

Let us consider a sample dataset have 1 input (x_i) and one output (y_i) and number of samples 4. Develop a simple linear regression model using momentum optimiser.

Sample (i)	x_i^a	y_i^a
1	0.2	3.4
2	0.4	3.8
3	0.6	4.2
4	0.8	4.6

• Do manual calculations for 2 iterations with 1st 2 samples.

Step-1: $[x, y]$ $m=1$, $c=-1$, $\eta=0.1$, epochs = 2, $\beta=0.9$, $v_m = v_c = 0$, $n_s = 2$

Step-2: $itr=1$

Step-3: Sample = 1

$$\begin{aligned} \text{Step-4: } g_m &= \frac{\partial E}{\partial m} = -(y_i - mx_i - c)x_i \\ &= -(3.4 - (1)(0.2) + 1)(0.2) \\ &= -0.84 \end{aligned}$$

$$\begin{aligned} g_c &= \frac{\partial E}{\partial c} = -(y_i - mx_i - c) \\ &= -(3.4 - 0.2 + 1) \\ &= -4.2 \end{aligned}$$

$$\begin{aligned} \text{Step-5: } v_m &= \beta v_m + \eta g_m \\ &= (0.9)0 + (0.1)(-0.84) \\ &= 0 - 0.084 \\ &= -0.084 \\ v_c &= \beta v_c + \eta g_c \\ &= 0.9 \times 0 + (-0.1)(-4.2) \\ &= -0.42 \end{aligned}$$

$$\begin{aligned} \text{Step-6: } m &= m + v_m \\ &= 1 + (-0.084) \\ &= 0.916 \\ c &= c + v_c \\ &= -1 - 0.42 \\ &= -1.42 \end{aligned}$$

(2)

Step-7: sample += 1

$$1+1=2$$

Step-8: if (sample > n_s)

goto step-9

2 > 2

else

goto step-4

$$\text{Step-9: } g_m = \frac{\partial E}{\partial m} = -(3.8 - 10.916)(0.4) + 1.12(0.4)$$

$$= -1.941$$

$$g_c = \frac{\partial E}{\partial c} = -4.853$$

$$\text{Step-5: } v_m = \delta v_m - \eta g_m$$

$$= (0.9)(-0.084) - [-0.1 \times -1.941]$$

$$= -0.2697$$

$$v_c = \delta v_c - \eta g_c$$

$$= (0.9)(-0.42) - [-0.1 \times -4.853]$$

$$= -0.863$$

$$\text{Step-6: } m = m + v_m$$

$$= 0.916 + (-0.2697)$$

$$= 0.6463$$

$$\text{Step-7: } c = c + v_c$$

$$= -1.42 - 0.863$$

$$= -2.283$$

$$\text{Step-7: } \text{sample} = \text{sample} + 1$$

$$= 2 + 1 = 3$$

Step-8: if (sample > n_s)

goto step-9

else

goto step-4

$$\text{Step-9: } \text{itr} += 1$$

$$1+1=2$$

Step-10: if (itr > epochs)

goto step-4

else

goto step-3

Step-3: sample = 1

(2)

$$\text{Step-4: } q_m = \frac{\partial E}{\partial m} = -(3.4 - (0.646)(0.2) + 2.283)(0.2) \\ = -1.110$$

$$q_c = \frac{\partial E}{\partial c} = -(3.4 - (0.646)(0.2) + 2.283) \\ = -5.553$$

$$\text{Step-5: } v_m = \eta v_m - \eta q_m \\ = (0.9)(-0.2697) - [-0.1 \times -1.110] \\ = -0.353$$

$$v_c = \eta v_c - \eta q_c \\ = (0.9)(-0.863) - [-0.1 \times -5.53] \\ = -1.332$$

$$\text{Step-6: } m = m + v_m \\ = 0.6463 + (-0.353) \\ = 0.293 \\ c = c + v_c \\ = -2.283 - 1.332 \\ = -3.615$$

Step-7: sample + 1
1 + 1 = 2

Step-8: if (sample > ns)
2 > 2
goto step-9
else
goto step-4

$$\text{Step-4: } q_m = -(3.8 - (0.293)(0.4) + 3.615)(0.4) \\ = -2.919 \\ q_c = -(3.8 - (0.293)(0.4) + 3.615) \\ = -7.297$$

$$\text{Step-5: } v_m = (0.9)(-0.353) - [-0.1 \times -2.919] \\ = ~~-0.353~~ = 0.6096 \\ v_c = (0.9)(-1.332) - [-0.1 \times -7.297] \\ = -1.9285$$

Step-6: $m+ = 2m$

$$0.293 - 0.609 = -0.316$$

$$c+ = 2c$$

$$-3.615 - 1.928 = -5.543$$

Step-7: sample + 1 = 1

$$2 + 1 = 3$$

Step-8: if (sample > ns)

goto step-9

else

goto step-4

Step-9: ite + 1

$$2 + 1 = 3$$

Step-10: if (ite > epochs)

goto step-11

else

goto step-3

Step-11: print m, c

$$m = -0.316, c = -5.543$$