Problem 1. **(2 points)**

The time T required to repair a machine is an exponentially distributed random variable with mean 8 (hours). What is the probability that a repair takes at least $12\frac{1}{2}$ hours given that its duration exceeds 12 hours?

Select the correct answer:

- A. $1-\frac{1}{8}e^{-\frac{1}{16}}$
- B. $1-e^{-8}$
- C. $e^{-\frac{1}{16}}$
- D. e^{-8}
- E. $1-e^{-\frac{1}{16}}$

Solution:

SOLUTION:

The correct answer is:
C

Answer(s) submitted:

- C

(correct)
Correct Answers:

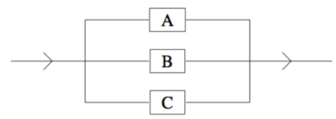
- C

Problem 2. **(2 points)**

Electronic components of a certain type have a length of life X, measured in hours with probability density given by

$$f(x)=\begin{cases}\frac{1}{150}e^{-\frac{x}{150}} & x>0\\ 0 & \text{otherwise}\end{cases}$$

Suppose that three such components operate independently and in parallel in a certain system as shown in the diagram below. Which statement best describes the distribution of Y, the length of life of the entire system?



Which statement best describes the distribution of Y, the length of life of the entire system?

- A. $Y=max(X_1,X_2,X_3), Y$ has an exponential distribution with mean of 450.
- B. $Y=max(X_1,X_2,X_3)$, Y is not an exponential random variable.
- C. $Y=max(X_1,X_2,X_3), Y$ has an exponential distribution with a mean of 50.
- D. $Y=min(X_1,X_2,X_3), Y$ has an exponential distribution with a mean of 50.
- E. $Y=min(X_1,X_2,X_3), Y$ has an exponential distribution with a mean of 450.

Answer(s) submitted:

- B

(correct)
Correct Answers:

- B

Problem 3. **(2 points)**
X is uniformly distributed over the interval $[-1,1]$.

Which of the following is the probability density function of $Y = e^X$?

- A.
$$f_Y(y) = \begin{cases} \frac{1}{2y} & \frac{1}{e} \le y \le e \\ 0 & \text{otherwise.} \end{cases}$$
- B.
$$f_Y(y) = \begin{cases} \frac{2e^2}{e^4-1}y & \frac{1}{e} \le y \le e \\ 0 & \text{otherwise.} \end{cases}$$
- C.
$$f_Y(y) = \begin{cases} 0.0729e^y & \frac{1}{e} \le y \le e \\ 0 & \text{otherwise.} \end{cases}$$
- D.
$$f_Y(y) = \begin{cases} \frac{e}{1-e^2} & \frac{1}{e} \le y \le e \\ 0 & \text{otherwise.} \end{cases}$$
- E.
$$f_Y(y) = \begin{cases} \frac{1}{2}\ln y + \frac{1}{2} & \frac{1}{e} \le y \le e \\ 0 & \text{otherwise.} \end{cases}$$

Answer(s) submitted:

- A

(correct)
Correct Answers:

- A

Problem 4. **(8 points)**

You are working on a programming project with your partner for a computer science course. The project is due in 48 hours. Together, you are to produce a computer program and each of you are assigned to write a portion of computer code. Both of you work simultaneously, but independently. The completion time of your task follows a uniform distribution between 30 and 50 hours. Your partner is stronger in programming and his task is more complex, and the completion time for his task follows a uniform distribution between 37 and 55 hours.

Part a) What is the expected completion time (in hours) for your partner's task?

- A. 51.5
- B. 46
- C. 53
- D. 40
- E. 48

Part b) What is the corresponding standard deviation for the completion time of your partner's task?

- A. 27.00
- B. 9.43
- C. 2.43
- D. 1.22
- E. 5.20

Part c) What is the probability that you and your partner are not able to hand in your project on time (that is, your team's project completion time exceeds 48 hours)?

- A. 0.6823
- B. 0.4888
- C. 0.5455
- D. 0.4500
- E. 0.5200

Part d) On the 48th hour when the project is due, you and your partner have not completed the project. You approach the course instructor for an extension. The course instructor grants you and your partner an extension of 4 hours to hand in the project starting from the 48th hour. What is the probability you and your partner are now able to meet the new deadline?

