5

Lecture 4

Present Worth Analysis

The Clock of the Long Now

The Clock of the Long Now, cont'd

- The Clock of the Long Now project is an attempt to build a clock that will run for 10,000 years.
- The clock's power cannot be fossil fuels or batteries, since a 10,000-year stockpile of either would be a valuable resource, likely to attract thieves as the Pyramids of Egypt attracted looters.
- Instead, it will get its power from the temperature difference between day and night.

The Clock of the Long Now, cont'd

- The intent of this project is to foster thinking and planning on very long time scales.
- The economic tools covered in this textbook do not appear to work well when extended to time periods beyond a century [as in our earlier climate change example].
- If we want to achieve long-term sustainability, quite different ways of thinking will have to be developed.
- The Long Now Foundation has begun to explore such ways in a series of lectures, *Seminars about Long-term Thinking*.

Learning Objectives

- Define the *present worth* and *future worth* criteria
- Use these criteria to choose between alternatives
- Apply the criteria in cases with equal, unequal, and infinite project lives
- Learn how bonds work

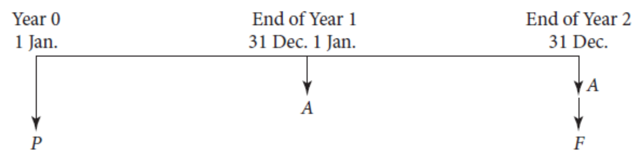
Key Summary: So Far, and What's Ahead

- Variables and parameters (puzzle pieces):
  - Different kinds of interest rates
  - Discount rates
  - Costs and cost savings or revenues, now and in the future
  - **Different expected lives of the possible project/purchases**
  - Salvage value
  - Taxes and tax savings
  - How these escalate
- Analysis methods (ways to put the pieces together):
  - **Present worth analysis**
  - Equivalent uniform annual cost analysis
  - Rate of return analysis
  - Benefit-cost ratio analysis
  - Payback period
  - Cost-effectiveness analysis

Reminder: Assumptions for Solving Economic Analysis Problems

- End-of-Year Convention
  - All cash flow amounts are calculated as amounts at the end of each period:
    - Now = end of period 0 (beginning of period 1)
    - Future amounts happen at the end of the period specified

A cash flow diagram of *P*, *A*, and *F* for the end-of-period convention is as follows:



- No Sunk Costs
  - Only the current situation and the potential future is considered.

Assumptions in Solving Economic Analysis Problems, cont'd

- Viewpoint of Economic Analysis Studies
  - Point of view of a firm *vs.* point of view of a department
  - Generally, we will want to take the point of view of a total firm when doing industrial economic analyses, and the point of view of broader society when doing public economic analyses.
- Borrowed Money Viewpoint
  - Each problem has two monetary aspects:
    - Financing—the obtaining of money
    - Investment—the spending of money
  - Money required to finance projects is borrowed at interest rate *i*.
  - *Discount rate vs interest rate*

Assumptions in Solving Economic Analysis Problems, cont'd

- Income Taxes
  - Income taxes must be considered in order to find the real payoff of a project.
  - However, taxes will often affect alternatives in the same way, allowing us to compare our choices without considering income taxes.
  - Will be looking at this later (Chapters 11 and 12)

- Effect of Inflation and Deflation
  - Inflation and deflation must be considered in order to find the real payoff of a project.
  - Will also be looking at this later (Chapter 14)

Economic Criteria

- Alternatives are judged based on economic efficiency
- How to use data about options to compare and make recommendations based on financial forecasted data

- To compare cash flows of alternatives, we must first move them to the same moment in time. Three possible techniques:
  - Present – *present worth analysis* (which we're covering today)
  - Future – *future worth analysis* (also today)
  - Series of equal cash flows at regular intervals – *annual cash flow analysis* (which we'll cover in Chapter 6)
- Each of these methods will always yield the same recommendation for choosing the best alternative(s).

Options Analysis Techniques

- We also also examine other alternative techniques:
  - Internal Rate of Return (IRR) analysis
  - Benefit-Cost analysis

Not necessarily the same answer.

