

## Today:

Business Casing example: UBC trees article  
Business Casing example for you to ponder  
Chapter 10

1

## Business Casing Example

Real life example - UBC story:

<https://www.cbc.ca/news/canada/british-columbia/ubc-trees-petition-1.6350023>

How could you conduct an appropriate analysis of the costs and benefits, and what would be included in scope?

2

Business casing question to ponder:

Fleet purchase analysis tool:  
Why is the cost 'lost interest on capital'  
included?

How does it impact the outcome?



10

## Lecture 12

### Uncertainty in Future Events

## Technological Uncertainty and the Consequence of Failure: Waste Disposal and Wild Game



## Technological Uncertainty and the Consequence of Failure: Waste Disposal and Wild Game , cont'd

- The Swan Hills Special Waste Treatment Centre was opened in 1987 to incinerate organic liquids and solids, treat inorganic liquids and solids, and **stabilize and decontaminate hazardous wastes** and landfill-contaminated bulk solids.
- Unfortunately, the technology was not as clean or as reliable as its designers had thought it would be.
- In October of 1996, there was a mechanical failure that resulted in the **release of toxic gases** into the atmosphere.

## Technological Uncertainty and the Consequence of Failure:

### Waste Disposal and Wild Game , cont'd

- As a consequence, the government of Alberta issued a public health warning to local First Nations communities not to eat wild game from the area around the plant.
- First Nations members became suspicious of their water, plants, medicines, and wild food, and many curtailed their traditional land use practices.
- They became more sedentary and reliant on store-bought foods, which are high in fats and sugar. Currently, these communities have high rates of diabetes and various social problems.

## Technological Uncertainty and the Consequence of Failure:

### Waste Disposal and Wild Game , cont'd

- Although these complex social and health issues cannot be attributed directly to the release of toxic gases, it is extremely difficult to rebuild confidence in the safety of traditional foods.
- Now, years after the release, the risk is still being monitored by analyzing wild game for traces of dioxins, furans, and PCBs.

## Learning Objectives

- Use a range of estimated variables to evaluate a project
- Describe possible outcomes with probability distributions
- Combine probability distributions for individual variables into joint probability distributions
- Use expected values for economic decision making
- Use economic decision trees to describe and solve more complex problems
- Measure and consider risk when making economic decisions
- Understand how simulation can be used to evaluate economic decisions
- Consider several 'real world' examples in which uncertainty affects decision making

## Two Possible Analysis Approaches

1. Fixed data → Unknown outcome

- Variables and parameters:
  - Different kinds of interest rates
  - Discount rates
  - Costs and cost savings or revenues, now and in the future
  - Different expected lives of the possible project/purchases
  - Salvage value
  - Taxes and tax savings
  - How these escalate
- Analysis methods:
  - Present worth analysis
  - Equivalent uniform annual cost analysis
  - Rate of return analysis
  - Benefit-cost ratio analysis

*Example problem: with known costs, revenues, interest rate, etc, does a potential project pay for itself?*

## Two Possible Analysis Approaches

- |                  |   |                 |
|------------------|---|-----------------|
| 1. Fixed data    | → | Unknown outcome |
| 2. Variable data | ← | Fixed outcome   |

- Variables and parameters:
  - Different kinds of interest rates
  - Discount rates
  - Costs and cost savings or revenues, now and in the future
  - Different expected lives of the possible project/purchases
  - Salvage value
  - Taxes and tax savings
  - How these escalate
- Analysis methods:
  - Present worth analysis
  - Equivalent uniform annual cost analysis
  - Rate of return analysis
  - Benefit-cost ratio analysis

*Example problem: How high would revenues need to be for a potential project, for it to pay for itself (break even)?*

## Break-Even Analysis

- Break-even analysis is a form of sensitivity analysis.
- It helps by answering the question “how much variability can a parameter have before the decision will be affected?”
- Typically presented as a break-even chart.
- It indicates the point at which two alternatives are equal to each other.
- Connection to data and variables?

