



The Polaris Missile Program, cont'd

- During World War II, scientists, mathematicians, and engineers often found themselves in charge of large and innovative projects.
- Following the war, and taking advantage of the new availability of computing machinery, they began to develop and formalize novel methods for project management.
- One of the best-known examples is the Project Evaluation and Review Technique, or PERT, developed during the genesis of the Polaris submarine-launched nuclear missile program.

The Polaris Missile Program, cont'd

- PERT's distinctive approach is to concentrate on the abstract structure of a project.
- Simple mathematical techniques can be used to determine how long it will take to reach the ultimate goal and how scarce resources can best be allocated to get there more quickly.
- The US Navy set up a new entity, the Special Projects Office (SPO), to manage the task of getting a newly developed nuclear missile into a newly developed nuclear submarine.

The Polaris Missile Program, cont'd

- PERT was used throughout the project and was subsequently credited with reducing the time to completion by several years.
- When the Royal Navy adopted Polaris years later, they also adopted PERT to guide its implementation.

Learning Objectives

- Plan complex projects and predict their duration
- Use scarce resources to reduce the completion time of a project
- Be aware of the tools offered by Operations Research

Scientific Management

- The development of scientific tools for management dates back to the beginning of the twentieth century and refers to engineer Frederick Winslow Taylor.
- Taylor's approach of applying scientific observation to the organization of work attracted widespread interest and emulation.
- The introduction of the moving assembly line allowed management to control the sequence of operations performed, as well as the rate at which operations were performed.

The Gantt chart

- A simple graphical method that shows the dependencies between the activities making up a project.
- To apply this technique, one needs to pick a project of the right size.
 - Too small vs. too complex projects
 - A suitable project will be one of intermediate size

The Gantt chart, cont'd

- The first step is to list the sub-tasks required to accomplish the project.
 - This list is known as the work breakdown structure (WBS).

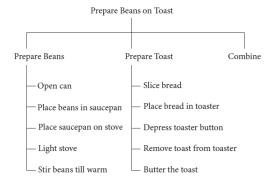


FIGURE 15-1 Work breakdown structure for preparing beans on toast.

The Gantt chart, cont'd

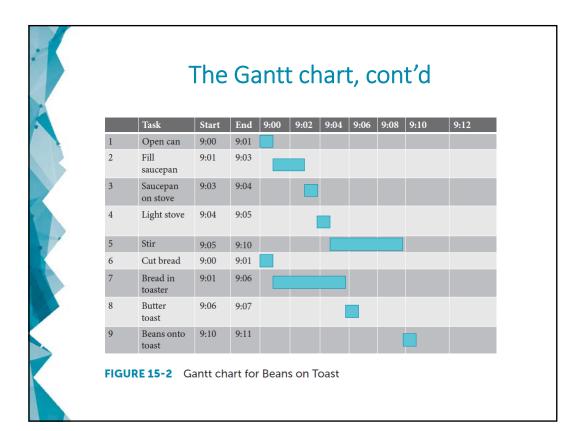
- Benefits obtained from the construction of the WBS.
 - Ensuring that no vital step is left out and that no step is duplicated
 - Making sure that everyone in the team knows which steps they are responsible for
 - Identifying particular material requirements

The Gantt chart, cont'd

- To convert the WBS into a Gantt chart, we must:
 - Make explicit the dependencies between the sub-goals
 - Estimate the duration of each activity listed

The Gantt chart, cont'd

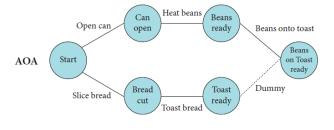
- After identifying the WBS, the Gantt chart is constructed as follows:
 - List the sub-activities of the WBS on the left-hand edge of a spreadsheet
 - Let the horizontal extent of the spreadsheet represent time
 - Choose a time to initiate the first sub-activity
 - Draw a bar extending to the right from the start time, corresponding to the time required for this activity
 - Initiate the second sub-activity at the earliest feasible time
 - Proceed in this manner until all the activities are completed
 - The time at which the final activity is complete gives the expected completion time for the project



Critical Path Method (CPM)

- A relatively simple extension of the Gantt chart.
- The first step is to represent the dependencies between the various sub-tasks as a network diagram
- Activity-on-arc (AOA)
 - Nodes represent states or milestones
 - Arcs represent activities
 - Use of dummy arcs
- Activity-on-node (AON)
 - Nodes represent activities
 - Arcs represent dependencies