- Today: Net Present Worth and Future Present Worth

Present Worth Techniques: Useful Life Options

- Careful consideration must be given to the time period covered by the analysis.

- Three potential analysis-period situations are possible when comparing alternatives:
  1. Useful life of each alternative equals the analysis period
  2. Useful lives different from the analysis period
  3. Infinite analysis period, $n = \infty$

1. Useful Lives Same as the Analysis Period

- When selecting between two alternatives using present worth analysis:
  - "Maximize":
    - Net Present Worth (NPW) = Present worth (PW) of benefits – Present worth (PW) of costs
  - The alternative with the higher NPW is selected
  - This criterion is called the *net present worth criterion* and is written simply as NPW
  - Economists call this criterion "Net Present Value"

1. Useful Lives Same as the Analysis Period, cont'd

- See Excel example

EXAMPLE 5-1

A city plans to build an aqueduct to bring water in from the mountains in the west. The aqueduct can be built now at a reduced size for \$300 million and enlarged 25 years hence for an additional \$350 million. An alternative is to construct the full-sized aqueduct now for \$400 million.

Both alternatives would provide the needed capacity for the 50-year analysis period. Maintenance costs are small and may be ignored. At 6% interest, which alternative should be selected?

SOLUTION

This problem illustrates staged construction. The aqueduct may be built in a single stage or in a smaller first stage followed many years later by a second stage to provide the additional capacity when needed.

For the Two-Stage Construction

$$\text{PW of cost} = \$300\text{ million} + \$350\text{ million}(P/F, 6\%, 25)$$
$$= \$300\text{ million} + \$81.6\text{ million}$$
$$= \$381.6\text{ million}$$

For the Single-Stage Construction

$$\text{PW of cost} = \$400\text{ million}$$

The two-stage construction has a smaller present worth of cost and is therefore the preferred construction plan.

1. Useful Lives Same as the Analysis Period, cont'd

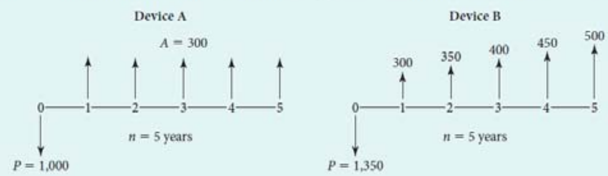
- See Excel example

EXAMPLE 5-2

A firm is considering which of two mechanical devices to install to reduce costs. Both devices have useful lives of five years and no salvage value. Device A costs $1,000 and can be expected to result in $300 savings annually. Device B costs $1,350 and will provide cost savings of $300 the first year, but the savings will increase by $50 annually, making the second-year savings $350, the third-year savings $400, and so forth. With interest at 7%, which device should the firm purchase?

SOLUTION

The analysis period can be conveniently chosen as the useful life of the devices, which is five years. The appropriate decision criterion is to maximize the net present worth of benefits minus costs.



$$PW_A = -1{,}000 + 300(P/A, 7\%, 5) = -1{,}000 + 300(4.100) = \$230$$

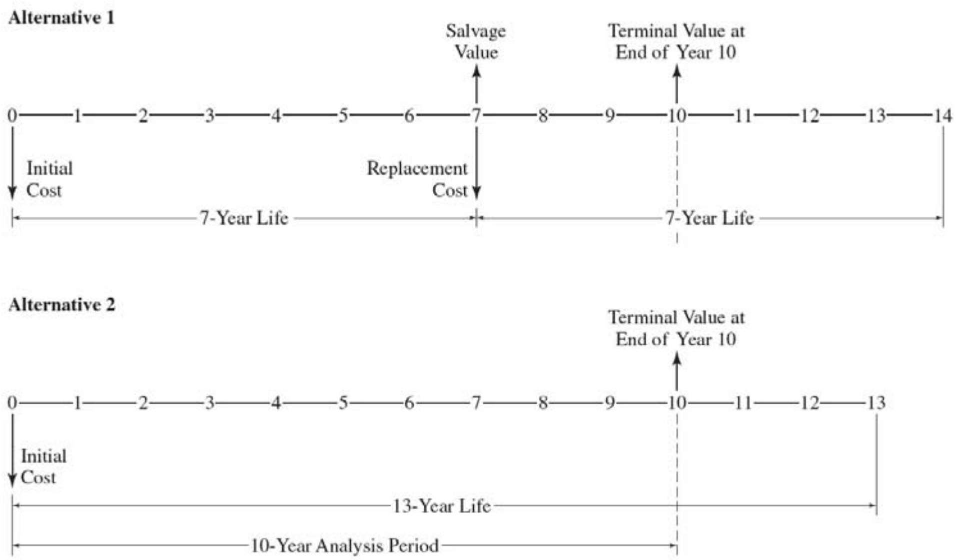
$$PW_B = -1{,}350 + 300(P/A, 7\%, 5) + 50(P/G, 7\%, 5)$$
$$= -1{,}350 + 300(4.100) + 50(7.647) = \$262.4$$

