PART A: TRUE OR FALSE

Circle your answers. Each question worths 2 marks.

1. Let A, B, and C be independent events where $P(A)$, $P(B)$ and $P(C)$ are all positive. Then $P(A|B) = P(A|C)$.

   TRUE FALSE

2. You toss a fair coin and roll a fair die. The chance of tossing a head or rolling a "6" is $\frac{1}{2} + \frac{1}{6}$.

   TRUE FALSE

3. Let X and Y be independent random variables. Then $V(XY) = V(X) \times V(Y)$.

   TRUE FALSE

4. Let X_i's be independent random variables each with mean μ and variance σ^2, and define $\overline{X}_n$ to be the random variable for the sample average of n X_i's. Then $\overline{X}_{25}$ has a larger variability than $\overline{X}_{16}$.

   TRUE FALSE

5. In hypothesis testing, the p-value is the probability that the null hypothesis is true.

   TRUE FALSE

6. You carry out a hypothesis test at the 5% significance level. Suppose the data provides sufficient evidence to reject the null hypothesis. If the significance level were 10% instead of 5%, the null hypothesis will still be rejected.

   TRUE FALSE

7. A linear regression line $\hat{y} = b_0 + b_1 x$ is fitted to some data. One of the data points (x_i, y_i) has a negative residual. This implies the data point falls below the regression line.

   TRUE FALSE

PART B: LONG ANSWERS

1. Consider two independent random variables X and Y. $X \sim N(6, 10)$ and Y has the following density function:

$$f(y) = \begin{cases} \frac{1}{18}(y-6) & 6 \leq y \leq 12 \\ 0 & \text{elsewhere} \end{cases}$$

   (a) Find $E(Y)$ and $V(Y)$. [5 marks]

(b) Find $E(3X - 5Y)$ and $V(3X - 5Y)$. [4 marks]

2. At a police spot check, 10% of cars stopped have defective headlights and a faulty muffler. 15% have defective headlights and a muffler which is satisfactory. If a car which is stopped has defective headlights, what is the probability that the muffler is also faulty? [5 marks]

3. A local car tire manufacturer guarantees the tires at 48,000 km such that only 2% of its tires will be replaced due to failure before the guaranteed number of kilometers.

   (a) What is the probability that the first tire that fails before the guaranteed distance is the $50th$ tire sold? Assume that the tires are independent. [3 marks]

   (b) One hundred tires are newly produced. Assuming that the tires are independent, use an approximation method to approximate the probability that less than 2 of the new tires will fail before the guaranteed distance. [5 marks]

4. The paint used to make lines on roads must reflect enough light to be clearly visible at night. An old type of paint has a mean reflectometer reading of 20. A new type of paint will be considered if its true mean reflectometer reading μ exceeds 20. A hypothesis test at the 10% significance level will be based on a random sample of size 25, which gives a sample mean reading of 22 and a sample standard deviation of 4. Should the new type of paint be considered?

   (a) Which of the following sets of hypotheses should be tested? Check your answer. [2 marks]

   ___ $H_0 : \mu = 20$ vs. $H_1 : \mu < 20$
   ___ $H_0 : \mu = 22$ vs. $H_1 : \mu < 22$
   ___ $H_0 : \mu = 20$ vs. $H_1 : \mu > 20$
   ___ $H_0 : \mu = 22$ vs. $H_1 : \mu > 22$

   (b) The test-statistic is (check your answer) [2 marks]

   ___ 2.5
   ___ 0.5
   ___ -2.5
   ___ -0.5

   (c) Do you reject the null hypothesis? Check your answer. [1 mark]
   ___ Yes ___ No

   Explain:

   (d) Draw conclusion in the context of the question. [1 mark]

5. An experiment to determine the effect of several methods of preparation for use in commercial yogurt was conducted by a food science research group. Four batches of yogurt were prepared using each of the three methods: traditional, ultra filtration and reverse osmosis. A trained expert then tasted each of the 12 batches, and judged them on a scale from 1 to 10 (1 indicates the best quality, 10 the worst). Assume constant variance and normality of the quality scores for the different preparation methods.

