3

Important

Lecture 2
Interest and Equivalence
Other Perspectives on Scoping
Electricity Rate Structures

Nuclear Waste Storage

Nuclear Waste Storage, cont’d

- Nuclear power offers a reliable supply of energy, with no greenhouse gas emissions.
- Spent fuel from reactors is highly radioactive and must be stored securely for long periods—hundreds of thousands of years—until the radioactivity has decayed to safe levels
- Although skilled engineers have been working on this problem for many decades, humanity as a whole has no experience in building anything that can last for such a long period

Nuclear Waste Storage, cont’d

- In October 2015 the Nuclear Waste Management Organization (NWMO) released a report, “Implementing Adaptive Phased Management, 2016–2020,” describing its plan to locate a suitable depository, deep underground in a suitable rock formation, where the fuel can be placed
- At the end of this chapter, we will look at the implications of engineering economic analysis for long-term projects of this kind

Learning Objectives

- Define and provide examples of the time value of money
- Distinguish between simple and compound interest, and use compound interest in engineering economic analysis
- Explain equivalence of cash flows
- Solve problems by using the single-payment compound interest formulas
- Distinguish and apply nominal and effective interest rates
- Use continuously compounded interest with single-payment interest factors

Key Summary: Course to date and coming soon

- Variables and parameters (puzzle pieces):
  - **Different kinds of interest rates**
  - **Discount rates**
  - Costs and cost savings or revenues, now and in the future
  - Different expected lives of the possible project/purchases
  - Salvage value
  - Taxes and tax savings
  - How these escalate
- Analysis methods (ways to put the pieces together):
  - Present worth analysis (Net Present Value)
  - Equivalent uniform annual cost analysis
  - Rate of return analysis
  - Benefit-cost ratio analysis
  - Payback period
  - Cost-effectiveness analysis

Computing Cash Flows

- We describe the benefits and costs as **receipts** (cash flowing in) and **disbursements** (cash flowing out) at different points in time
- The foundation of engineering economic analysis is a set of techniques for comparing the value of money at different dates

Time Value of Money

- When monetary consequences of an project we're considering doing will occur over a period of time into the future, we can't simply add up the various sums of money. Why?
- Money
  - It has value over time
  - It is a valuable asset that people are willing to pay to have available for use
  - It can be rented in roughly the same way other things can (like an apartment)
    - The charge is called **interest** instead of rent

Now is better than later.

Value of money

Value of money: same now and future?

- Why?
- Implications? (benefits, costs)

- Has fmla sheet

Discounting

- method of comparing benefits and costs at different times
- future benefits and costs are "discounted" at rate (r)

Present Value (PV)

- measures current worth of future benefits and costs

$$PV(\$_n) = \frac{(\$_n)}{(1+r)^n}$$

arithmetic sum

time equivalence.

- inflation to come.

Example of present value

time period	0	1	2	3	4
interest earned (10 %)	✓	\$ 10	\$ 11	\$ 12.10	\$ 13.31
new account balance	\$ 100	\$ 110	\$ 121 ↓10%	\$ 133.10	\$ 146.41

choice.

$$\text{present value after 4 years } (PV_4) = \frac{(\$_n)}{(1+r)^n} = \frac{\$\,146.41}{(1+0.1)^4} = \$\,100$$

- Don't care.

- Current / later costs, diffⁿ times.

- standardized

Time Value of Money, cont'd

- Our preference for having money now rather than money in the future differs from person to person
  - The preference for having money now rather than later has nothing to do with inflation
- The bank expresses the time value it puts on money by publishing the interest rate that it charges to borrow money

- reflects time value.

e.g.

Time Value of Money, cont'd

- Simple Interest
  - Interest that is computed only on the original sum and not on accrued interest (so, no compounding)
  - Total interest earned = $P \times i \times n = Pin$
    - ➢ Where: P = present sum, i = interest rate/period, and n = # of time periods (e.g. years)
  - Total money after n periods (F) = $P + Pin$ or $F = P(1+in)$
    - ➢ Where: F = future sum

- other financial instruments.
  - drawing interest off endowment.
  - does not pile up and earn more.
  - not more interest

Time Value of Money, cont'd

- Simple Interest, cont'd

EXAMPLE 3-3

You have agreed to lend a friend \$5,000 for five years at a simple-interest rate of 8% a year. How much interest will you receive from the loan? How much will your friend pay you at the end of five years?

SOLUTION

$$\text{Total interest earned} = Pin = (\$5{,}000)(0.08)(5\text{ yr}) = \$2{,}000$$
$$\text{Amount due at end of loan} = P + Pin = 5{,}000 + 2{,}000 = \$7{,}000$$

5000 + 0.08 x5.

