Chapter 8 - Statistical Modeling and Inference

STAT 251

Lecture 29

Examples
Confidence Intervals for the mean,
Hypothesis Testing about Mean,
Type I and Type II Errors

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Chapter 8 - Learning Outcomes

- Point Estimation for μ ans σ
- Bias of an estimator
- Confidence Interval for μ
- Testing of Hypotheses about μ
- One sample problems
- Two sample problems

Example: 4

Viscosity characteristics of Rubber-modified asphalts

Suppose that for a particular application it is required that the true average viscosity be 3000 *cp*s. For a random sample of size 5, it produced the sample mean 2887.6 *cps* and sample standard deviation 84 *cps*.

Does this requirement appear to have been satisfied? State and test the appropriate hypotheses. Assume that the population is normal.

Example: 4 - Solution

Let μ be the true average viscosity

Hypotheses

$$H_0 : \mu = 3000 \qquad \Rightarrow \mu_0 = 3000$$
$$H_a : \mu \neq 3000 \qquad \text{this is a two-tail test}$$

We know that $n = 5,\ \bar{x} = 2887.6,\ s = 84, \quad df = n - 1 = 4$

α is not given; consider $\alpha = 0.05$

Test statistic

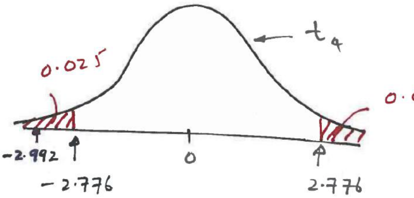
$$\begin{aligned} t &= \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \sim t_{n-1} \\ &= \frac{2887.6 - 3000}{84/\sqrt{5}} \sim t_4 \\ &= -2.992 \end{aligned}$$

Example: 4 - Solution

Method 1: Critical value approach

$$\alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.025$$

need to consider $\frac{\alpha}{2}$ because this is a two-tailed test



Reject H_0 if,

$$|t| > t_{0.025,4} = 2.7776$$

$$|-2.992| > 2.776 \Rightarrow \text{Therefore, reject } H_0 \text{ at } \alpha = 0.05$$

Conclusion:

The requirement of true average viscosity be $3000cps$ is not satisfied.

Example: 4 - Solution

Method 2: p-value approach

p-value $= 2 \times P$(observing data as extreme or more extreme than what we observed towards alternative hypothesis, given H_0 is true)

*** We need to multiply by 2 since this is a two-tailed test

$$\begin{aligned} p\text{-value} &= 2 \times P(\bar{x} \leq 2887.6 \text{ when } \mu = 3000) \\ &= 2 \times P\left(\frac{\bar{x} - \mu_0}{s/\sqrt{n}} \leq \frac{2887.6 - 3000}{84/\sqrt{5}}\right) \\ &= 2 \times P(t_4 \leq -2.992) \end{aligned}$$

$\Rightarrow P(t_4 \leq -2.992)$ is between 0.02 and 0.025

$\Rightarrow$ Therefore p-value is between 0.04 and 0.05

$\Rightarrow p\text{-value} < \alpha = 0.05 \Rightarrow$ Reject H_0

Conclusion: The requirement of true average viscosity be 3000cps is not satisfied.

Example: 4 - Solution

Confidence Interval approach also can be used to test Hypotheses about population mean μ

$\Rightarrow$ Reject H_0 if the $(1-\alpha)100\%$ CI for μ does not include the hypothesized mean μ_0

For the previous example, consider 95% CI for μ

(We consider 95% CI because it is a two-tailed test & $\alpha = 0.05$)

95% CI for μ is

$$\bar{x} \pm t_{0.025.4} \frac{s}{\sqrt{n}}$$

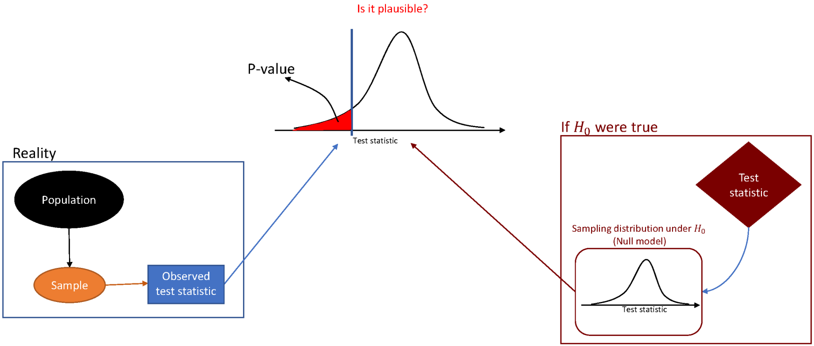
$$\Rightarrow \quad 2887.6 \pm 2.776 \, \frac{84}{\sqrt{5}}$$
$$\Rightarrow \quad 2887.6 \pm 104.28$$
$$\Rightarrow \quad [2783.32, \; 2991.88]$$

This interval does not include the true mean $\mu = 3000$. Therefore, reject H_0 and conclude that the requirement of true average viscosity be 3000 is not satisfied.

$p-$value - Explanation

For example consider, $H_0 : \mu \geq 10$ vs $H_a : \mu < 10$

p-value: The probability of getting a value at least as "extream" as the observed one.



Activity

More questions (clicker questions) will be discussed.

Before the next class ...

Visit the course website at canvas.ubc.ca

- Review Lecture 29 (Questions and Answers) and related sections in the text book
- Topic of next class: **Chapter 8: Hypothesis Testing about difference of two Population Means , Examples**