## Optimistic and Pessimistic Estimates

- We can't foretell precisely the costs and benefits for future years.
- It is more realistic to describe a range of possible values.
  - For example a range could include:
    - "Optimistic," "most likely," and "pessimistic" estimates
    - The analysis could then determine whether the decision is sensitive to the range of values.
    - Mean value =  $(\text{Optimistic value} + 4 \times (\text{Most likely value}) + \text{Pessimistic value}) / 6$

## Optimistic and Pessimistic Estimates, cont'd

### EXAMPLE 10-3

A firm is considering an investment. The most likely data values were found during the feasibility study. Analyzing past data from similar projects shows that optimistic values for the first cost and the annual benefit are 5% better than most likely values. Pessimistic values are 15% worse. The firm's most experienced project analyst has estimated the values for the useful life and salvage value.

	Optimistic	Most Likely	Pessimistic
Cost	\$950	\$1,000	\$1,150
Net annual benefit	\$210	\$200	\$170
Useful life, in years	12	10	8
Salvage value	\$100	\$0	\$0

Compute the rate of return for each estimate. If a 10% before-tax minimum attractive rate of return is required, is the investment justified under all three estimates? If it is only justified under some estimates, how can these results be used?

- <see Excel example>

## Optimistic and Pessimistic Estimates, cont'd

### SOLUTION

#### Optimistic Estimate

$$\begin{aligned} PW &= 0 = -\$950 + 210(P/A, \text{IRR}_{\text{opt}}, 12) + 100(P/F, \text{IRR}_{\text{opt}}, 12) \\ \text{IRR}_{\text{opt}} &= i(n, A, P, F) = i(12, 210, -950, 100) \\ &= 19.8\% \end{aligned}$$

#### Most Likely Estimate

$$\begin{aligned} PW &= 0 = -\$1,000 + 200(P/A, \text{IRR}_{\text{most likely}}, 10) \\ (P/A, \text{IRR}_{\text{most likely}}, 10) &= 1,000/200 = 5 \rightarrow \text{IRR}_{\text{most likely}} = 15.1\% \end{aligned}$$

or

$$\text{IRR}_{\text{most likely}} = i(n, A, P, F) = i(10, 200, -1,000, 0) = 15.1\%$$

#### Pessimistic Estimate

$$\begin{aligned} PW &= 0 = -\$1,150 + 170(P/A, \text{IRR}_{\text{pess}}, 8) \\ (P/A, \text{IRR}_{\text{pess}}, 8) &= 1,150/170 = 6.76 \rightarrow \text{IRR}_{\text{pess}} = 3.9\% \end{aligned}$$

or

$$\text{IRR}_{\text{pess}} = i(n, A, P, F) = i(8, 170, -1,150, 0) = 3.9\%$$

From the calculations we conclude that the rate of return for this investment is most likely to be 15.1% but might range from 3.9% to 19.8%. The investment meets the 10% MARR criterion for two of the estimates. These estimates can be considered to be scenarios of what may happen with this project. Since one scenario suggests that the project is not attractive, we need to have a method of weighting the scenarios or considering how likely each is.

## Probability

- Probability of flipping a coin:
  - Head (50%), Tail (50%)
- This is the long range frequency and the single trial likelihood.
- Probabilities can also be based on past data, expert judgement, or some combination of both.
- Mathematically:
  - Between 0 and 1
    - 0 (can never happen) – 0%
    - 1 (will always happen) – 100%
    - 0.5 (half the time—as in the above example of flipping a coin—50%)



## Probability, cont'd

- The sum of all probabilities must equal 1.
  - Head (0.5), Tail (0.5)
  - $0 \leq \text{Probability} \leq 1$
  - $\sum P(\text{outcome}_j) = 1$ , where there are  $K$  outcomes
- As opposed to large distributions, it is more common in economic analysis to use 2 to 5 discrete possibilities.
  - This is because:
    - Each outcome requires more analysis.
    - Expert judgement should be limited for accuracy.
    - Why not choose and evaluate more possibilities?

Choosing to use more possibilities implies we actually can be accurate at that level of precision, which usually isn't true

## Joint Probability Distributions

- The product of two independent events
  - $P(A \text{ and } B) = P(A) \times P(B)$
- Example: Flipping a coin and rolling a die
  - Turning up a head and rolling a 4
    - $1/2 \times 1/6 = 1/12$  joint probability
- Joint probabilities increase the number of possibilities and can become arithmetically burdensome.