- See excel for examples

Time Value of Money, cont'd

- Compound Interest
  - In practice, interest is almost always determined using the compound interest method.
  - Simple interest is normally not used unless specifically stated otherwise
  - Interest is calculated on the accumulated amount and not simply on the original amount
    - Interest on top of interest

Time Value of Money, cont'd

- Compound Interest, cont'd
  - Below, consider a $25,000 loan at 10%/year

Year "*n*"	Total in Year "*n*"	Interest accumulated at end of year "*n*"	Amount accumulated at end of year "*n*"
1	$25,000	$2,500	$27,500
2	$27,500	$2,750	$30,250
3	$30,250	$3,025	$33,275
4	$33,275	$3,327.50	$36,602.50

Simple and Compound Interest: Problem

- If you borrowed $150,000 now, how much would you owe at the end of 4 years using simple interest at 9% per year, compared to how much you would owe using compound interest at 9% per year?

$$150{,}000 \times 1.09^4 = F$$

Simple and Compound Interest: Problem, cont'd

Solution

Simple Interest

$F = P + Pin = 150{,}000 + 150{,}000(0.09)(4)$

$F = \$204{,}000$

Compound Interest

$F = P(1 + i)^n$

$F = 150{,}000(1.09)^4$

$F = \$211{,}737.24$

Difference:

$211{,}737.24 - 204{,}000 = \$7737.24$

You would owe $7737.24 more with a compound interest arrangement.

Time Value of Money, cont'd

- Repaying a Debt
  - There are many ways in which debts are repaid, including the following three plans:
    1. Constant principal
    2. Interest only
    3. Constant payments

- Predictability is better
- Know payment amount

Equivalence

- Equivalence with respect to the "time value of money" implies that a sum of money in one time period may have the same "value" to a different sum in another time period with respect to an interest rate.

- Example:
  - $1000 now is equivalent to:
    - $1100 one year from now at 10% per year
    - $1050 one year from now at 5% per year
    - $1210 two years from now at 10% per year
    - $1102 two years from now at 5% per year

Example of present value: Equivalence

time period	0	1	2	3	4
interest earned (10 %)		$ 10	$ 11	$ 12.10	$ 13.31
new account balance	$ 100	$ 110	$ 121	$ 133.10	$ 146.41

present value after 4 years (PV_4) $= \dfrac{(\$_n)}{(1 + r)^n} = \dfrac{\$\ 146.41}{(1 + 0.1)^4} = \$\ 100$

Equivalence, cont'd

- Equivalence is dependent on interest rate.
- Equivalence is useful when:
  - There are cash flows (positive and/or negative) or other non-cash flow benefits or costs in future time periods that need to be compared
  - There are alternative comparisons of multiple cash flows
    - Alternative projects with cash flows over "n" time periods

Single-Payment Compound Interest Formulas

- Notation:
  - i = interest rate per interest period
  - n = number of interest periods
  - P = a present sum of money
  - F = a future sum of money at the end of the nth interest period, which is equivalent to P at the interest rate i

Single-Payment Compound Interest Formulas, cont'd

- If the interest rate's period is in years:
  - After one year, the future amount at the end of year one would be:

$$F = P(1 + i)$$

  - After two years, the future amount at the end of year two would be the additional interest on year one's total:

$$F = P(1 + i) + iP(1 + i)$$

  - By rearranging, we get:

$$F = P(1 + i)(1 + i) \text{ or } P(1 + i)^2$$

Single-Payment Compound Interest Formulas, cont'd

- Generalizing the previous slide:

$$F = P\boxed{(1 + i)^n}$$

- The above formula is the "single payment compound amount formula," which is written in functional notation as:

$$F = P\boxed{(F/P\,, i, n)}$$

  - The notation in brackets meaning future sum "*F*," given present sum "*P*" at interest rate "*i*" per interest period for "*n*" periods.
  - Functional notation is written algebraically correct so that:

$$F = \cancel{P}(F/\cancel{P}) \text{ so, } F = F$$

Single-Payment Compound Interest: Problem

- If you deposit \$3,000 in a bank account and earn 7% per year interest, how much money would be in the account after four years?

