Chapter 6 - Some Probability Models

STAT 251

Lecture 16

Binomial Distribution - Examples

Geometric Distribution

Dr. Lasantha Premarathna

Chapter 6 - Learning Outcomes

- Bernoulli Experiments
- Bernoulli and Binomial Random Variables
- Geometric Distribution
- Poisson process and associated random variables
- Poisson Approximation to the Binomial
- Heuristic Derivation of the Poisson and Exponential Distributions

Example: 1

A biased coin is tossed 5 times. The probability of heads on any toss is 0.7. Let X denote the number of heads observed out of 5 tosses.

(a) What is the probability we observe exactly 4 heads?

(b) Find the mean and the standard deviation of the random variable X.

(a) $\binom{5}{4}(0.7)^4(0.3)^{5-4} =$

$5 \times (0.2401) \times 0.3 = 0.36015$

(b) Mean & s.d.

Mean $= 0.7 \times 5 = 3.5$

Var $= (0.7)(0.3)(5)$

sd $= \sqrt{1.05} = 1.025$

Example: 2

Suppose that only 20% of all drivers come to complete stop at an intersection having flashing red lights in all directions when no other vehicles are visible. What is the probability that of 10 randomly chosen drivers coming to such an intersection under these conditions?

(a) exactly 2 will come to complete stop? $\binom{10}{2}(0.2)^2(0.8)^8$

(b) less than 2 will come to complete stop? $\binom{10}{0} + \binom{10}{1}\left(0.2^2 \cdot 0.8^8\right)$

save time

(c) at most 3 will come to complete stop? $\binom{10}{0}+\binom{10}{1}+\binom{10}{2}+\binom{10}{3}$

(d) at least 2 will come to complete stop? $\binom{10}{2}+\binom{10}{3}+\binom{10}{4}+\binom{10}{5}\cdots$

(e) How many of the randomly chosen 10 drivers you expect to come to complete stop? $0.2 \times 10 = 2$ drivers. $Var = n \times p(1-p)$

(f) What is the standard deviation of the number of drivers come to complete stop out of 10 drivers? $\sqrt{0.2 \times 0.8 \times 10} =$

Geometric Distribution

- A Geometric random variable counts the number of independent trials needed until the first success occurs where each trial has successes probability p. (i.e. Geometric distribution is the probability distribution of the number X of Bernoulli trails needed to get one success.)

$$X \sim Geo(p) \quad ; p \text{ is the probability of success}$$

<u>*pmf*</u>: $\quad P(X = x) = p(1-p)^{x-1} \qquad ; x = 1, 2, 3, \cdots$

$$\text{mean} = \mu = E(X) = \frac{1}{p}$$

$$\text{variance} = \sigma^2 = Var(X) = \frac{1-p}{p^2}$$

The expected value of the number of trials before the first occurrence of a certain event is called the **return period** of that event.

cdf of Geometric random variable

$$\text{cdf of } X: \quad F(x) = 1 - (1-p)^x \qquad ; x \geq 1$$

- We use the formula for the sum of a geometric series for the proof.

$$1 + r + r^2 + r^3 + \cdots = \frac{1}{1-r} \qquad \text{where } 0 < r < 1$$

- Proof:

$$\begin{aligned}
F(x) &= P(X \leq x) \\
&= 1 - P(X > x) = 1 - \{P(X = x+1) + P(X = x+2) + \cdots\} \\
&= 1 - \{p(1-p)^x + p(1-p)^{x+1} + p(1-p)^{x+2} + \cdots\} \\
&= 1 - p(1-p)^x\{1 + (1-p) + (1-p)^2 + \cdots\} \\
&= 1 - p(1-p)^x \frac{1}{1-(1-p)} = 1 - p(1-p)^x \frac{1}{p} \\
&= 1 - (1-p)^x \qquad ; x = 1, 2, 3, \cdots
\end{aligned}$$

Example: 3

From past experience, it is known that 2% of accounts in a large accounting population are in error   success = error.

(a) What is the probability that 10th account audited will be the first account in error found? $p(1-p)^{x-1} = 0.02\,(0.98)^9$
$= 0.0167.$

(b) What is the probability that at least 50 accounts are audited before an account in error is found? $1-(1-(1-p)^{50}) = 0.3642.$

(c) How many accounts should be expected to audit to get the first error? Find the variance.

$\frac{1}{0.02} = 50$ // $\frac{1-p}{p^2} = 2450$

Before the next class ...

Visit the course website at canvas.ubc.ca

- Review Lecture 16 and related sections in the text book
- Topic of next class:
  - Chapter 6: Poisson Distribution, More Examples