

1) Find global minimum point and value for the function $f(x) = x^4 + 3x^2 + 10$

$$f(x) = x^4 + 3x^2 + 10$$

Let $x = 2$ $\eta = 0.01$ (learning rate)

$$\frac{\partial f(x)}{\partial x} = 4x^3 + 6x$$

\Rightarrow for 1 iteration: $\left. \frac{\partial f(x)}{\partial x} \right|_{x=2} = 4(2)^3 + 6(2)$
 $= 32 + 12 = 44$

$$\Delta x = -\eta \times \frac{\partial f(x)}{\partial x}$$

$$\Delta x = -(0.01)(44) = -0.44$$

$$\boxed{x = x + \Delta x}$$

$$x = 2 - 0.44 = 1.56$$

\Rightarrow for 2 iteration:

$$\left. \frac{\partial f(x)}{\partial x} \right|_{x=1.56} = 4(1.56)^3 + 6(1.56)$$
$$= 24.54$$

$$\Delta x = -\eta \times \frac{\partial f(x)}{\partial x}$$

$$\Delta x = -(0.01)(24.54) = -0.2454$$

$$\boxed{x = x + \Delta x}$$

$$x = 1.56 - 0.24 \Rightarrow x = 1.314$$

Repeats until the gradient is close to zero.