Assignment Assignment-05 due 10/23/2022 at 11:59pm PDT

Problem 1. **(1 point)**

The length of a coil of copper wire is a random variable with mean 150 m and standard deviation 6.5 m.

If we choose five coils of wire at random, what is the variance of the total length of the wire in the coils?

- A. $32.5m^2$
- B. $1056.25m^2$
- C. $0.8m^2$
- D. $211.25m^2$
- E. $162.5m^2$

Answer(s) submitted:

- D

(correct)

Correct Answers:

- D

Problem 2. **(1 point)**

Which of the following normal distributions has the widest spread?

- A. A normal distribution with mean 2 and standard deviation 1
- B. A normal distribution with mean 0 and standard deviation 2
- C. A normal distribution with mean 1 and standard deviation 3
- D. A normal distribution with mean 3 and standard deviation 2
- E. None of the above

Answer(s) submitted:

- C

(correct)

Correct Answers:

- C

Problem 3. **(1 point)**

A random variable X follows a Normal distribution with mean $\mu = 36$ and standard deviation $\sigma = 4$.

Which of the following gives the expectation $E(X^2)$?

- A. 1312
- B. 1280
- C. 1600
- D. 1296
- E. Insufficient information to calculate $E(X^2)$

Solution: Note that $\text{Var}(X) = E(X^2) - \mu^2$. Hence $E(X^2) = \text{Var}(X) + \mu^2 = \sigma^2 + \mu^2 = 4^2 + 36^2 = 1312$ (choice A).

Answer(s) submitted:

- A

(correct)

Correct Answers:

- A

Problem 4. **(1 point)**

The lengths of a certain type of chain are approximately Normally distributed with a mean of 3.8 cm and a standard deviation of 0.3 cm.

Find the value of ℓ such that $P(L > \ell) = 0.01$

- A. 0.30 cm
- B. 3.93 cm
- C. 3.80 cm
- D. 4.50 cm
- E. 9.15 cm

Solution: We want a length ℓ such that $P(L < \ell) = 0.99$. Transforming this to a standard normal distribution, this gives $P(Z < \frac{\ell-3.8}{0.3}) = 0.99$. Looking up the inverse probability for the standard normal distribution, we have that $\frac{\ell-3.8}{0.3} = 2.33$. Thus $\ell = 4.50$ cm (choice D).

Answer(s) submitted:

- D

(correct)

Correct Answers:

- D

Problem 5. **(1 point)**

A random variable X is normally distributed, with a mean of 21 and a standard deviation of 5.4.

Which of the following is the appropriate interquartile range for this distribution?

- A. $40.68 - 1.32 = 39.36$
- B. $22.35 - 19.65 = 2.70$
- C. $30.48 - 11.52 = 18.96$
- D. $22.75 - 19.25 = 3.50$
- E. $24.64 - 17.36 = 7.28$

Solution:

SOLUTION:

To find the value q_1 such that $P(X < q_1) = 0.25$, convert to a standard Normal distribution and use the Z-score: $0.25 = P(X < q_1) = P(Z < \frac{(q_1 - \mu)}{\sigma}) = P(Z < \frac{(q_1 - 21)}{5.4})$. The value z for which $P(Z < z) = 0.25$ is -0.675, so solving for q_1 we find $\frac{q_1 - 21}{5.4} = -0.675$ so $q_1 = 21 + 5.4 \cdot (-0.675) = 17.36$.

Similarly, solve for q_3 such that $P(X < q_3) = 0.75$ to find the upper boundary of the third quartile. Then the interquartile range is $q_3 - q_1 = 24.64 - 17.36 = 7.28$.

Hence E is the correct choice.

Answer(s) submitted:

- `E`

(correct)

Correct Answers:

- `E`

Problem 6. **(6 points)**

You purchase a chainsaw, and can buy one of two types of batteries to power it, namely Duxcell and Infinitycell. Batteries of each type have lifetimes before recharge that can be assumed independent and Normally distributed. The mean and standard deviation of the lifetimes of the Duxcell batteries are 10 and 2 minutes respectively, the mean and standard deviation for the Infinitycell batteries are 11 and 3 minutes respectively.

