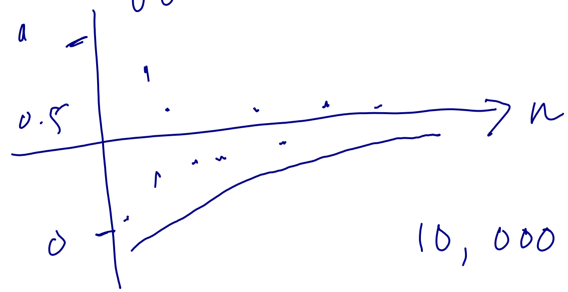
Chapter 3 – Probability

Conditional Probability, Independence and applications

Probability of long term freq.



Lecture 6

$\frac{4900}{10\,000} = 0.49.$

Converges to 0.5

Dr. Lasantha Premarathna

$$P(A \cup B \cup C) = P(A) + P(B) - P(A \cap B) + P(C) - P(A \cap C) - P(B \cap C) - P(A \cap B \cap C)$$

Conditional Probability

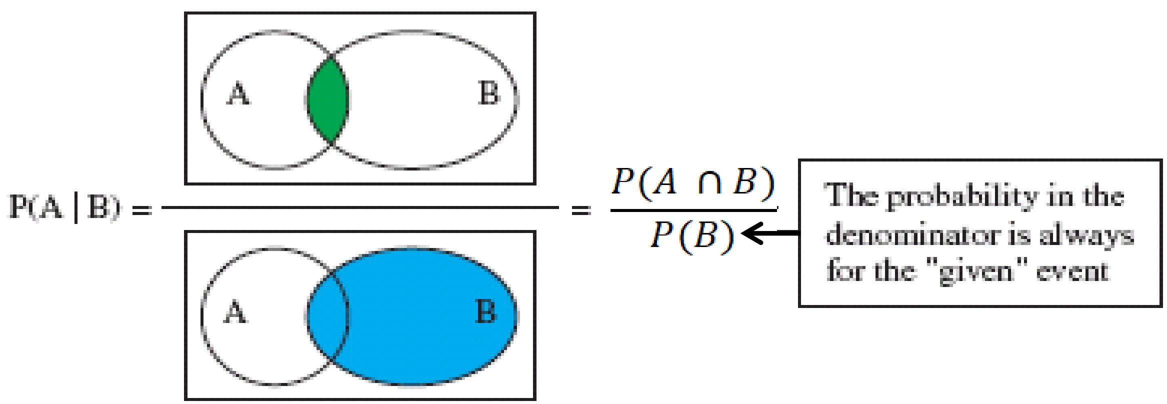
- Conditional probability is used to determine how two events are related; that is, we can determine the probability of one event given the occurrence of another related event.

- For any two events, A and B with P(B) > 0, the conditional probability of A given that B has occurred written as P(A | B) and read as "the probability of A given B" and is calculated by

- B Already happened

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

A given B .



Example:

For two events A and B, $P(A) = 0.60$, $P(B) = 0.40$, and $P(B|A) = 0.6$. Find $P(A|B)$.

$$P(B|A) = \frac{P(A \cap B)}{0.6} = 0.6.$$

$$P(A \cap B) = 0.36.$$

$$P(A|B) = \frac{0.36}{0.4} = 0.9$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Notice that if we rearrange the equation above we get the **Multiplication Rule:**

$$P(A \cap B) = P(B) \times P(A|B) \quad \text{and}$$

$$P(A \cap B) = P(A) \times P(B|A)$$

Independent Events Defined Using Conditional Probabilities

Two events, A and B are independent if knowing that one occurs does not change the probability that the other occurs. That is:

$$P(A|B) = P(A)$$

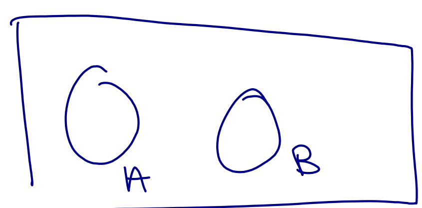
The probability of A is the same when we are given that B has occurred. Equivalently, A and B are independent if

$$P(B|A) = P(B)$$

Two events *A* and *B* are **independent** if and only if

$$P(A \cap B) = P(A)\,P(B)$$

Therefore, to obtain the probability that two independent events will occur, we simply find the product of their individual probabilities.



← Not Indp.

$P(A \cap B) = 0 = P(A) \times P(B)$

– so one of $P(A)$ or $P(B) = 0$.

→ Not Indp.

Note: Do not confuse the notion of mutually exclusive (disjoint) events with that of independent events.

Example: Suppose that you flip a fair coin and roll a fair die. What is the probability of obtaining a Tail and an even number? Show that obtaining a Tail and an even number are independent.

$A =$ coin, tail.

$B =$ die, even

$A = \{T1, T2, T3, T4, T5, T6\}$

$B = \{H2, T2, H4, T4, H6, T6\}$

$A \cap B = \{T2, T4, T6\}$

Since equally likely. $H1, H_2 \cdots H_6, T_1, \cdots T_6$.

$P(A \cap B) = 3/12 = 1/4$

$P(A) \times P(B) = 6/12 \times \frac{6}{12} = 1/4$ $=$ thus independent.

Example: A machine has 8 switches. The probability that any particular switch work properly is 0.99. Assuming independent operation of the switches, calculate the probability that at least one switch fails to work properly.

P(All switches work) = 0.99^8.

P(not all switches work) = $1 - 0.99^8$.

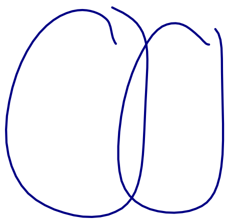
Example:

If events A and B are independent then

(a) A^c and B are also independent

(b) A and B^c are also independent

(c) A^c and B^c are also independent



(a) $P(A^c \cap B) = P(B) - P(A \cap B)$

$= P(B) - P(A) \times P(B)$

$= P(B) \times (1 - P(A))$

$= P(B) \times P(A^c)$

D

(b) WLOG, see (a)

(c)

$P(A^c \cap B^c) = 1 - P(A \cap B)$

$= 1 - P(A) \times P(B)$

$= 1 - ((1 - P(A^c)) \times (1 - P(B^c)))$

$= 1 - P(A^c) - P(B^c) + P(A^c) \times P(B^c)$

Example: Consider two events D and E with probabilities

$$P(D) = 0.5,\ P(E) = 0.6 \text{ and } P(D \cup E) = 0.65$$

(i) Find $P(E|D)$

(ii) Are E and D independent?

$$0.65 = 0.5 + 0.6 - P(D \cap E)$$

$$0.65 - 1.1 = -P(D \cap E)$$

$$P(D \cap E) = 0.45$$

$$0.45 \stackrel{?}{=} 0.5 \times 0.6$$

$$0.45 \neq 0.3.$$

There fore not indp.

Next: Lecture 7

- Chapter 3 : Bayes Theorem