Chapter 3 – Probability

Lecture 7

Bayes Theorem

Tree Diagram

Applications

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Bayes Theorem

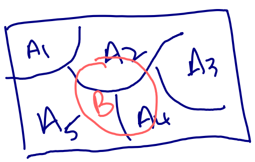
- Often we begin probability analysis with initial or prior probabilities.
- Then, from a sample or a special report we obtain some additional information
- Given this information, we calculate revised or posterior probabilities
- Bayes' theorem provides the means for revising the prior probabilities.

e.g. health problem
↳ risk diff for age grp.

Prior Probabilities → New Information → Application of Bayes' Theorem → Posterior Probabilities

Bayes Theorem

- Let $A_1, A_2, \ldots, A_n$ be mutually exclusive (disjoint) events that together form the sample space *S*. Let B be any event from the same sample space, such that P(B) > 0. Then
- Posterior probability that event $\boldsymbol{A_i}$ will occur given that event ***B*** has occurred, we apply Bayes' theorem



$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{\sum_{k=1}^{n} P(A_k)P(B|A_k)}$$

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)}$$

Some are 0.

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(A_1 \cap B) + P(A_2 \cap B) + \ldots + P(A_n \cap B)}$$ → find all P(B)

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \ldots + P(A_n)P(B|A_n)}$$

given fmla

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{\sum_{k=1}^{n} P(A_k)P(B|A_k)}$$

Example: A company has three plants. Plant 1 produces 35% of the car output, plant 2 produces 20% and plant 3 produces the remaining 45% of cars. 1% of the output of plant 1 is defective, 1.8% of the output of plant 2 is defective and 2% of the output of plant 3 is defective. The annual total production of the company is 1,000,000 cars. A car chosen at random from the annual output and is found defection. What is the probability that it came from plant 2?

$P(D|1) = 0.01 \quad P(D|2) = 0.018 \quad P(D|3) = 0.02$

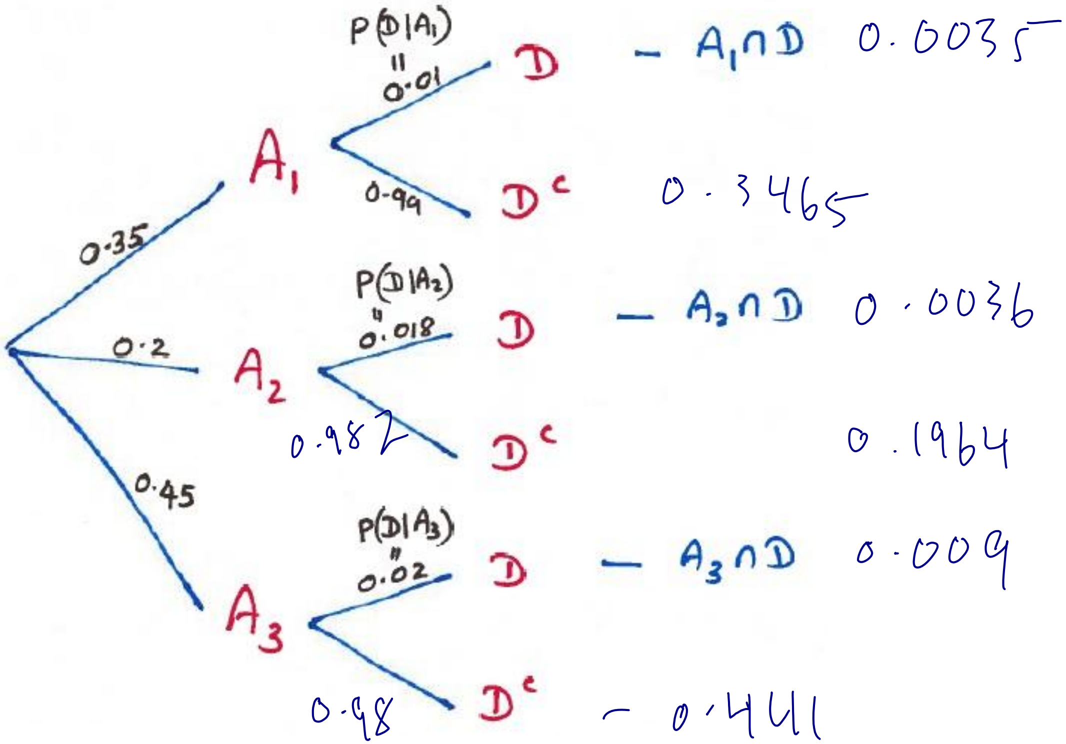
$P(1) = 0.35, \quad P(2) = 0.2 \quad P(3) = 0.45 \Rightarrow$ m.e.