Device B has the larger present worth and is the preferred alternative.

2. Useful Lives Different from the Analysis Period

- It is NOT correct to analyze alternatives using NPW if the alternatives have different expected lives.
- Methods to handle this problem:
  - Examine the alternatives using a "least common multiple" (LCM) of lives.
    - For example: Option 1 lasts 3 years, and Option 2 lasts 4 years. LCM=12 years
  - Decide on an reasonably comparable analysis period if the LCM method is too onerous or doesn't make sense.
    - For example: 7 and 13 years gives an LCM = 91 years
    - Let's unpack this one next.

Example: Selecting an Analysis Period



2. Useful Lives Different from the Analysis Period, cont'd

- See Excel example

EXAMPLE 5-3

A purchasing agent is considering the purchase of some new equipment for the mailroom. Two different manufacturers have provided quotations. An analysis of the quotations shows the following:

Manufacturer	Cost	Useful Life (years)	End-of-Useful-Life Salvage Value
Speedy	$1,500	5	$200
Allied	1,600	10	325

The equipment of both manufacturers is expected to perform at the desired level of (fixed) output. Which manufacturer's equipment should be selected? Assume 7% interest and equal maintenance costs.

SOLUTION

The Speedy and the Allied equipment will both provide the same benefit, but the Allied equipment will provide it for ten years, while the Speedy will only provide it for five. One way of making a fair comparison would be to suppose that, if we buy the Speedy, we can replace the equipment after five years at the same cost. That is, we select an analysis period that is the *least common multiple* of the lives of the two alternatives. Our two alternatives now both offer the same benefit over a ten-year period, so we can simply choose the one that does this more cheaply.

Speedy

$$\begin{aligned}\text{PW of cost} &= \$1{,}500 + (1{,}500 - 200)(P/F, 7\%, 5) - 200(P/F, 7\%, 10)\\ &= 1{,}500 + 1{,}300(0.7130) - 200(0.5083)\\ &= 1{,}500 + 927 - 102 = \$2{,}325\end{aligned}$$

Allied

$$\begin{aligned}\text{PW of cost} &= 1{,}600 - 325(P/F, 7\%, 10) = 1{,}600 - 325(0.5083)\\ &= 1{,}600 - 165 = \$1{,}435\end{aligned}$$

Since it is only the *differences between alternatives* that are relevant, maintenance costs may be left out of the economic analysis. So we should buy the Allied equipment.

2. Useful Lives Different from the Analysis Period, cont'd

EXAMPLE 5-4

A diesel manufacturer is considering the two alternative production machines depicted in Figure 5-1. Specific data are as follows:

	Alt. 1	Alt. 2
Initial cost	\$50,000	\$75,000
Estimated salvage value at end of useful life	\$10,000	\$12,000
Useful life of equipment, in years	7	13

The manufacturer uses an interest rate of 8% and wants to use the PW method to compare these alternatives over an analysis period of 10 years.

