Chapter 8 - Statistical Modeling and Inference

STAT 251

Lecture 27

Confidence Interval for μ when σ unknown - Example
Hypothesis Testing about Mean
Type I and Type II errors

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Chapter 8 - Learning Outcomes

- Point Estimation for μ and σ
- Bias of an estimator
- Confidence Interval for μ
- Testing of Hypotheses about μ
- One sample problems
- Two sample problems

Example:

A reporter for a student newspaper is writing an article on the cost of off-campus housing. A sample of 16 studio apartments within 3 km of campus resulted the sample mean $1,000 per month and the sample standard deviation $150. Assuming the population distribution of rent of studio apartments is normal, calculated the 95% confidence interval for the population mean rent per month.

$\bar{x} = 1000$, $s = 150$, $n = 16$

For large

Pop$|^n$ mean = \$1000

$95\% = z_{0.025} \frac{150}{\sqrt{16}}$

for larger sizes, via approximate.

t-distr method. For small.

$t_{0.025, 15} \times \frac{150}{4}$

$\pm 2.131 \times \frac{150}{4} = \pm 79.91$

$\Rightarrow (920.09, 1079.91)$

Hypothesis Testing about Population Mean, μ

Hypothesis testing can be used to determine whether a statement about the value of a population parameter should or should not be rejected.

Null and alternative hypotheses

- The null hypothesis, denoted by H_0 is a tentative assumption about a population parameter.
- The alternative hypothesis, denoted by H_a (or H_1) is the opposite of what is stated in the null hypothesis.
- The alternative hypothesis is what the test is attempting to establish.
- The equality part of the hypotheses always appears in the null hypothesis.

Hypothesis Testing about Population Mean, μ

Hypothesis test about the value of a population mean μ must be take one of the following 3 forms

- $H_0 : \mu \geq \mu_0$ vs $H_a : \mu < \mu_0$ $\Leftarrow$ one-tailed test (lower tail test)
- $H_0 : \mu \leq \mu_0$ vs $H_a : \mu > \mu_0$ $\Leftarrow$ one-tailed test (upper tail test)
- $H_0 : \mu = \mu_0$ vs $H_a : \mu \neq \mu_0$ $\Leftarrow$ two-tailed test

  where μ_0 is the hypothesized value of the population mean.

- The hypotheses should be formulated before viewing or analyzing the data.

Hypothesis Testing

Test Procedures: A test procedure is specified by the following

- A **test statistic** is a function of the data on which the decision (reject H_0 or do not reject H_0) is to be based.
- A **rejection region** is the set of all test statistics values for which H_0 will be rejected

- The null hypothesis will be rejected if and only if the observed or computed test statistic value falls in the rejection region.
- We also use the test statistic to assess the evidence against the null hypothesis by giving a probability, $p-$value.

$p-$value

- The $p-$value summarizes the evidence
- It describes how unusual the data would be if H_0 were true.
- $p-$value is defined as the probability of observing a result as extreme or more extreme towards the alternative hypothesis than what we observed given that H_0 is true.

Test Statistic

A **test statistic** is constructed assuming the null hypothesis is correct.

- Case 1: σ is known

$$\text{test statistic} \;\Rightarrow\; Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \;\sim\; N(0,1)$$

- Case 2: σ is unknown

$$\text{test statistic} \;\Rightarrow\; t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \;\sim\; t_{n-1}$$

When n is large

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \;\sim\; N(0,1)$$

Significance level α

- The significance level is a predetermined number such that we reject H_0 if the $p-$value is less than or equal to that number
- In practice, the most common significance level is $\alpha = 0.05$
- When we reject H_0, we say the results are **statistically significant**.

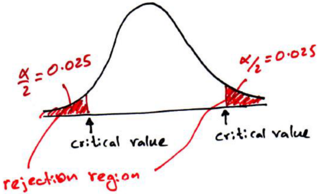
$$\begin{aligned} \text{If } p\text{−value} \leq \alpha &\;\Rightarrow\; \text{Reject } H_0 \\ \text{If } p\text{−value} > \alpha &\;\Rightarrow\; \text{Do not reject } H_0 \end{aligned}$$

Rejection Region and Critical value

Example 1: Consider two tailed test with $\alpha = 0.05$

$$H_0 : \mu = \mu_0$$

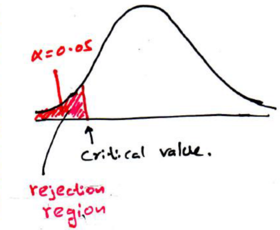
$$H_a : \mu \neq \mu_0$$



Example 2: Consider left-tailed test (one-tailed test) with $\alpha = 0.05$

$$H_0 : \mu \geq \mu_0$$

$$H_a : \mu < \mu_0$$



Steps of Hypothesis Testing

1. Develop the null and alternative hypotheses
2. Specify the level of significance α
3. Collect the sample data and compute the test statistic

- **$p-$value approach**
  4. use the value of the test statistic to compute the $p-$value
  5. Reject H_0 if $p-$value $\leq \alpha$
  6. Conclusion

- **critical value approach**
  4. use the level of the significance to determine the critical value and rejection rule
  5. use the value of the test statistic and rejection rule to determine whether to reject H_0
  6. Conclusion

Explain these with Examples...

Decisions and Types of errors in Hypothesis Testing

		Reality (population condition)	
		H_0 True	H_0 False
Decision	Reject H_0	Type I Error	Correct Decision
	Do Not Reject H_0	Correct Decision	Type II Error

Two possible errors can be made in any test

Type I and Type II errors

- Type I error is rejecting H_0 when H_0 is true
- Type II error is not rejecting H_0 when H_0 is false

- What test is a good test

  A test that rarely makes type I and type II errors

- There are probabilities associated with each type of error

$$P(\text{Type I error}) = \alpha$$

$$P(\text{Type II error}) = \beta$$

Before the next class ...

Visit the course website at canvas.ubc.ca

- Review Lecture 27 and related sections in the text book
- Topic of next class: **Chapter 8: more on Hypothesis Testing about the Mean, Type I and Type II errors, Examples**