Assignment 2

EECE/CPEN 481

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Selected problems listed below are based on textbook problems or directly drawn from the textbook (Engineering Economic Analysis: Fourth Canadian Edition).

Problems are drawn mainly from material in Chapters 4 and 5.

1. Problem 1 (0.5x value)
2. Problem 2 (1.5x value)
3. Problem 3 (1.5x value)
4. Problem 4 (0.5x value)
5. Problem 5 (0.5x value)
6. Problem 6
7. Problem 7
8. Problem 8 (1.5x value)

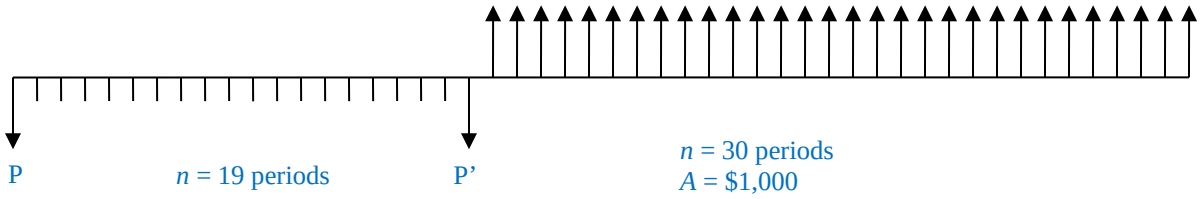
Solutions, which will be distributed to the students, given in blue type.

Assignment 2 Solutions

1. **Problem 1**

What initial amount will be necessary in order to purchase, on an engineer's 40th birthday, an annuity to provide him with 30 equal semi-annual payments of $1,000 each, the first to be received on his 50th birthday, if nominal interest is 4% compounded semi-annually? Round to the nearest dollar.

Solution:



To calculate how much money you need at the start of his 50th birthday, you can use an A to P conversion:

P' = \$1,000 (P/A, 2%, 30) = \$1,000 (22.396)
= \$22,396

Then, you can use an F to P conversion to figure out how much money is needed right now, to build up enough earnings to have that amount ready at the start of his 50th birthday.

Note: There are 19 interest periods between P(40th birthday) and P' (6 months prior to 50th birthday)

P = P' (P/F, 2%, 19) = \$22,396 (0.6864)
= \$15,374 [Cost of Annuity]

2. **Problem 2**

To pay for a university education for her son, a woman opened a bank account in which equal deposits were made. The first deposit was made on 1 January 2000, and the last deposit was made on 1 January 2017. The yearly university expenses, including tuition, were estimated to be $8,000 for each of four years. Assuming the interest rate to be 5.75%, how much did the mother have to deposit each year into the account for her son to draw $8,000 per year for four years, beginning 1 January 2017? Round your answer to the nearest dollar.

Solution:

The four flows of $8,000 for 4 years from 1 January 2017 have a present worth on 1 January 2017 of *P1*. We can't use the standard annuity formula, since that formula assumes that the first payment of the annuity occurs a year in the future. So instead we can write:

$P1 = 8{,}000 + 8{,}000(P/A, 5.75\%, 3) = 8{,}000(1+(P/A, 5.75\%, 3)) = 8{,}000(3.673) =$ **$29,483**

So we want the annual amounts, A, that the woman has been depositing between 2000 and 2017 to have this value on 1 January 2017. Over this period she's made 18 payments, so we can write:

Thus, $A = \$29{,}483\ (A/F, 5.75, 18)$
$(A/F, 5.75\%, 18) = i/[(1+i)^n - 1] = 0.0331$
Thus, $\boldsymbol{A = \$977}$

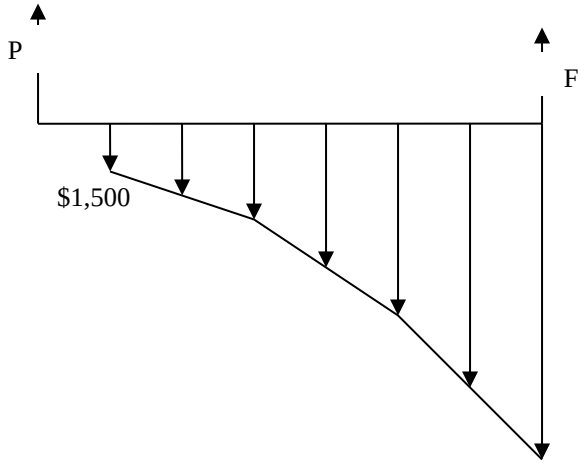
3. Problem 3

Mark saves a fixed percentage of his salary at the end of each year. This year he saved $2,000. For the next five years, he expects his salary to rise at an 8% annual rate, and he plans to increase his savings at the same 8% annual rate. He invests his savings in the stock market. Thus there will be six end-of-year investments (the initial deposit, which occurs right away (in year zero), plus five more).