## Expected Value

- Each outcome is weighted by its probability and the results are summed:
  - Expected value =  $\text{Outcome}_A \times P(A) + \text{Outcome}_B \times P(B) + \dots$
- Not a simple average: it's a weighted average, with the weight reflecting the probability of that outcome occurring.

## Probability, cont'd

### EXAMPLE 10-5

What are the probability distributions for the annual benefit and life for the following project?

The most likely value of the annual benefit is \$8,000 with a probability of 60%. There is a 30% probability that it will be \$5,000, and the highest value that is likely is \$10,000. A life of six years is twice as likely as a life of nine years.

- <see Excel example>

## Probability, cont'd

### SOLUTION

Probabilities are given for only two of the possible outcomes for the annual benefit. The third value is found from the fact that the probabilities for the three outcomes must sum to 1 (Equation 10-3).

$$1 = P(\text{benefit is } \$5,000) + P(\text{benefit is } \$8,000) + P(\text{benefit is } \$10,000)$$

$$P(\text{benefit is } \$10,000) = 1 - 0.6 - 0.3 = 0.1$$

The probability distribution can then be summarized in a table. Figure 10-1 shows the histogram, or relative frequency diagram.

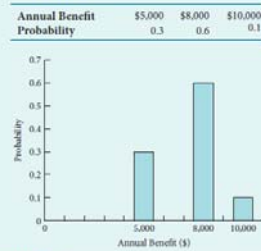


FIGURE 10-1 Probability distribution for annual benefit.

For the probability distribution of project life, the problem statement tells us

$$P(\text{life is 6 years}) = 2P(\text{life is 9 years})$$

If we assume that these are the only two possible lifespans, Equation 10-3 can be used to write a second equation for their likelihoods:

$$P(6) + P(9) = 1$$

Combining these, we write

$$2P(9) + P(9) = 1$$

$$P(9) = 1/3$$

$$P(6) = 2/3$$

The probability distribution for the life is  $P(6) = 66.7\%$  and  $P(9) = 33.3\%$ .

- <see Excel example>

## Expected Value, cont'd

### EXAMPLE 10-7

The first cost of the project in Example 10-5 is \$25,000. Use the expected values for annual benefits and life to estimate the present worth. Use an interest rate of 10%.

- <see Excel example>

## Expected Value, cont'd

### EXAMPLE 10-7

The first cost of the project in Example 10-5 is \$25,000. Use the expected values for annual benefits and life to estimate the present worth. Use an interest rate of 10%.

#### SOLUTION

$$EV_{\text{benefit}} = 5,000(0.3) + 8,000(0.6) + 10,000(0.1) = \$7,300$$

$$EV_{\text{life}} = 6(2/3) + 9(1/3) = 7 \text{ years}$$

Using these values, the PW is

$$PW(EV) = -25,000 + 7,300(P/A, 10\%, 7) = -25,000 + 7,300(4.868) = \$10,536$$

(Note: This is *not* the expected value of the present worth,  $EV(PW)$ ; rather, it is the present worth of the expected values,  $PW(EV)$ . This is an easy value to calculate and one that approximates the  $EV(PW)$ , which will be computed from the joint probability distribution found in Example 10.8.)

## Expected Value: Problem 1

The proposed projects have the potential uniform annual benefits and associated probability of occurrence shown below. Which project is more desirable based on these data?

Project A		Project B	
<i>EUAB</i>	<i>Probability</i>	<i>EUAB</i>	<i>Probability</i>
\$1000	0.10	\$1500	0.20
\$2000	0.30	\$2500	0.40
\$3000	0.40	\$3500	0.30
\$4000	0.20	\$4500	0.10

- <see Excel example>

## Expected Value: Problem 1, cont'd

### Solution

Determine which project looks better based on expected values.

$$\begin{aligned}\text{Expected Value}_A &= 1000(0.1) + 2000(0.3) + 3000(0.4) + 4000(0.2) \\ &= \$2700\end{aligned}$$

$$\begin{aligned}\text{Expected Value}_B &= 1500(0.2) + 2500(0.4) + 3500(0.3) + 4500(0.1) \\ &= \$2800\end{aligned}$$

**Project B** has the greatest expected value and should be selected.

## Expected Value, cont'd

Example 10-8 (building on Examples 10-5 through 10-7):




Use the probability distribution function of the PW that was derived in Example 10-6 to calculate the EV(PW). Does this indicate an attractive project?

