7

Lecture 6
Internal Rate of Return
Minimum Acceptable Rate of Return

Pay Now or Pay Later—
the Story of the Giant Mine

Pay Now or Pay Later—
the Story of the Giant Mine , cont’d

- From its start-up, the Giant Mine went through a series of owners until 1999, when then-owner Royal Oak Mines went bankrupt.
- The refractory gold at the mine was “roasted” out of the ore. This process converted naturally occurring arsenic into arsenic trioxide powder.
- By the time this practice was discontinued, the mine had produced some 237,000 tonnes of poisonous arsenic trioxide dust.
- Originally dumped into tailings ponds or released to go up the stack, this product, in the early 1950s, started to be disposed of in underground, mined-out chambers, where it remains to this day.

Pay Now or Pay Later—the Story of the Giant Mine, cont'd

- Today, it is the responsibility of the federal government to deal with the environmental legacy of the mine.
- The current government proposal is to freeze the arsenic permanently in place with an active thermosiphon system around the chambers, convert to a passive thermosiphon system after a period of time, and monitor the arsenic into the distant future.
- Estimates of cost for the Frozen Block method are in the range \$900–\$1,000 million, with an ongoing \$2,000,000 per year to monitor and maintain the site.

- And then the mining company goes bankrupt and reforms into new company.

Learning Objectives

- Evaluate project cash flows with the internal rate of return (IRR) measure
- Use an incremental rate of return analysis to evaluate competing alternatives
- Conduct a sensitivity analysis by plotting the present worth (PW) of a project against the interest rate
- Recognize when to calculate the modified internal rate of return (MIRR)

Key Summary: Update

- Variables and parameters (puzzle pieces):
  - Different kinds of interest rates
  - Discount rates
  - Costs and cost savings or revenues, now and in the future
  - Different expected lives of the possible project/purchases
  - Salvage value
  - Taxes and tax savings
  - How these escalate

- Analysis methods (ways to put the pieces together):
  - Present worth analysis
  - Equivalent uniform annual cost analysis
  - **Rate of return analysis**
  - Benefit-cost ratio analysis
  - Payback period
  - Cost-effectiveness analysis

Two Possible Analysis Approaches

1. Fixed data → Unknown outcome

- Variables and parameters:
  - Different kinds of interest rates
  - Discount rates
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Two Possible Analysis Approaches

1. Fixed data → Unknown outcome
2. Variable data ← Fixed outcome

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Introduction

Rarely in government.
- Most proj. don't generate revenue
- for public good.

- Rate of return analysis is the most frequently used measure in industry.
- It provides a measure of a project's desirability in terms that are easily understood.
- In rate of return analysis, we compute a rate of return (more precisely called *internal rate of return*) from the cash flow.
- The calculated rate of return can be compared with a pre-selected Minimum Attractive Rate of Return (MARR).

Internal Rate of Return (IRR)

- A project's rate of return is called the IRR.
  - The Internal Rate of Return (IRR) is the interest rate (or discount rate) at which the benefits are equivalent to the costs.
  - Another way to say this: Internal Rate of Return (IRR) is the interest rate at which the present worth and equivalent uniform annual worth are equal to zero.
  - Similar to but not same as Return On Investment (ROI)

Analysis Approach to calculate IRR

1. Fixed data → Unknown outcome
2. Variable data ← Fixed outcome

Variables and parameters:

- **Variable** discount rate / interest rate
- Fixed costs and cost savings or revenues, now and in the future
- Fixed salvage value

Analysis method:

Rate of return analysis:

Discounted benefits = discounted costs

OR

Net present worth of costs = 0

OR

EUAW = 0

Same concept, just differing by problem context

Calculating Rate of Return

- Given a cash flow, we want to solve for the interest rate that will cause the project to break even.
- There are several equivalent ways to state the conditions under which we will solve for the unknown interest rate (IRR).
  - Present worth = Net present worth = 0
  - PW of benefits – PW of costs = 0
  - PW of benefits/PW of costs = 1
  - EUAW = EUAB – EUAC = 0
  - PW of costs = PW of benefits

Rate of Return: Problem 1

With an initial investment of $6549.32 in a new machine, you will provide your company with $4000 more incoming dollars over the next four years. However, over those four years, the maintenance of the machine will cost $800. Also, in Year 2 a refit of the machine will cost $5,100.

