Version- A

[1] (7 marks) Assume that the helium porosity (in percentage) of coal samples taken from any particular seam is normally distributed with true standard deviation 0.75.

(a) (4marks) Compute a 98% confidence interval for true average porosity of a certain seam if the average porosity for 16 specimens from the seam was 4.56.

Let μ be the true average porosity

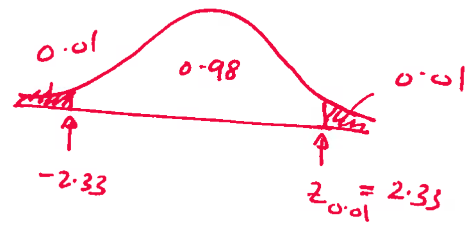
$n = 16$, $\bar{x} = 4.56$

$\sigma = 0.75$

98% CI for μ

$\alpha = 0.02$

$\alpha/2 = 0.01$



$\bar{x} \pm z_{0.01} \frac{\sigma}{\sqrt{n}}$ ← 1 mark

$4.56 \pm 2.33 \frac{0.75}{\sqrt{16}}$ — 1 mark

4.56 ± 0.437 1 mark for correct z value

$(4.123, 4.997)$ Values are in percentages.

← (1 mark)

(b) (3 marks) What sample size is necessary to estimate true average porosity with margin of error 0.2 and 99% confidence.

99% CI for μ.

$\bar{x} \pm \underbrace{z_{0.005} \frac{\sigma}{\sqrt{n}}}_{m.e.}$

$m.e = 0.2 = z_{0.005} \cdot \frac{\sigma}{\sqrt{n}}$

$0.2 = 2.57 \cdot \frac{0.75}{\sqrt{n}}$ ← (1 mark)

$n = \left(\frac{2.57 \times 0.75}{0.2}\right)^2$

1 mark for correct z-value from the table.

$n = 92.88$

$\Rightarrow$ $\underline{n = 93}$ ← (1 mark)

If we consider.

$z_{0.005} = 2.58$

then $n = 93.60$

$\Rightarrow$ $\underline{n = 94}$

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[2] (8 marks) **Circle the best answer. There is only one correct answer to each question.**

(i) Suppose we wish to test

H_0: $\mu \leq 52$ versus H_a: $\mu > 52$.

What will result if we conclude that the mean is less than 52 when its true value is really 54?

Not reject H_0 whe H_0 false.

A) We have made a correct decision
B) We have made a Type I error
C) We have made a Type II error
D) None of the above are correct

(ii) Suppose a 95% confidence interval for μ turns out to be (1000, 2100). Give a definition of what it means to be "95% confident" as an inference.

A) 95% of the observations in the entire population fall in the given interval
B) In repeated sampling, the population parameter would fall in the given interval 95% of the time
C) 95% of the observations in the sample fall in the given interval
D) In repeated sampling, 95% of the intervals constructed would contain the population mean

(iii) A professor receives, on average, 14.7 e-mails from students the day before the midterm exam. To compute the probability of receiving at least 10 e-mails on such a day, he will use what type of probability distribution?

A) Normal distribution
B) Poisson distribution
C) Binomial distribution
D) none of the above

(iv) Which of the following is **true** ?

A) The standard deviation of the sampling distribution of the sample mean is always σ
B) The shape of the sampling distribution of the sample mean is always approximately normal
C) The mean of the sampling distribution of the sample mean is always μ
D) All of the above are true

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[3] (9 marks) Suppose small aircraft arrive at a certain airport according to a Poisson process with rate $\alpha = 8$ per hour.

(a) (3 marks) What is the probability that at least 2 small aircraft arrive during a 1-hour period?

X : # of small aircraft arrive per hour.

$X \sim \text{Poisson}(8)$

$X \sim \text{Poisson}(\alpha)$

$\Rightarrow P(X=x) = \dfrac{\alpha^x e^{-\alpha}}{x!}$; $x = 0, 1, 2 \ldots$

$$P(X \geq 2) = 1 - P(X < 2)$$
$$= 1 - P(X=0) - P(X=1)$$
$$= 1 - \frac{8^0 e^{-8}}{0!} - \frac{8^1 e^{-8}}{1!}$$
$$= 1 - e^{-8} - 8e^{-8} = 1 - e^{-8}(1+8)$$
$$= \underline{0.99698} \approx 0.997$$

method - 2 marks

Answer - 1 mark.

(b) (3 marks) A small aircraft just arrived. What is the probability of waiting time greater than 20 minutes until the next small aircraft arrive at the airport?

Let Y = waiting time till next small aircraft (in hours)

$Y \sim \exp(8)$

1 mark

$Y \sim \exp(\lambda)$

$\Rightarrow f(x) = \lambda e^{-\lambda x}, \quad x > 0$

$\Rightarrow F(x) = 1 - e^{-\lambda x}$

$F(x) = \int_0^x \lambda e^{-\lambda t}\, dt$

$= \lambda \left. \dfrac{e^{-\lambda t}}{-\lambda} \right|_0^x = -e^{-\lambda x} - (-1)$

$= 1 - e^{-\lambda x}$; $x > 0$

$$P(Y > 20 \text{ min}) = P(Y > \tfrac{1}{3} \text{ hours})$$
$$= 1 - P(Y < \tfrac{1}{3})$$
$$= 1 - F(\tfrac{1}{3})$$
$$= 1 - [1 - e^{-8 \cdot \frac{1}{3}}]$$
$$= e^{-8/3}$$
$$= \underline{\underline{0.0695}}$$

method 1 mark

Answer 1 mark

(c) (3 marks) What are the expected value and the standard deviation of the number of small aircraft that arrive during a 2.5-hour period?