<switch to Excel>

Potential break  
point



## Economic Decision Trees

- Decision Trees can be created for complex decision making:
  - Symbols:
    - Decision Node 
    - Chance Node 
    - Outcome Node 

## Economic Decision Trees, cont'd

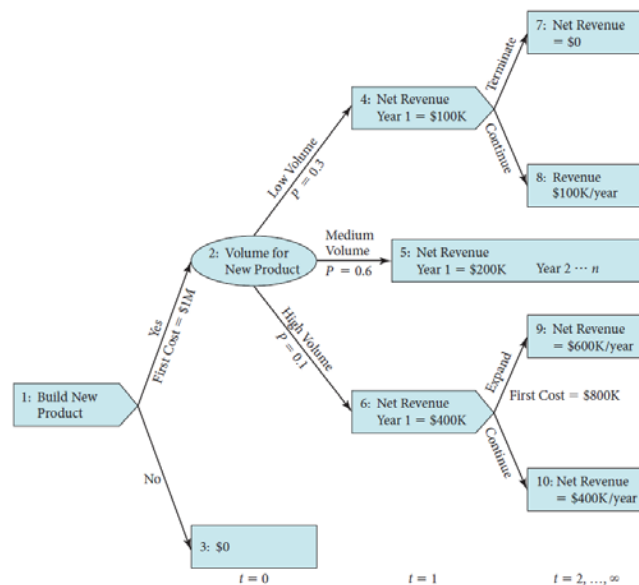


FIGURE 10-3 Economic decision tree for new product.

## Risk

- Risk is the chance of getting an outcome other than the expected value.
- A common measure is standard deviation:

$$\sigma = \sqrt{[EV(X - \text{mean})^2]}$$

- Standard deviation is used instead of variance because standard deviation is in the same unit as the expected value.
  - Variance would be in “squared dollars.”

## Risk, cont'd

### EXAMPLE 10-13

Using the probability distribution for the PW from Example 10-6, calculate the standard deviation of the PW.

- <see Excel example>

## Risk, cont'd

### EXAMPLE 10-13

Using the probability distribution for the PW from Example 10-6, calculate the standard deviation of the PW.

#### SOLUTION

The following table adds a column for  $(PW^2)$  (probability) to calculate the  $EV(PW^2)$ .

Annual Benefit	Probability	Life (years)	Probability	Joint Probability	PW	PW × Probability	PW <sup>2</sup> × Probability
\$ 5,000	30%	6	66.7%	20.0%	-\$ 3,224	-\$ 645	\$ 2,079,480
8,000	60	6	66.7	40.0	9,842	3,937	38,747,954
10,000	10	6	66.7	6.7	18,553	1,237	22,950,061
5,000	30	9	33.3	10.0	3,795	380	1,442,100
8,000	60	9	33.3	20.0	21,072	4,214	88,797,408
10,000	10	9	33.3	3.3	32,590	1,086	35,392,740
					EV	\$10,209	\$189,409,745

$$\text{Standard deviation } (\sigma) = \sqrt{\{EV(X^2) - [EV(X)]^2\}}$$

$$\sigma = \sqrt{\{189,409,745 - [10,209]^2\}} = \sqrt{85,182,064} = \$9,229$$



## Risk versus Return

- Graphing risk versus return is one way to consider these items together.
- Risk measured by standard deviation is put on the x axis.
- Return measured by expected value is put on the y axis.
- Normally done with internal rate of return of the alternatives.
- Rule of Thumb:
  - If the expected present worth is at least double the standard deviation of the present worth, then the project is relatively safe.
    - In a normal distribution about 2.5% of the values are less than two standard deviations below the mean.
    - Implications
    - Opportunities

This implies that this rule of thumb is really cautious, and therefore will mean many good opportunities are not acted on : being cautious comes at a price too, just a different price

Opportunities to do something beneficial probably should be evaluated very differently than are projects necessary to protect safety and human health

## Risk versus Return: Problem 2

A city engineer has compiled the following data on a flood damage project. To control the flood, the construction of a small dam will cost \$100,000. Based on the estimated damage data collected, determine if it is worth building a dam.