What is the rate of return?

Rate of Return: Problem 1, cont'd

Solution

set to 0.

$$0 = -6549.32 + 4000(P/A, i, 4) - 800(P/A, i, 4) - 5100(P/F, i, 2)$$

initial

annual

annual

one time

$i = 6\%/\text{year}$

How do we solve for this interest rate (the IRR)?

Turn to Excel examples.

- Cannot use formula here.

Rate of Return: Problem 2

Calculate the interest rate that is required for accumulating $20,000 if $1517 per year will be saved over a 10-year period.

See excel

Rate of Return: Problem 2, cont'd

Solution

Net PW= $0.0 = -1517(F/A, i, 10) + 20{,}000$

$(F/A, i, 10) = 20{,}000/1517 = 13.18$

$i = 6\%$

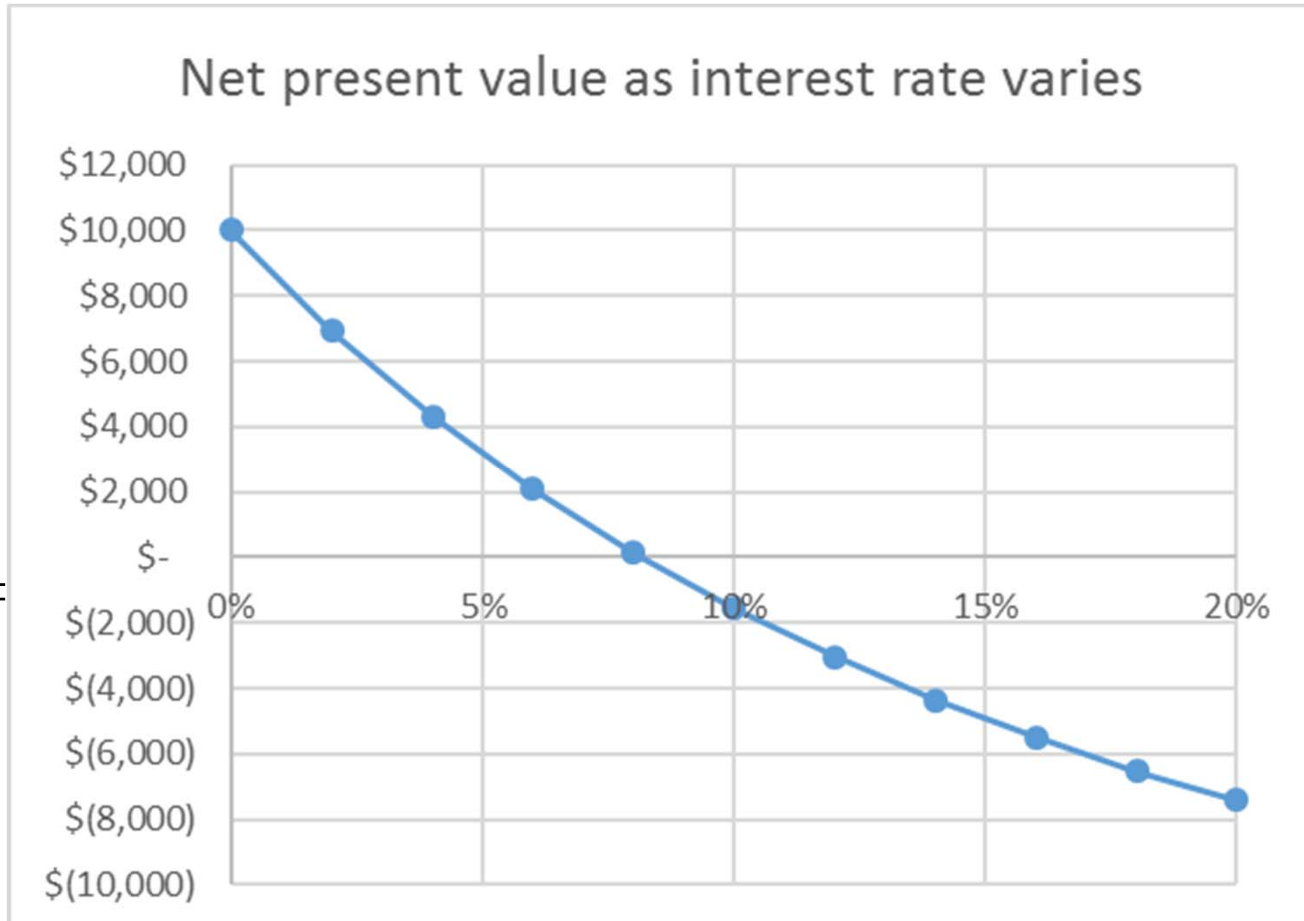
<see Excel example>

Plot of NPV versus Interest Rate *i*

- Return on an investment:

  fixed loan, fixed revenue stream, under different interest rates

- Non-linear due to non-linearity of discounting (compounding)
- The IRR is located where the plot crosses the NPW = 0 point.
- See Excel example



The IRR is where the plot crosses the NPW = 0 point

Interest Rates When There Are Fees or Discounts

- The internal rate of return is affected by fees or discounts.

- Example:
- .An electronics retailer offers a 2% interest rate on purchases. However, they charge a financing fee of $200.00 to provide the service.
    - The fee increases the true internal interest rate of the purchase.
-

Interest Rates When There Are Fees or Discounts, cont'd

