6

Lecture 5
Annual Cash Flow Analysis
Salvage Value
Mortgages

NINJA Loans

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NINJA Loans, cont'd

- Bankers discovered that, rather than lending money to house buyers and waiting to be repaid, they could instead sell the debt to someone else as a "mortgage-backed security."
- Since getting repaid by the borrower was no longer the bank's problem, banks relaxed their lending standards so that even people with no regular job or income could qualify for a loan.
- These loans became known as "No Income, No Job, and no Assets," or NINJA, loans.

NINJA Loans, cont'd

- Unfortunately, though perhaps not surprisingly, it turned out that many of the NINJA borrowers were not able to keep up their mortgage payments and defaulted on their loans.
- It became clear that the acronym NINJA was particularly appropriate, since, like a ninja, the borrowers were stealthily vanishing.
- A large number of repossessed houses now appeared on the market.
- As a result of this oversupply, house prices dropped dramatically.

NINJA Loans, cont'd

- Many of the borrowers now found that the amount they still owed on their house was more than the house's market value, an undesirable condition known as being "underwater."
- This crisis then spread to the rest of the economy, creating the crash of 2008.

Learning Objectives

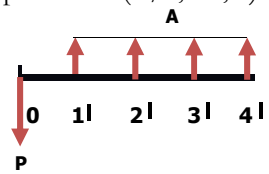
- *Equivalent uniform annual cost (EUAC)*
  - Resolve any series of cash flows into its annual cash flow equivalent
  - Use *annual costs* to compare alternatives with equal, common multiple, or continuous lives, or over some fixed study period
  - This is an alternative way to analyze a set of costs and revenues or cost savings
  - Sometimes it's easier to grasp numbers in annual terms rather than totals, and easier to compare them with other annual numbers
- Add another element to our analysis: salvage value
- How mortgages work

Key Summary: Course to date and coming soon

- Variables and parameters (puzzle pieces):
  - Different kinds of interest rates
  - Discount rates
  - Costs and cost savings or revenues, now and in the future
  - Different expected lives of the possible project/purchases
  - **Salvage value**
  - Taxes and tax savings
  - How these escalate
- Analysis methods (ways to put the pieces together):
  - Present worth analysis
  - **Equivalent uniform annual cost analysis**
  - Rate of return analysis
  - Benefit-cost ratio analysis
  - Payback period
  - Cost-effectiveness analysis

Annual Cash Flow Calculations:
Equivalent Uniform Annual Cost Analysis

- The objective is to compare alternatives based on annual cash flows.
- This requires converting present values and one-time values on the timeline to their equivalent uniform annual costs (EUAC).
  - Using annual worth factors: F
    - For example: A = P(A/P, i%, 4)



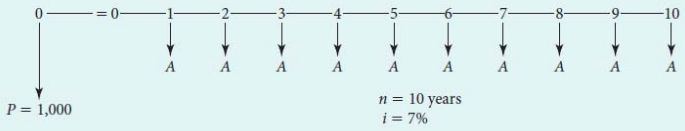
- EUAC is also known as the "capital recovery cost" of a project: you buy capital, and need a stream of savings to pay for it.

Equivalent Uniform Annual Cost Analysis, cont'd

See Excel example. Several possible ways to solve this.

EXAMPLE 6-1

A student bought $1,000 worth of home furniture. If it is expected to last 10 years, what will the equivalent uniform annual cost be if interest is 7%? (The student might, for example, need this information in order to compare the annual lease costs of a furnished versus an unfurnished apartment.)



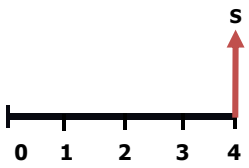
SOLUTION

$$\begin{aligned}\text{Equivalent uniform annual cost} &= P(A/P, i, n)\\ &= 1{,}000(A/P, 7\%, 10)\\ &= \$142.40\end{aligned}$$

Treatment of Salvage Value

- When there is a salvage value at the end of the life of an asset, it is represented as a one-time cash flow benefit (revenue) at the end of the asset's life

Example: Salvage value (S) of asset with a four-year life



- Salvage value is included in nearly all analysis methods.
- When salvage value exists, it lowers the equivalent uniform annual cost.

Adding Salvage Value to EUAC analysis

- When there is an initial cost (P) followed by a salvage value (S) the equivalent uniform annual worth (EUAC) can be computed by:
  - EUAC = P(A/P, i, n) – S(A/F, i, n)

  Or, equivalently:
  - EUAC = (P – S)(A/F, i, n) + Pi

  Or, equivalently:
  - EUAC = (P – S)(A/P, i, n) + Si

  The first is the most common formula (and perhaps the most intuitive).