Damage Estimate	Probability
\$200,000	0.30
\$100,000	0.50
\$50,000	0.10
\$0	0.10

- <see Excel example>

## Risk versus Return: Problem 2, cont'd

### Solution

$$\begin{aligned}\text{Expected damage} &= \$200,000(0.30) + 100,000(0.50) + \\ &50,000(0.10) + 0.0(0.10) \\ &= \mathbf{\$115,000}\end{aligned}$$

Since the dam will cost only \$100,000, it is worth building the dam.

The expected savings in one flood season will take care of the cost.

Damage estimation:

## Differential NPW Analysis and Risk Analysis

- Differential analysis
- Sensitivity analysis
- Timing: “Value Engineering”: How would delaying the project affecting the business case?

## Analysis example

- Implications on fleet selection tool example:
  - How could uncertainties and sensitivities be incorporated to this kind of analysis?

Potential break  
point



## Risk versus Return, cont'd

### EXAMPLE 10-14

A large firm is discontinuing an older product, and thus some facilities are becoming available for other uses. The following table summarizes eight new projects that would use the facilities. Considering expected return and risk, which projects are good candidates? The firm believes it can earn 4% on a risk-free investment in government securities (labelled as Project F).

Project	IRR	Standard Deviation
1	13.1%	6.5%
2	12.0	3.9
3	7.5	1.5
4	6.5	3.5
5	9.4	8.0
6	16.3	10.0
7	15.1	7.0
8	15.3	9.4
F	4.0	0.0

*continued*

## Risk versus Return, cont'd

### SOLUTION

It is far easier to answer the question if we use Figure 10-7. Since a larger expected return is better, we want to choose projects that are as high up on the graph as possible. Since a lower risk is better, we want to choose projects that are as far left as possible. The graph lets us examine the trade-off of accepting more risk for a higher return.

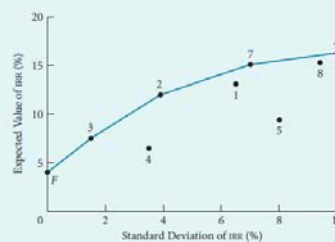


FIGURE 10-7 Risk-versus-return graph.

First, we can eliminate Projects 4 and 5. They are dominated projects. Project 4 is dominated by Project 3, which has a higher expected return and a lower risk. Project 5 is dominated by Projects 1, 2, and 7. All three have a higher expected return and a lower risk.

Second, we look at the efficient frontier. This is the line in Figure 10-7 that connects Projects F, 3, 2, 7, and 6. Depending on the trade-off that we want to make between risk and return, any of these could be the best choice.

Project 1 appears to be inferior to Projects 2 and 7. Project 8 appears to be inferior to Projects 7 and 6. Projects 1 and 8 are inside and not on the efficient frontier.

There are models of risk and return that can allow us to choose between Projects F, 3, 2, 7, and 6; but those models are beyond what is covered here.

## Risk Preference

Risk preference options:

- Risk-averse
- Risk-neutral
- Risk-seeking

Basis of your preference is how you internalize expected value comparisons

## Risk Preference

Example:

- Options available:
  - A: Guaranteed \$50 payment, or
  - B: 50% chance of \$100, 50% chance of \$0

Which would you prefer?

Let's take a poll.

## Risk Preference

Example:

• Options available:

- A: Guaranteed \$50 payment, or
- B: 50% chance of \$100, 50% chance of \$0

If you prefer A to B: risk averse

If you are indifferent: risk neutral

If you prefer B to A: risk seeking

## Risk Preference

Example:

• Options available:

- A: Guaranteed \$50 payment, or
- B: 50% chance of \$100, 50% chance of \$0

If you prefer A to B: risk averse

If you are indifferent: risk neutral

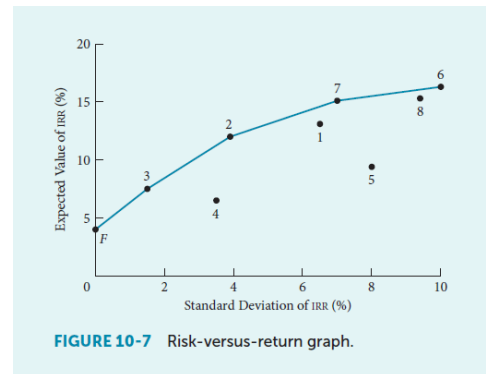
If you prefer B to A: risk seeking

Would your answer change if the numbers were  
1000x higher?

