# Chapter 5 - Normal Distribution

## STAT 251

Lecture 14

Notations and Important properties

Examples

Dr. Lasantha Premarathna

# Normal Distribution

- Normal Distribution as a mathematical function

  $X \sim N(\mu, \sigma^2)$

  $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad ; \; -\infty < x < \infty, \;\; -\infty < \mu < \infty, \;\; \sigma > 0$

- Any given Normal random variable $X \sim N(\mu, \sigma^2)$ can be transformed into a standard normal random variable $Z = \dfrac{X - \mu}{\sigma}$ then $Z \sim N(0, 1)$

- The standard normal density is denoted by the Greek letter $\phi$ (Phi) and the standard normal distribution function is denoted by the corresponding upper case Greek letter $\Phi$
  - pdf of $Z \longrightarrow \phi(z) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{z^2}{2}}$
  - cdf of $Z \longrightarrow \Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}}\, e^{-\frac{t^2}{2}} dt$

# Some important facts about the Normal Distribution

- *Fact 1*: If $X \sim N(\mu, \sigma^2)$ and $Y = aX + b$ where $a$ and $b$ are two constants with $a \neq 0$, then
$$Y \sim N(a\mu + b,\ a^2\sigma^2)$$

- *Fact 2*: Suppose that $X_1, X_2, \ldots, X_n$ are independent normal random variables such that $X_i \sim N(\mu_i, \sigma_i^2)$

  Let $Y$ be a linear combination of the $X_i$,that is
$$Y = a_1X_1 + a_2X_2 + \ldots + a_nX_n,$$
  where $a_1(i = 1, 2, \ldots, a_n)$ are some constants

  then
$$Y \sim N(a_1\mu_1 + a_2\mu_2, \ldots + a_n\mu_n,\ a_1^2\sigma_1^2 + a_2^2\sigma_2^2 + \ldots a_n^2\sigma_n^2)$$

# Some important facts about the Normal Distribution

- *Fact 2*: contd...

  If $X_i \sim N(\mu, \sigma^2) \longleftarrow X_i$'s are identically distributed
  then,
  $$Y \sim N\left((a_1 + a_2, \ldots + a_n)\mu,\ (a_1^2 + a_2^2 + \ldots a_n^2)\sigma^2\right)$$

- *Fact 3*: When $X_1, X_2, \ldots, X_n$ is a normal sample, that is, when the $X_1, X_2, \ldots, X_n$ are independent and identically distributed normal random variables with mean $\mu$ and variance $\sigma^2$, then the sample mean $\bar{X}$ also follows a Normal distribution and

$$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$$

# Some important facts about the Normal Distribution

- *Fact 3*: contd...

$$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$$

- $\bar{X} = \frac{X_1+X_2+...+X_n}{n}$

$$\begin{aligned} E(\bar{X}) &= E(\tfrac{X_1+X_2+...+X_n}{n}) = \tfrac{1}{n}\{E(X_1)+E(X_2)+\ldots+E(X_n)\} \\ &= \tfrac{1}{n}\{\mu+\mu+\ldots+\mu\} = \tfrac{1}{n}\{n\mu\} = \mu \end{aligned}$$

$$\begin{aligned} V(\bar{X}) &= V(\tfrac{X_1+X_2+...+X_n}{n}) = \tfrac{1}{n^2}\{V(X_1)+V(X_2)+\ldots+V(X_n)\} \\ &= \tfrac{1}{n^2}\left\{\sigma^2+\sigma^2+\ldots+\sigma^2\right\} = \tfrac{1}{n^2}\left\{n\sigma^2\right\} = \tfrac{\sigma^2}{n} \end{aligned}$$

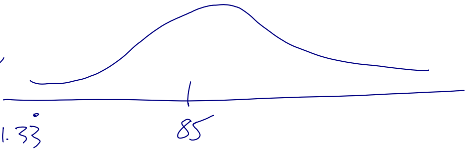
- $\bar{X}$ is a linear combination of normal random variables. Therefore $\bar{X}$ is also normal and

$$\bar{X} \ \sim N(\mu, \tfrac{\sigma^2}{n})$$

# Example

$z_{low} = \frac{73-85}{15} = \frac{-12}{15} = -0.8$

$z_{hi} = \frac{105-85}{15} = \frac{20}{15} = \frac{4}{3} = 1.3\dot{3}$




Assume that the weight of airline adult passenger is reasonably normally distributed with mean=85 kg and standard deviation=15 kg.

(a) If a passenger is selected randomly, what is the probability that the passenger weight is between 73kg and 105kg?

$0.9082 - 0.2119 = 0.6963$

(b) Commuter plane carries 50 passengers. What is the probability that the total weight of the passengers exceeds 4350kg?

$\mu = 50 \times 85 = 4250$ kg

$\text{var} = 106.066$

$z = \frac{100}{106.066}$ 0.942

$1 - 0.8264 = 0.1736$

# Before the next class ...

Visit the course website at canvas.ubc.ca

- Review Lecture 14 and related sections in the text book
- Topic of next class:
  - Chapter 6: Bernoulli and Binomial Random Variables