Chapter 6 - Some Probability Models

STAT 251

Lecture 15

Chapter 6 - Bernoulli and Binomial Distributions

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Chapter 6 - Learning Outcomes

- Bernoulli Experiments
- Bernoulli and Binomial Random Variables
- Geometric Distribution
- Poisson process and associated random variables
- Poisson Approximation to the Binomial
- Heuristic Derivation of the Poisson and Exponential Distributions

Bernoulli Random Variable

- A random variable that takes value 1 in case of *Success* and 0 in case of *Failure*
- Bernoulli trials has only two possible outcomes. One is called *Success* and the other one is called *Failure*.

$$X(\text{Success}) = 1$$

$$X(\text{Failure}) = 0$$

where

$$P(\text{Success}) = p$$

$$P(\text{Failure}) = 1 - p$$

Bernoulli Distribution

$$X \sim \text{Bernulli}(p)$$

When the random variable X follows a Bernoulli distribution with probability of success p, then it's has the following pmf

$$P(X = x) = p^x(1-p)^{1-x}\ , \qquad x = 0, 1$$

- Mean and Variance of Bernoulli random Variable

$$\text{mean} = \mu = E(X) = p$$

$$\text{variance} = \sigma^2 = Var(X) = p(1-p)$$

Binomial Distribution

- The Binomial model is also based on the idea of Bernoulli trials. A Binomial random variable is the number of success for n independent trials.
- When a discrete random variable follows a Binomial distribution then it is denoted by , $X \sim \text{Bin}(n, p)$ and it has the pmf

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \;, \qquad x = 0, 1, 2, 3, \;\ldots\;, n$$

where n is the number of trials and p is the probability of success

- Mean and Variance of Binomial random Variable

$$\text{mean} = \mu = E(X) = np$$

$$\text{variance} = \sigma^2 = Var(X) = np(1-p)$$

Motivation for Binomial Distribution

- Consider performing an experiment n times where the probability of success in every trial is p and the n trials are independent. We are interested in the probability of x successes.
- The probability of observing x successes (S) and $(n-x)$ failures (F) in the specific order

$$\underbrace{SS\ldots S}_{x}\quad \underbrace{FF\ldots F}_{n-x}$$

is

$$\underbrace{pp\ldots p}_{x}\quad \underbrace{(1-p)(1-p)\cdots(1-p)}_{n-x} = p^{x}(1-p)^{n-x}$$

- since there are $\binom{n}{x} = \frac{n!}{x!(n-x)!}$ ways that the trial outcome can be ordered

$$P(x \text{ successes}) = \binom{n}{x} p^x (1-p)^{n-x}\,, \qquad x = 0, 1, 2, 3, \ \ldots\ , n$$

Combinations

- Each different order in which we can have x successes in n trials is combination. The total number of ways that can be happen is written $\binom{n}{x}$.

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

$$\text{where } n! = n \times (n-1) \times \ \ldots \ \times 2 \times 1$$

- Note: $0! = 1$
- Ex:

$$\binom{4}{3} = \frac{4!}{3!(4-3)!} = \frac{4!}{3! \times 1!} = \frac{4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 1} = 4$$

$$\binom{5}{0} = \frac{5!}{0!(5-0)!} = \frac{5!}{1 \times 5!} = 1$$

Binomial Distribution

You must be able to to determine when a Binomial(n, p) distribution is appropriate for modeling. This is done by assessing whether the three assumptions of the Binomial distribution are satisfied or approximately satisfied. 3 assumptions are

- There are n number of trials each resulting in either success or failure.
- Trials are independent of one another
- Each trial has the same probability of success p

Binomial random variable is

$$X\text{: number of successes in } n \text{ trials}$$

Critical Assumption of Independence: Binomial Model

- One of the important requirements for Bernoulli trials is that the trials be independent.
- When we don?t have an infinite population, the trials are not independent. But, there is a rule that allows us to pretend we have independent trials:
- The 10% condition: Bernoulli trials must be independent. When you randomly select more than 10% of a population, the independence assumption fails and you cannot use the Binomial model.

Before the next class ...

Visit the course website at canvas.ubc.ca

- Review Lecture 15 and related sections in the text book
- Topic of next class:
  - Chapter 6: Examples-Binomial distribution, Geometric Distribution