

Find the global minimum point and value for the function.

$$f(x, y) = x^2 + y^2 + 10$$

- Do manual calculations for two iterations.
- Find the optimal solution using python programming.

Step-1: $x = -1$ $y = +1$ $\eta = 0.1$ $epochs = 2$

Step-2: $itr = 1$

Step-3: $\frac{\partial f}{\partial x} = 2x = -2$

$$\frac{\partial f}{\partial y} = 2y = 2$$

Step-4: $\Delta x = -\eta \frac{\partial f}{\partial x} = -2(-0.1)$
 $= 0.2$

$$\Delta y = -\eta \frac{\partial f}{\partial y} = -(0.1)(2) = -0.2$$

Step-5: $x = x + \Delta x = -1 + 0.2 = -0.8$

$$y = y + \Delta y = 1 - 0.2 = 0.8$$

Step-6: $itr = itr + 1$
 $= 1 + 1 = 2$

Step-7: if ($itr > epochs$)
 goto step-5
 else
 goto step-3

Step-3: $\frac{\partial f}{\partial x} = 2x = 2(-0.8) = -1.6$

$$\frac{\partial f}{\partial y} = 2y = 2(0.8) = 1.6$$

Step-4: $\Delta x = -\eta \frac{\partial f}{\partial x}$
 $= -(0.1)(-1.6) = 0.16$


$$\Delta y = -\eta \frac{\partial f}{\partial y}$$

 $= -(0.1)(1.6) = -0.16$

Step-5: $x = x + \Delta x$

$$= -0.8 + 0.16$$

$$= -0.64$$

 $y = y + \Delta y$

$$= 0.8 - 0.16$$

$$= 0.64$$

Step-6: $itr = itr + 1$

$$= 2 + 1 = 3$$

Step-7: if ($itr > epochs$)

$$3 > 2$$

goto step-8

else

goto step-3

Step-8: $x = -0.64$

$$y = 0.64$$

$$f(x, y) = x^2 + y^2 + 10$$

$$= (-0.64)^2 + (0.64)^2 + 10$$

$$= 0.4 + 0.4 + 10$$

$$= 10.8$$