
Problem 1. **(3 points)**

The times (in seconds) between fifteen consecutive eruptions of a geyser were as follows:

914, 900, 883, 919, 940, 949, 933, 880, 918, 928, 855, 899, 926, 883, 907

By entering the data into R (some software packages use slightly different rules to find quartiles thus may give slightly different results), find the following (giving answers to two decimal places):

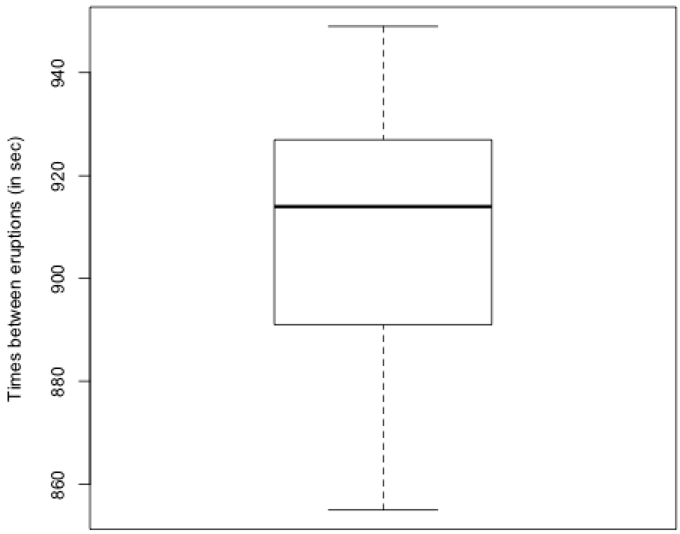
Part a) The sample mean is ___ (sec).

Part b) The sample variance is ___ (sec^2).

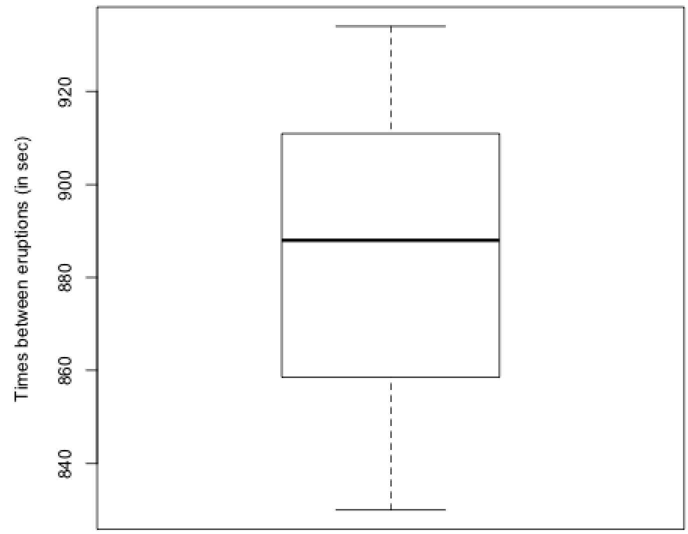
Part c) The sample median is ___ (sec).

Part d) The sample IQR is ___ (sec).

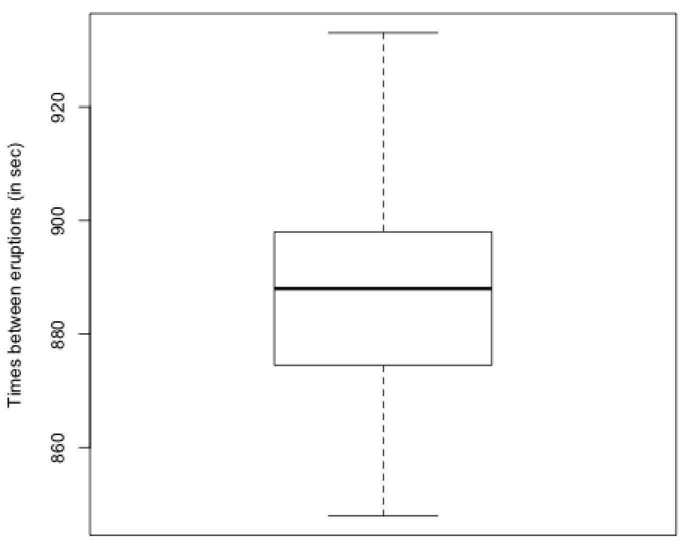
Part e) Which of the following is a boxplot of the data?
[?/A/B/C/D]



