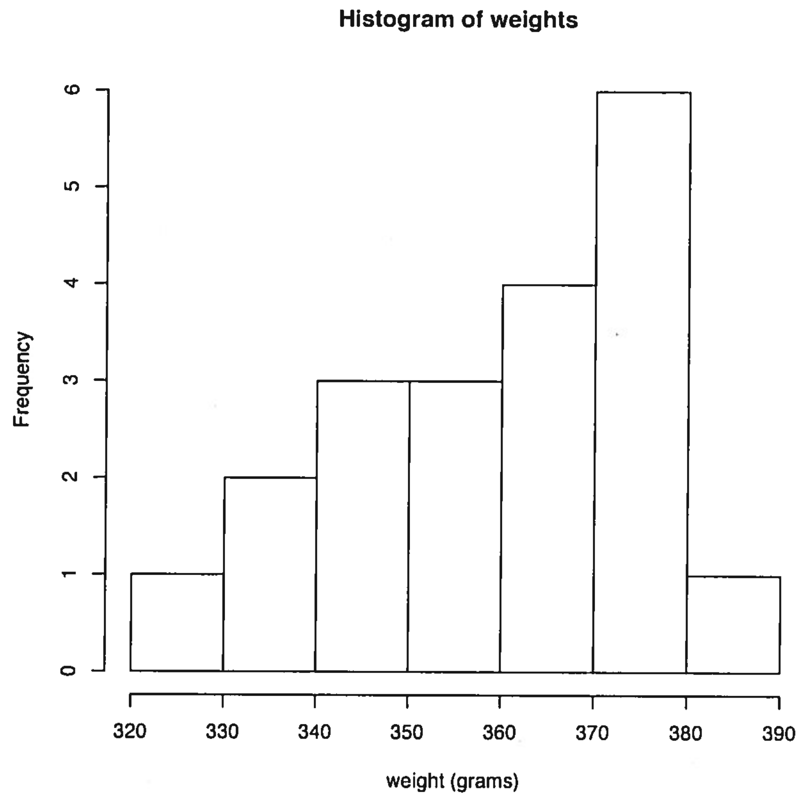
1. **Multiple Choice.**

   (a) The weights of a certain item are shown in the histogram below. Which statement(s) below is(are) true about the distribution of weight data?




Check all that apply.

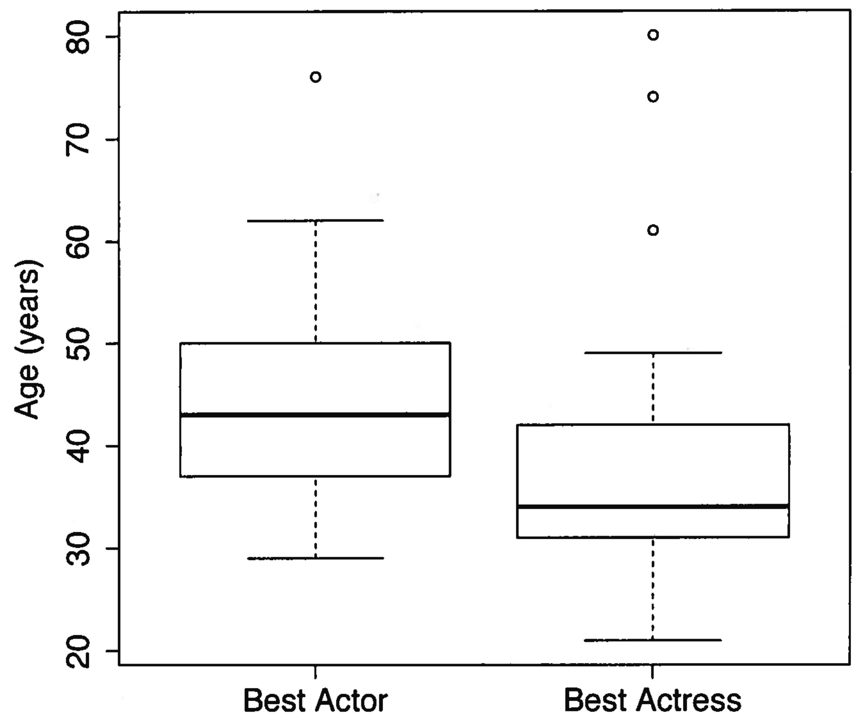
_____ More than 75% of the weights fall between 330 and 370 grams

_____ The mean is between 370 and 380 grams

__✓__ The value of the $\frac{median}{mean}$ ratio is greater than 1

_____ None of the above

(b) The side-by-side boxplots below show the distributions of ages for male and female Academy Award winners from 1977 to 2009. Which statement(s) below is(are) true about the distribution of age?




