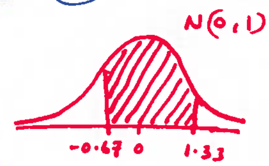
Version A

[1] (9 marks) The breakdown voltage of a randomly chosen diode of a certain type is known to be normally distributed with mean value 40 V and standard deviation 1.5 V.

(a) What is the probability that the voltage of a single diode is between 39 and 42?

$X \equiv$ breakdown voltage

$\mu = 40, \ \sigma = 1.5$



$$P(39 < X < 42) = P\left(\frac{39-40}{1.5} < Z < \frac{42-40}{1.5}\right) \leftarrow (1)$$

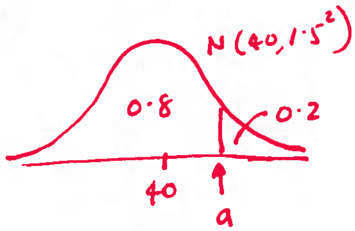
$$= P(-0.67 < Z < 1.33)$$

$$= P(Z < 1.33) - P(Z < -0.67) \leftarrow (1)$$

$$= 0.9082 - (1 - 0.7486)$$

$$= \underline{\underline{0.6568}} \leftarrow (1)$$

(b) What value is such that only 20% of all diodes have voltage exceeding that value?



we desire the 80^{th} percentile.

$$P\left(Z \le \frac{a-40}{1.5}\right) = 0.8 \leftarrow (1)$$

$$\Rightarrow \frac{a-40}{1.5} = 0.84 \leftarrow (1) \text{ table value}$$

$$\Rightarrow \underline{\underline{a = 41.26}} \leftarrow (1)$$

(c) If four diodes are independently selected, what is the probability that at least one has a voltage between 39 and 42?

Let $Y \equiv$ number of diodes with voltage between 39 and 42.
out of 4 diodes

(0.5)

$$\boxed{\text{Part(a)} \Rightarrow P(39 < X < 42) = 0.6568 = p}$$

$$Y \sim Bin(4, p) \leftarrow (0.5)$$

$$P(Y \ge 1) = 1 - P(Y=0)$$
$$= 1 - \binom{4}{0}(0.6568)^0(1-0.6568)^4 \leftarrow (1) \text{ for procedure.}$$

$$= \underline{\underline{0.9861}} \leftarrow (1)$$

Version A

[2] (6 marks)

Using a long rod that has length μ, you are going to lay out a square plot in which the length of each side is μ. Thus the area of the plot will be μ^2. However you do not know the value of μ, so you decide to make n independent measurements $X_1, X_2, \ldots, X_n$ of the length. Assume that each X_i has mean μ (unbiased measurements) and variance σ^2.

(a) Show that $\bar{X}^2$ is not an unbiased estimator for μ^2.

If $\bar{x}^2$ is an unbiases estimator for μ^2.

then $E[\bar{x}^2] = \mu^2$.

We know that

$$\text{Var}(\bar{x}) = E[\bar{x}^2] - (E(\bar{x}))^2. \quad — (1)$$

X_i's iid with mean μ & variance σ^2

Therefor.

$E[\bar{x}] = \mu$

$\text{Var}(\bar{x}) = \frac{\sigma^2}{n}$

$$\frac{\sigma^2}{n} = E[\bar{x}^2] - \mu^2$$

$$\Rightarrow E[\bar{x}^2] = \frac{\sigma^2}{n} + \mu^2. \quad — (1)$$

Therefore $E[\bar{x}^2] \neq \mu^2$. — (1)

$\Rightarrow \bar{x}^2$ is not an unbiased estimator for μ^2.

(b) For what value of k is the estimator $\bar{X}^2 - kS^2$ unbiased for μ^2.
(hint: S^2 is an unbiased estimator of σ^2)

$$E[\bar{x}^2 - ks^2] = E[\bar{x}^2] - k\,E[s^2] \quad — (1)$$

$$= \frac{\sigma^2}{n} + \mu^2 - k\sigma^2$$

; since s^2 is an unbiased estimator for σ^2

$E[s^2] = \sigma^2$

$$E[\bar{x}^2 - ks^2] = \mu^2 + \sigma^2\left(\frac{1}{n} - k\right). \quad (1)$$

If $k = \frac{1}{n}$, $E[\bar{x}^2 - ks^2] = \mu^2$ — (1)

Therefore when $k = \frac{1}{n}$, $\bar{x}^2 - ks^2$ is unbiased for μ^2

Version A

[3] (8 marks) **Circle the best answer.**

(i) Suppose a 95% confidence interval for μ turns out to be (1000, 2100). To make more useful inferences from the data, it is desired to reduce the width of the confidence interval. Which of the following will result in a reduced interval width?