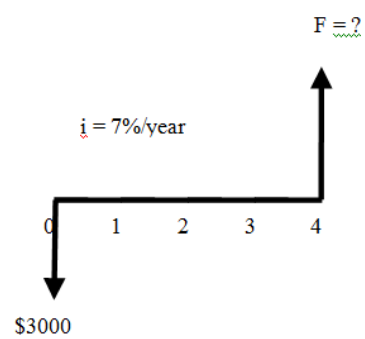
Single-Payment Compound Interest: Problem, cont'd

Solution

$F = P(F/P, 7\%, 4)$

$F = 3{,}000(1+0.07)^4$

$F = \$3{,}932.39$



Single-Payment Compound Interest: Problem, cont'd

- Suppose you want to find an equivalent value now for a future value.

$$F = P(1+i)^n$$

  - Rearranging:

$$P = F\frac{1}{(1+i)^n} = F(1+i)^{-n}$$

  - The notation becomes:

$$P = F(P/F, i, n)$$

Single-Payment Compound Interest: Problem, cont'd

- If you want to have \$3,000 in the bank after four years at 7% per year interest, what would you have to deposit now?

$$x + (1.07)^4 = 3000$$

$$x = \frac{3000}{1.07^4}$$

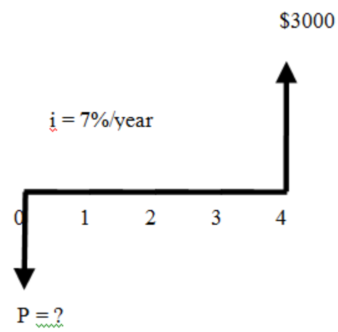
Single-Payment Compound Interest: Problem, cont'd

Solution

$P = F(P/F, 7\%, 4)$

$P = 3{,}000(1+0.07)$^-4

$P = \$2{,}288.69$



Nominal and Effective Interest

- When specifying an interest rate, we implicitly mention two time periods
  - We pay an interest rate of *i* per *time_period1*, compounded every *time_period2*
- **Effective interest rate**: when these time periods are both the same – line up pay per. e.g. cash adv.
- **Nominal interest rate**: when the two time periods don't match
- When you are given an interest rate without a specified period, assume it is annual (per year).

Interest Rates: Nominal, Effective, Effective Annual

Example 3-8

Consider a person depositing \$100 in bank that pays 5% interest, compounded semi-annually. How much would be in the savings account at the end of one year?

- Assume the 5% is per year since it doesn't list a period.
- Nominal r = declared rate, not accounting for compounding = 5%
- Effective rate = declared rate / # of compounding periods per year m = 2.5%
- Effective annual $i_a = (1+\frac{r}{m})^m - 1 = (1+\frac{.05}{2})^2 - 1 = 5.06\%$
- At the end of one year: \$100 x (1+5.06%) = \$105.06

- Adjust for more freq. compounding.

Interest Rates: Nominal, Effective, Effective Annual

If you know the effective annual rate i_a and want to calculate effective rate (r/m):

- $i_a = (1+\frac{r}{m})^m - 1 \rightarrow r = m\left[(i_a+1)^{\frac{1}{m}} - 1\right]$

Example 3-36: What annual interest rate, compounded quarterly, is equivalent to a 9.31% effective annual interest rate?

$r = m\left[(i_a+1)^{\frac{1}{m}} - 1\right] = 4\left[(.0931+1)^{\frac{1}{4}} - 1\right] = 4(.022504) = 9\%$

of Compounding Periods -> Continuous Compounding

- The number of compounding periods depends on the number of subdivisions.
- A nominal interest rate of 12%/year compounded:
  - m = 1: yearly (equals 12% effective yearly)
  - m = 2: semi-annually (equals 12.360% effective yearly)
  - m = 4: quarterly (equals 12.551% effective yearly)
  - m = 52: weekly (equals 12.734% effective yearly)
  - m = 365: daily (equals 12.747% effective yearly)
  - m = 525600: hourly (equals 12.749% effective yearly)
- Note: we are approaching a limit!

For infinite (continuously) compounding:

- Effective annual $i_a = \lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^m - 1 = e^r - 1$ = 12.7497%

- Not on exam.

Continuous Compounding

- To find compound amount and present worth for continuous compounding and a single payment, we write:
  - Compound amount F = P(e^{rn}) = P[F/P, r, n]
  - Present worth P = F(e^{-rn}) = F[P/F, r, n]
- Square brackets around the factors denote continuous compounding.

Computing $ Flows – extra notes

- Not every benefit or cost will involve a real change to the physical system you're making choices about
  - Example: tax consequences of depreciation
- Not every benefit or cost will involve cash flow
  - Example: avoided environmental damages, depreciation of assets
- We'll handle how tax implications are included in analysis in the second half of the course, and how societally-based analyses (which include broader impacts on society) late in the first half

- no cash change hand, but taxes!