True False The median age for winning male actors is higher than the median age for winning female actresses.

True False The distribution for winning female actresses is left skewed.

True False Approximately 75% of winning male actors are between 37 and 50 years old.

For the following questions, please circle the most appropriate response.

(c) Let A = {Draw a red card from a regular deck of 52 cards}, and B = {Draw an ace from a regular deck of 52 cards}. Then the events A and B are:

I. disjoint
II. independent
III. complements
IV. none of the above

(d) The number of defective parts produced each hour by a certain production line has the following probability distribution:

| Number of defective parts ($x$) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = x)$ | 0.15 | 0.30 | 0.25 | 0.20 | 0.10 |

Suppose it is known that there were more than 2 defective parts produced in a particular hour. What is the probability that the number of defective parts was fewer than 4?

I. 0.10
II. 0.36
III. 0.67
IV. 0.90
V. None of the above

(e) The length of a metal rod is a random variable with mean 150m and standard deviation 2m. The mean (in m) and variance (in $m^2$) of the total length of five randomly chosen metal rods will be:

I. Mean = 150, Variance = 10
II. Mean = 750, Variance = 10
III. Mean = 150, Variance = 20
IV. Mean = 750, Variance = 20
V. Mean = 750, Variance = 100

(f) Consider tossing a coin 3 times, and define the following events: A = {Toss 3 heads in a row} and B = {Toss a head, then a tail, then a head}. Choose one of the following answers.

I. $P(A) > P(B)$
II. $P(A) = P(B)$
III. $P(A) < P(B)$
IV. Not enough info to tell

**Short Answer**. Please show all your work. Be sure to define variables, state models used and check assumptions where appropriate.

2. A process for making a particular type of alloy yields up to 1 ton of alloy a day. The actual amount produced, $Y$ is a random variable because of machine breakdowns and various slowdowns. Suppose $Y$ has the following pdf

$$f(y) = \begin{cases} 2y & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

The company is paid \$300 per ton of alloy, but there is also a fixed overhead cost of \$100 per day. Let $U$ be the company's daily profit (in hundreds of dollars).

(a) Find the probability density function of $U$.

$U = 3Y - 1 \qquad 0 \le y \le 1 \Rightarrow -1 \le u \le 2$

$F_Y(y) = \int_0^y 2t\,dt = t^2 \Big|_0^y = y^2$

$G_U(u) = P(U \le u) = P(3Y - 1 \le u) = P\left(Y \le \frac{u+1}{3}\right)$

$= F_Y\left(\frac{u+1}{3}\right)$

$G_U(u) = \left(\frac{u+1}{3}\right)^2$

$g_U(u) = \frac{d}{du}\left(\frac{u+1}{3}\right)^2 = 2\left(\frac{u+1}{3}\right) \cdot \frac{1}{3}$

$$g_U(u) = \begin{cases} \frac{2}{9}(u+1), & -1 \le u \le 2 \\ 0, & \text{otherwise} \end{cases}$$

(b) What is the company's expected daily profit?

$$E(u) = \int_{-\infty}^{\infty} u\, g(u)\, du$$

$$= \int_{-1}^{2} u \cdot \frac{2}{9}(u+1)\, du$$

$$= \frac{2}{9}\int_{-1}^{2} u^2 + u \quad du$$

$$= \frac{2}{9}\left[\frac{u^3}{3} + \frac{u^2}{2}\right]_{-1}^{2} = \frac{2}{9}\left[\frac{8}{3} + \frac{4}{2} - \left(-\frac{1}{3} + \frac{1}{2}\right)\right]$$

$$= \frac{2}{9}\left[\frac{9}{2}\right] = 1$$

The expected daily profit is \$100

3. A manufacturing company performs risk assessments to try to prevent worker injuries. Workers' tasks are classified as low risk, medium risk and high risk. From previous records, 32% of tasks are low risk, 47% are medium risk and 21% are high risk. In a given year, the probability of a worker having an accident is 0.11 for a low risk task, 0.23 for a medium risk task and 0.44 for a high risk task.

   (a) What is the probability that a randomly selected worker will have an accident?