	Alt. 1	Alt. 2
Estimated market value, end of 10-year analysis period	\$20,000	\$15,000

continued

2. Useful Lives Different from the Analysis Period, cont'd

- See Excel example

SOLUTION

In this case, the decision maker is setting the analysis period at 10 years rather than accepting a common multiple of the lives of the alternatives. This is a legitimate approach—perhaps the diesel manufacturer will be phasing out this model at the end of the 10-year period. In any event, we need to compare the alternatives over the 10 years.

As illustrated in Figure 5-1, we may assume that Alternative 1 will be replaced by an identical machine after its seven-year useful life. Alternative 2 has a 13-year useful life. The diesel manufacturer has provided an estimated market value of the equipment at the time of the analysis period. We can compare the two choices over 10 years as follows:

$$\begin{aligned}\text{PW (Alt 1.)} &= -50{,}000 + (10{,}000 - 50{,}000)(P/F, 8\%, 7) + 20{,}000(P/F, 8\%, 10)\\ &= -50{,}000 - 40{,}000(0.5835) + 20{,}000(0.4632)\\ &= -\$64{,}076\end{aligned}$$

$$\begin{aligned}\text{PW (Alt 2.)} &= -75{,}000 + 15{,}000(P/F, 8\%, 10)\\ &= -75{,}000 + 15{,}000(0.4632)\\ &= -\$68{,}052\end{aligned}$$

To minimize PW of costs the diesel manufacturer should choose Alternative 1.

3. Infinite Analysis Period: Capitalized Cost

- The need for large-scale infrastructure projects (bridges, pipelines, etc.) is considered to be permanent.
- These types of projects are considered to have an infinite analysis period.
- Capitalized cost is the present sum that is required to provide the service indefinitely.

- Never used at Metro Vanc.

3. Infinite Analysis Period: Capitalized Cost, cont'd

- For any initial present sum P, there can be an end-of-period withdrawal of A which is equal to $P(i)$:
  - These withdrawals will never decrease the original principal
  - $A = Pi$ for n = infinity
  - Capitalized Cost = $P = A/i$
    - ➢ The money set aside that can provide the funds for the project forever

3. Infinite Analysis Period: Capitalized Cost, cont'd

EXAMPLE 5-5

How much should you set aside to pay \$50 a year for maintenance on a gravesite if interest is assumed to be 4%? For perpetual maintenance, the principal sum must remain undiminished after the annual disbursement is made.

SOLUTION

$$\text{Capitalized cost } P = \frac{\text{Annual disbursement } A}{\text{Interest rate } i}$$

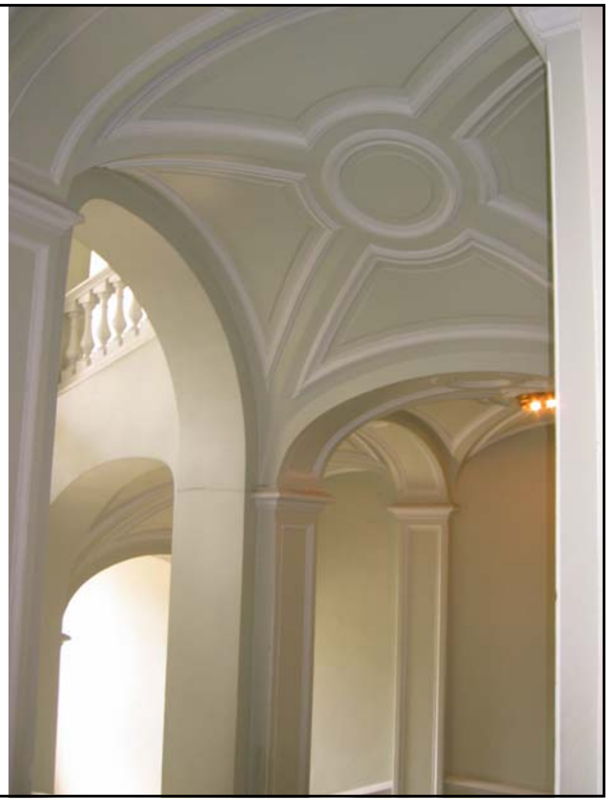
$$P = \frac{50}{0.04} = \$1{,}250$$

You should set aside \$1,250.