See Excel examples 7-4 and 7-5

EXAMPLE 7-5

The corporate bond in Example 7-4 was part of a much larger offering that the firm arranged with the underwriter. Each of the bonds had a face value of $1,000 and a life of 10 years. Since $40 at 4% of the face value was paid in interest every six months, the bond had a nominal interest rate of 8% a year. If the firm paid the underwriter a 1% fee to sell the bond, what is the effective annual interest rate that the firm is paying on the bond?

SOLUTION

From the firm's perspective, it receives $1,000 minus the fee at Time 0, then it pays interest every six months for 10 years, and then it pays $1,000 to redeem the bond. The 1% fee reduces to $990 what the firm receives when the bond is sold. The interest payments are $40 every six months. This is easiest to model with 20 six-month periods.

$$\text{PW}(i) = 990 - 40(P/A, i, 20) - 1{,}000(P/F, \text{i}, 20)$$

Since the nominal interest rate is 4% every six months, we know that the fee will raise this somewhat. So let us use the next higher table of 4.5%.

$$\begin{aligned}\text{PW}(4.5\%) &= 990 - 40(P/A, 4.5\%, 20) - 1{,}000(P/F, 4.5\%, 20)\\ &= 990 - 40(13.008) - 1{,}000(0.4146) = \$55.08\end{aligned}$$

We know that the PW of the interest and final bond payoff is $1,000 at 4%.

$$\text{PW}(4\%) = 990 - 1{,}000 = -10$$

Now we interpolate to find the interest rate for each six-month period.