## Risk Preference

This kind of analysis can be applied to more complicated probability distribution functions.

That would allow us to compare the dominant options (in this example: options 5, 3, 2, 7, and 6)



## Simulation

- Simulation is a more advanced approach to considering risk in engineering economy problems.
- The form of economic simulation known as the Monte Carlo method uses random sampling from the probability distributions of one or more variables to analyze an economic model for many iterations.

## Simulation, cont'd

- Simulation can be done by hand using a table of random numbers and only a few iterations.
- More commonly, it is done with a computer using a spreadsheet random function or some other simulation package.
- Using a computer allows for larger numbers of iterations to be calculated and/or more variables to be randomized.
- As well, using a computer allows for the evaluation of a number of different types of probability distributions.
- This can be done in Excel, using the RAND() function to generate random numbers.

## Simulation, cont'd

### EXAMPLE 10-15

ShipM4U is considering installing a new, more-accurate scale, which will reduce the error in calculating postage charges and save \$250 a year. The useful life of the scale is believed to be distributed uniformly over 12, 13, 14, 15, and 16 years. The initial cost of the scale is estimated to be distributed normally with a mean of \$1,500 and a standard deviation of \$150.

Use Excel to simulate 25 random samples of the problem, and compute the rate of return for each sample. Construct a graph of rate of return versus frequency of occurrence.

### SOLUTION

This problem is simple enough that a table with the values of the life and the first cost of each iteration can be constructed. From these values and the annual savings of \$250, the IRR for each iteration can be calculated with the RATE function. These are shown in Figure 10-8. The IRR values are summarized in a relative frequency diagram in Figure 10-9.

*continued*



## Simulation, cont'd

	A	B	C	D
1	250	Annual Savings		
2		Life	First Cost	
3	Min	12	1,500	Mean
4	Max	16	150	Std dev
5				
6	Iteration			IRR
7	1	12	1,277	16.4%
8	2	15	1,546	13.9%
9	3	12	1,523	12.4%
10	4	16	1,628	13.3%
11	5	14	1,401	15.5%
12	6	12	1,341	15.2%
13	7	12	1,683	10.2%
14	8	14	1,193	19.2%
15	9	15	1,728	11.7%
16	10	12	1,500	12.7%
17	11	16	1,415	16.0%
18	12	12	1,610	11.2%
19	13	15	1,434	15.4%
20	14	12	1,335	15.4%
21	15	14	1,468	14.5%
22	16	13	1,469	13.9%
23	17	14	1,409	15.3%
24	18	15	1,484	14.7%
25	19	14	1,594	12.8%
26	20	15	1,342	16.8%
27	21	14	1,309	17.0%
28	22	12	1,541	12.1%
29	23	16	1,564	14.0%
30	24	13	1,590	12.2%
31	25	16	1,311	17.7%
32				
33	Mean	14	1,468	14.4%
34	Std dev	2	135	2.2%

FIGURE 10.8 Excel spreadsheet for simulation ( $N = 25$ ).

(Note: Each time Excel recalculates the spreadsheet, different values for all the random numbers are generated. Thus the results depend on the set of random numbers, and your results will be different if you create this spreadsheet.)

## Simulation, cont'd

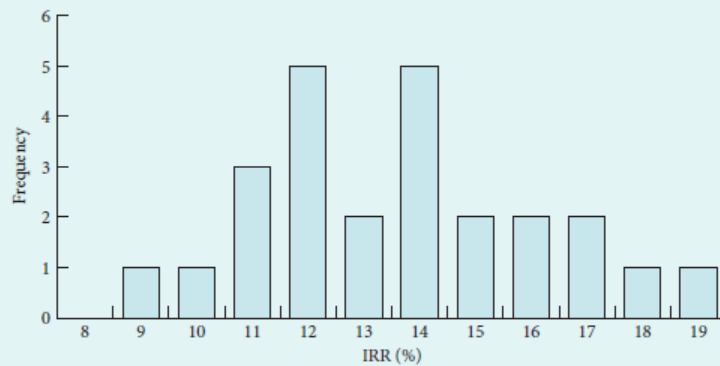


FIGURE 10.9 Graph of IRR values.