Time Step for Present Worth Analysis

- Sometimes monthly analysis is valuable and appropriate: For example, electrical costs for air conditioning will vary seasonally considerably.
- Other times, annual average conditions are sufficient for the purposes of comparing options
- Time step decision is somewhat independent of how you choose to solve them: spreadsheets and formulas can both work, but formulas are not well adapted for non-uniform or complex (e.g. sinusoidal) patterns

Potential break point



Present Worth Analysis: Problem 1

Two outdoor facilities are being considered for an upcoming Olympic baseball event in three years. The ticket price is fixed for the event at $150/person payable in the event year. Facility A requires a non-refundable deposit of $250,000 and will hold 15,000 people for the event. Facility B does not require a deposit but holds only 13,000 people. If the event sells out in either facility, which facility should be chosen based on a present worth analysis, if the interest rate is 10%?

Present Worth Analysis: Problem 1, cont'd

Solution

Present worth of Facility A

$$P = -250{,}000 + 15{,}000(150)(P/F, 10\%, 3)$$
$$P = -250{,}000 + 15{,}000(150)(0.7513)$$
$$P = \$1{,}440{,}425$$

Present worth of Facility B

$$P = 13{,}000(150)(P/F, 10\%, 3)$$
$$P = 13{,}000(150)(0.7513)$$
$$P = \$1{,}465{,}035$$

Choose **Facility B** because the NPW of B is more profitable based on a present worth analysis.

Present Worth Analysis: Problem 2

A municipal contractor has agreed to construct an electric power plant and to deposit sufficient money in a perpetual trust fund to pay a \$10,000/year operating cost and to perform a major renovation to the plant every 15 years at a cost of \$200,000. The plant itself will initially cost \$500,000 to construct. If the trust fund earns 10% interest per year (compounded annually), what is their capitalized cost to construct the plant, to make the future periodic renovations, and to pay the annual operating costs forever?

Present Worth Analysis: Problem 2, cont'd

Solution

A = \$10,000 /year

Renovation cost every 15 years = \$200,000

Initial cost = \$500,000.   Interest rate = 10%

Capitalized cost, P_0 = 500,000 + (10,000/0.10) + {200,000 (A/F, 10%, 15)/0.10} = 500,000 + 100,000 + {200,000(0.0315)/0.10} = **\$662,948**

Multiple Alternatives

- Present worth analysis can be applied to any number of alternatives.
- The NPW can be evaluated for each project.
- Selection is made based on the criteria for the project (minimizing or maximizing the present worth of the alternatives).

Multiple Alternatives, cont'd

- See Excel example

EXAMPLE 5-7

A contractor has been awarded the contract to construct a 6-km tunnel in the mountains. During the five-year construction period, the contractor will need water from a nearby stream. She will construct a pipeline to carry the water to the main construction yard. An analysis of costs for various pipe sizes is as follows:

	Pipe Sizes (cm)			
	5	7	9	11
Installed cost of pipeline and pump	$22,000	$23,000	$25,000	$30,000
Cost per hour for pumping	$1.20	$0.65	$0.50	$0.40

The pipe and pump will have a salvage value at the end of five years equal to the cost of removing them. The pump will operate 2,000 hours a year. The lowest interest rate at which the contractor is willing to invest money is 7%. Select the alternative with the least present worth of cost.

SOLUTION

We can compute the present worth of cost for each alternative. For each size of pipe, the present worth of cost is equal to the installed cost of the pipeline and pump plus the present worth of five years of pumping costs.

	Pipe Size (cm)			
	5	7	9	11
Installed cost of pipeline and pump	$22,000	$23,000	$25,000	$30,000
$1.20 \times 2{,}000$ hr $\times$ $(P/A, 7\%, 5)$	9,840			
$0.65 \times 2{,}000$ hr $\times$ 4.100		5,330		
$0.50 \times 2{,}000$ hr $\times$ 4.100			4,100	
$0.40 \times 2{,}000$ hr $\times$ 4.100				3,280
Present worth of cost	$31,840	$28,330	$29,100	$33,280

Thus, to minimize the present worth of cost, choose the 7-cm pipe.