Adding Salvage Value to EUAC analysis, cont'd

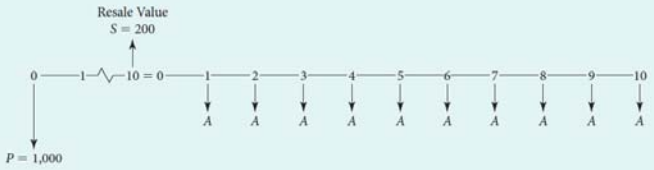
- Direct relationship exists between present worth cost and equivalent uniform cost:
  - EUAC = PW Cost(A/P, i, n)
  - Expenditure of money increases EUAC, whereas receipt of money decreases EUAC.
  -
  - Direct relationship also exists between an arithmetic gradient and its equivalent uniform annual cost, using the factor (A/G, i, n)

Adding Salvage Value to EUAC analysis, cont'd

See Excel example. A couple possible ways to solve this.

EXAMPLE 6-2

The student in Example 6-1 now believes the furniture can be sold at the end of 10 years for $200. Under these circumstances, what is the equivalent uniform annual cost?



SOLUTION

For this situation, the problem may be solved by any of three different calculations.

SOLUTION 1

$$\begin{aligned} \text{EUAC} &= P(A/P, i, n) - S(A/F, i, n) \qquad (6\text{-}1)\\ &= 1{,}000(A/P, 7\%, 10) - 200(A/F, 7\%, 10)\\ &= 1{,}000(0.1424) - 200(0.0724)\\ &= 142.40 - 14.48 = \$127.92 \end{aligned}$$

$$\begin{aligned} \text{EUAC} &= \text{PMT}(i, n, \text{P}, \text{F})\\ &= \text{PMT}(7\%, 10, -1000, 200)\\ &= \$127.90 \end{aligned}$$

Adding Salvage Value to EUAC analysis, cont'd

SOLUTION 2

Equation 6-1 describes a relationship that may be modified by an identity presented in Chapter 4:

$$(A/P, i, n) = (A/F, i, n) + i \qquad (6\text{-}2)$$

Substituting this into Equation 6-1 gives

$$\begin{aligned} \text{EUAC} &= P(A/F, i, n) + Pi - S(A/F, i, n)\\ &= (P - S)(A/F, i, n) + Pi \qquad (6\text{-}3)\\ &= (1{,}000 - 200)(A/F, 7\%, 10) + 1{,}000(0.07)\\ &= 800(0.0724) + 70 = 57.92 + 70\\ &= \$127.92 \end{aligned}$$

This method computes the equivalent annual cost due to the unrecovered $800 when the furniture is sold, and it adds annual interest on the $1,000 investment.

SOLUTION 3

If the value for $(A/F, i, n)$ from Equation 6-2 is substituted into Equation 6-1, we obtain

$$\begin{aligned} \text{EUAC} &= P(A/P, i, n) - S(A/P, i, n) + Si\\ &= (P - S)(A/P, i, n) + Si \qquad (6\text{-}4)\\ &= (1{,}000 - 200)(A/P, 7\%, 10) + 200(0.07)\\ &= 800(0.1424) + 14 = 113.92 + 14 = \$127.92 \end{aligned}$$

Example 6-3

Betty owned a car for five years. One day she wondered what her uniform annual cost for maintenance and repairs had been *(ed: as one does…)* She assembled the following data:

Year	Maintenance and Repair Cost for Year
1	$45
2	90
3	180
4	135
5	225

Switch to Excel examples.

Compute the equivalent uniform annual cost (EUAC), assuming 7% interest and end-of-year disbursements.

Cash Flow Calculations: Problem 1

- A university student looking for new tires has located the following alternatives:

Expected Tire Life	Price/Tire
12 Months	$30.95
24 Months	$44.95
36 Months	$53.95
48 Months	$59.95

- If she figures that money is worth 12%, which tires should she choose?

Cash Flow Calculations: Problem 1 , cont'd

$$\text{EUAC} = P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$$

Notice that this is the same formula we used earlier: EUAC just replaced 'A', the annual uniform payment.