$$i = 4\% + (4.5\% - 4\%)(10)/(10 + 55.08) = 4.077\%$$

The effective annual rate is

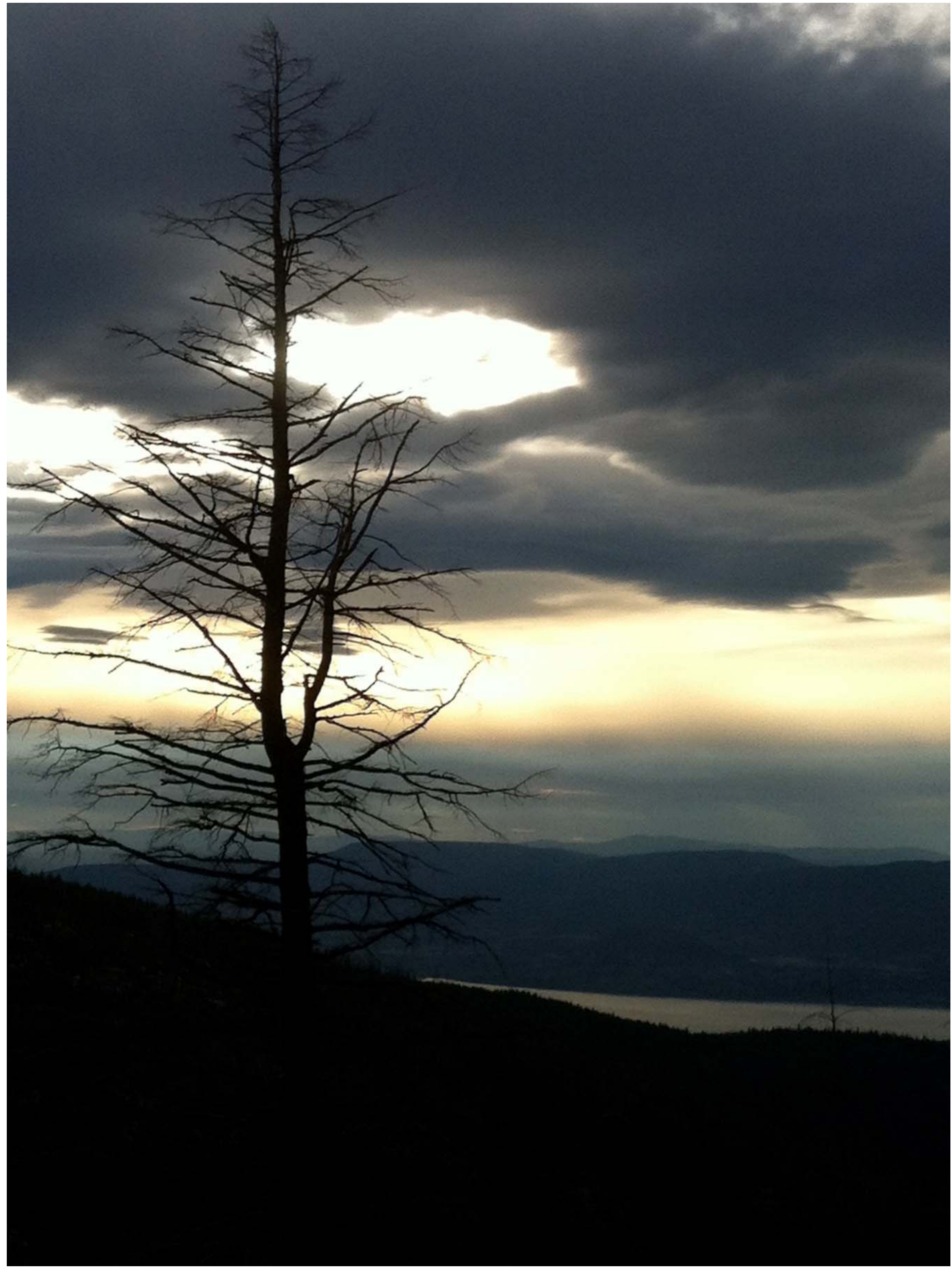
$$i_a = 1.04077^2 - 1 = 0.0832 = 8.32\%$$

IRR vs ROI

Return on Investment (ROI)

- ROI (sometimes called rate of return, ROR) is the percentage increase or decrease in an investment over a set period.
- ROI = $\frac{F-P}{P}$
- ROI is NOT an annual rate, it is total return over entire time frame → Does not tell you how fast. - time value of money.
- When we refer to "Rate of Return" in this course, we mean IRR.
- *Return to Example 7-2 to see example of the difference*

Potential break point



Minimum Attractive Rate of Return

experiential. diffᶜ than borrowing rate,

Is a possible project the best possible way to use your money?