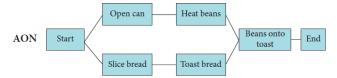
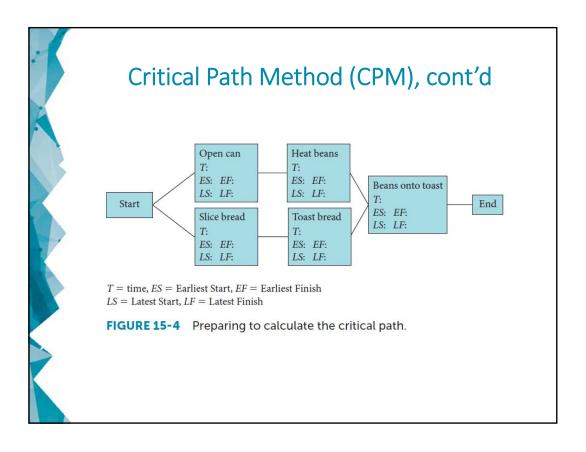
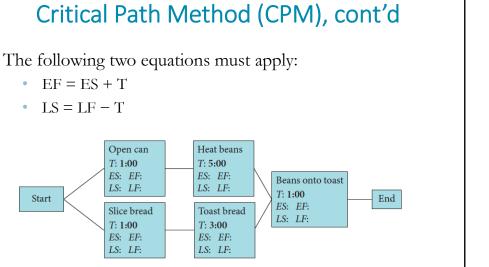


FIGURE 15-3 Two conventions for CPM diagrams.

- Labels are added to activities in the AON diagram to show its expected duration T (estimated from past similar activities).
- Four other labels are also added to each node:
 - Earliest Start (ES), the earliest time at which the activity can start;
 - Earliest Finish (EF), the earliest time at which the activity can be completed;
 - Latest Start (LS), the latest time at which we can start the activity without delaying the completion of the project as a whole; and
 - Latest Finish (LF), the time at which the activity will be completed if we begin at *LS*.

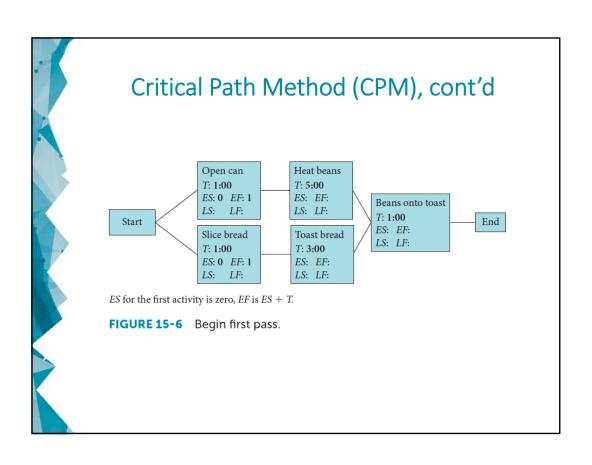




Fill in the times.

FIGURE 15-5 Activity times estimated.

- Having filled in our estimates for T, we begin our first pass through the diagram, working from left to right.
- The ES for our first activity is, by definition, Time Zero.
- The EF must therefore be Time Zero plus the time required for the activity itself (T).



- We now proceed one step to the right.
- The ES for the "Heat beans" activity coincides with the EF for the "Open can" activity.
- Given that heating beans is estimated to take 5 minutes, the EF for heating beans is 6 minutes after Time Zero.

Critical Path Method (CPM), cont'd Heat beans Open can T: 1:00 T: 5:00 ES: 0 EF: 1 ES: 1 EF: 6 Beans onto toast LS: LF: LS: LF: T: 1:00 Start End ES: 6 EF: 7 Slice bread Toast bread LS: LF: T: 1:00 T: 3:00

ES: 1 EF: 4

LS: LF:

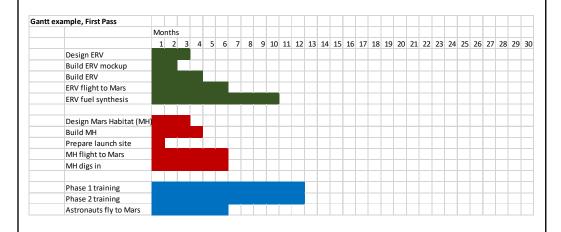
For subsequent activities, EFS is max (EF of all upstream activities), EF is ES + T.

