Assignment 6

EECE/CPEN 481

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Selected problems, drawing in part on material from the textbook (Engineering Economic Analysis: Fourth Canadian Edition).

Problems are drawn mainly from material in Chapters 10 and 14.

Point values for the problems are listed below.

1. Based on Problem 8-19 (2x value)
2. Based on Problem 14-24
3. Problem 14-34
4. Based on Problem 14-42
5. Based on Problem 10-5
6. Based on Problem 10-19
7. Problem 10-30
8. Based on Problem 10-47 (2x value)

Solutions given in blue type.

1. **Based on Problem 8-19**

The provincial highway department is analyzing the reconstruction of a mountain road to Tofino. The vehicle traffic increases each year; hence the benefits to the motoring public also increase. Based on a traffic count, the benefits are projected as follows:

Year	End-of-Year Benefit
2025	$10,000
2026	$12,000
2027	$14,000
2028	$16,000
2029	$18,000
2030	$20,000
etc	And so on, increasing $2,000 per year

The reconstructed pavement will cost $230,000 if it is installed in 2024. If it is installed in a future year, the cost will be higher: construction costs are expected to rise 7.5% per year. It will have a 15-year useful life. In each potential situation where you are considering beginning the project, the construction will always occur in one year, and the benefits will begin the following year. Assume a 4% discount rate. The reconstruction, if done at all, must be operational no later than 2030 (so, the last possible year of construction is 2029). Based on NPW analysis, should the project be done, and if so, in what year should it be constructed?

Solution:

We must decide which year construction should take place.
To answer this, we compute the NPW for each possibility. In each case, values are discounted to the same year. I recommend that be the present year (2022).
(It is also possible to choose another common year to use as a 'base' year: 2024 would be plausible, since that's the first year construction could occur. The textbook uses this approach on similar problems. I recommend always discounting to the present year. Good news though: You get the same end conclusions either way.)

NPW each year = PW of future escalated construction cost + PW of future stream of benefits
= (230,000 (1+7.5%)^n')(P/F, 4%, n') + starting benefits (P/A, 4%, 15) + 2,000 (P/G, 4%, 15) (P/F, 4%, n')

NPW for 2024 construction:
= [-230,000 *(1+f)^(2024-2024) + 10,000 (*P/A*, 4%, 15) + $2,000 (*P/G*, 4%, 15)] (P/F, 4%, 2)
= [-230,000 * 1.075^0 + 10,000 (11.118) + 2,000 (69.735)] (0.925)
= [-212,648 + 231,745]
= **$19,097**

NPW for 2025 construction:
= [-230,000 * (1+f)^(2025-2024) + 12,000 (*P/A*, 4%, 15) + $2,000 (*P/G*, 4%, 15)] (P/F, 4%, 3)
= [-230,000 * 1.075^1 + 10,000 (11.118) + 2,000 (69.735)] (0.889)
=[-219,804 + 242,600]
= **$22,795**

NPW for 2026 construction = $25,075
NPW for 2027 construction = $26,003

NPW for 2028 construction = $25,641
NPW for 2029 construction = $24,047

NPW is greatest when construction takes place in 2027.

2. **Based on Problem 14-24**

Assume your salary is $55,000 in 2020 and is expected to be $110,000 in 2050. Inflation is expected to average 1.8% a year during that time.

(a) What will the annual average market rate of your salary increase be during that period, rounded to the one decimal place?
(b) What will the annual average real rate of salary increase be, adjusted to account for inflation, rounded to one decimal place?
(c) What would your salary in 2050 be, if your salary instead increased at the rate of inflation? Round your answers to the nearest dollar.

Solution:
Determine the market rate and then back out the real interest rate.

a)
$\$110{,}000 = \$55{,}000\ (F/P, i\%, 30)$
$(1 + i)^{30} = \$110{,}000/\$55{,}000 = 2.00$
b)
$i = 2^{1/30} - 1 = 0.0234 =$ **2.3%**
c)
$i' = (i - f)/(1 + f) = (0.0234 - 0.018)/(1 + 0.018) = 0.0053 =$ **0.5%**

Future salary if only inflationary increases: 55,000 * (1+.018)^30 = **$93,928**

3. **Problem 14-34**

The cost of solid waste management services in Green Gulch is $4.5 million for year 1. The population is increasing by 3%, the nominal cost per tonne is increasing 6% per year, and the inflation rate is predicted to be 4% per year. Estimate the year 3 cost of solid waste management services (a) in actual market dollars, and (b) in inflation-adjusted ("real") dollars. In both cases, round your answer to $x.xx million.