Future Worth Analysis

- In present worth analysis, alternatives are compared in terms of their present equivalent value.
- We can also choose between alternatives by finding their equivalent value at some future date. This is called **future worth analysis**.
- Future worth analysis is a more natural way of thinking about certain types of problems.
  - For example: if we are setting aside money regularly to provide ourselves with a pension on retirement, we're interested in how much money we'll have when we retire. We don't care so much what that amount of money would be worth now, since we're not going to be spending it now.

Future Worth Analysis, cont'd

EXAMPLE 5-10

Ron, a 20-year-old university student, smokes about a carton of cigarettes a week. He wonders how much money he could accumulate by age 65 if he quit smoking now and put his cigarette money into a savings account. Cigarettes cost \$85 a carton. Ron expects that a savings account would earn 5% interest, compounded semi-annually. Compute the future worth of Ron's savings at age 65.

SOLUTION

$$\text{Semi-annual saving} = (\$85/\text{carton})(26\text{ weeks}) = \$2{,}210$$

$$\text{Future worth (FW)} = A(F/A, 2.5\%, 90) = 2{,}210(329.2) = \$727{,}532$$

Bond Pricing (Sidebar Topic)

- A Bond is a way for firms or governments to borrow money from others. For the buyer:
- When Bonds are purchased, the fixed items are:
  - Face Value (e.g., \$1000)
    - Amount paid when bond matures
  - A nominal interest rate (e.g., 8% semi-annually)
    - Coupon rate aka.
    - Amount of interest paid to bond holder per comp. per.
- The purchase price can vary depending on the current market interest rate as the present worth of a bond is determined from the fixed values above (the interest paid out per compounding period and the final face value paid out in the future).
-

Bond Pricing, cont’d

- See Excel example

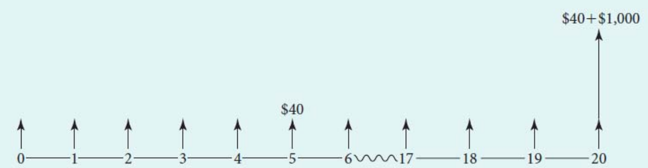
EXAMPLE 5-9

A 15-year municipal bond was issued five years ago. Its coupon interest rate is 8%, interest payments are made semi-annually, and its face value is $1,000. If the current market interest rate is 12.36%, what should the price of the bond be? *Note*: The issuer of the bond (a government or company) makes interest payments to the bondholder (at the coupon rate), as well as a final value payment.

SOLUTION

The first five years are past, and there are 20 more semi-annual payments. The coupon interest rate is the nominal annual rate or APR. Half of that, 4% of $1,000 = $40, is paid at the end of each six-month period.

The price of the bond is the PW of the cash flows that will be received if the bond is purchased. The cash flows are $40 at the end of each of the 20 semi-annual periods and the face value of $1,000 at the end of period 20.



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Bond Pricing, cont’d

Since the $40 in interest is received semi-annually, the market or effective annual interest rate (i_a) must be converted to a semi-annual rate. Using Equation 3-8, we obtain

First step

$$(1 + i)^2 = 1 + i_a = 1.1236$$
$$(1 + i) = 1.06$$
$$i = 6\% \text{ effective semi-annual interest rate}$$

Second step

$$PW = 40(P/A, 6\%, 20) + 1{,}000(P/F, 6\%, 20)$$
$$= 40(11.470) + 1{,}000(0.3118) = \$770.6$$

The $770.6 is the discounted price, that is, the PW at 12.36% of the cash flows from the $1,000 bond. The $229.4 discount raises the rate of return on the investment from a nominal 8% for the face value to 12.36% on an investment of $770.6.

This example also illustrates why it is better to state cash flows separately. At the end of period 20 from now, there are two cash flows, $40 and $1,000. The $40 is part of the 20-period uniform series, and the $1,000 is a single cash flow. All of these numbers come directly from the problem statement. If the two final cash flows are combined into $1,040, then the $40 uniform series has only 19 periods—and it is easy to err and forget that change.