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Assignment Assignment-06 due 11/05/2022 at 11:59pm PDT

Problem 1. **(6 points)**

The life times, Y in years of a certain brand of low-grade lightbulbs follow an exponential distribution with a mean of 0.55 years. A tester makes random observations of the life times of this particular brand of lightbulbs and records them one by one as either a success if the life time exceeds 1 year, or as a failure otherwise.

Part a)
Find the probability to 3 decimal places that the first success occurs in the fifth observation. ___

Part b)
Find the probability to 3 decimal places of the second success occurring in the 8th observation given that the first success occurred in the 3rd observation. ___

Part c) Find the probability to 2 decimal places that the first success occurs in an odd-numbered observation. That is, the first success occurs in the 1st or 3rd or 5th or 7th (and so on) observation.___

Solution:

Part a)

$$\begin{aligned} P(\text{success}) &= P(Y \geq 1) \\ &= \int_1^\infty \frac{1}{0.55} e^{\frac{-1}{0.55}y}\, dy \\ &= \left[-e^{\frac{-1}{0.55}y} \right]_1^\infty \\ &= 0 - (-e^{\frac{-1}{0.55}}) \\ &= 0.1623 \end{aligned}$$

Now let X be a random variable representing the number of observations until the first success.

$$X \sim \text{Geometric(p = 0.1623)}$$

Hence

$$\begin{aligned} P(X = 5) &= (1 - 0.1623)^4 (0.1623) \\ &= 0.0799 \end{aligned}$$

Part b) The setup here is as for part a). We wish to compute

$$\begin{aligned} &P(\text{ 2nd success occurs on 8-th observation given that first success occurred} \\ &= \frac{P(\text{ 2nd success occurs on 8th observation} \cap \text{ first success occurs o}}{P(\text{ first success occurs on 3rd observation })} \\ &= \frac{P(X = 8-3) \cdot P(X = 3)}{P(X = 3)} = P(X = 5) \\ &= (1 - 0.1623)^4 (0.1623) \\ &= 0.0799 \end{aligned}$$

Part c) We have $X \sim \text{Geometric(p = 0.1623)}$, and we wish to compute $P(X \text{ is odd})$.. This is

$$\begin{aligned} P(X \text{ is odd}) &= P(X = 1 \text{ or } X = 3 \text{ or } X = 5 \text{ or } X = 7 \cdots) \\ &= p + (1-p)^2 \cdot p + (1-p)^4 \cdot p \cdots \\ &= p \left(1 + (1-p)^2 + (1-p)^4 \cdots\right) \end{aligned}$$

This is p times a geometric series with term $(1-p)^2$. Using the expression for the sum of a geometric series, we have

$$P(X \text{ is odd}) = \frac{p}{1-(1-p)^2}$$

Substituting the value of p which we computed in part a), we obtain a numeric answer of 0.5442.

Answer(s) submitted:

- `0.080`
- `0.080`
- `0.544`

(correct)

Correct Answers:

- `0.0799`
- `0.0799`
- `0.5442`

Problem 2. (9 points)

The number of major faults on a randomly chosen 1 km stretch of highway has a Poisson distribution with mean 1.4. The random variable X is the distance (in km) between two successive major faults on the highway.

Part a) What is the probability of having at least one major fault in the next 2 km stretch on the highway? Give your answer to 3 decimal places. ___

Part b)
Which of the following describes the distribution of X, the distance between two successive major faults on the highway?

- A. $X \sim \text{Exponential}(\text{mean} = \frac{1}{2 \cdot 1.4})$
- B. $X \sim \text{Poisson}(2 \cdot 1.4)$
- C. $X \sim \text{Exponential}(\text{mean} = 2 \cdot 1.4)$
- D. $X \sim \text{Poisson}(1.4)$
- E. $X \sim \text{Exponential}(\text{mean} = \frac{1}{1.4})$

Part c)
What is the mean distance (in km) and standard deviation between successive major faults?

- A. mean = 2.8000; standard deviation = 2.8000
- B. mean = 1.4; standard deviation = 1.4
- C. mean = 0.7143; standard deviation = 0.7143
- D. mean = 0.7143; standard deviation = 0.5102
- E. mean = 0.3571; standard deviation = 0.3571

Part d) What is the median distance (in km) between successive major faults? Give your answer to 2 decimal places. ___

Part e) What is the probability you must travel more than 3 km before encountering the next four major faults? Give your answer to 3 decimal places. ___

Part f) By expressing the problem as a sum of independent Exponential random variables and applying the Central Limit Theorem, find the approximate probability that you must travel more than 25 km before encountering the next 33 major faults? Give your answer to 3 decimal places. Please use R to obtain probabilities and keep at least 6 decimal places in intermediate steps. ___

Solution:

Answer(s) submitted:

- `0.939`
- `E`
- `C`
- `0.50`
- `0.395`
- `0.364`

(correct)

Correct Answers:

- `0.9392`
- `E`
- `C`
- `0.5`
- `0.395`
- `0.364`

Problem 3. (5 points)

A Statistical Tutorial Centre has been designed to handle a maximum of 25 students per day. Suppose that the number X of students visiting this centre each day is a normal random variable with mean 15 and variance 16.

Part a) What is the return period for this centre rounded to the nearest day? ___

Part b) What is the probability that the designed number of visits will not be exceeded before the 10th day? Leave your answer in 3 decimal places. ___

Answer(s) submitted:

- `161`
- `0.945`

(correct)

Correct Answers:

- `161`
- `0.9455`