Chapter 3 – Probability

3.1 Sets and Probability

Lecture 5

Basic concepts of probability

Set theory for events using Venn Diagrams

Addition Rule, complement rule

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Chapter 3 – Probability

Learning Outcomes

Demonstrate an understanding of the basic concepts of probability and random variables.

- Recall rudimentary mathematical properties of probability.
- Describe the sample space for certain situations involving randomness.
- Explain probability in terms of long-term relative frequencies in repetitions of experiments.
- Recall what are meant by the terms independent, mutually exclusive (disjoint) and complementary events.
- Apply the definition of independence to attempt to determine whether an assumption of independence is justifiable in a given situation.

Chapter 3 Learning Outcomes

- Find probabilities of single events, complementary events and the unions and intersections of collections of events.
- Use Venn diagrams where appropriate to solve probability problems
- Apply the definitions of independence and conditional probability to solve probability problems
- Calculate posterior probabilities through tree diagrams or Bayes theorem.
- Use the law of total probability where appropriate to solve probability problems
- Compute the reliability (that is, the probability that a system works) in simple circuits of independent components connected in series and/or parallel given the reliability of each component.

Introduction to Probability

Random experiments

→ expect reactions to be the same
↳ Can predict what happens next time.

- In statistics, the notion of an experiment differs somewhat from that of an experiment in the physical sciences.
- In statistical experiments, probability determines outcomes.
- Even though the experiment is repeated in exactly the same way, an entirely different outcome may occur. Outcome cannot be determined beforehand.

↳ Cannot predict coin flips outcome.
↳ Can only tell probability.

Sample Space (denoted with *S*)

Sample space is the set of all possible outcomes of a random experiment.

↳ e.g. $S = \{H, T\}$ for coin flip.

Event

An event is a subset of the sample space.

Usually denoted with capital letters e.g. A, B, C.

Example 1: Flipping a coin 3 times

$$S = \left\{ \begin{array}{ll} HHH & HTT \\ HHT & THT \\ HTH & TTT \\ THH & \\ TTH & \end{array} \right\}$$

Event A: Observe 2 or more tails in 3 trials

$A = \{TTT, TTH, THT, HTT\}$

Event B: Observe 2 heads in 3 trials.

$B = \{HHT, HTH, THH\}$

Example 2: Total auto accidents in BC in a year

$0 - \infty$

$S = \{0, 1, 2, 3 \cdots\}$

let A be the event of more than 100 accidents.

$A = \{101, 102, 103 \cdots\}$

Example 3: lifespan in hours of 2 components

$S = \{ (x_1, x_2) : x_1 \geq 0, x_2 \geq 0 \}$

Note S is bivariate & continuous.

Assume a system works if both comp. work

$A = \{ (x_1, x_2) : 0 \leq x_1 < 10 \text{ or } 0 \leq x_2 < 10 \}$.

Probabilities for a sample space

- Each outcome in a sample space has a probability
- The probability of each individual outcome is between 0 and 1
- The total of all the individual probabilities equals 1.

Probability of an Event

- The probability of an event A, denoted by P(A), is obtained by adding the probabilities of the individual outcomes in the event.
- $0 \leq P(A) \leq 1$
- P(A) = 0 implies that event A is impossible and
- P(A) = 1 implies that event A always occurs

- When all the possible outcomes are **equally likely:**

$$P(A) = \frac{\text{number of outcomes in event A}}{\text{number of outcomes in the sample space}}$$

e.g. Flipping a fair coin 3 times

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

These are equally likely outcomes

N° of outcomes $= 2^3$ — 3 opportunities

↑ 2 options

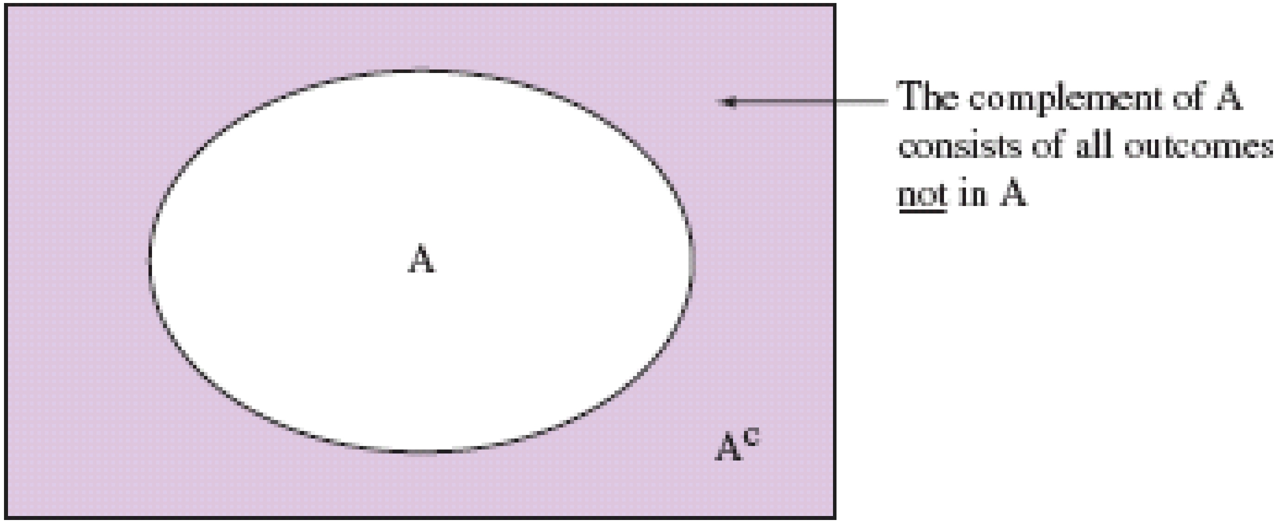
event A = 2 or more tails in 3 trials → A = {TTH, THT, HTT, TTT}