   (a) Complete the ANOVA table below. [6 marks]

Source of Variation	df	Sum of Squares	Mean Squares	F-ratio
Treatment		17.30		
Error				
Total		24.05		

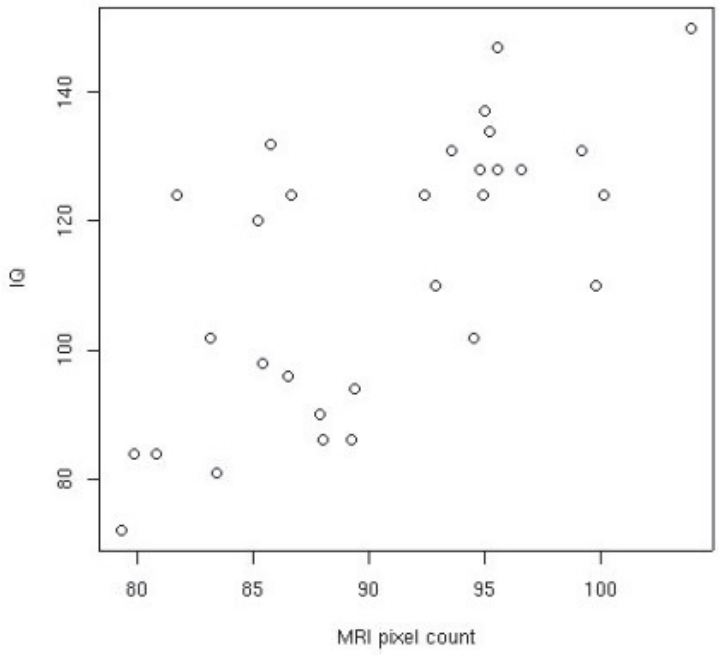
   (b) Do the data suggest that the mean quality scores are different among the three methods of preparation? Test the null hypothesis that the population mean scores are all equal at the 5% significance level. You need <u>not</u> perform multiple comparisons regardless of the results you obtain. [4 marks]

(c) Give an estimate for the common variance of the quality scores for the different preparation methods. [2 marks]

6. The monthly rents for studio apartments in downtown Vancouver have a mean of $750 and a standard deviation of $56. Find the probability that the sample average monthly rent of 64 randomly chosen apartments will be between $729 and $764. [6 marks]

7. A study was conducted to investigate whether the brain size is an indicator of mental capacity. The study researcher drew a random sample of 30 students, and used magnetic resonance imaging (MRI) to determine the brain size of the students. The total pixel count ($\times 10^4$) from a fixed number of MRI scans served as an index for brain size (a larger count indicates a larger brain size). The students' IQ scores and gender information were also collected.

   The following shows the scatterplot of IQ (Y) versus MRI pixel count (X) for the 30 students.



   (a) Check the most appropriate answer for the value of the correlation coefficient r. [2 marks]

   ___ -0.48 ___ -0.03 ___ 0.68 ___ 0.89

(b) The least squares regression line that predicts IQ from MRI pixel count was found to be

$$\hat{y} = -88.8 + 2.2x$$

It is also given that: $\sum x = 2716, \sum(x^2) = 247183,$
Residual sum of squares (RSS)= $\sum(y - \hat{y})^2 = 7107$

i. Construct a 90% confidence interval for the population slope β_1. [5 marks]

ii. Is brain size (measured by MRI pixel count) useful for predicting IQ? Test the appropriate hypotheses on the population slope β_1 at $\alpha = 0.01$. [6 marks]

(c) The researcher also wanted to compare the brain size between the two genders. The MRI pixel count data are summarized as follows:

gender	number of students	sample mean	sample SD
male	13	92.2	7.4
female	17	89.3	5.8

Is there any evidence that the mean brain size is different between male and female students? Test the appropriate hypotheses at $\alpha = 0.05$. State any assumption(s) that you make to validate the test. [10 marks]