Chapter 8 - Statistical Modeling and Inference

STAT 251

Lecture 28

Hypothesis Testing about Mean - Examples
Type I and Type II Errors

Dr. Lasantha Premarathna

Chapter 8 - Learning Outcomes

- Point Estimation for μ and σ
- Bias of an estimator
- Confidence Interval for μ
- Testing of Hypotheses about μ
- One sample problems
- Two sample problems

Decisions and Types of errors in Hypothesis Testing

		Reality (population condition)	
		H_0 True	H_0 False
Decision	Reject H_0	Type I Error	Correct Decision
	Do Not Reject H_0	Correct Decision	Type II Error

Four possible outcomes

- Reject H_0
  - In reality null is false: we've made the **correct decision!**
  - In reality null is true: we've made an **error**
- Fail to reject H_0
  - In reality null is false: we've made an **error**
  - In reality null is true: we've made the **correct decision!**

Type I and Type II errors

- Type I error is rejecting H_0 when H_0 is true
- Type II error is not rejecting H_0 when H_0 is false

- What test is a good test

  A test that rarely makes type I and type II errors

- There are probabilities associated with each type of error

  $$P(\text{Type I error}) = \alpha$$

  $$P(\text{Type II error}) = \beta$$

Type I and Type II errors

- We can control the probability of type I error by our choice of the significance level, α
- It's difficult to control the probability of making type II error
- Statisticians avoid the risk of making a type II error by using "Do not reject H_0" and NOT "accept H_0"
- $1-\beta$ referred to as the power of a test

$$1-\beta = 1 - P(\text{Type II error}) = \text{power}$$ higher is better.

- We want the power to be large
- α, β are test properties, independent of data

Power of a Test

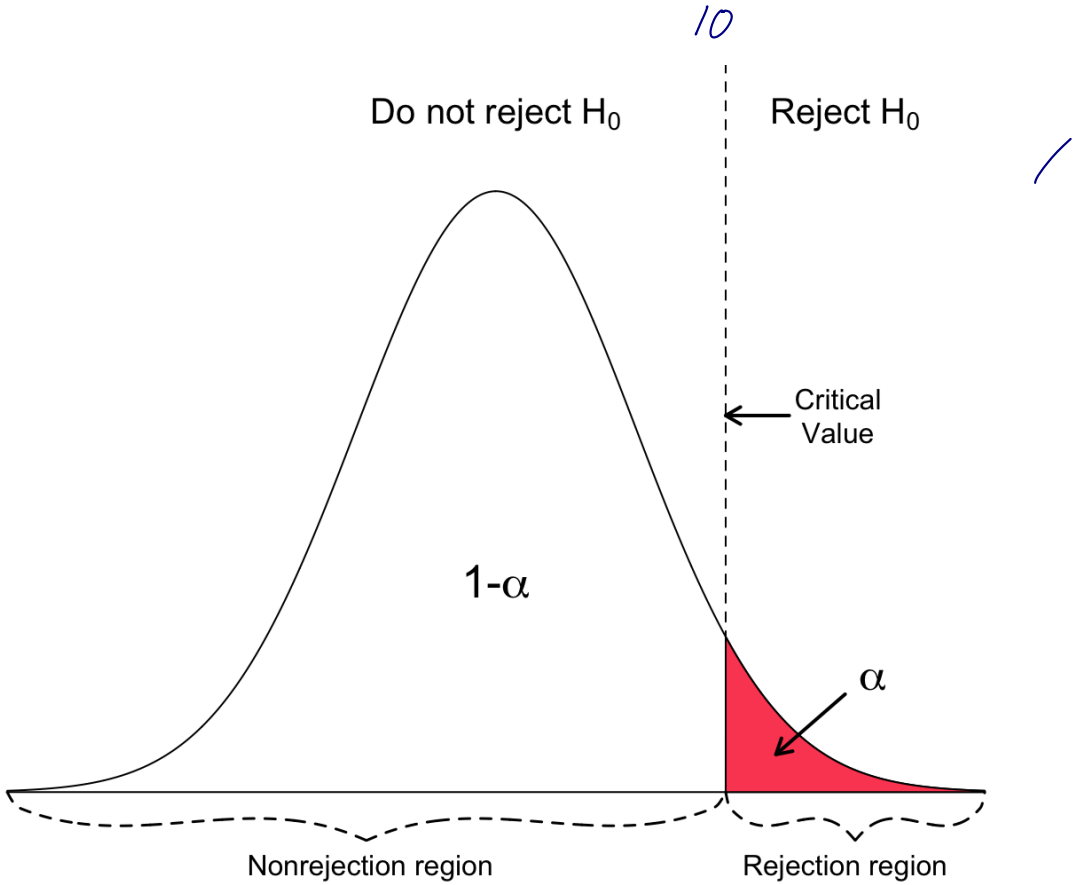
Power is the probability of correctly rejecting the null hypothesis H_0, when H_0 is false

$$\begin{aligned} \text{power} &= P(\text{Reject } H_0 \text{ when } H_0 \text{ is false}) \\ &= 1 - \beta \end{aligned}$$

Type I error

Suppose $H_0 : \mu = 10$ vs. $H_1 : \mu > 10$ and $\alpha = 5\%$.