Solution:

Given these parameters, the overall increase in market rate would be: $i = 0.06 + 0.03 + 0.06(0.03) = 0.0918 = 9.2\%$
Real i' = (i-f)/(1+f) = 5.0%

We need to convert the Year 1 value to nominal value and real value in Year 3.
F (nominal) = $\$4{,}500{,}000\ (F/P, 9.2\%, 2) = \$4{,}500{,}000(1.0918)^2 = \$5{,}364{,}123 =$ **$5.36M**
F (real) = $\$4.5M\ (F/P, 5.0\%, 2) = \$4.5M\ (1.0498)^2 = \$4{,}959{,}433 =$ **$4.96M**

4. **Based on Problem 14-42**

Sam bought a house for $150,000 with some creative financing. The bank, which agreed to lend Sam $120,000 for six years at 15% interest, took a first mortgage on the house. The Joneses, who sold Sam the house, agreed to lend him the remaining $30,000 for six years at 12% interest. They received a second mortgage on the house. Thus Sam became the owner without paying any cash at the beginning. He pays $1,500 a month on the first mortgage and $300 a month on the second mortgage. In both cases these are "interest-only" loans, meaning that the principal is due at the end of the loan.

Sam rented the house to a different couple. After receiving their monthly rent and paying the taxes, insurance, and so on, he must pay $1,000 a month of his own money to cover the remainder of the monthly mortgage payments. At the end of three years, Sam sold the house for $205,000. After paying off all costs (such as the two loans and the real estate broker), he had $40,000 left. (Ignore taxes for this problem.)

(a) What rate of return did Sam receive on this investment (rounded to 1 decimal point)?
(b) After the 6% annual inflation rate is taken into account, what was his rate of return (same rounding)?

Solution:

(a)

For 36 months, the cash outflow = $1,000

After 36 months, the cash inflow = $40,000

To make the cash outflow equal to the cash inflow:

1,000 $(F/A, i\%, 36 \text{ months}) = \$40{,}000$

$(F/A, i\%, 36 \text{ months}) = \$40{,}000$

This can be solved by interpolation, or by using Goal Seek or an IRR calculation in Excel.

$i = 0.593\%$ per month

Equivalent annual interest rate:

i (per year) $= (1 + 0.00593)^{12} - 1 = 0.0735 =$ **7.4%**

(b)

Nominal effective annual rate = 7.35%

Inflation rate = 8%

Real rate of return = ?

Real rate of return = (Nominal interest rate – Inflation)/(1 + Rate of inflation)

= (0.0735 – 0.08)/(1 + 0.08)

= (–0.0065)/1.08

= –0.00599 or **–0.6%**

5. **Based on Problem 10-5**

A new two-lane road is needed in a part of town that is growing. At some point, the road will need to have four lanes to handle the anticipated traffic. The city has three estimates of when the expansion will be needed: pessimistically (if town growth is lower than expected), in 12 years; most likely, in 8 years; and optimistically, in 5 years. The estimated expansion cost in today's dollars is $20 million. Construction costs are expected to grow by 3% per year. Assume a borrowing rate (and therefore a discount rate) of 5%.

(a) What is the present worth of each scenario, rounded to one decimal place (x.x million)? (In other words, how much money would the city need to set aside now to pay for the road in the future, in each different scenario?)

(b) If the pessimistic scenario has a 35% chance of occurring, the most likely has a 40% chance, and the optimistic scenario has a 25% chance of occurring, what is the average of the present worth of cost across the scenarios (rounded to one decimal place)?

Solution:

(a) Pessimistic Estimate (12 years)

PW = $20M *(1+f)^12 * (*P/F*, 5%, 12) = $20M (1.426) (0.557) = $15.9M

Most Likely Estimate (8 years)

PW = $20M *(1+f)^8 * (*P/F*, 5%, 8) = $20M (1.267) (0.677) = $17.1M

Optimistic Estimate (5 years)

PW = $20M *(1+f)^5 * (*P/F*, 5%, 5) = $20M (1.159) (0.784) = $18.2M

(b) Mean value = (25% x $15.9M) + (40% x $17.1M) + (35% x $18.2M) = $17.0M

6. **Based on Problem 10-19**

Craps is a popular casino gambling game that uses two (six-sided) dice. One of the many bets available is the "Hard-way 8", which occurs if both dice come up fours when rolled. If a "hard-way 8" is rolled before a 7 is rolled or any other combination that adds to 8 is rolled, a $1 bet will typically win the player $9 (and they will get their original $1 bet back as well). If any other total number is rolled, the bet neither wins nor loses. What is the expected return for such a $1 bet, if you assume the gambler will keep playing until they either win or lose? Round to the nearest cent.

Solution:

There are 36 possible combinations of two six-sided dice.
6 of those combinations add to 7, and will cause the bet to be lost.
4 of those combinations add to 8 but are NOT two fours (2,6; 3,5; 5,3; 6,2), and will cause the bet to be lost.
1 combination (4 and 4) causes the bet to win.
All the other combinations don't cause the bet to be decided (the bet 'rides'), so can be ignored.

