[redacted]

Assignment Assignment-03 due 10/08/2022 at 11:59pm PDT

Problem 1. **(2 points)**

A game of chance involves rolling an unevenly balanced 4-sided die. The probability that a roll comes up 1 is 0.16, the probability that a roll comes up 1 or 2 is 0.43, and the probability that a roll comes up 2 or 3 is 0.45 . If you win the amount that appears on the die, what is your expected winnings? (Note that the die has 4 sides.)

Answer = ____________ dollars.

Answer(s) submitted:

- 2.8

(correct)

Correct Answers:

- 2.8

Problem 2. **(3 points)**

The mean and standard deviation of a random variable x are -8 and 1 respectively. Find the mean and standard deviation of the given random variables:

(1) $y = x + 4$
$\mu =$ ___
$\sigma =$ ___

(2) $v = 3x$
$\mu =$ ___
$\sigma =$ ___

(3) $w = 3x + 4$
$\mu =$ ___
$\sigma =$ ___

Answer(s) submitted:

- -4
- 1
- -24
- 3
- -20
- 3

(correct)

Correct Answers:

- -4
- 1
- -24
- 3
- -20
- 3

Problem 3. **(4 points)**

A robot fires 3 shots at a moving target. For the first shot, the probability of hitting the moving target is $1/3$. For subsequent shots beyond the first shot, the probability of hitting the moving target is $1/2$ if the previous shot is a hit (for example, the probability of hitting the moving target on the 3rd shot is $1/2$ if the 2nd shot is a hit) and the probability of hitting the moving target is $1/4$ if the previous shot is a miss.
What is the mean and variance of the number of hits?

Mean (rounded to the nearest whole number): ___

Variance (correct to 2 decimals): ___

Solution:

Mean: We list out all 8 possible outcomes with their probabilities as shown below.

Sequence	No. Hits	Probability
hit/hit/hit	3	$\frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} = \frac{4}{48}$
hit/hit/miss	2	$\frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} = \frac{4}{48}$
hit/miss/hit	2	$\frac{1}{3} \times \frac{1}{2} \times \frac{1}{4} = \frac{2}{48}$
hit/miss/miss	1	$\frac{1}{3} \times \frac{1}{2} \times \frac{3}{4} = \frac{6}{48}$
miss/hit/hit	2	$\frac{2}{3} \times \frac{1}{4} \times \frac{1}{2} = \frac{4}{48}$
miss/hit/miss	1	$\frac{2}{3} \times \frac{1}{4} \times \frac{1}{2} = \frac{4}{48}$
miss/miss/hit	1	$\frac{2}{3} \times \frac{3}{4} \times \frac{1}{4} = \frac{6}{48}$
miss/miss/miss	0	$\frac{2}{3} \times \frac{3}{4} \times \frac{3}{4} = \frac{18}{48}$

We then compute the probabilities for each value of X by adding the appropriate entries in the table above.

X	0	1	2	3
Probability	18/48 = 3/8	16/48 = 1/3	10/48 = 5/24	4/48 = 1/12

Finally, we find $E(X) = \sum_{i=0}^{3} iP(X = i) = 1$.

Variance: by computing $E(X^2) = \frac{3}{8} \times 0^2 + \frac{1}{3} \times 1^2 + \frac{5}{24} \times 2^2 + \frac{1}{12} \times 3^2$ using the probabilities above, we find the variance by $Var(X) = E(X^2) - E(X)^2 = 0.92$.

Answer(s) submitted:

- `1`
- `0.92`

(correct)

Correct Answers:

- `1`
- `0.92`

Problem 4. **(2 points)**

The following density function describes a random variable X.

$$f(x) = \frac{x-1}{8} \text{ if } 1 < x < 5$$

A. Find the probability that X lies between 2 and 4.

Probability = ________

B. Find the probability that X is less than 3.

Probability = ________

Answer(s) submitted:

- `0.5`
- `0.25`

(correct)

Correct Answers:

- `0.5`
- `0.25`

Problem 5. (3 points)

The time that a butterfly lives after emerging from its chrysalis can be modelled by a random variable T, the model here taking the probability that a butterfly survives for more than t days as

$P(T > t) = \frac{36}{(6+t)^2}$, $t \geq 0$.
For these problems, please ensure your answers are accurate to within 3 decimals.

Part a)

What is the probability that a butterfly will die within 6 days of emerging?

___

Part b)

If a large number of butterflies emerge on the same day, after how many days would you expect only 6 % to be alive?

___

Part c)

Calculate the mean lifetime of a butterfly after emerging from its chrysalis.

___

Solution:

Part a)

We require

Part b)

Now we wish to find t such that $P(T > t) = 0.06$ (i.e. the lifetime exceeds t with probability 6 %). We solve

$\frac{36}{(6+t)^2} = 0.06,$

that is,

$(6+t)^2 = 600.000.$

Solving this quadratic and taking the positive root, we find $t = 18.495$.

Part c)

The c.d.f. satisfies

$F(t) = P(T \leq t) = 1 - P(T > t)$

and so

$F(t) = \begin{cases} 0 \text{ if } t < 0 \\ 1 - \frac{36}{(6+t)^2} \text{ if } t \geq 0. \end{cases}$

Hence the p.d.f. can be found by differentiation,

and so

$f(t) = \begin{cases} 0 \text{ if } t < 0 \\ \frac{72}{(6+t)^3} \text{ if } t \geq 0. \end{cases}$

Hence, the expectation is given by

$E(T) = \int_0^\infty \frac{72t}{(6+t)^2} dt.$

Putting $x = 6 + t$, the integral above becomes

$\int_6^\infty \frac{72(x-6)}{x^3} dx = \int_6^\infty (\frac{72}{x^2} - \frac{432}{x^3}) dx = 6.$

Answer(s) submitted:

- `0.75`
- `18.495`
- `6`

(correct)

Correct Answers:

- `0.75`
- `18.495`
- `6`

Problem 6. **(6 points)**

The continuous random variable X has cumulative distribution function given by

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ x - \frac{1}{4}x^2 & \text{for } 0 \leq x \leq 2 \\ 1 & \text{for } x > 2 \end{cases}$$

Part(a) Find $P(X > \frac{1}{2})$. Give your answer as a decimal, correct to 2 decimal places. ___

Part(b) Two independent observations of X are taken. Find the probability correct to 2 decimal places that one is less than $\frac{1}{2}$ and the other is greater than $\frac{1}{2}$. The order in which we take the observations matters.___

Part(c) Find the median of X, correct to 2 decimal places.___

Part(d) Find $E(\sqrt{X})$, correct to 2 decimal places.___

Part(e) Find the value of q such that $P(X < q) = \frac{1}{4}$. Give your answer as a decimal correct to 3 decimal places. ___

Part(f) Find the value of c correct to one decimal place given that $E(X+c) = 4E(X-c)$. ___

Answer(s) submitted:

- `0.56`
- `0.49`
- `0.59`
- `0.75`
- `0.268`
- `0.4`

(correct)

Correct Answers:

- `0.56`
- `0.49`
- `0.59`
- `0.75`
- `0.268`
- `0.4`