step 7: } \text{sample} = \text{sample} + 1 = 1 + 1 = 2$$

$$\text{Step 8: } \text{if}(\text{sample} > n_s)$$

$$2 > 2 \quad \text{false}$$

$$\text{goto step 4}$$

$$\text{Step 4: } g_m = \frac{\partial E}{\partial m} = -(3.8 - (+0.293)(0.4) + 3.615)(0.4)$$

$$g_m = -2.919$$

$$g_c = \frac{\partial E}{\partial c} = -(3.8 - (0.293)(0.4) + 3.615)$$

$$g_c = -7.297$$

$$\text{Step 5: } v_m = \eta v_m - \eta g_m$$

$$= (0.9)(-0.353) - (-0.1)(-2.919)$$

$$v_m = -0.609$$

$$v_c = \eta v_c - \eta g_c$$

$$= (0.9)(-1.332) - (-0.1)(-7.297)$$

$$v_c = -1.9285$$

$$\text{Step 6: } m = m + v_m$$

$$m = 0.293 - 0.609$$

$$m = -0.316$$

$$c = c + \frac{V_c}{\Delta} = -3.615 - 1.9285$$

$$c = -5.5435$$

$$\text{Step 7: sample} = \text{sample} + 1 = 2 + 1 = 3$$

Step 8 :- if (sample > ns)

$$3 > 2 \text{ True}$$

⇒ goto next step

else goto step 4

$$\text{Step 9: iter} = \text{iter} + 1$$

$$= 2 + 1 = 3$$

Step 10: if (iter > epochs)

$$3 > 2 \text{ True}$$

goto step 11

Step 11: print m, c values

$$m = -0.316, c = -5.543$$