Equivalence and Sustainability

- The formulas and methods we use give reasonable results when applied to time spans of less than a century, but can be misleading when applied to longer time periods
- Many of the problems humanity currently faces require us to plan on a time scale of centuries or longer

long time scale - 30 yrs

60 - 100 - break down

Long-term implications: example

years from now	0	100	Total sum
Cost or benefit	- $ 100M		
Present value	- $ 100M		

- good for near term decisions
- bad for intergenerational equity

· Banker: bad investment | current

if children, greatly impact.

Long-term implications: example

years from now	0	100	Total sum
Cost or benefit	- $ 100M	$ 10,000M	
Present value	- $ 100M	$ 29M	- 71M

$$\text{present value after 100 years} = \frac{\$\ 10{,}000\text{M}}{(1 + 0.06)^{100}} = \$\ 29\text{M}$$

Is Arbitrary

(borrowing rate at bank)

- uncert.
- borrowing now, certainty

Long-term implications: example

years from now	0	100	Total sum
Cost or benefit	- $ 100M	$ 10,000M	
Present value	- $ 100M	$ 29M	-$ 71M

$$\text{present value after 100 years} = \frac{\$\ 10{,}000M}{(1 + 0.06)^{100}} = \$\ 29M$$

discount rate

Other perspectives: First Nations

"In our every deliberation, we must consider the impact of our decisions on the next seven generations."

The Constitution of the Iroquois Nation (The Great Binding Law) explains "seventh generation" philosophy as follows: *"The thickness of your skin shall be seven spans — which is to say that you shall be proof against anger, offensive actions and criticism. Your heart shall be filled with peace and good will and your mind filled with a yearning for the welfare of the people of the Confederacy. With endless patience you shall carry out your duty and your firmness shall be tempered with tenderness for your people. Neither anger nor fury shall find lodgement in your mind and all your words and actions shall be marked with calm deliberation. In all of your deliberations in the Confederate Council, in your efforts at law making, in all your official acts, self interest shall be cast into oblivion. Cast not over your shoulder behind you the warnings of the nephews and nieces should they chide you for any error or wrong you may do, but return to the way of the Great Law which is just and right. Look and listen for the welfare of the whole people and have always in view not only the present but also the coming generations, even those whose faces are yet beneath the surface of the ground — the unborn of the future Nation."*

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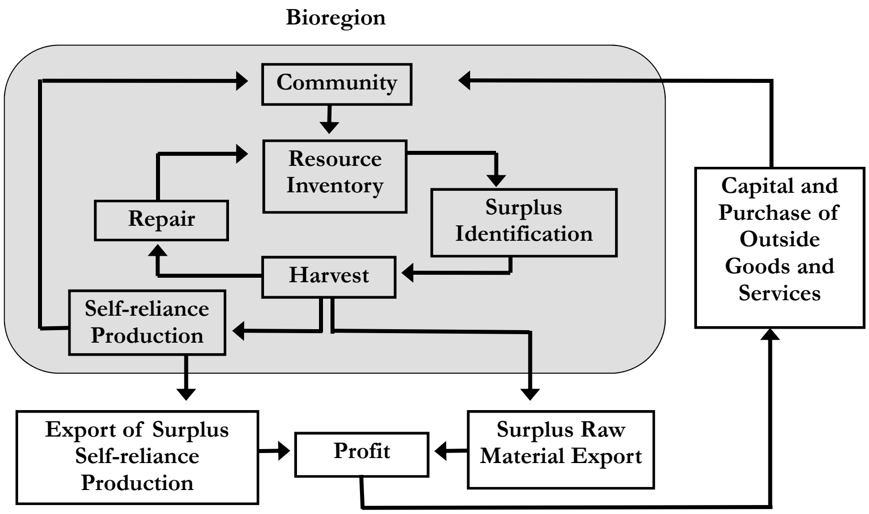
Other perspectives: First Nations



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Other perspectives: First Nations

- Implications on decision making frameworks and analyses?

- Thinking Qs on final.

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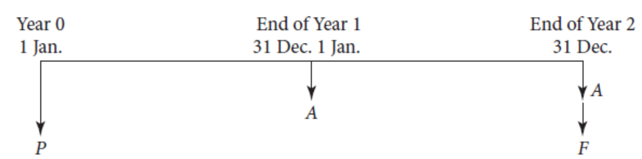
Assumptions in Solving Economic Analysis Problems

- End-of-Year Convention
  - All cash flow amounts are calculated as amounts at the end of each period:
    - ➢ Now = end of period 0 (beginning of period 1)
    - ➢ Future amounts happen at the end of the period specified

A cash flow diagram of *P*, *A*, and *F* for the end-of-period convention is as follows:



- No Sunk Costs
  - Only the current situation and the potential future is considered.