Solution

EUAC (12 month tire) = \$30.95 (A/P, 12%, 1) = \$34.66

EUAC (24 month tire) = \$44.95 (A/P, 12%, 2) = \$26.60

EUAC (36 month tire) = \$53.95 (A/P, 12%, 3) = \$22.46

EUAC (48 month tire) = \$59.95 (A/P, 12%, 4) = \$19.74

Choose the 48 Month Tire for lowest annual costs among these options.

Annual Cash Flow Analysis

- Just like in other analysis methods, differences are most important for choosing among alternatives. When comparing alternatives, we can always ignore any cash flows common to each alternative.
- So:
- Conversely:

Annual Cash Flow Analysis: Problem 2

- The following data are available for three different alternatives:

	Alternative A	**Alternative B**	**Alternative C**
Initial Cost	$1000	$1500	$2000
Uniform Annual Benefits	$200	$276.20	$654.80
Useful Life in Years	--	20	5
Interest Rate	15%	15%	15%

- Alternatives B and C are replaced at the end of their useful lives with identical replacements. Using annual cash flow analysis find the most attractive alternative.

Annual Cash Flow Analysis: Problem 2

- Switch to Excel example

Annual Cash Flow Analysis: Problem 2, cont'd

(Solution summary as described and calculated in Excel)

Solution

Alternative A

EUAB – EUAC = 200 – 1000(0.15) = $50

Alternative B

EUAB – EUAC = 276.2 – 1500(A/P, 15%, 20) = 276.2 – 1500(0.1598) = $36.5

Alternative C

EUAB – EUAC = 654.8 – 2000(A/P, 15%, 5) = 654.8 – 2000(0.2983) = $58.2

Choose Alternative C.

EUAC: Impact of Analysis Period

- Alternatives have equal lives.
  - If the lives are equal, the analysis period is based on the same lifetime.
- Alternatives have unequal lives.
  - If the lives are unequal, the analysis period is based on alternate lifetimes.
    - **No correction is required** as is necessary in present worth analysis (using lowest common multiple).
    - Multiples of service life are equivalent to one service life with annual worth analysis—therefore, it doesn't matter!

Example 6-5 in the textbook shows this to be the case.

Analysis Period, cont'd

Example 6-5:

Two possible pumps. Interest rate 7%

	Pump A	Pump B
Initial cost	$7,000	$5,000
Salvage value	$1,500	$1,000
Useful life (years)	12	6

Turn to Excel example for 6-5.

Analysis Period, cont'd

EXAMPLE 6-6

Pump B in Example 6-5 is now believed to have a nine-year useful life. Assuming the same initial cost and salvage value, compare it with Pump A, using the same 7% interest rate.

SOLUTION

If we assume that the need for A or B will exist for some continuing period, the comparison of costs per year for the unequal lives is an acceptable technique. For 12 years of Pump A:

$$\text{EUAC} = (7{,}000 - 1{,}500)(A/P, 7\%, 12) + 1{,}500(0.07) = \$797$$

For nine years of Pump B:

$$\text{EUAC} = (5{,}000 - 1{,}000)(A/P, 7\%, 9) + 1{,}000(0.07) = \$684$$

For minimum EUAC, choose Pump B.

Return to Excel example for 6-6.

Infinite Analysis Period

- Since multiples of finite service lives are equivalent to one service life, an infinite analysis of finite service lives yield:
  - $EUAC_{\text{infinite analysis period}} = EUAC_{\text{for limited life } n}$
- However, when an alternative with an infinite life is evaluated over an infinite analysis period:
  - $EUAC_{\text{infinite analysis period}} = P(A/P, i, \infty)$ + Any other annual costs
- When n = ∞, A = Pi, therefore:
  - $EUAC_{\text{infinite analysis period}} = Pi$ + Any other annual costs

Infinite Analysis Period, cont'd

- The difference in annual cost between a long life and an infinite life is normally small, unless an unusually low interest rate is used.
- Example 6-7 demonstrates this:

Infinite Analysis Period, cont'd

See Excel example.

EXAMPLE 6-7

In the construction of an aqueduct to expand the water supply of a city, there are two alternatives for a particular portion of the aqueduct. Either a tunnel can be constructed through a mountain, or a pipeline can be laid to go around the mountain. If there is a permanent need for the aqueduct, should the tunnel or the pipeline be chosen for this particular portion of the aqueduct? Assume a 6% interest rate.