FIGURE 15-7 Completing the first pass.

ES: 0 EF: 1

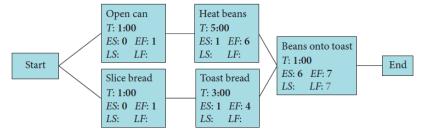
LS: LF:





- The activity "Beans onto toast" has two prerequisites:
 - The beans must have been warmed
 - The bread must have been toasted
- So, the ES for this activity must be the later of the two completion times for the two prior activities.
- As such, the earliest we can start putting the beans onto the toast is at six minutes, since the beans will not be sufficiently warm before that.
- So, the earliest the beans on toast are going to be ready is seven minutes after Time Zero.





The earliest finish of the final activity is also the latest acceptable finish for the final activity, assuming we don't want the project completion to be delayed.

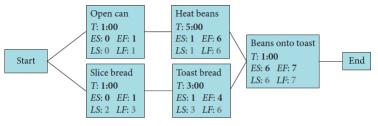
FIGURE 15-8 Begin the second pass.

- Now we work from right to left.
- To avoid delaying breakfast, we must finish the "Beans onto toast" activity by the seven-minute mark.
- Since it takes one minute to get the beans onto the toast, we cannot start this activity any later than six minutes after Time Zero.

- So, we write a "6" in the LS slot for this node and a "6" in the LF slot of both the "Heat beans" and "Toast bread" nodes.
- Then, we write a "1" in the LS slot of the "Heat beans" node and a "3" in the LS slot of the "Toast bread" node.

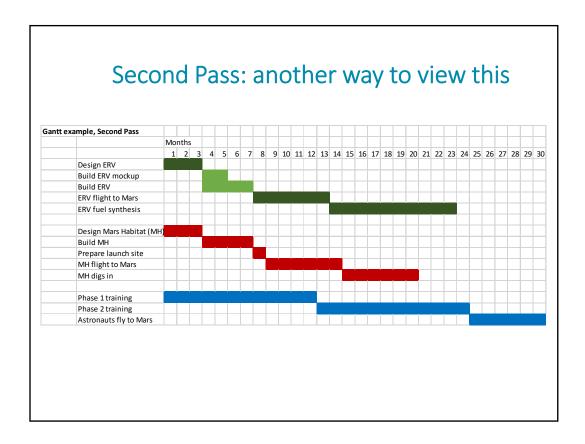
Critical Path Method (CPM), cont'd

• Assign to each node a LF equal to the ES of any of the nodes to its right, and a LS that precedes the LF by the time required for the activity.



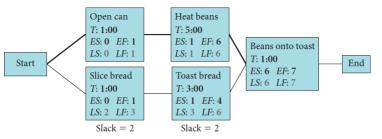
Then we do a second, backwards, pass through the network, where for each node, LF is min (LS of all downstream nodes) and LS is LF - T

FIGURE 15-9 Completing the second pass.



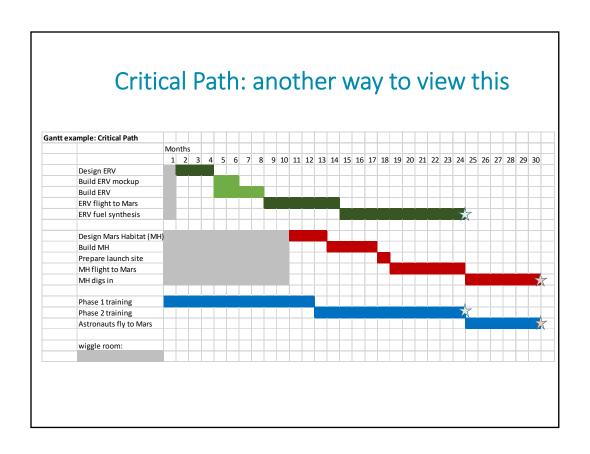
- For some nodes, such as "Heat beans," the ES coincides with the LS.
- Nodes with this feature are said to be on the "critical path."
- Team members responsible for these activities cannot afford to delay them as this will delay the completion of the entire project.