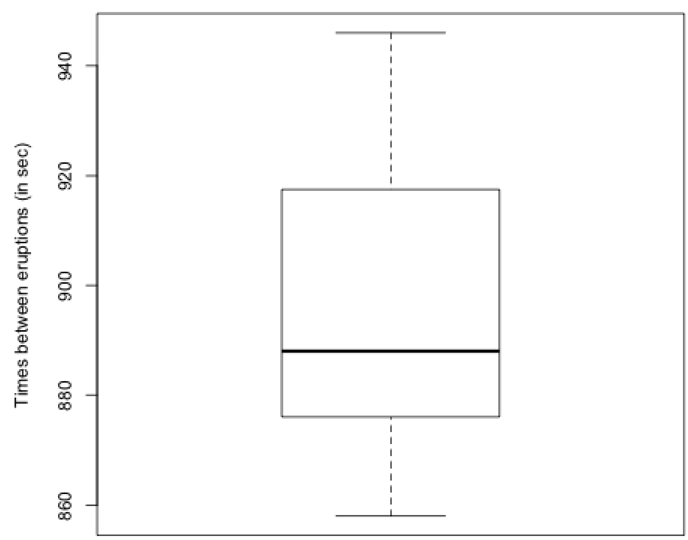
A



B



C



D

(Click on a graph to enlarge it.)

Solution:

Suppose you have entered the data in R as follows:

```
x <-  c( 914, 900, 883, 919, 940, 949, 933, 880, 918, 928, 855, 899, 926, 883, 907 )
```

Then the sample mean, sample variance, sample median, and sample IQR are obtained as follows:

```
mean(x)
var(x)
median(x)
IQR(x)
```

You can obtain a boxplot by typing

```
boxplot( x, ylab = "Times between eruptions (in sec)", id.method="y")
```

For the data as given, the sample mean is 908.933 sec, the variance is 663.352 $\min^2$, the median is 914 min and the IQR is 36 min (from R's default type; it may be quite different if you used a different type.)

Answer(s) submitted:

- 908.93
- 663.35
- 914
- 36
- A

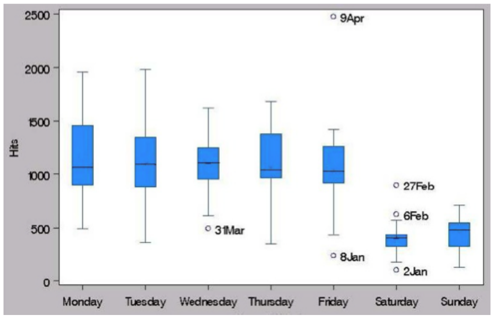
(correct)

Correct Answers:

- 908.933
- 663.352
- 914
- 36
- A

Problem 2. **(1 point)**

The following are boxplots of the numbers of hits at a certain website for the different days of the week.



True or false? Less than 25 percent of Thursdays had a higher number of hits than the busiest Saturday.

- A. True
- B. False

Answer(s) submitted:

- B

(correct)

Correct Answers:

- B

Problem 3. **(1 point)**

Events A and B are mutually exclusive. $P(A) = 0.7$ and $P(B) = 0.1$. Find $P(A \cup B)$ to one decimal place.

$P(A \cup B) =$ ________

Hint: What is the probability $P(A \cap B)$?

Solution: Solution

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Since the events are mutually exclusive, $P(A \cap B) = 0$. Hence $P(A \cup B) = P(A) + P(B) = 0.7 + 0.1 = 0.8$

Answer(s) submitted:

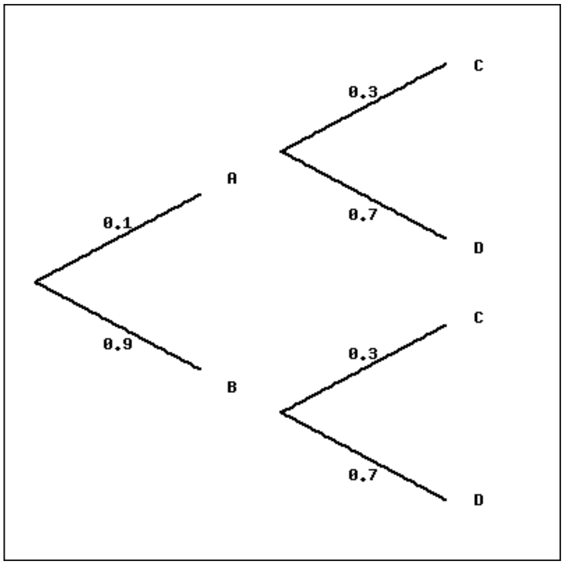
- 0.8

(correct)

Correct Answers:

- 0.8

Problem 4. **(2 points)**



Find each probability by referring to the tree diagram above.

(a) $P(C|A) =$ ______

(b) $P(D|B) =$ ______

(c) $P(A \cap C) =$ ______

(d) $P(B \cap D) =$ ______

(e) $P(C) =$ ______

(f) $P(D) =$ ______

Answer(s) submitted:

- 0.3
- 0.7
- 0.03
- 0.63
- 0.3
- 0.7

(correct)

Correct Answers:

- 0.3
- 0.7
- 0.03
- 0.63
- 0.3
- 0.7

Problem 5. **(1 point)**

Events A and B are independent. $P(A)=0.1$ and $P(B)=0.9$. Find $P(A\cup B)$ to two decimal places.

$P(A\cup B) =$ ________

Hint: What is the probability $P(A\cap B)$?

Solution: Solution

Recall that $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

Since the events are independent, $P(A\cap B)=P(A)P(B)$. Hence $P(A\cup B)=P(A)+P(B)-P(A)P(B)=0.1+0.9-0.09=0.91$.

Answer(s) submitted:

- `0.91`

(correct)

Correct Answers:

- `0.91`

Problem 6. **(2 points)**

If A and B are mutually exclusive events with P(A) = 0.70, then P(B):

- A. can be any value between 0 and 1
- B. cannot be smaller than 0.30
- C. can be any value between 0 and 0.70
- D. cannot be larger than 0.30

If P(A) = 0.20, P(B) = 0.30 and P(A and B) = 0.06, then A and B are:

- A. mutually exclusive events
- B. dependent events
- C. independent events
- D. complementary events

Answer(s) submitted:

- `D`
- `C`

(correct)

Correct Answers:

- `D`
- `C`

Problem 7. **(2 points)**

Items in your inventory are produced at three different plants: 50% from plant A1, 30% from plant A2 and 20% from plant A3. You are aware that your plants produce at different levels of quality: A1 produces 5 percent defectives, A2 produces 7 percent defectives and A3 yields 8 percent defectives. You randomly select an item from your inventory and it turns out to be defective. Which plant is the item most likely to have come from?

- Plant A1
- Plant A2
- Plant A3

Solution:

SOLUTION:

The correct answer is Plant A1

Draw a decision tree. If a sample is chosen at random, it could have come from A1 (probability 50%), A2 (30%), or A3 (20%). Now consider if the sample is defective. If it came from A1, the probability that it is defective is 5%. So the probability that the sample came from A1 AND is defective is 50% times 5%, or 2.5%. If you calculate the probability for the sample being defective in the other two cases, you will see that the first case has the highest probability.

Answer(s) submitted:

- `Plant A1`

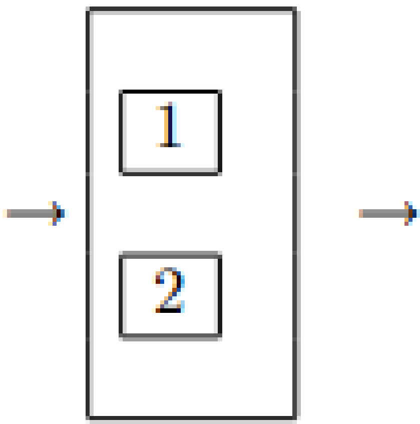
(correct)

Correct Answers:

- `Plant A1`

Problem 8. (2 points)

An electrical system consists of two components in parallel, as indicated in the diagram. At least one component must work for the system to work. During a day, let F_1 be the event that component 1 fails and F_2 be the event that component 2 fails.



