

Step:1

Given

$$y = x^4 + 3x^2 + 10$$

calculate derivative of  $y$  with respect to  $x$ .

$$\Rightarrow \frac{\partial y}{\partial x} = \frac{d}{dx}(y) = \frac{\partial}{\partial x}(x^4 + 3x^2 + 10)$$

$$\frac{\partial y}{\partial x} = 4x^3 + 6x$$

Step:2 - Initializing  $x$  value, number of ~~values~~ maximum iterations and

learning rate  $\eta$

$$\Rightarrow x = 1$$

$$\eta = 0.1$$

$$\text{no. of iters} = 1$$

$$\text{max iters} = 2$$

Step:3 - Calculate  $\frac{\partial y}{\partial x}$  when  $x=1$

$$\therefore \frac{\partial y}{\partial x} \Big|_{x=1} = 4x^3 + 6x \Big|_{x=1}$$

$$= 4(1)^3 + 6(1) = 4 + 6 = 10$$

Step: 4 Calculate change in  $x$  i.e.  $\Delta x$   
 $\Delta x$  can be calculated using formula.

$$\Delta x = -\eta \frac{dy}{dx}$$

$$= -(0.1)(10)$$

$$= -\frac{1}{10} \times 10 = -1$$

$$\therefore \Delta x = -1$$

Step: 5 add change in  $x$  to  $x$  i.e.  
 perform  $x + \Delta x$

$$\rightarrow x + \Delta x = 1 - 1 = 0$$

$$\text{no. of iters} = \text{no. of iters} + 1$$

Step: 6 If no. of iters  $>$  max\_iters

Stop calculations else repeat steps with  
 updated  $x$  value i.e.  $x = 0$

$$\therefore 2 > 2 \Rightarrow \text{false}$$

$\therefore$  repeat step 3 with  $x = 0$



step: 7

$$\frac{\partial y}{\partial x} \Big|_{x=0} = 4x^3 + 6x \Big|_{x=0}$$

$$= 4(0) + 6(0) = 0$$

step: 8 calculate change in  $x$  ( $\Delta x$ ) when  $x=0$ .

$$\therefore \Delta x = -\eta \frac{\partial y}{\partial x}$$

$$= -(0.1)(0)$$

$$= 0$$

step: 9 - update  $x$  as adding  $\Delta x$  to  $x$

$$\Rightarrow x + \Delta x = 0 + 0 = 0$$

step: 10 :- increment no. of iterations

$$\text{no. of iters} = \text{no. of iters} + 1$$

step: 11 if no of iters  $>$  max-iters , stop process else repeat step 3

Here  $3 > 2$

Hence stop the process

Step: 12  $\therefore \text{slope} = 0$

point at which slope = 0  $\Rightarrow y(0)$

$$= x^4 + 3x^2 + 10 \Big|_{x=0}$$

$$= 0 + 3(0) + 10$$

$$= 0 + 10$$

point at slope  $\rightarrow 0 \Rightarrow 10 \Rightarrow \underline{\text{GLOBAL minimum}}$