**Assignment Assignment-08 due 11/26/2022 at 11:59pm PST**

---

**Problem 1.** **(2 points)**

In a study to estimate the proportion of residents in a city that support the construction of a new bypass road in the vicinity, a random sample of 1225 residents were polled. Let $X$ denote the number in the sample who supported the proposal. To estimate the true proportion in support of the plan, we can compute $\hat{p} = \frac{X+\sqrt{1225}/2}{1225}$. The estimator $\hat{p}$ has bias

- A. $1/35$.
- B. $35/2$.
- C. $1/1225$.
- D. $1/(2\times 35)$.
- E. 0, so unbiased.

**Solution:** Taking expectations, since we can assume $X \sim B(n,p)$,

$E(\hat{p}) = \frac{E(X)+\sqrt{1225}/2}{1225}$

$= \frac{1225\times p+\sqrt{1225}2}{1225}$

$= p + \frac{1}{2\times\sqrt{1225}}$.

Hence the bias is $1/(2\times 35)$.

*Answer(s) submitted:*

- D

(correct)

*Correct Answers:*

- D

---

**Problem 2.** **(4 points)**

The time in hours for a worker to repair an electrical instrument is a Normally distributed random variable with a mean of $\mu$ and a standard deviation of 50. The repair times for 12 such instruments chosen at random are as follows:

183 222 303 262 178 232 268 201 244 183 201 140

**Part a)** Find a 95( ___ , ___ ).

**Part b)** Find the least number of repair times needed to be sampled in order to reduce the width of the confidence interval to below 26 hours. ___

**Solution:**

**Part a)**

The answer is $\underbrace{\bar{x}}_{\text{averaged from data}} \pm 1.96\times\frac{\sigma}{\sqrt{n}}$

$= 218.08 \pm 1.96\times\frac{50}{\sqrt{12}}$

$= [189.8, 246.4]$.

**Part b)**

We wish for the width $= 2\times 1.96\times\frac{50}{\sqrt{n}} < 26$

so we find

$2\times 1.96\times 50 < 26\sqrt{n}$

$\implies \sqrt{n} > 7.54$

$\implies n > 56.83$

$\implies n = 57$

*Answer(s) submitted:*

- 189.8
- 246.4
- 57

(correct)

*Correct Answers:*

- 189.793
- 246.373
- 57

**Problem 3.** **(6 points)**

Traffic police monitor the speed of vehicles as they travel over a new bridge. The average speed for a sample of 27 vehicles was 91.29 km/h, with the sample standard deviation being 4.94 km/h. We will assume that the speeds are Normally distributed, and the police are interested in the mean speed.

**Part a)** Since the variance of the underlying Normal distribution is not known, inference here would involve the t distribution. How many degrees of freedom would the relevant t distribution have? ___

**Part b)** Create a 95 % confidence interval for the mean speed of vehicles crossing the bridge. Give the upper and lower bounds to your interval, each to 2 decimal places. ( ___,___)

**Part c)**
The police hypothesized that the mean speed of vehicles over the bridge would be the speed limit, 80 km/h. Taking a significance level of 5 %, what should infer about this hypothesis?

- A. We should not reject the hypothesis since 80 km/h is in the interval found in (b).
- B. We should reject the hypothesis since the sample mean was not 80 km/h.
- C. We should reject the hypothesis since 80 km/h is in the interval found in (b).
- D. We should not reject the hypothesis since the sample mean is in the interval found in (b).
- E. We should reject the hypothesis since 80 km/h is not in the interval found in (b).

**Part d)**
Decreasing the significance level of the hypothesis test above would (select all that apply)

- A. either increase or decrease the Type I error probability.
- B. decrease the Type I error probability.
- C. increase the Type I error probability.
- D. not change the Type I error probability.
- E. not change the Type II error probability.

**Solution:**
**Part a)**

The answer is $27-1$ degrees of freedom.

**Part b)**

The confidence interval is of the form $\bar{x} \pm t_{n-1}(0.975)\frac{s}{\sqrt{n}}$ where the 97.5 % point of $t_{n-1}$ is denoted $t_{n-1}(0.975)$.

Here, the interval is $91.29 \pm 2.06 \times \frac{4.94}{\sqrt{27}} = (89.34, 93.24)$.