Part a)

The event the system fails during a day is

- A. $(F_1 \cup F_2)^c$
- B. $F_1^c \cap F_2^c$
- C. $F_1 \cap F_2$
- D. $F_1 \cup F_2$
- E. $F_1^c \cup F_2^c$

Part b)

The event the system works throughout the day is

- A. $F_1 \cup F_2$
- B. $(F_1 \cup F_2)^c$
- C. $F_1^c \cap F_2^c$
- D. $F_1 \cap F_2$
- E. $F_1^c \cup F_2^c$

Solution:

Part a:

The system fails only when both components fail, which is $F_1 \cap F_2$.;

Part b:

The system is working on the complement of the event in (a), which is $(F_1 \cap F_2)^c = F_1^c \cup F_2^c$ by De Morgan's Laws (or otherwise).;

Answer(s) submitted:

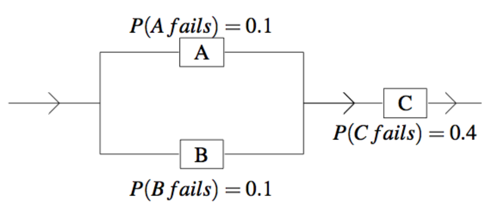
- `C`
- `E`

(correct)
Correct Answers:

- `C`
- `E`

Problem 9. (1 point)

Calculate the reliability (to three decimal places) of the system described in the following figure. The probabilities of failure for each component is given. Note that the components work independently of one another.



Reliability of system = ____

Solution:

SOLUTION:

The reliability of the given network is (1- 0.1 * 0.1) * (1- 0.4) = 0.594

Answer(s) submitted:

- `0.59421`

(correct)
Correct Answers:

- `0.594`

Problem 10. **(1 point)**

Events A and B are such that $P(A) = 0.1$, $P(B) = 0.7$, and $P(A \cap B) = 0.1$.
Find $P(A|B^c)$. You should type a fraction.

$P(A|B^c) =$ ________

Hint: What is the probability $P(B^c)$?

Solution: Solution

$P(A|B^c) = \frac{P(A \cap B^c)}{P(B^c)}$

$P(A \cap B^c) = P(A) - P(A \cap B)$; also $P(B^c) = 1 - P(B)$. Hence $P(A|B^c) = \frac{0.1-0.1}{1-0.7} = 0$.

Answer(s) submitted:

- 0

(correct)

Correct Answers:

- 0

Problem 11. **(3 points)**

A careless university student leaves her iClicker device behind with probability 1/4 each time she attends a class. She sets out with her iClicker device to attend 5 different classes (each class is in a different lecture theatre).

Part 1)
If she arrives home without her iClicker device (after attending 5 classes), what is the probability (to 3 SIGNIFICANT figures) that she left it in the 5th class?

Probability = ____

Solution:

SOLUTION:

The correct solution is: 0.104

Part 2)
What is the probability (to 3 significant figures) that she will leave her iClicker device in the 5th class?

Probability = ____

Solution:

SOLUTION:

The correct solution is: 0.0791

Part 3)
If she arrives home without her iClicker device and she is sure she has the iClicker device after leaving the first class, what is the probability (to 3 SIGNIFICANT figures) that she left it in the 5th class?

Probability = ____

Solution:

SOLUTION:

The correct solution is: 0.154

Part 4)
She arrives home without the iClicker device and rushes back to the university to retrieve the device. She has enough time to get to only one lecture theatre before the theatres are locked up for the day. Which class should she try so that she has the best chance of retrieving her device?

- First class
- Second class
- Third class
- Fourth class
- Fifth class

Solution:

SOLUTION:

The correct solution is: First class

Answer(s) submitted:

- `0.104`
- `0.0791`
- `0.154`
- `First class`

(correct)

Correct Answers:

- `0.104`
- `0.0791`
- `0.154`
- `First class`

Problem 12. **(1 point)**

Factories A and B produce computers. Factory A produces 3 times as many computers as factory B. The probability that an item produced by factory A is defective is 0.021 and the probability that an item produced by factory B is defective is 0.04.
A computer is selected at random and it is found to be defective. What is the probability it came from factory A?
Answer:__________

Answer(s) submitted:

- `63/103`

(correct)

Correct Answers:

- `0.611650485436893`