Part a)

What is the probability that a Duxcell battery will last longer than an Infinitycell battery? Give your answer to four decimal places. ___

Part b)

What is the probability that an Infinitycell battery will last more than twice as long as a Duxcell battery? Give your answer to four decimal places. ___

Part c)

You are going to cut down a large tree and do not want to break off from the job to recharge your chainsaw battery. You buy two Duxcell batteries, and plan to use one until it runs out of power, after which you immediately replace it with the second battery. How long (in minutes) can the job last so that with probability 0.75 you can complete the job using the two Duxcell batteries in sequence?

Provide your answer to 1 decimal place. ___

Solution: The correct answers are:

Part a) The probability is 0.39
Part b) The probability is 0.04
Part c) 18.1 minutes

Answer(s) submitted:

- `0.3908`
- `0.0359`
- `18.1`

(correct)

Correct Answers:

- `0.3908`
- `0.0359`
- `18.1`

Problem 7. **(6 points)**

The time it takes Alice to walk to the bus stop from her home is Normally distributed with mean 12 minutes and variance 2 minutes^2. The waiting time for the bus to arrive is Normally distributed with mean 5 minutes and standard deviation 2 minutes. Her bus journey to the UBC bus loop is a Normal variable with mean 24 and standard deviation 5 minutes. The time it take Alice to walk from the bus loop to the lecture theatre to attend STAT 251 is Normally distributed with mean 18 minutes and variance 4 minutes^2. The total time taken for Alice to travel from her home to her STAT 251 lecture is Normally distributed. Please use R to find probabilities (R's pnorm() function).

Part a)

What is the mean travel time (in minutes)? ___

Part b)

What is the standard deviation of Alice's travel time (in minutes, to 2 decimal places)? ___

Part c)

The STAT 251 class starts at 8 am sharp. Alice leaves home at 7 am. What is the probability (to 2 decimal places) that Alice will not be late for her class? ___

Solution:

Part a)

Let T_1 be the time to walk from home to the bus stop. Then $T_1 \sim N(12,2)$. If T_2 is the walking time for the bus, $T_2 \sim N(5,2^2)$. The time on the bus is $T_3 \sim N(24,5^2)$, and the walk to the lecture threatre is $T_4 \sim N(18,4)$. The mean walking time is the mean of

$S = T_1 + T_2 + T_3 + T_4$.

In computing we find $E(S) = 12+5+24+18 = 59$ minutes.

Part b)

Assuming the four times are independent (which is not explicitly stated but seems reasonable),

$\mathrm{Var}(S) = \sum_{i=1}^{4} \mathrm{Var}(T_i) = 2+4+25+4 = 35$

and the standard deviation of the total travel time is $\sqrt{35} = 5.92$ minutes.

Part c)

To find the probability Alice arrives on time, if $Z \sim N(0,1)$ we require

$P(S \leq 60) = P(Z \leq \frac{60-(12+5+24+18)}{\sqrt{2+4+25+4}})$.

The result is 0.57.

Answer(s) submitted:

- 59
- 5.92
- 0.57

(correct)

Correct Answers:

- 59
- 5.92
- 0.57

Problem 8. **(3 points)**

If X is Normally distributed with a mean of 3 and a variance of 4, find $P(|X-3| > 1.3)$ to 2 decimal places.

The probability is: ___

Solution:

If $X \sim N(3,4)$ and $Z \sim N(0,1)$, we have

$P(|X-3| > 1.3)$
$= P(X > 4.3) + P(X < 1.7)$
$= P(Z > \frac{4.3-3}{2}) + P(Z < \frac{1.7-3}{2})$
$= P(Z > 0.65) + P(Z < -0.65)$
$= (1-P(Z < 0.65)) + P(Z < -0.65)$
$= 0.2578461 + 0.2578461$
$= 0.52$
to 2 decimal places.;

Answer(s) submitted:

- 0.52

(correct)

Correct Answers:

- 0.5156922