W : # of aircraft arrive per 2.5 hour period.

$W \sim \text{Poisson}(8 \times 2.5) = \text{Poisson}(20)$

1 mark

$W \sim \text{poisson}(\lambda)$

$E[W] = \lambda$

$Var(W) = \lambda$

Expected value $= E[W] = 20$ ← 1 mark

$Var(W) = 20$

Standard deviation $= \sqrt{20} = \underline{4.472}$ ← 1 mark

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[4] (8 marks) A drug company is considering marketing a new local anesthetic. The effective time of the anesthetic the drug company is currently producing has a normal distribution with an average of 7.4 minutes. The chemistry of the new anesthetic is such that the effective time should be normal with the same standard deviation as the current one, but the mean effective time may be lower. If it is lower, the drug company will market the new anesthetic; otherwise, they will continue to produce the older one. A sample of size 16 of the new anesthetic results in a sample mean of 7.1 and sample standard deviation 1.2 . A hypothesis test will be done to help make the decision. Give the null and alternative hypotheses. Test the hypotheses at $\alpha = 0.05$ and state the conclusions

Let μ be the mean effective time of the new anesthetic.

Hypotheses: $H_0: \mu \geq 7.4$, $\mu_0 = 7.4$

$H_a: \mu < 7.4$

(2 mark)

$n = 16$

$\bar{x} = 7.1$

$s = 1.2$

$\alpha = 0.05$

$d.f = n-1 = 15$

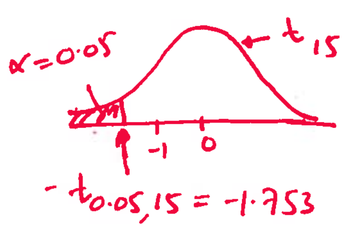
$t_{0.05,15} = 1.753$

Test statistic $= t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} \sim t_{15}$ ← (1 mark)

$$t_{obs} = \frac{7.1 - 7.4}{1.2/\sqrt{16}}$$

$$= -1$$ ← (1 mark)

Correct t value from the table. (1 mark)



$|t_{obs}| = |-1| < t_{0.05,15} = 1.753$

$\Rightarrow$ Do not reject H_0 at $\alpha = 0.05$

1 mark for Comparision (1 mark)

Conclusion: Do not have enough evidence to conclude that the mean effective time for new anesthetic is lower than 7.4 at the confidence level 0.05

(1 mark)

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[5] (8 marks) The wait time (after a scheduled arrival time) in minutes for a train to arrive is Uniformly distributed over the interval [0,12]. You observe the wait time for the next 90 trains to arrive. Assume wait times are independent.

(a) (4 marks) What is the approximate probability that the average of the 90 wait times exceeds 7 minutes?

X = wait time for a train to arrive

$X \sim \text{Uniform}[0,12]$ ← 0.5

$X \sim \text{uniform}[a,b]$

$E[X] = \frac{a+b}{2}$

$Var(X) = (b-a)^2/12$

$F(x) = (x-a)/(b-a)$

$E[X] = \frac{0+12}{2} = 6 = \mu$

$Var(X) = \frac{(12-0)^2}{12} = 12 = \sigma^2$

$n = 90$

By CLT $\bar{X} \overset{approx}{\sim} N\left(\mu = 6, \frac{\sigma^2}{n} = \frac{12}{90}\right)$ ← 1.5 mark

{ 0.5 for correct μ
{ 0.5 for correct σ^2

$P(\bar{X} > 7) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{7-6}{\sqrt{12}/\sqrt{90}}\right)$ ← 0.5

$= P(Z > 2.74)$ ← 0.5

$= 1 - P(Z < 2.74)$

Correct probability value from the table ⇒ 0.5

$= 1 - 0.9969 = \underline{0.0031}$ ← answer ⇒ 0.5

(b) (4 marks) Use the Normal approximation to the Binomial distribution to find the probability that 60 or more of the 90 wait times recorded exceed 4 minutes.

wait time exceed 4 min $= P(X > 4) = 1 - P(X \le 4)$

$= 1 - F_X(4)$

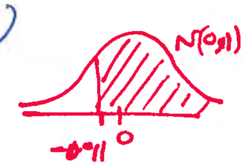
$= 1 - 4/12 = 2/3$ ← 0.5

1 { Let Y be the number of wait times out of 90, that exceed 4 minutes,
$Y \sim \text{Binomial}(90, 2/3)$ where $n = 90$, $p = 2/3$

normal approximation to Binomial ⇒ $Y \sim N(\underset{\mu}{np = 60}, \underset{\sigma^2}{np(1-p) = 20})$ ← 1

$P(Y \geq 60) = P(Y \geq 59.5)$: Continuity Correction ← 0.5

$= P\left(\frac{Y-\mu}{\sigma} \geq \frac{59.5-60}{\sqrt{20}}\right) = P(Z \geq -0.11)$ ← 0.5



$= 1 - P(Z < -0.11) = P(Z < 0.11)$

$= \underline{0.5438}$ ← 0.5