Chapter 7 - Normal Probability Approximations

STAT 251

Lecture 23

Central Limit Theorem (CLT) - Examples

Normal Approximation to the Binomial distribution

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Chapter 7 - Learning Outcomes

- Statistic and parameter
- Sampling distribution
- Central Limit Theorem (CLT)
- Normal Approximation to the Binomial distribution
- Normal Approximation to the Poisson distribution

Example 1

Closing prices of stocks have a right-skewed distribution with mean μ of \$25 and standard deviation of \$20. What is the probability that mean of a random sample of 40 stocks will be less than \$20?

$\mu = 25, \quad sd = 20$

$$P\left(Z \leq \frac{20-25}{20/\sqrt{20}}\right) \quad \leftarrow \text{By normal distr.}$$

Example: 2

$$\bar{X} = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{n} \qquad \mu = 10,\ \sigma = 2,\ n = 5$$

$$P(\bar{X} \le 11)$$

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

The time taken by a randomly selected applicant for a mortgage to fill out a certain form has a normal distribution with mean value 10 minutes and standard deviation 2 minutes. If five individuals fill out a form on one day, what is the probability that the sample average amount of time taken on that day is at most 11 minutes.

$$P(\bar{X} \le 11) = P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \le \frac{11 - 10}{2/\sqrt{5}}\right)$$

$$= P(Z \le 1.12)$$

$$= 0.8686$$

Normal Approximation to the Binomial Distribution

- Let $X \sim Bin(n, p)$. When n is large so that both $np \geq 5$ and $n(1-p) \geq 5$, we can use the normal distribution to get an approximate answer

$$X \sim N\left(np,\ np(1-p)\right)\ , \qquad approximately$$

$$\mu = E(X) = np \text{ and } \sigma^2 = Var(X) = np(1-p)$$
$$\text{because } X \sim Bin(n, p)$$

- Wen we use normal approximation to the Binomial distribution, the **continuity correction** should be used because we are approximating a discrete random variable with a continuous random variable.

Example 3

Let $X \sim Bin(10,\, 0.5)$. Obtain $P(X \leq 2)$

(a) exactly

(b) using the normal approximation

(c) using the normal approximation with a continuity correction

(a) $\binom{10}{0}(0.5)^0(0.5)^{10} + \binom{10}{1}(0.5)^1(0.5)^9 + \binom{10}{2}\, 0.5^2 \times 0.5^8$

(b) $\mu = 10 \times 0.5 = 5, \quad \sigma = 5(0.5) = 2.5 \Rightarrow$ Chk $np = 5 \geq 5$, $n(1-p) \geq 5$.

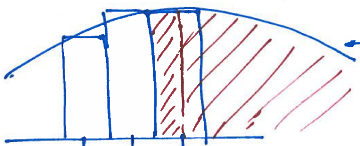
$X \sim N(5, 2.5) \rightsquigarrow P(X \leq 2) = 0.0287$

Not sample, no need n.

(c)

Continuity Correction

Continuity Correction.



- $P(X > 4) = P(X \geq 5) = P(Y \geq 4.5)$ → Add or Subtr. 0.5
- $P(X \geq 4) = P(Y \geq 3.5)$
- $P(X < 4) = P(X \leq 3) = P(Y \leq 3.5)$
- $P(X \leq 4) = P(Y \leq 4.5)$
- $P(X = 4) = P(4 - 0.5 \leq Y \leq 4 + 0.5)$

Before the next class ...

Visit the course website at canvas.ubc.ca

- Review Lecture 23 and related sections in the text book
- Topic of next class: **Activity-CLT and Normal approximation to Poisson distributions**