All of the combinations above are equally likely. 1 of the 11 combinations wins, and 10 of the 11 lose. We can ignore the other 25 combinations, because you will keep playing if any of those combinations occur.

Expected value of $1 bet = (10/11)*(-$1) + (1/11)*(+$9) = $0.91

This means an expected $0.09 loss on a $1 bet.

Note: if the question had given different conditions, and you would make only one single bet and would stop, then the set of outcomes and the expected value would have been different:

Expected value of $1 bet = (10/36)*(-$1) + (1/36)*(+$9) + (25/36)*($0) = $0.97
This means an expected $0.03 loss on a $1 bet.

Note that the problem as listed in the book is mathematically correct but not correct in how such a bet actually is played.

7. Problem 10-30

The tree in Figure P10-30 in the textbook has probabilities after each chance node and present worth values for each terminal node. What is the expected value of each decision option at D1 (Pick A or Pick B) and at D2 (Pick A1 or Pick A2)? What decision should be made?

Solution:

At decision node D2 we must decide between PickA1 and PickA2 based on PW:
PW(C3) = 0.4($12,000) + 0.6($8,100) = **$9,660**
Or **$10,000**
PickA2 is the better choice (greater PW).

At decision node D1 we must decide between Pick A and Pick B based on PW:
PW(C1) = 0.4($10,000) + 0.6($4,000) = **$6,400**
PW(C2) = 0.4($9,000) + 0.6($5,000) = **$6,600**

"Pick B" is the better choice.

8. **Problem 10-47**

Your firm is considering investing in a power generator to meet its own power needs and avoid paying power bills.

The first cost (initial capital cost) of a power generator is uncertain: the cost is distributed normally with a mean value of $180,000 and a standard deviation of $35,000. The power generator will have no salvage value.

The life of the generator is also uncertain: it may last 3, 4, 5, 6, or 7 years. (Assume a discrete uniform distribution applies.)

Currently, the power bill is $60,000 per year. If you make this investment, you will save this money every year (since you will no longer have a power bill). The borrowing interest rate (and therefore discount rate) is 5%. The annual operating cost of the power generator is expected to be $10,000 per year.

Draw 30 different random sets from the distributions above to serve as data for your analysis.

(a) For your set of samples, what is the expected value of the net present worth of the plant investment?
(b) For your set of samples, what is the standard deviation of the net present worth?
(c) Would you recommend making this investment? Why or why not?

Show your work for these calculations. You can use Excel or other software capable of statistical analysis to solve this problem.

Solution:

Steps:

Draw the 30 random samples from the normalized distribution (representing capital cost) and the uniform discrete distribution (for the number of years of life for the equipment, N). (30 samples are enough to a reasonable amount of the diversity across the distributions.)

Avoided power costs can be converted to a Prevent Value. So can the Operating Costs.
The formula (P/A, i, n) will vary with N (sampled above).

For each possible set of N and capital cost P (both sampled above), calculate the NPW, by adding up capital cost P, PW of avoided costs, and PW of operating costs.

Average NPW over the set of outcomes will be the expected value of NPW, because each of the samples drawn are equally likely.

Standard deviation can be calculated using Equation 10-7. Calculations shown below. Note that Excel formula STDEV.P calculates this value (but you need to calculate it manually in this case).

Range of uniform distribution:		3	7			i	5.0%	
Mean capital cost			180,000			Avoided power costs per year A	$	60,000
Stdev of capital cost			35,000			Op Costs A	$	(10,000)
sample set from the joint probability distribution			(Values-only paste of same)					
N	P		N	P				
4.00	$ (173,472)		4	$ (121,601)				
3.00	$ (144,641)		4	$ (164,284)				
5.00	$ (166,822)		6	$ (129,791)				
3.00	$ (188,345)		4	$ (202,285)				
4.00	$ (173,439)		4	$ (184,450)				
3.00	$ (205,820)		4	$ (168,811)				
7.00	$ (185,298)		6	$ (162,723)				
6.00	$ (206,013)		6	$ (196,208)				
5.00	$ (190,463)		7	$ (203,622)				
6.00	$ (141,008)		3	$ (155,025)				
3.00	$ (178,728)		3	$ (197,845)				
4.00	$ (202,427)		4	$ (176,969)				
5.00	$ (188,630)		4	$ (102,266)				
6.00	$ (194,832)		6	$ (150,708)				
7.00	$ (233,199)		6	$ (168,841)				
3.00	$ (163,204)		6	$ (142,249)				
7.00	$ (156,029)		4	$ (148,331)				
7.00	$ (201,869)		5	$ (182,237)				
6.00	$ (175,421)		5	$ (136,534)				
4.00	$ (179,348)		7	$ (174,543)				
4.00	$ (207,402)		6	$ (175,494)				
6.00	$ (199,993)		7	$ (151,010)				
5.00	$ (168,142)		5	$ (179,710)				
5.00	$ (128,521)		3	$ (164,862)				
5.00	$ (206,829)		7	$ (198,914)				
5.00	$ (170,769)		3	$ (151,346)				
3.00	$ (204,670)		7	$ (187,529)				
4.00	$ (156,463)		6	$ (170,602)				
3.00	$ (196,984)		6	$ (199,044)				
6.00	$ (189,929)		6	$ (219,714)				