• Activities on the lower part of the diagram, the slicing and toasting of the bread, have two minutes of slack time.



The difference between *EF* and *LF* for a node is the *slack* for that activity. Activities with no slack are on the *critical path*.

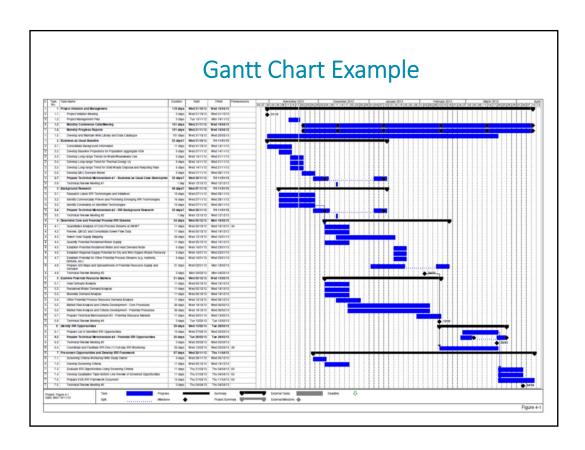
FIGURE 15-10 Slack and critical path.

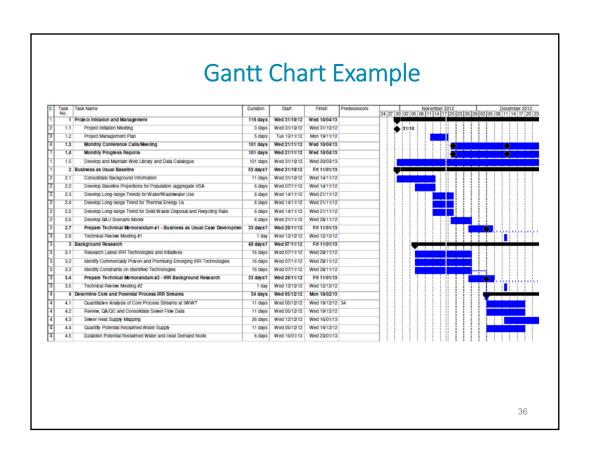


- It would be desirable to extend the Critical Path Method (CPM) to find the most economical way of reducing project completion time.
- We know that we should concentrate on those activities that lie on the critical path.

Project Evaluation and Review Technique (PERT)

- The CPM approach can be extended systematically, giving us the Project Evaluation and Review Technique (PERT).
- To apply PERT, we need to make additional assumptions about our project.
- At least some of the activities must be capable of being accelerated, or "crashed"; otherwise no reduction in completion time is possible.
- We will further assume that the speed-up is a linear function of cost between certain limits.
- These assumptions are not universally valid.





Linear Programming and the Simplex Method

- We are producing several different product lines
 - Pies
- Each item in each product line requires a certain amount of several limited resources
 - An apple pie requires 0.5 kg of pastry and two apples
 - A steak pie requires 0.7 kg pastry and 0.8 kg of horsemeat
- What combination of apple and steak pies should we produce to maximize our profits, assuming we make:
 - A dollar on each apple pie sold; and
 - Two dollars on each steak pie?

Linear Programming and the Simplex Method, cont'd

 Assuming linearity, x apple pies and y steak pies make a profit of:

$$P = x + 2y$$

- Also, we are constrained by the limits on our resources as follows:
 - A total of 50 kg of pastry;
 - A barrel of 120 apples; and
 - A dead horse of 100 kg.

Linear Programming and the Simplex Method, cont'd

• Then we can write:

$$0.5x + 0.7y \le 50 \text{ (pastry)}$$

 $2x \le 120 \text{ (apples)}$
 $0.8y \le 100 \text{ (horse)}$

- The simplex method can be used to determine the most profitable mix of apple and steak pies.
- See also Excel solution example.

Queuing Theory

- Queuing means standing in line waiting for something.
- Suppose we manage a supermarket. We need to decide how many cashiers we are going to employ.
- Queuing theory allows us to predict the average length of the checkout queue as a function of the number of cashiers, and thus decide on the optimum number of cashiers to employ.

Queuing Theory, cont'd

- There are many situations that can also be described as queues, even when no one is standing in line.
- For example, telephone calls coming into an exchange and awaiting a connection, or repair jobs coming into a workshop and awaiting a technician.