SOLUTION

	Tunnel through Mountain	Pipeline around Mountain
Initial cost	$5.5 million	$5 million
Maintenance	0	0
Useful life	Permanent	50 years
Salvage value	0	0

Tunnel

For the tunnel, with its permanent life, we want $(A/P, 6\%, \infty)$. For an infinite life, the capital recovery is simply the interest on the invested capital. So $(A/P, 6\%, \infty) = i$, and we write

$$\begin{aligned} \text{EUAC} &= Pi = \$5.5 \text{ million}(0.06) \\ &= \$330{,}000 \end{aligned}$$

Pipeline

$$\begin{aligned} \text{EUAC} &= \$5 \text{ million}(A/P, 6\%, 50) \\ &= \$5 \text{ million}(0.0634) = \$317{,}000 \end{aligned}$$

For fixed output, minimize EUAC. Choose the pipeline.

Analysis complexities

- Other costs and revenues, like the salvage value or operational and maintenance costs, may also need to be included, so – as always – make sure to include all relevant items.

Potential break point

Mortgages in Canada

- Although technically a **mortgage** is a legal document, most people use the word to mean a long-term amortized loan that is used for buying real property such as a house or land.
- If the mortgage payments are not made, the lender can take the property and sell it to recover the outstanding debt.

Mortgages in Canada, cont'd

- A mortgage document:
  - Outlines the terms and conditions for repaying the money borrowed: the amount borrowed, the interest rate, the first and last payment dates, the repayment period, and the date the balance is due (the renewal date or term).
  - Prepayment options and penalties may also be included.
- Amortization is the process of paying off a debt over time.
- Amortization period is the length of time it takes to pay off the mortgage assuming:
  - Payments are made on time with no additional payments
  - Interest rate doesn't change

Mortgages in Canada, cont'd

- Amortization periods are typically between 5 years and 40 years
  - Norms: 20 - 25 yrs CA, 15 / 30 yrs US
- Terms
  - In Canada: Amort of mortgage made of smaller periods called terms. Term is period in which interest rt. "term" is established
  - Interest rates: can be fixed for term or var.
  - At the end of the term: Can be renewed for another term @ curr. int. rate.
- Rates
  - Interest rates are usually stated as the nominal annual rate, but are applied differently (more on this soon).

Building an Amortization Schedule

- An amortization schedule lists the following for each payment period:
  - Loan payment
  - Interest paid
  - Principal paid
  - Remaining balance
- For each period the interest paid equals the interest rate times the balance remaining from the period before.
- Then, the principal payment equals the payment minus the interest paid.
- Finally, this principal payment is applied to the balance remaining from the preceding period to calculate the new remaining balance.

Building an Amortization Schedule, cont'd

EXAMPLE 6-9

An engineer wanted to celebrate graduating and getting a job by buying $2,400 worth of new furniture. Luckily the store was offering six-month financing at the low interest rate of 6% per year nominal (really 0.5% per month). Calculate the amortization schedule.

SOLUTION

	A	B	C	D	E
1	2,400	Initial balance			
2	0.50%	*i*			
3	6	*N*			
4	$407.03	Payment	= −PMT(A2,A3,A1)		
5					
6			Principal	Ending	
7	Month	Interest	Payment	Balance	
8	0			2,400.00	=A1
9	1	12.00	395.03	2,004.97	=D8−C9
10	2	10.02	397.00	1,607.97	
11	3	8.04	398.99	1,208.98	
12	4	6.04	400.98	807.99	
13	5	4.04	402.99	405.00	
14	6	2.03	405.00	0.00	
15				=$A$4−B14	
16				=Payment−Interest	
17			=$A$2*D13		
18			=rate*previous balance		

FIGURE 6-1 Amortization schedule for furniture loan.

The first step is to calculate the monthly payment:

$$A = 2{,}400(A/P, 0.5\%, 6) = 2{,}400(0.1696) = \$407.03$$

With this information the engineer can use the spreadsheet in Figure 6-1 to obtain the amortization schedule.

See Excel example.

Mortgage Compounding Periods

- In the US and many other countries, effective annual rate is listed, and monthly equivalent can be calculated directly.
  - See Excel example 'mortgage ex 1'
- In Canada: rates are compounded semi-annually by law.
  - But payments are monthly (or biweekly), so lenders determine monthly interest rates that are equivalent to the semi-annual declared rate.
  - For example:
    - 6% rate quoted
    - Semi-annual rate = 6%/2 = 3%
    - Effective annual rate = $(1+6\%/2)^{\wedge 2} = 6.09\%$
    - Equivalent monthly rate = $(1+6.09\%)^{\wedge(1/12)} = 0.493862\%$
  - See also Excel example 'mortgage ex 2'

Types of Mortgages Available

- "Conventional":
  - For 80% or less of the appraised value of the property, and as such they require the purchaser to make a down payment of at least 20%
- "High-ratio" mortgages:
  - Higher than 80% and usually require an outside agency such as the CMHC (Central mortgage and housing corporation) to insure the mortgage.
- Some others:
  - Open, variable rate, ARM (adjustable rate morgage), capped rate, closed, convertible rate, second, reverse, CHIP.