(a) How much will the investments be worth at the end of the fifth year if their value increases at a 10% annual rate?

(b) How much will Mark have at the end of the fifth year if his stock market investments increase only at 8% annually? In both cases, round your answer to the nearest dollar.

Solution:



(a) We only have a formula for converting A to P, so we must use two steps: convert A (including g) to P, then convert P to F.

$P = ?$ $n = 6$ $i = 10\%$ $g = 8\%$

$P = A_1 (P/A, g\%, i\%, n)$

$(P/A, g\%, i\%, n) = [(1 - (1 + g)^n (1 + i)^{-n})/(i - g)] = [(1 - (1.08)^6 (1.10)^{-6})/(0.10 - 0.08)] = 5.212$

$P = \$2{,}000\ (5.212) = \$10{,}425$

$F = P\ (F/P, i\%, n) = \$10{,}425\ (F/P, 10\%, 6) = \$18{,}469$

As a check, solve with single payment factors (rounding to the dollar):

$2,000.00 (F/P, 10%, 5) = $2,000.00 (1.611) = $3,221
$2,160.00 (F/P, 10%, 4) = $2,160.00 (1.464) = $3,162
$2,332.80 (F/P, 10%, 3) = $2,332.80 (1.331) = $3,105
$2,519.42 (F/P, 10%, 2) = $2,519.42 (1.210) = $3,049
$2,720.98 (F/P, 10%, 1) = $2,720.98 (1.100) = $2,993
$2,938.66 (F/P, 10%, 0) = $2,938.66 (1.000) = $2,939

Total Amount = $18,469

Here, i% = g%, hence the geometric gradient to present worth equation is:

$P = A_1\ n\ (1 + i)^{-1} = \$2{,}000\ (6)\ (1.08)^{-1} = \$11{,}111$

$F = P\ (F/P, 8\%, 6) = \$8{,}333\ (1.587) =$ **$17,632**

Note that n=6. This may seem confusing, because we are calculating the result at the end of year 5. But there are 6 deposits, so year zero counts as a year too.

4. **Problem 4**

Pete borrows $50,000 to buy a car. He must repay the loan in 48 equal end-of-period monthly payments. Interest is calculated at 0.35% a month. Determine the following:

(a) The nominal annual interest rate, rounded to the nearest tenth of a percent
(b) The effective annual interest rate, rounded to the nearest tenth of a percent
(c) The amount of the monthly payment, rounded to the nearest dollar

Solution:

(a) $r = i \times m$
$= (0.35\%)\ (12)$
$= 4.2\%$

(b) $i_a = (1 + 0.0035)^{12} - 1$
$= 4.3\%$

(c) $A = \$50{,}000\ (A/P, 0.35\%, 48)$
$= \$50{,}000\ (0.0227)$
$= \$1{,}133$

5. **Problem 5**

IBP Inc. is considering establishing a new machine to automate a meat-packing process. The machine will save $50,000 in labour annually. The machine can be bought for $200,000 today and will be used for 10 years. It has a salvage value of $10,000 at the end of its useful life. The new machine will require an annual maintenance cost of $9,000. The corporation has a minimum rate of return of 10%. Do you recommend automating the process?

Solution:

The objective is to determine if the Net Present Worth is non-negative.

NPW of Benefits $= \$50{,}000\ (P/A, 10\%, 10) + \$10{,}000\ (P/F, 10\%, 10)$
$= \$50{,}000\ (6.145) + \$10{,}000\ (0.3855)$
$= \$311{,}084$

PW of Costs $= \$200{,}000 + \$9{,}000\ (P/A, 10\%, 10)$
$= \$200{,}000 + \$9{,}000\ (6.145)$
$= \$255{,}301$

NPW $= \$311{,}084 - \$255{,}301$
$= \$55{,}783$

Since NPW is positive, the process should be automated.

6. **Problem 6**

A man had to have the muffler replaced on his car. The repairman offered two alternatives. For $450 he would install a muffler, which would be guaranteed to last for two years. For $750 he would install a muffler guaranteed to last for four years. Assuming the present owner expects to keep the car for four more years, which muffler would you advise him to have installed? Use an interest rate of 8% (apply this as a discount rate). Assume that the less expensive muffler would last only two years, and could be replaced by a muffler at the same cost.