If our null hypothesis $H_0 : \mu = 10$ was actually true, what percent of the time would we wrongly reject H_0?



Type I and Type II errors

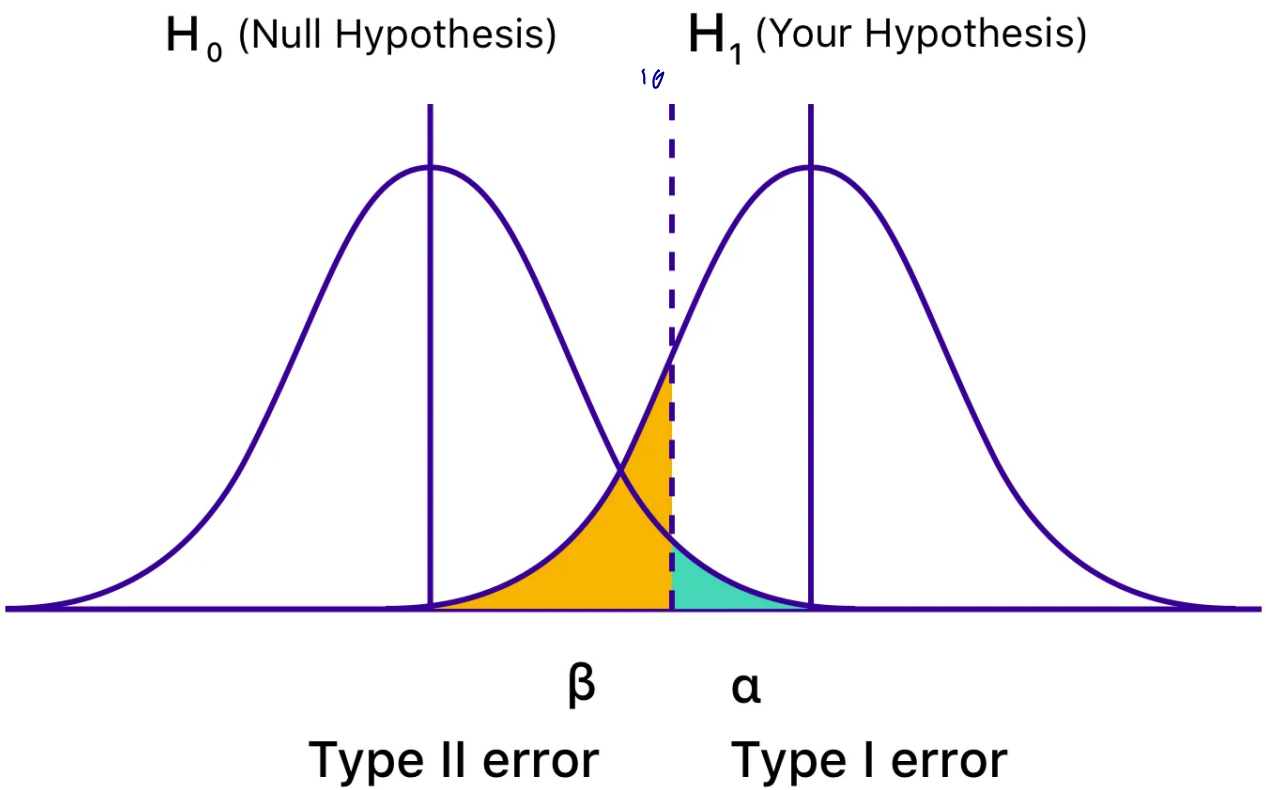
- The Type II error occurs when the null hypothesis is false, but we do not reject it.

- We control the Type I Error by specifying the significance level. However, the probability of Type II Error will depend on:
  - Effect size (i.e., the difference between the null hypothesis and reality)
  - The sample size
  - The probability of Type I Error

Type I and Type II errors

Suppose $H_0 : \mu = 10$ vs. $H_1 : \mu > 10$

The Type II error occurs when the null hypothesis is false, but we do not reject it.



Example: 3

A department store manager determines that the new billing system will be cost-effective only if the mean monthly account is more than \$170.

A random sample of 400 monthly accounts is drawn and the sample mean was found to be \$178. Assume that monthly accounts are approximately normally distributes with $\sigma =$ \$65.

default $H_0: \mu \leq 170$ $H_a: \mu > 170$ $n = 400$, $\bar{x} = 178$, $\sigma = 65$

$$z_{obs} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{178 - 170}{65/\sqrt{400}} = 2.46$$

(a) Can we conclude that the new system will be cost-effective? Use $\alpha = 0.05$

$z_{obs} = 2.46 > z_{0.05} = 1.645$

$\Rightarrow$ Reject H_0. - is effective.

(b) Describe type I and type II errors in the context of this problem

type1 - system is not effective when it is accepted type2 - system is effective when it is rejected

(c) Considering the test procedure, find the rejection region of $\bar{x}$.

(d) When $\mu = 180$, find the probability of type II error.

(e) Evaluate the power of the test when $\mu = 180$.

Before the next class ...

Visit the course website at canvas.ubc.ca

- Review Lecture 28 and related sections in the text book
- Topic of next class: **Chapter 8: more on Hypothesis Testing about the Mean, Examples**