How Do Electricity Rates Work?

- Covering this because the textbook explanations and problems aren't correct.

- Methane capture

- Do not care about sunk cost

-

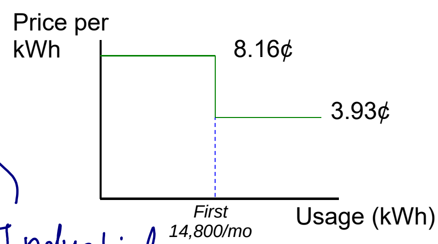
Want businesses to come to B.C.

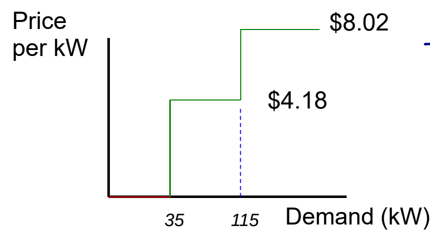
Nowadays flat.

Example Electricity Pricing Structure

- Small, medium and large general service rates (12xx):

No incentive to redu. elec.





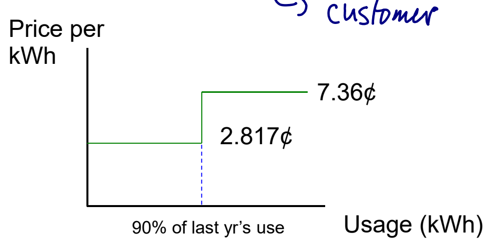
highest rate.

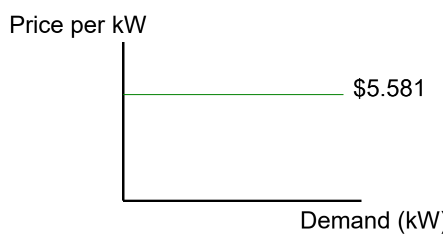
Industrial.

- Promote less electrical use.

- CBL rate (1823):

customer

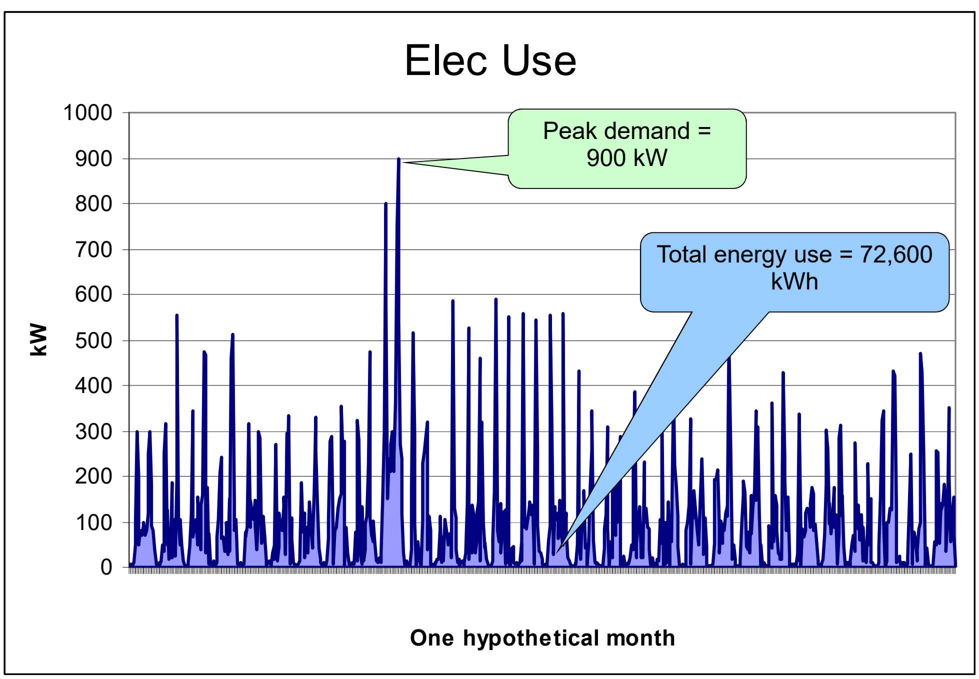




As of Apr 2010

Conservation rate.

How accounts are charged: hypothetical example



Example Electricity Pricing Structure

$$blended\ rate = \left[\frac{total\ usage\ cost + demand\ charge\ cost}{usage\ amount\ (kWh)}\right]$$

- Can be calculated over any time period of interest
- Consequent cost per kWh is a “blended” rate that accounts for all variable costs
- Note that fixed administrative costs are not included (although they can be significant), because they won’t change if a project increases energy use or (peak) demand.