MARR is a benchmark: an estimation of how much you think you could earn through other investments.

- The purpose of rate of return analysis is to allow us to decide between different possible projects.
- Once we have calculated that the rate of return on a particular project is, for example, 23%, we need some form of benchmark against which we can compare this number.
- The average rate at which we have to recompense our creditors and investors sets a lower bound on the rate of return at which a proposed project becomes attractive.
- The highest of these lower bounds is the Minimum Attractive Rate of Return (MARR).

- typically set at borrowing rt. for young companies, then rises w/ better opportunities.

IRR Caution

- Internal rate of return can be deceiving.
- Comparing projects strictly on the IRR of each project can provide incorrect results and disagree with present worth or annual worth analysis.
  - Note: Projects can include the "do-nothing" alternative if the selection of an alternative is not "required."
- The objective is to maximize the "return" NOT the "rate of return."
- This is why we use a different technique:

  Incremental Analysis (ΔIRR)

Incremental Analysis

- In rate of return analysis, two or more alternatives can be compared by using IRR analysis on the **differences** in cost and revenue streams. This is called the incremental rate of return (ΔIRR).

Incremental IRR Analysis Algorithm

- Calculate the rate of return for each alternative, and discard any for which IRR < MARR.
- Arrange the remaining alternatives in ascending order of first cost.
- The alternative with lowest first cost is your current champion.
- Calculate the incremental IRR of upgrading from the current champion to the alternative with next-lowest first cost.
- If the incremental IRR > MARR, upgrade; otherwise, stick with the current champion.
- Repeat steps 4 and 5 until you run out of alternatives.

Incremental IRR Analysis Algorithm

Textbook example:

Year	Alt 1	Alt 2
0	-$10,000	-$20,000
1	+$15,000	+$28,000

- Calculate the rate of return for each alternative, and discard any for which IRR < MARR of 6%.
  - Alt 1 IRR = 50%, Alt 2 IRR = 40%. Both exceed MARR.
- Arrange the remaining alternatives in ascending order of first cost.
- The alternative with lowest first cost is your current champion.
- Calculate the incremental IRR of upgrading from the current champion to the alternative with next-lowest first cost.
- If the incremental IRR > MARR, upgrade; otherwise, stick with the current champion.
- Repeat last two steps until you run out of alternatives.

$\frac{13\ 000}{10\ 000}$ 30%

30% incremental IRR > MARR

Year	Alt 1	Alt 2	Alt 2 – Alt 1
0	-$10,000	-$20,000	-$20,000 – (-$10,000) = -$10,000
1	+$15,000	+$28,000	+$28,000 – (+$15,000) = +$13,000

IRR Example

- It may be more intuitive/simpler to do this by just comparing the IRRs of the alternatives, with the scope set properly.
  - See Excel example Ex 7-7.

EXAMPLE 7-7

If the computations above do not convince you, and you still think Alternative 1 would be preferable, try this problem.
You have $20 in your wallet and two different ways of lending Bill some money.

(a) Lend Bill $10 with his promise of a 50% return. That is, he will pay you back $15 at the agreed time.
(b) Lend Bill $20 with his promise of a 40% return. He will pay you back $28 at the same agreed time.

You can choose whether to lend Bill $10 or $20. This is a one-time situation, and any money not lent to Bill will remain in your wallet. Which alternative do you choose?

SOLUTION

A 50% return on the smaller sum is less rewarding to you than 40% on the larger sum. Since you would prefer to have $28 than $25 ($15 from Bill plus $10 remaining in your wallet) after the loan is paid, lend Bill $20.

Spreadsheets and Rate of Return Analysis

- IRR function solves for IRR, so you don’t need to use tables or Goal Seek.
- IRR (range: undiscounted set of costs and revenues)
- Demonstrated in each of the examples stored in Excel examples file
-

Potential break point

Analysis Period

- In rate of return analysis, the analysis period needs to be the same if we are examining the increments between alternatives.

- A common multiple of the alternative service lives may be used, assuming identical replacement.

