Midterm Exam
Version A -Solutions

Question 1:

(a) (*5 marks : 1 mark for each question*)

i) **False**

ii) **True**

iii) **False**

iv) **False**

v) **False**

(b) (*2 marks*)

(**F**) **0.9648**

Question 2:

(3 marks: 0.5 for defining events, 1 for finding $P(A)$, 1 for correct use of methods, 0.5 for the final answer)

Let A be the First component functions satisfactory
and B be the second component functions satisfactory

$$P(B) = 0.8$$
$$P(A \cup B) = 0.98$$
$$P(A \cap B) = 0.7$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.98 = P(A) + 0.8 - 0.7$$
$$P(A) = 0.88$$

Need to find $P(B|A)$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$
$$P(B|A) = \frac{0.7}{0.88}$$
$$P(A) = 0.0.795$$

Question 3:

a) *(3 marks: 1 mark for identifying random variable and the distribution, 1.5 for showing the method, 0.5 for the answer)*

Let X be the number of patients tested to get the first patient with positive result. Then $X \sim \text{Geo}(p = 0.1)$.

$$\begin{aligned} P(X > 15) &= 1 - P(X \leq 15) \\ &= 1 - [1 - (1-p)^{15}] = (1-p)^{15} \qquad \text{; using cdf of geometric distribution} \\ &= 0.2059 \end{aligned}$$

b) *(3 marks: 1.5 for identifying the distribution with correct parameter values, 1 for showing the method, 0.5 for the answer)*

Let Y be the number of patients that test positive. We know that $Y \sim \text{Bin}(10, 0.1)$.

$$\begin{aligned} P(3 \leq Y \leq 4) &= P(Y = 3) + P(Y = 4) \\ &= \left(\binom{10}{3} \times 0.1^3 \times 0.9^7\right) + \left(\binom{10}{4} \times 0.1^4 \times 0.9^6\right) \\ &= 0.06855. \end{aligned}$$

Question 3: (Contd.)

c) *(3 marks: 2 mark for the method, 1 mark for the answer)*

Let Y be the number of patients that test positive. Assume that there are n patients. We have

$$\begin{aligned} P(Y \geq 1) &= 1 - P(Y = 0) \\ &= 1 - \binom{n}{0} p^0 (1-p)^n \\ &= 1 - (1-p)^n. \end{aligned}$$

When $P(Y \geq 1) > 0.4$

$$\begin{aligned} 1 - (1-p)^n &> 0.4 \\ (1-p)^n &< 0.6 \\ (0.9)^n &< 0.6 \end{aligned}$$

This gives

$$n > \frac{\log 0.6}{\log(1 - 0.05)} = 4.848.$$

Therefore, at least 5 patients are needed.

Question 4: [6 marks - 2 marks for each question]

No need to show your work for the following questions. Provide the correct Answer in the given spaces. You can use scrap papers to do the calculations.

a) **Answer:** $\frac{12}{32} = \frac{3}{8} = \mathbf{0.375}$

b) **Answer:** **6.6**

(c) **Answer:** **0.5252**

Question 5:

a) *(3 marks: 1 mark for standardized values, 1 mark for the correct probability values read from the Z-table, 1 mark for the answer)*

Let the random variable X be the breakdown voltage
$X \sim N(\mu = 50, \sigma^2 = 1.5^2)$

$$
\begin{aligned}
P(49 < X < 52) &= P\left(\frac{49-50}{1.5} < \frac{X-\mu}{\sigma} < \frac{52-50}{1.5}\right) \\
&= P(-0.67 < Z < 1.33) \\
&= P(Z < 1.33) - P(Z < -0.67) \\
&= P(Z < 1.33) - P(Z > 0.67) \quad \text{; by symmetry} \\
&= P(Z < 1.33) - [1 - P(Z < 0.67)] \\
&= 0.9082 - [1 - 0.7486] \\
&= 0.6568
\end{aligned}
$$

b) *(3 marks: 1 mark for probability statement, 1 mark for the correct z-value read from the Z-table, 1 mark for the answer)*

we need to find 85th percentile. Let 85th percentile is a

$$P(Z < \frac{a-50}{1.5}) = 0.85$$

$$\Rightarrow \frac{a-50}{1.5} = 1.04$$

$$\Rightarrow a = 51.56$$

Question 6: (*5 marks*)

Let A be the event that number 4 appears when a die is rolled
then $P(A) = 1/6$ and $P(A^c) = 5/6$

If Bob wins ,the number of rolls should be 2, 4, 6, 8, ...

2st roll $\Rightarrow$ P(Bob wins) $= P(A^c)P(A) = (5/6) \times 1/6$
4rd roll $\Rightarrow$ P(Bob wins) $= P(A^c)^3 P(A) = (5/6)^3 \times 1/6$
6th roll $\Rightarrow$ P(Bob wins) $= P(A^c)^5 P(A) = (5/6)^5 \times 1/6$
8th roll $\Rightarrow$ P(Bob wins) $= P(A^c)^7 P(A) = (5/6)^7 \times 1/6$
and so on.....

$$\begin{aligned}
P(\text{Bob wins}) &= [(5/6) \times 1/6] + [(5/6)^3 \times 1/6] + [(5/6)^5 \times 1/6] + [(5/6)^7 \times 1/6] + \cdots \\
&= [(5/6) \times 1/6] \left\{1 + (5/6)^2 + (5/6)^4 + (5/6)^4 + (5/6)^6 + \cdots \right\} \\
&= \frac{5}{36} \sum_{k=0}^{\infty} (5/6)^{2k} \\
&= \frac{5}{36} \times \frac{1}{1 - \left(\frac{5}{6}\right)^2} \\
&= \frac{5}{36} \times \frac{36}{11} \\
&= \frac{5}{11}
\end{aligned}$$

4 marks for the method with correct intermediate steps, 1 mark for the answer