$$P(2|D) = \frac{P(2) \cdot P(D|2)}{P(1) \cdot P(D|1) + P(2) \cdot P(D|2) + P(3) \cdot P(D|3}$$

$$= \frac{0.2 \times 0.018}{0.35 \times 0.01 + 0.2 \times 0.018 + 0.45 \times 0.02}$$

$$= \frac{0.0036}{0.0035 + 0.0036 + 0.009} = \frac{0.0036}{0.0161} = 0.45$$

Method 2: Using tree diagram

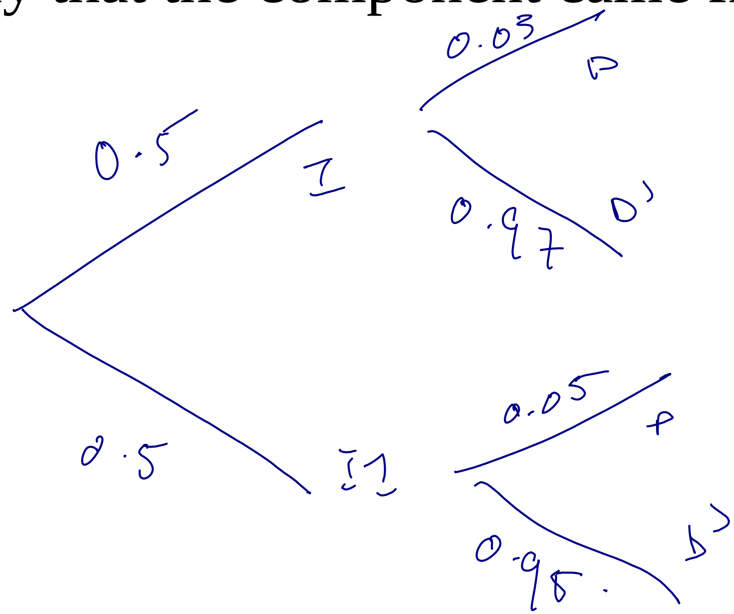


$$\frac{0.0036}{0.0036 + 0.009 + 0.0035} = \frac{0.0036}{0.0161} = 22.3\%$$

- can pick baye's thm or tree diagram.

↳ represent correctly

Two shipments of components were received by a factory and stored in two separate bins. Shipment I has 3% of its contents defective, while shipment II has 5% of its contents defective. Given that a randomly selected component is defective, what is the probability it came from shipment I? Assume that it is equally likely that the component came from shipment I as from shipment II.



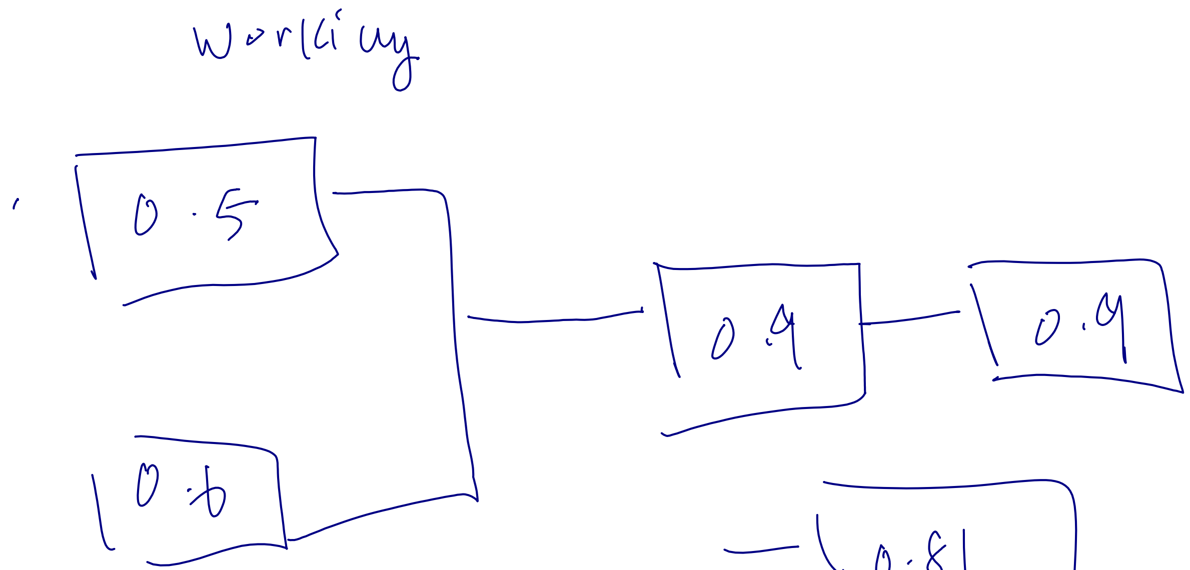
$$\frac{0.5 \times 0.03}{0.5 \times 0.03 + 0.05 \times 0.5}$$

$$\frac{0.015}{0.015 + 0.025}$$

$$\frac{0.015}{0.04} = 37.5$$

Next Class:

- Chapter 4: Radom Variables and Distributions



Both fail = 0.5×0.4

$0.2.$

$= 0.8 \times 0.81 = 0.648$