$P(A) = \frac{4}{8} = 0.5$

Set theory for events using Venn Diagrams

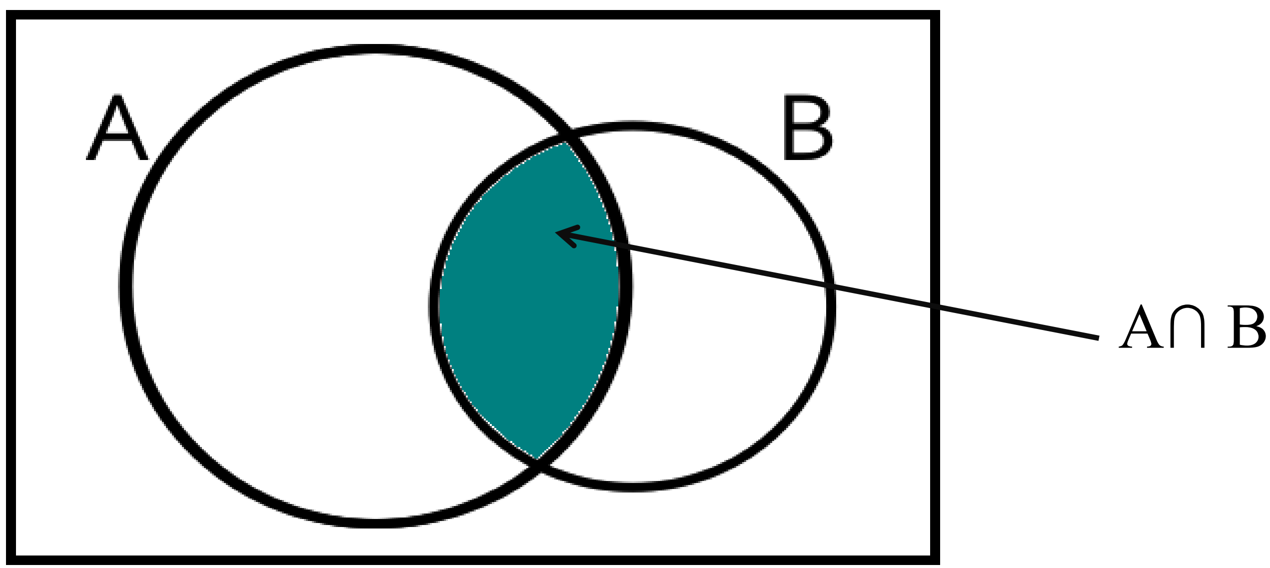
Complement of an event

- The complement of an event A consists of all outcomes in the sample space that are not in A.
- The probabilities of A and of A^c add to 1
  ↳ or A'
- $P(A^c) = 1 – P(A)$



Intersection of two events

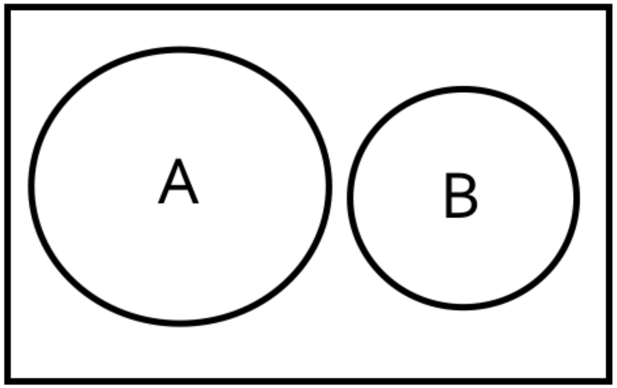
- The **intersection** of A and B, is the set of all elements that are common to A and B.
- Probability of intersection of A and B is denoted by P(A and B) or P(A∩ B)