Let $L$ = worker's task is low risk.
$M$ = " " medium " "
$H$ = " " high " "
$A$ = worker has accident.

$$P(A) = P(A \cap L) + P(A \cap M) + P(A \cap H)$$
$$= P(A|L)P(L) + P(A|M)P(M) + P(A|H)P(H)$$
$$= 0.11 \times 0.32 + 0.23 \times 0.47 + 0.44 \times 0.21 = 0.2357.$$

   (b) What is the probability that a randomly selected worker performs a medium risk task and does not have an accident?

$$P(M \cap A^c) = P(A^c|M)P(M)$$
$$= (1 - P(A|M))P(M)$$
$$= (1 - 0.23)0.47$$
$$= 0.3619$$

   (c) If a randomly selected worker is known to have had an accident, what is the probability that they were performing a low risk task?

$$P(L|A) = \frac{P(L \cap A)}{P(A)} = \frac{P(A|L)P(L)}{P(A)} = \frac{0.11 \times 0.32}{0.2357} = 0.149$$

4a) Let $X$ be the time to failure $X \sim exp(\frac{1}{4})$

$$F_X(x) = \int_0^x \frac{1}{4} e^{-\frac{t}{4}} \, dt = \frac{1}{4} \cdot (-4) \, e^{-t/4} \Big|_0^x = 1 - e^{-x/4} \quad , x \geq 0$$

Set $F_X(x) = 0.5$

$$1 - e^{-x/4} = 0.5$$

$$x = 2.77 \text{ years.}$$

b) $P(X \leq 1) = F_X(1) = 1 - e^{-(1)(\frac{1}{4})} = 0.2212$.

22.12% of computers will fail within warranty period.

5. $X \sim U(5, 10)$

$$f_X(x) = \begin{cases} \frac{1}{5} & 5 \le X \le 10 \\ 0 & \text{o.w} \end{cases}$$

$$F_X(x) = \frac{x - a}{b - a} = \frac{x-5}{5} \quad , \quad 5 \le x < 10$$

$Y \sim U(7, 10)$

$$f_Y(y) = \begin{cases} \frac{1}{3} & 7 \le y \le 10 \\ 0 & \text{o.w} \end{cases}$$

$$F_Y(y) = \frac{y-7}{7} \quad , \quad 7 \le y < 10$$

Let $W = \min(X, Y)$

$$\begin{aligned} F_W(w) = P(W \le w) &= 1 - P(W > w) \\ &= 1 - P(\{X > w\} \cap \{Y > w\}) \\ &= 1 - P(X > w) P(Y > w) \quad X, Y \text{ independent} \\ &= 1 - [1 - F_X(w)][1 - F_Y(w)] \\ &= 1 - \left(1 - \frac{w-5}{5}\right)\left(1 - \frac{w-7}{3}\right) \\ &= 1 - \left(\frac{10-w}{5}\right)\left(\frac{10-w}{3}\right) \end{aligned}$$

$$F_W(w) = \begin{cases} 0 & w < 5 \\ \dfrac{w-5}{5} & 5 \le w \le 7 \\ 1 - \left(\dfrac{10-w}{5}\right)\left(\dfrac{10-w}{3}\right) & 7 \le w \le 10 \\ 1 & w > 10 \end{cases}$$

$$f_W(w) = \frac{d}{dw}\left(F_W(w)\right)$$

$$f_W(w) = \begin{cases} 0 & w < 5 \\ \dfrac{1}{5} & 5 \le w \le 7 \\ \dfrac{20-2w}{15} & 7 \le w \le 10 \\ 0 & w > 10 \end{cases} \rightarrow f_W(w) = \begin{cases} \dfrac{1}{5} & 5 \le w \le 7 \\ \dfrac{20-2w}{15} & 7 \le w \le 10 \\ 0 & \text{o.w.} \end{cases}$$

$$E(W) = \int_5^7 \frac{w}{5}\, dw + \int_7^{10} w\left(\frac{20-2w}{15}\right) dw$$

$$= \frac{w^2}{10}\Big|_5^7 + \frac{2}{15}\left(5w^2 - \frac{w^3}{3}\right)\Big|_7^{10}$$

$$= \frac{1}{10}\left(49-25\right) + \frac{2}{15}\left[\left(5\cdot 100 - \frac{1000}{3}\right) - \left(5\times 49 - \frac{343}{3}\right)\right]$$

$$= \frac{12}{5} + \frac{24}{5} = \frac{36}{5}$$