

ASSIGNMENT-9

Let consider a sample dataset have one input (x_i^a) and one output (y_i^a) and number of samples 4. Develop a simple linear regression model using momentum optimizer.

Sample(i)	x_i^a	y_i^a
1	0.2	3.4
2	0.4	3.8
3	0.6	4.2
4	0.8	4.6

→ Do Manual calculations for two iterations with first two samples

Step 1:- $[x, y]$, $m=1$, $c=-1$, epochs=100,
 $\eta=0.1$, $\gamma=0.9$, $V_m=V_c=0$

Step 2: iter = 1

Step 3: sample = 1

[Note: considering stochastic Gradient descent.]

$$\text{step 4: } g_m = \frac{\partial E}{\partial m} = -(y_i - mx_i - c) x_i$$

$$= -(3.4 - 1(0.2) + 1) 0.2 = -0.84$$

$$g_c = \frac{\partial E}{\partial c} = -(y_i - mx_i - c)$$

$$= -(3.4 - 0.2 + 1) = -4.2$$

$$\text{step 5: } \Delta m = \eta \Delta V_m - \eta g_m$$

$$= (0.9) 0 - (-0.1) (-0.84)$$

$$= 0 - 0.084$$

$$\boxed{\Delta m = -0.084}$$

$$\Delta c = \eta \Delta V_c - \eta g_c = 0.9 \times 0 - (-0.1) (-4.2)$$

$$\boxed{\Delta c = -0.42}$$

$$\text{step 6: } m = m + \Delta m = 1 + (-0.084) = 0.916$$

$$c = c + \Delta c = -1 - 0.42 = -1.42$$

$$\text{step 7: } \text{sample} + 1 = 1$$

$$= 1 + 1 = 2$$

step 8: if (sample > ns) goto step 9

else: goto step 4.

$$\text{step 4: } g_m = \frac{\partial E}{\partial m} = -(3.8 - (0.916)(0.4) + (1.42)(0.4))$$

$$= -1.04$$

$$\text{step 5: } g_c = \frac{\partial E}{\partial c} = -4.853$$

$$\begin{aligned}\text{step 5: } v_m &= \beta v_m - \eta g_m \\ &= (0.9)(-0.084) - [-0.1 \times -1.941] \\ &= -0.2697\end{aligned}$$

$$\begin{aligned}v_c &= \beta v_m - \eta g_m \\ &= (0.9)(-0.42) - [-0.1 \times -4.853]\end{aligned}$$

$$v_c = -0.863$$

$$\begin{aligned}\text{step 6: } m &= m + v_m = 0.916 + (-0.2697) \\ &= 0.6463\end{aligned}$$

$$\begin{aligned}c &= c + v_c = -1.42 - 0.863 \\ &= -2.283\end{aligned}$$

$$\begin{aligned}\text{step 7: } \text{sample} &= \text{sample} + 1 \\ &= 2 + 1 \Rightarrow 3\end{aligned}$$

$$\begin{aligned}\text{step 8: } &\text{if (sample} > \text{ns)} : \text{goto step 9} \\ &\text{else: goto step 4}\end{aligned}$$

$$\begin{aligned}\text{step 9: } \text{iter} &+ = 1 \\ &1 + 1 = 2\end{aligned}$$

step 10 : if ($iter > epochs$)

goto step 1

else :

goto step 3

step 3 : sample = 1

$$\text{step 4: } g_m = \frac{\partial E}{\partial m} = -(3.4 - (0.646)(0.2) + 2.283)(0.2)$$

$$= -1.110$$

$$g_c = \frac{\partial E}{\partial c} = -(3.4 - (0.646)(0.2) + 2.283)$$

$$= -5.553$$

$$\text{step 5: } v_m = \eta v_m - \eta g_m$$

$$= (0.9)(-0.2697) - [0.1 \times -1.110]$$

$$= -0.353$$

$$v_c = \eta v_c - \eta g_c$$

$$= (0.9)(-0.863) - [-0.1 \times -5.53]$$

$$= -1.332$$

$$\text{step 6: } m = m + v_m \Rightarrow 0.6463 + (-0.353)$$

$$= 0.293$$

$$c = c + v_c \Rightarrow -2.283 - 1.332 = -3.615$$

step 7: sample + 1 = 1

$$\Rightarrow 1 + 1 = 2$$

step 8: if (sample > ns) goto step 9

else; goto step 4

$$\begin{aligned}\text{step 4: } f_m &= -(3.8 - (0.293)(0.4) + 3.615)(0.4) \\ &= -2.919\end{aligned}$$

$$\begin{aligned}f_c &= -(3.8 - (0.293)(0.4) + 3.615) \\ &= -7.297\end{aligned}$$

$$\begin{aligned}\text{step 5: } V_m &= (0.9)(-0.353) = -[-0.1x - 2.919] \\ &= -0.6096\end{aligned}$$

$$\begin{aligned}V_c &= (0.9)(-1.332) - [+0.1x - 7.297] \\ &= -1.9285\end{aligned}$$

$$\text{step 6: } m = m + V_m \Rightarrow 0.293 - 0.609 = -0.316$$

$$c = c + V_c \Rightarrow -3.615 - 1.928 = -5.543$$

step 7: sample + 1 = 1

step 8: if (sample > ns) ; goto step 9

else goto step 4

step 9: $iter + 1$

$$\Rightarrow 2 + 1 = 3$$

step 10: if ($iter > epochs$) goto step 11

else goto step 3

step 11: print m, c

$$m = -0.316, c = -5.543$$