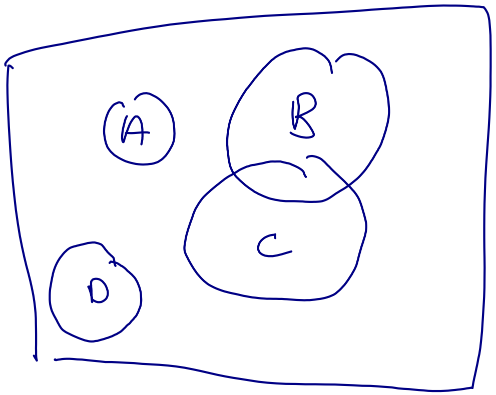


Disjoint or Mutually Exclusive Events:

- when events have no outcomes in common they are said to be disjoint.
- They cannot occur simultaneously

  i.e. P(A and B occur simultaneously) = 0





e.g. Roll a fair die

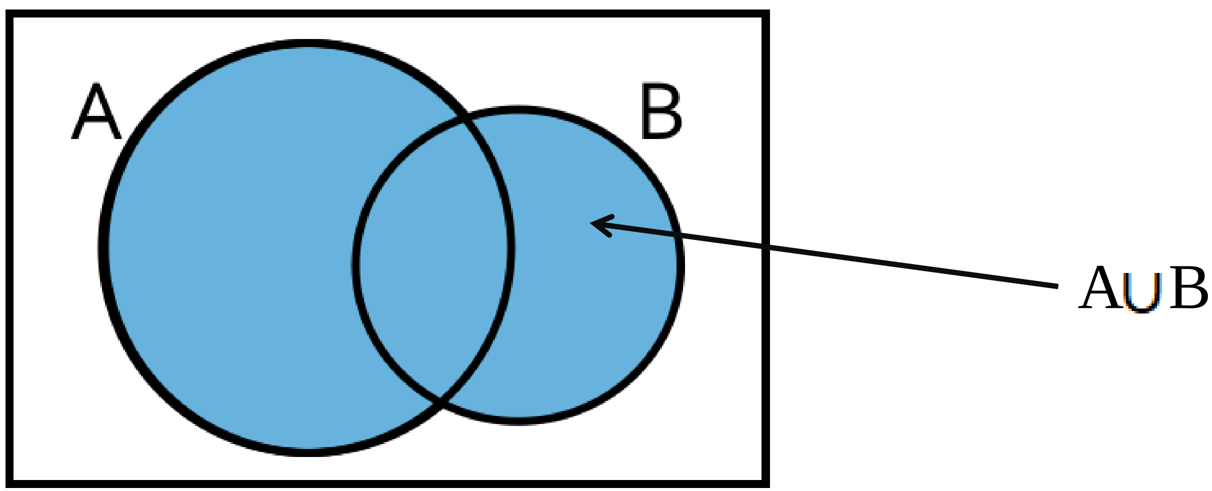
event A: obtain an odd number, Event B: obtain an even number

Then A and B disjoint

Union of two events

- The union of A and B consists of outcomes that are in A or B or in both A and B.
- Probability of union of A and B is denoted by

  P(A or B) or P(A ⋃ B)



Definition

The **probability** of an event A is the sum of the weights of all sample points in A. Therefore,

$$0 \leq P(A) \leq 1, \quad P(\phi) = 0, \quad \text{and} \quad P(S) = 1.$$

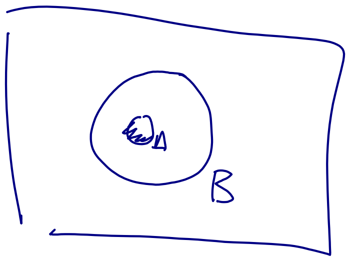
Furthermore, if $A_1, A_2, A_3, \ldots$ is a sequence of mutually exclusive events, then

$$P(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots.$$

(Countable additivity)

Some properties of probability

- General Addition Rule:
  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Notice that if A and B and disjoint (mutually exclusive) events then
  $P(A \cup B) = P(A) + P(B)$
- Complement Rule: $P(A^c) = 1 - P(A)$
- If $A \subseteq B$ then $P(A \cap B) = P(A)$
- If $A \subseteq B$ then $P(A) \leq P(B)$



Example

If 85% of Canadian like either baseball or hockey, 45% like baseball and 65% like hockey, what is the probability that a randomly chosen Canadian likes baseball and hockey?

$$P(B \cup H) = P(B) + P(H) - P(B \cap H)$$

$$85\% = 45\% + 65\% - P(B \cap H)$$

$$P(B \cap H) = \underline{25\%}.$$

Ex:

Verify that for any three events A, B, and C

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

- Explain probability in terms of long-term relative frequencies in repetitions of experiments.

Summary

- Basic concepts of probability
- Set theory for events using Venn Diagrams

Before the next class

- Review the lecture 5 and related sections in the text book

Next Class:

- Chapter 3 : Conditional Probability and Independence