- In this respect, IRR has the same limitations that present worth has.

Analysis Period, cont'd

See Excel example

Some numbers are missing in the table in the textbook

EXAMPLE 7-10

Two machines are being considered for purchase. If the MARR is 10%, which machine should be bought? Use an IRR analysis comparison.

	Machine X	Machine Y
Initial cost	$200	$700
Uniform annual benefit	95	120
End-of-useful-life salvage value	50	150
Useful life, in years	6	12

SOLUTION

The solution is based on a 12-year analysis period and a replacement Machine X that is identical to the present Machine X. The cash flow for the differences between the alternatives is as follows:

Year	Machine X	Machine Y	Difference between Alternatives Machine Y − Machine X
0	−$200	−$700	−$500
1	+95	+120	+25
2	+95	+120	+25
3	+95	+120	+25
4	+95	+120	+25
5	+95	+120	+25
6		+120	+25 +150
7	+95	+120	+25
8	+95	+120	+25
9	+95	+120	+25
10	+95	+120	+25
11	+95	+120	+25
12		+120 +150	+25 +100

Analysis Period, cont'd

But numbers included in this equation in the textbook are correct

$$\text{PW of cost (differences)} = \text{PW of benefits (differences)}$$
$$500 = 25(P/A, i, 12) + 150(P/F, i, 6) + 100(P/F, i, 12)$$

The sum of the benefits over the 12 years is $550, which is only a little greater than the $500 additional cost. This suggests that the rate of return is quite low. Try $i = 1\%$.

$$25(\text{P/A}, 1\%, 12) + 150(\text{P/F}, 1\%, 6) + 100(\text{P/F}, 1\%, 12)$$
$$= 25(11.255) + 150(0.942) + 100(0.887) = 511$$

The interest rate is too low. Try $i = 1.5\%$:

$$25(\text{P/A}, 1.5\%, 12) + 150(\text{P/F}, 1.5\%, 6) + 100(\text{P/F}, 1.5\%, 12)$$
$$= 25(10.908) + 150(0.914) + 100(0.836) = 494$$

The internal rate of return on the $Y - X$ increment is about 1.3%, far below the 10% minimum attractive rate of return. The additional investment to obtain Machine Y yields an unsatisfactory rate of return; therefore Machine X is the preferred alternative.

Sensitivity Analysis

- It can often happen in engineering problems that we need to make tentative conclusions about a project while one or more important parameters remain undetermined.
- We can compute how much an estimate can change and the effect on a particular decision.
- This is "sensitivity analysis."
- It is often possible to construct a graph showing what performance we would expect for a range of plausible values of the undetermined parameter(s).

Sensitivity Analysis in Context of Assessment

1. Fixed data → Unknown outcome

- Variables and parameters:
  - Different kinds of interest rates
  - Discount rates
  - Costs and cost savings or revenues, now and in the future
  - Different expected lives of the possible project/purchases
  - Salvage value
  - Taxes and tax savings
  - How these escalate

- Analysis methods:
  - Present worth analysis
  - Equivalent uniform annual cost analysis
  - Rate of return analysis
  - Benefit-cost ratio analysis

Uncertainty about the data leads to trying different values, and seeing whether they affect the outcome

Sensitivity Analysis for MARR

- In order to select projects based on incremental rate of return, we needed to know the MARR.
- If we don't know the MARR, we can still obtain a basis for decision by plotting the project's net present worth as a function of the unknown MARR.
- The general method involves the following steps:
  - Write down an expression for the PW or EUAC of each alternative, with the unknown MARR as a free parameter.
  - Plot PW (or EUAC) versus MARR for each alternative on the same graph.
  - Note which alternative gives the maximum PW (or minimum EUAC) at each value of MARR, and the crossover points between alternatives.