- A. 0.6888
- B. 0.3175
- C. 0.775
- D. 0.4375
- E. 0.6296

Solution: Let X be the time I take to complete my task. Let Y be the time it takes my partner to complete her task.

Part a) The question asks for the expectation $E(Y)$ which for a uniform random variables $Y \sim \text{uniform}(37,55)$ is given by $\frac{37+55}{2}$. Hence the answer is B;

Part b) Here we are asked for the standard deviation, which is the square root of the variance $\text{Var}(Y)$. Since Y is uniform, the variance is given by $\frac{(55-37)^2}{12}$. Hence the standard deviation is 5.20 which is choice E;

Part c) Let W be our project's completion time. Here we assume that X and Y are independent and my partner and I start at the same time. Then $W = \max(X,Y)$. We're asked to find $F_W(w)$ for $37 \leq w \leq 55$. $P(W \leq w) = P(X \leq w, Y \leq w) = P(X \leq w) \cdot P(Y \leq w)$ since X,Y are independent. But $P(X \leq w) = F_X(w)$ and $P(Y \leq w) = F_Y(w)$ so the quantity we wish to compute is $1 - F_X(w) \cdot F_Y(w) = 1 - \frac{w-30}{20} \cdot \frac{w-37}{55-37}$, given by choice D;

Part d) First we need to write down $F_W(w)$. This is given by

$$F_W(w) = \begin{cases} 0 & w < 37 \\ \frac{w-30}{20} \cdot \frac{w-37}{55-37} & 37 \leq w < 50 \\ \frac{w-37}{55-37} & 50 \leq w < 55 \\ 1 & w \geq 55. \end{cases}$$

Then

$$\begin{aligned} P(W < 48+4 \mid W > 48) &= \frac{P(W < 52 \cap W > 48)}{P(W > 48)} \\ &= \frac{P(48 < W < 52)}{P(W > 48)} \\ &= \frac{F_W(52) - F_W(48)}{1 - F_W(48)} \\ &= \frac{(\frac{52-37}{55-37}) - (\frac{48-30}{20} \cdot \frac{48-37}{55-37})}{1 - (\frac{48-30}{20} \cdot \frac{48-37}{55-37})} \\ &\approx 0.6296 \end{aligned}$$

(choice E);

Answer(s) submitted:

- B
- E
- D
- E

(correct)

Correct Answers:

- B
- E
- D
- E

Problem 5. **(2 points)**

The amount of electricity consumed (in kWh) on a randomly chosen day in a warehouse can be modelled by the random variable with probability density function given by

$$f(x) = \begin{cases} \frac{75}{2x^2} & \text{for } 30 \le x \le 150 \\ \\ 0 & \text{otherwise.} \end{cases}$$

Find the probability (correct to 2 decimals) that on a randomly chosen day more than 115 kWh of electricity will be consumed.

Probability (rounded to two decimals): ___

Solution:

Let X denote the amount of electricity (in kWh) used in the day. Then the probability we need to compute is $P(X > 115) = \int_{115}^{150} \frac{75}{2x^2}\,dx = 0.08$.

Answer(s) submitted:

- `0.08`

(correct)

Correct Answers:

- `0.08`

Problem 6. **(4 points)**

Suppose X has the Uniform $(0,1)$ distribution. Find the median of the distribution of e^X correct to 2 decimals.

Median (rounded to two decimals): ___

Solution:

Let $X \sim U(0,1)$, then the c.d.f. of X is

$$F_X(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } 0 \le x \le 1 \\ 1 & \text{for } x > 1. \end{cases}$$

Let $Y = e^X$, then the c.d.f. of Y is $F_Y(y) = P(Y \le y) = P(e^X \le y) = P(X \le \log y) = \log y$.

The median of Y is the value of y so that $F_Y(y) = 1/2$. Since $P(X \le 1/2) = 1/2$, we solve $\log y = 1/2$ so $y = 1.65$ (to 2 decimals).

Answer(s) submitted:

- `1.65`

(correct)

Correct Answers:

- `1.65`