Interest Rate Considerations

- Fixed mortgage rates are influenced mainly by the bond market, starting with government-issued bonds (which set the tone for the market).
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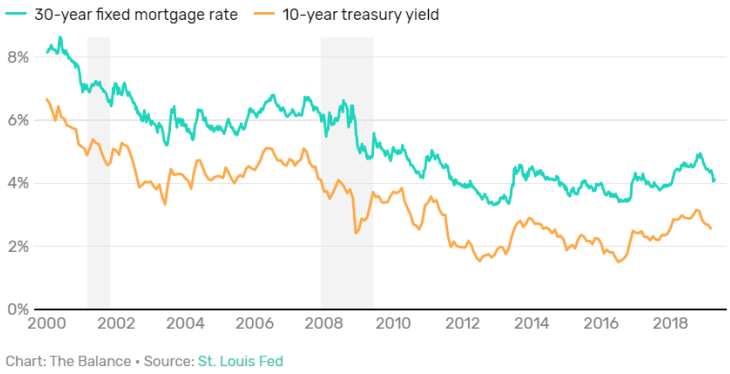

Interest Rate Considerations

- Variable mortgage rates are tied to lending rates that National banks (like the Bank of Canada or the Federal Reserve) set, for loaning money to financial institutions
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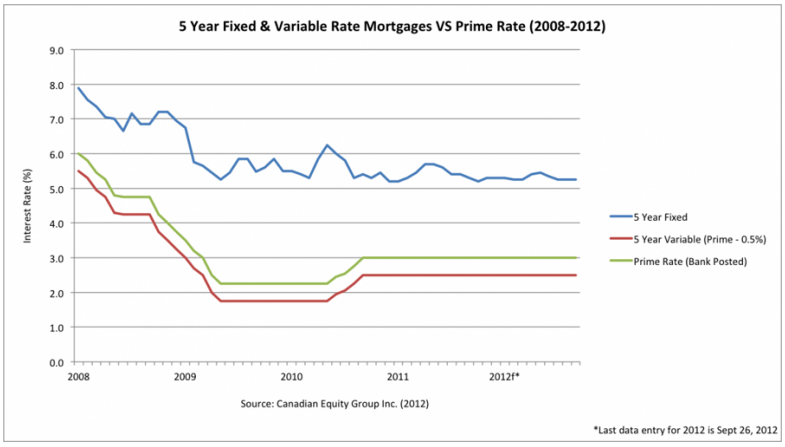
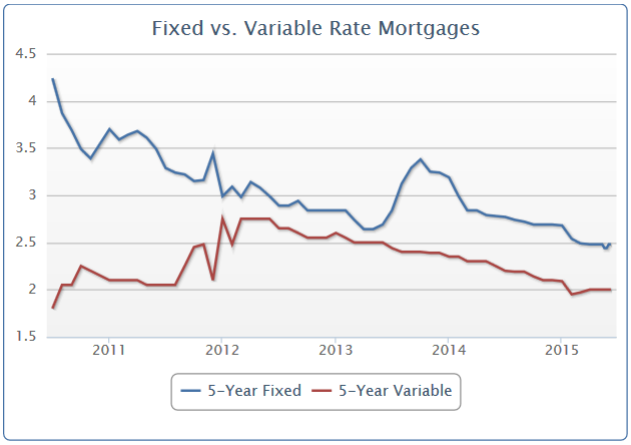


Interest Rate Considerations

- Most people are risk averse: that means they would prefer less mortgage risk than more, all else the same.
- Fixed rates mean less uncertainty, which therefore usually cost more
- These markets can move in different directions, leading to variability in the gap between them



Equity

- Equity
  - The value remaining in a property after all mortgage and loans registered against the title are subtracted from its value.
  - Another way to say it: the amount you actually have paid off.
  - For example:

Appraised value $210,000

minus mortgage $150,000

minus second mortgage $25,000

equals equity $35,000