Sensitivity Analysis, cont'd

See Excel example

EXAMPLE 7-11

A pressure vessel can be made out of brass, stainless steel, or titanium. The first cost and expected life for each material are as follows:

	Brass	Stainless Steel	Titanium
Cost	$100,000	$175,000	$300,000
Life, in years	4	10	25

The pressure vessel will be in the non-radioactive portion of a nuclear power plant that is expected to have a life of 50 to 75 years. The public utility commission and the power company have not yet agreed on the interest rate to be used for making decisions and setting rates. Build a choice table to determine the best alternative at each interest rate.

SOLUTION

The pressure vessel will be replaced repeatedly during the life of the facility, and each material has a different life. Thus, the best way to compare the materials is by using EUAC (see Chapter 6). This assumes identical replacements.

Figure 7-5 graphs the EUAC for each alternative. In this case the best alternative at each interest rate is the material with the *lowest* EUAC.

The factor equation is

$$\text{EUAC} = \text{first cost}(A/P, i, \text{life})$$

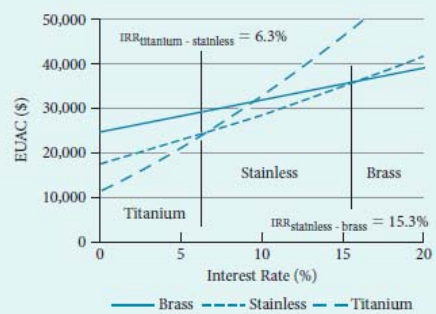


FIGURE 7-5 EUAC comparison of alternatives.

The choice table for each material is

Interest Rate	Best Choice
$0\% \le i \le 6.3\%$	Titanium
$6.3\% \le i \le 15.3\%$	Stainless steel
$15.3\% \le i$	Brass

Example 7-12

The following information if for five mutually exclusive alternatives that have 20-year useful lives. The decision maker may choose any one of the options or reject them all. MARR = 10.5%.

Prepare a choice table.

	Alternatives				
	A	B	C	D	E
Cost	$4,000	$2,000	$6,000	$1,000	$9,000
Uniform annual benefit	$639	$410	$761	$117	$785

Switch over to Excel example.

Potential break point

IRR Formula Limitations

- When the set of costs and revenues become too complex, the IRR function can fail.
- In that situation, other analysis techniques probably should be used.
- See Excel example worksheet 'Ex 7 – IRR limitations'.
- But there is one option for sticking with IRR-type analysis: using a Modified IRR analysis.

Modified Internal Rate of Return (MIRR)

- The IRR is determined by the cash flows of a given project and solving for the roots of the resulting equation.

- This makes the assumption that:
  - Positive cash flows can be reinvested at the IRR rate (which is not necessarily true).

- External interest rates are used to establish a MIRR.

- walk thru briefly

Modified Internal Rate of Return (MIRR), cont'd

- The MIRR is a measure of the attractiveness of the cash flows, but it is also a function of the two external rates of return.
- Use of external rates:
  - Investing rate (e_{inv}) is used for positive cash flows.
  - Financing rate (e_{fin}) is used for negative cash flows.
  - The positive cash flows are moved to the end of the project's time period using $(F/P, e_{\text{inv}}, n)$.
  - The negative cash flows are moved to the beginning of the project's time period using $(P/F, e_{\text{fin}}, n)$.
    - ➢ Then: $0 = (1 + \text{MIRR})^n (\text{PW}) + \text{FW}$

- Not in final exam

- Archaic.

Modified Internal Rate of Return (MIRR), cont'd

Approach:

1. Combine cash flows in each period into one value.
2. Find the present worth of the expenses using the financing rate.
3. Find the future worth of the receipts using the investing rate.
4. Move the present worth of the expenses into the future using the MIRR, and set this equivalent to the future worth calculated in Step 3.

- These steps result in the formula:

$(F/P, MIRR, n) \sum_t E_t (P/F, e_{fin}, t) = \sum_t R_t (F/P, e_{inv}, n\text{-}t)$

Finally… ”Spreadsheets for IRR Analysis”

- Spreadsheet analysis offers advantages over formulas, for IRR analysis.
  - No interpolation needed for interest rates
  - Easy to calculate a repayment schedule