Using values-only, to provide one answer at bottom:					
(P/A, 0.06, N)	PW of Avoided power costs	PW of Op Costs	NPW	NPW * probability	NPW^2 * probability
3.55	$ 212,757	$ (35,460)	$ 55,696	$ 1,856.55	$ 103,403,232
3.55	$ 212,757	$ (35,460)	$ 13,014	$ 433.79	$ 5,645,220
5.08	$ 304,542	$ (50,757)	$ 123,993	$ 4,133.11	$ 512,478,241
3.55	$ 212,757	$ (35,460)	$ (24,988)	$ (832.93)	$ 20,813,229
3.55	$ 212,757	$ (35,460)	$ (7,152)	$ (238.41)	$ 1,705,173
3.55	$ 212,757	$ (35,460)	$ 8,486	$ 282.88	$ 2,400,625
5.08	$ 304,542	$ (50,757)	$ 91,062	$ 3,035.40	$ 276,410,053
5.08	$ 304,542	$ (50,757)	$ 57,576	$ 1,919.21	$ 110,500,556
5.79	$ 347,182	$ (57,864)	$ 85,696	$ 2,856.55	$ 244,795,850
2.72	$ 163,395	$ (27,232)	$ (18,862)	$ (628.75)	$ 11,859,663
2.72	$ 163,395	$ (27,232)	$ (61,683)	$ (2,056.10)	$ 126,826,525
3.55	$ 212,757	$ (35,460)	$ 328	$ 10.95	$ 3,594
3.55	$ 212,757	$ (35,460)	$ 75,032	$ 2,501.06	$ 187,659,299
5.08	$ 304,542	$ (50,757)	$ 103,077	$ 3,435.90	$ 354,161,686
5.08	$ 304,542	$ (50,757)	$ 84,944	$ 2,831.45	$ 240,513,787
5.08	$ 304,542	$ (50,757)	$ 111,535	$ 3,717.84	$ 414,669,882
3.55	$ 212,757	$ (35,460)	$ 28,967	$ 965.55	$ 27,968,767
4.33	$ 259,769	$ (43,295)	$ 34,237	$ 1,141.23	$ 39,072,420
4.33	$ 259,769	$ (43,295)	$ 79,940	$ 2,664.66	$ 213,012,179
5.79	$ 347,182	$ (57,864)	$ 114,775	$ 3,825.84	$ 439,111,968
5.08	$ 304,542	$ (50,757)	$ 78,291	$ 2,609.70	$ 204,315,880
5.79	$ 347,182	$ (57,864)	$ 138,308	$ 4,610.27	$ 637,638,334
4.33	$ 259,769	$ (43,295)	$ 36,764	$ 1,225.47	$ 45,053,254
2.72	$ 163,395	$ (27,232)	$ (28,700)	$ (956.66)	$ 27,455,880
5.79	$ 347,182	$ (57,864)	$ 90,404	$ 3,013.48	$ 272,431,292
2.72	$ 163,395	$ (27,232)	$ (15,184)	$ (506.12)	$ 7,684,674
5.79	$ 347,182	$ (57,864)	$ 101,790	$ 3,392.99	$ 345,371,955
5.08	$ 304,542	$ (50,757)	$ 83,182	$ 2,772.74	$ 230,642,467
5.08	$ 304,542	$ (50,757)	$ 54,741	$ 1,824.69	$ 99,884,691
5.08	$ 304,542	$ (50,757)	$ 34,071	$ 1,135.69	$ 38,693,561
				$ 50,978	$ 5,242,183,935
			Standard deviation		$ 51,414
			$ 50,978	Average NPW	
			$ 51,414	Standard deviation of the NPW (STDEV.P)	
			$ 52,293	Standard deviation of the NPW (STDEV.S)	
	Note that the lower value SHOULD match the manually calculated value, but it doesn't.				

Your answer will be different than this one, because your sample set of 30 will be different than mine.

Part c:

We know that the average NPW is positive, so, at first glance, it looks like a good investment. However, whether this is a good investment or not depends on your (and your firm's) risk tolerance, and whether other opportunities to use the same capital funds would be more profitable than this opportunity. Notice that the standard deviation is greater than the average NPW: there is a fair chance that this investment won't have a positive NPW, so if the firm is very risk averse, this may not be an appealing investment.