Solution:

Add up total discounted costs for each and compare.

Option 1:

PW Cost = 450 + 450 /(1.08)^2 = $450 + $386 = $ 836

Option 2:

PW Cost = $750 $ 750

Difference in cost: $ 86

Option 2 is less expensive.

7. Problem 7

A cost analysis is to be made to determine what, if anything, should be done in a situation offering three "do-something" and one "do-nothing" alternatives. Estimates of the cost and benefits are as follows:

Alternatives	Cost ($)	Uniform Annual Benefit ($)	End-of-Useful-Life Salvage Value ($)	Useful Life (years)
1	500	135	0	5
2	600	100	250	5
3	700	100	180	10
4	0	0	0	0

Use a 10-year analysis period for the four mutually exclusive alternatives. At the end of five years, Alternatives 1 and 2 may be replaced with identical alternatives (with the same cost, benefits, salvage value, and useful life).

(a) If an 8% interest rate is used, which alternative should be chosen?

(b) If a 12% interest rate is used, which alternative should be chosen?

Solution:

Choose the alternative to maximize NPW.

a) 8% interest

$NPW_1 = \$135\ (P/A, 8\%, 10) - \$500 - \$500\ (P/F, 8\%, 5)$
$= +\$65.55$

$NPW_2 = -\$600 + (\$100)\ (P/A, 8\%, 10) + (-\$600+\$250)\ (P/F, 8\%, 5)$
$= -\$51.41$

$NPW_3 = \$100\ (P/A, 8\%, 10) - \$700 + \$180\ (P/F, 8\%, 10)$
$= +\$54.38$

$NPW_4 = \$0$

Choose Alternative 1.

b) 12% interest

$NPW_1 = \$135\ (P/A, 12\%, 10) - \$500 - \$500\ (P/F, 12\%, 5)$
$= -\$20.95$

$NPW_2 = (\$100 + \$250)\ (P/A, 12\%, 10) - \$600 - \$350\ (P/F, 12\%, 5)$
$= -\$153.09$

$NPW_3 = \$100\ (P/A, 12\%, 10) - \$700 + \$180\ (P/F, 12\%, 10)$
$= -\$77.04$

$NPW_4 = \$0$

Choose Alternative 4.

8. Problem 8

A steam boiler is needed as part of the design of a new plant. The boiler can be fired by natural gas, fuel oil, or coal. A decision must be made on which fuel to use. An analysis of the costs shows that the installed cost, with all controls, would be least for natural gas at $35,000; for fuel oil it would be $75,000; and for coal it would be $270,000. If natural gas is used rather than fuel oil, the annual fuel cost will increase by $3,500. If coal is used rather than fuel oil, the annual fuel cost will be $16,000 per year less. Assuming 7% interest, a 20-year analysis period, and no salvage value. which is the most economical installation?

Solution:

There are two ways to solve this.

One way to solve it uses differential analysis.

Notice that the operating costs are only given as differentials. You don't know the operating cost of fuel oil.

Start by converting the capital cost to differentials, to match the operating costs:

Boiler options	capital cost differential	operating cost annual differential	(compared to fuel oil)
natural gas	$ (40,000)	+$3,500	
fuel oil	$ -		
coal	$ 195,000	-$16,000	

Use formulas or spreadsheets to calculate.

Using formulas:

Natural Gas

Delta PW of Cost = ($35,000-$75,000) + $3,500 ($P/A$, 7%, 20)

compared with fuel oil = **-$2,921**

Coal

Delta PW of Cost = ($270,000-$75,000) − $16,000 ($P/A$, 7%, 20)

compared with fuel oil = **+$25,496**

PW of Natural gas costs less than Fuel Oil.

PW of Coal costs more than Fuel Oil.

Install the natural gas-fired steam boiler.

The other way (which the textbook uses) adds up totals rather than differentials:
For fixed output, minimize PW of Cost:

Natural Gas

PW of Cost = \$35,000 + \$3,500 (P/A, 7%, 20) + PW of Fuel Oil Cost
= \$35,000 + \$3,500 (10.594) + PW of Fuel Oil Cost
= \$72,079 + PW of Fuel Oil Cost

Fuel Oil

PW of Cost = \$75,000 + PW of Fuel Oil Cost

Coal

PW of Cost = \$270,000 − \$16,000 (P/A, 7%, 20) + PW of Fuel Oil Cost
= \$270,000 − \$16,000 (10.594) + PW of Fuel Oil Cost
= \$100,496 + PW of Fuel Oil Cost

Install natural gas-fired steam boiler.