**Part c)**

We check to see if 80 is contained within the confidence interval above. If it is, then we do not reject the hypothesis; if it is not, then we reject the hypothesis. In this case, the correct answer is:

We should reject the hypothesis since 80 km/h is not in the interval found in (b).

**Part d)**

Typically decreasing the significance level will increase the Type II error probability.

*Answer(s) submitted:*

- `26`
- `89.34`
- `E`
- `B`

(correct)
*Correct Answers:*

- `26`
- `89.34; 93.24`
- `E`
- `B`

## Problem 4. (6 points)

Some car tires can develop what is known as "heel and toe" wear if not rotated after a certain mileage. To assess this issue, a consumer group investigated the tire wear on two brands of tire, A and B, say. Fifteen cars were fitted with new brand A tires and thirteen with brand B tires, the cars assigned to brand at random. (Two cars initially assigned to brand B suffered serious tire faults other than heel and toe wear, and were excluded from the study.) The cars were driven in regular driving conditions, and the mileage at which heal and toe wear could be observed was recorded on each car. For the cars with brand A tires, the mean mileage observed was 31.86 (in $10^3$ miles ) and the variance was 9.98 (in $10^6$ miles$^2$). For the cars with brand B, the corresponding statistics were 24.62 (in $10^3$ miles) and 9.16 (in $10^6$ miles$^2$ ) respectively. The mileage before heal and toe wear is detectable is assumed to be Normally distributed for both brands.

**Part a)** Calculate the pooled variance $s^2$ to 3 decimal places. During intermediate steps to arrive at the answer, make sure you keep as many decimal places as possible so that you can achieve the precision required in this question. ___ $\times 10^6$ miles $^2$

**Part b)** Determine a 95% confidence interval for $\mu_A - \mu_B$, the difference in the mean $10^3$ mileages before heal and toe wear for the two brands of tire. Leave your answer to 2 decimal places. ( ___,___)

**Part c)**
Based on the 95% confidence interval constructed in the previous part, which of the following conclusions can be drawn when we test $H_0 : \mu_A = \mu_B$ vs. $H_a : \mu_A \neq \mu_B$ with $\alpha = 0.05$.

- A. Do not reject $H_0$ since 7.24 is within the interval found in part (b).
- B. Do not reject $H_0$ since 0 is within the interval found in part (b).
- C. Reject $H_0$ since 0 is not within the interval found in part (b).
- D. Reject $H_0$ since 0 is in the interval found in part (b).
- E. Do not reject $H_0$ since 0 is not in the interval found in part (b).

**Solution:**
**Part a)**

$$s^2 = \frac{(n_A-1)s_A^2+(n_B-1)s_B^2}{n_A+n_B-2}$$
$$= \frac{(15-1)9.98+(13-1)9.16}{15+13-2}$$
$$= 9.602 \text{ miles }^2.$$

**Part b)**

The 95

$$(\bar{x}_A - \bar{x}_B) \pm \underbrace{t_{0.025,df=15+13-2}}_{=2.056} \times \text{SE}$$
$$= (31.86 - 24.62) \pm 2.056(s\sqrt{1/n_A + 1/n_B})$$
$$= (4.83, 9.65).$$

**Part c)**

We check to see if 0 is contained within the confidence interval above. If it is, then we do not reject the null hypothesis; if it is not, then we reject the null hypothesis. In this case, the correct answer is:

Reject $H_0$ since 0 is not within the interval found in part (b).

*Answer(s) submitted:*

- `9.602`
- `4.83`
- `C`

(correct)

*Correct Answers:*

- `9.60154`
- `4.8259; 9.6541`
- `C`

## Problem 5. (2 points)

When one changes the significance level of a hypothesis test from 0.10 to 0.05, which of the following will happen? Check all that apply.

- A. The chance of committing a Type I error changes from 0.10 to 0.05.
- B. It becomes harder to prove that the null hypothesis is true.
- C. The chance that the null hypothesis is true changes from 0.10 to 0.05.
- D. The test becomes more stringent to reject the null hypothesis (i.e., it becomes harder to reject the null hypothesis).
- E. The test becomes less stringent to reject the null hypothesis (i.e. it becomes easier to reject the null hypothesis).
- F. It becomes easier to prove that the null hypothesis is true.
- G. The chance of committing a Type II error changes from 0.10 to 0.05.

*Answer(s) submitted:*

- `( A, D )`

(correct)

*Correct Answers:*

- `AD`