

作业 4

题目一

$$a = 2, b = 4, n^{\log_b a} = n^{\frac{1}{2}}$$

$$1. f(n) = 1 \quad f(n) = \mathcal{O}(n^{\log_b a - \epsilon}) \quad \epsilon < \frac{1}{2}$$

$$T(n) = \Theta(n^{\frac{1}{2}})$$

$$2. f(n) = n^{\frac{1}{2}} \quad f(n) = \Theta(n^{\log_b a} (\lg n)^k) = \Theta(n^{\frac{1}{2}} (\lg n)^0)$$

$$T(n) = \Theta(n^{\frac{1}{2}} \lg n)$$

$$3. f(n) = n^{\frac{1}{2}} (\lg n)^2 \quad f(n) = \Theta(n^{\log_b a} (\lg n)^k) = \Theta(n^{\frac{1}{2}} (\lg n)^2)$$

$$T(n) = \Theta(n^{\frac{1}{2}} (\lg n)^3)$$

$$4. f(n) = n \quad f(n) = \Omega(n^{\log_b a - \epsilon}) \quad \epsilon < \frac{1}{2}$$

$$af(n/b) = 2 * (n/4) = \frac{n}{2} <= cf(n) \quad c <= \frac{1}{2}$$

$$T(n) = \Theta(n)$$

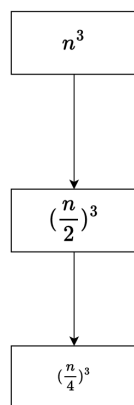
$$5. f(n) = n^2 \quad f(n) = \Omega(n^{\log_b a - \epsilon}) \quad \epsilon < \frac{3}{2}$$

$$af(n/b) = 2 * (n/4)^2 = \frac{n}{8} <= \frac{n^2}{8} <= cf(n) \quad c <= \frac{1}{8}, n >= 1$$

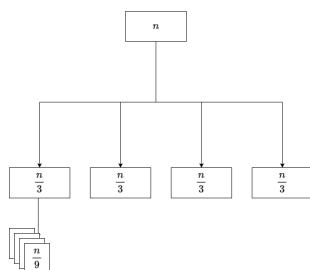
$$T(n) = \Theta(n^2)$$

题目二

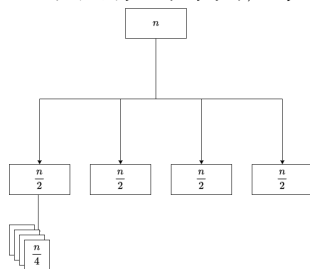
1. 每一层是上一层的 $\frac{1}{8} < 1$ $T(n) = \mathcal{O}(f(n)) = \mathcal{O}(n^3)$



2. 第 i 层有 4^i 个节点，每个节点的值是 $\frac{n}{3^i}$ 所以每一层是上一层的 $\frac{4}{3} > 1$ $T(n) = \mathcal{O}(n^{\log_b a}) = \mathcal{O}(n^{\frac{4}{3}})$



3. 第 i 层有 4^i 个节点，每个节点的值是 $\frac{n}{2^i}$



4.第 i 层有 3^i 个节点，每个节点的值是1 第 i 层总和为 3^i $T(n) = \sum_{i=0}^{n-1} 3^i = \mathcal{O}(n^3)$

