Part A

1. True
2. False
3. False
4. False
5. False
6. True
7. True

Part B

1. (a)

$$\begin{aligned} E(Y) = \int_6^{12} y f(y) dy &= \int_6^{12} \frac{1}{18}(y^2 - 6y) dy \\ &= \frac{1}{18}\left[\frac{y^3}{3} - 3y^2\right]\Bigg|_6^{12} \\ &= \frac{1}{18}\left[\frac{1728}{3} - 3 \times 144 - \frac{216}{3} + 3 \times 36\right] \\ &= \frac{180}{18} \\ &= 10 \end{aligned}$$

$V(Y) = E(Y^2) - E(Y)^2$

$$\begin{aligned} E(Y^2) = \int_6^{12} y^2 f(y) dy &= \int_6^{12} \frac{1}{18}(y^3 - 6y^2) dy \\ &= \frac{1}{18}\left[\frac{y^4}{4} - 2y^3\right]\Bigg|_6^{12} \\ &= \frac{1}{18}[5184 - 2 \times 1728 - 324 + 432] \\ &= \frac{1836}{18} \\ &= 102 \end{aligned}$$

$\therefore V(Y) = 102 - 10^2 = 102 - 100 = 2$

(b)

$$\begin{aligned} E(3X - 5Y) &= 3E(X) - 5E(Y) = 3(6) - 5(10) = -32 \\ V(3X - 5Y) &= 9V(X) + 25V(Y) = 9(10) + 25(2) = 140 \end{aligned}$$

2. Let H = having a defective headlight
Let M = having a faulty muffler

$$P(H \cap M) = 0.10$$
$$P(H \cap M^C) = 0.15$$
$$P(M|H) = \frac{P(M \cap H)}{P(H)} = \frac{P(M \cap H)}{P(M \cap H) + P(M^C \cap H)} = \frac{0.10}{0.10 + 0.15} = 0.40$$

3. (a) $X = \#$ tires before the first that fails before the guaranteed distance
$X \sim Geom(p = 0.02)$

$$P(X = 50) = (1 - 0.02)^{49} \times 0.02 = 0.007432$$

(b) $Y = \#$ tires out of 100 that fails before the guaranteed distance
$Y \sim Bin(n = 100, p = 0.02)$

$$\left.\begin{aligned} \therefore min(np, n(1-p)) < 5 \\ n > 20 \end{aligned}\right\} \Rightarrow \text{use Poisson approximation } Poisson(\alpha = np = 2)$$

$$\begin{aligned} P(Y < 2) &= P(Y = 0) + P(Y = 1) \\ &= \frac{2^0 exp(-2)}{0!} + \frac{2^1 exp(-2)}{1!} \\ &= 3exp(-2) = 0.406 \end{aligned}$$

4. $H_0 : \mu = 20$
$H_A : \mu > 20$ (the new paint will be considered)
This is a right-tailed test. $\mu_0 = 20$, $\alpha = 0.10$
$n = 25 > 20$, σ^2 unknown, distribution of y is unknown $\rightarrow$ CASE 2
The test-statistic is

$$\frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{22 - 20}{4/\sqrt{25}} = 2.5$$

Since the test statistic falls in the critical region, we reject H_0 and conclude that the true mean reflectometer reading of the new paint is significantly larger than 20. The new paint should be considered.

5. (a)

Source of Variation	df	Sum of Squares	Mean Squares	F-ratio
Treatment	$3 - 1 = 2$	17.3	$\frac{17.3}{2} = 8.65$	$\frac{8.65}{0.75} = 11.53$
Error	$12 - 3 = 9$	6.75	$\frac{6.75}{9} = 0.75$	
Total	$12 - 1 = 11$	24.05		

(b) F-ratio = 11.53
Critical F-value = $F_{0.05,2,9} = 4.26$
F-ratio> 4.26, so reject H_0 and conclude that the mean quality scores are significantly different among the 3 methods of preparation.

(c) MSE = 0.75

6. Let X = monthly rent for a studio apartment
$E(X) = \mu = 750$, $V(X) = 56^2$
$n = 64 > 20$, distribution of X unknown
By CLT,

$$\overline{X} \overset{approx.}{\sim} N\left(750, \frac{56^2}{64}\right) = N(750, 49)$$

$$\begin{aligned}
P(729 < \overline{X} < 764) &= P\left(\frac{729-750}{\sqrt{49}} < Z < \frac{764-750}{\sqrt{49}}\right) \\
&= P(-3 < Z < 2) \\
&= \Phi(2) - \Phi(-3) \\
&= \Phi(2) - [1 - \Phi(-3)] \\
&= 0.9772 - (1 - 0.9987) \\
&= 0.9759
\end{aligned}$$

7. (a) 0.68

(b) i.

$$s_e = \sqrt{\frac{RSS}{n-2}} = \sqrt{\frac{7107}{28}} = 15.93$$
$$n = 30$$

$$SE(b_1) = s_e \Bigg/ \left[\sum(x^2) - \frac{[\sum x]^2}{n}\right]^{1/2} = \frac{15.93}{\sqrt{247183 - \frac{2716^2}{30}}} = \frac{15.93}{35.9} = 0.44$$

90% CI for β_1:

$$b_1 \pm t_{(28)}(0.10) \times SE(b_1) = 2.2 \pm 1.70 \times 0.44 = (1.45, 2.95)$$

ii. $H_0 : \beta_1 = 0$ vs. $H_1 : \beta_1 \neq 0$
$\alpha = 0.01$
We construct a $100(1-\alpha)\% = 99\%$ t-CI for β_1.

$$b_1 \pm t_{(28)}(0.01) \times SE(b_1) = 2.2 \pm 2.76 \times 0.44 = (0.9856, 3.4144)$$

Because 0 is not contained in the CI, we reject H_0 and conclude that brain size is useful for predicting IQ.

(c) Let X = brain size for a male student
Let Y = brain size for a female student
$H_0 : \mu_X = \mu_Y$ vs $H_1 : \mu_X \neq \mu_Y$
$n_1 = 13$, $n_2 = 17$, $both < 20$
Assume:
(1) normality of brain size
(2) constant variance $\sigma_X^2 = \sigma_Y^2 = \sigma^2$
use 2-sample t-test

$$S_{pooled} = \sqrt{\frac{(13-1) \times 7.4^2 + (17-1) \times 5.8^2}{13+17-2}} = 6.53$$

We construct a $100(1-\alpha)\% = 95\%$ CI for $\mu_X - \mu_Y$:

$$(\overline{x} - \overline{y}) \pm t_{(n_1+n_2-2=28)}(\alpha = 0.05) \times S_{pooled} \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$
$$\Rightarrow (92.2 - 89.3) \pm 2.05 \times 6.53 \times \sqrt{\frac{1}{13} + \frac{1}{17}}$$
$$\Rightarrow (-2.032, 7.832)$$

Because 0 is inside the CI, we do not reject H_0 and conclude that the 2 genders do not have significantly different mean brain sizes.