A) Increase the sample mean.
B) Increase the confidence level.
C) Increase the population mean.
D) Increase the sample size.
E) Decrease the sample size.

(ii) A researcher wishes to estimate the mean resting heart rate for long-distance runners. A random sample of 10 long distance runners yields the following heart rates, in beats per minute.

71	62	65	60	69
78	79	73	65	60

Use the data to obtain a point estimate of the mean resting heart rate for all long distance runners.

A) 66.4 beats per minute
B) 69.2 beats per minute
C) 62.8 beats per minute
D) 75.6 beats per minute
E) 68.2 beats per minute

$\bar{x} = 68.2$

$\bar{x}$ is a point estimate for μ

(iii) On the average, 1.8 customers per minute arrive at any one of the checkout counters of a grocery store. What type of probability distribution can be used to find out the probability that there will be no customer arriving at a checkout counter?

A) Poisson distribution.
B) Binomial distribution.
C) Normal distribution.
D) Geometric distribution
E) none of the above.

(iv) Suppose we wish to test $H_0: \mu \le 57$ versus $H_1: \mu > 57$. What will result if we conclude that the mean is greater than 57 when its true value is really 55?

A) We have made a correct decision
B) We have made a Type I error.
C) We have made a Type II error.
D) None of the above are correct.

Reject H_0 when H_0 true

Version A

[4] (9 marks) 70% of the employees in a specialized department of a large software firm are computer science graduates.

(a) I meet 25 employees at random. What is the probability that the 4th employee I meet is the first one who is a computer science graduate? Give the exact distribution related to this problem and then calculate the required probability.

1 { X ≡ number of people I meet until I meet the first Computer science graduate.

$X \sim \text{Geometric}(0.7)$ }

$P(X=4) = (0.7)(1-0.7)^3$ ← 1

$= \underline{\underline{0.0189}}$ ← 1

(b) I meet 60 employees at random on a daily basis.

i. What is the mean number of computer science graduates among the 60 that I meet?

1 { Let $Y \equiv$ number of computer science graduate I meet daily.

$Y \sim \text{Bin}(60, 0.7)$, $n=60$, $p=0.7$ }

$E[Y] = np$ ← 1

$= 60 \times 0.7$

$= \underline{\underline{42}}$ ← 1

ii. Find the probability that 32 or more of the 60 employees I meet in a day are computer science graduates.

$Y \sim \text{Bin}(60, 0.7)$

$E[Y] = 60 \times 0.7 = 42$

$\text{Var}(Y) = 60 \times 0.7 \times 0.3 = 12.6$

{ $np = 60 \times 0.7 = 42 \geq 5$ ✓

$n(1-p) = 60 \times 0.3 = 18 \geq 5$ ✓

Therefore Binomial distribution can be approximated by Normal distribution.

$Y \overset{\text{approx}}{\sim} N(42, 12.6)$ } 1

$P(Y \geq 32) = P(Y \geq 31.5)$; use continuity correction. ← 0.5

$Y \sim \text{Bin}(60, 0.7)$

$= P\left(Z \geq \frac{31.5 - 42}{\sqrt{12.6}}\right)$ — 0.5

$= P(Z \geq -2.96) = P(Z \leq 2.96)$ 0.5

$= \underline{\underline{0.9985}}$ ← 0.5

Version A

[5] (8 marks) A sample of 50 radon detectors of a certain type was selected, and each was exposed to 100 pCi/L of radon. Sample mean and sample standard deviation are 98.3 and 4.1, respectively. Does this data suggest that the population mean reading under these conditions differs from 100?

a) State appropriate hypotheses

Let $\mu \equiv$ population mean reading.

$H_0: \mu = 100 = \mu_0$

$H_a: \mu \neq 100$

2

b) Calculate the test statistic and test the hypotheses using critical value approach at the significance level $\alpha = 0.05$

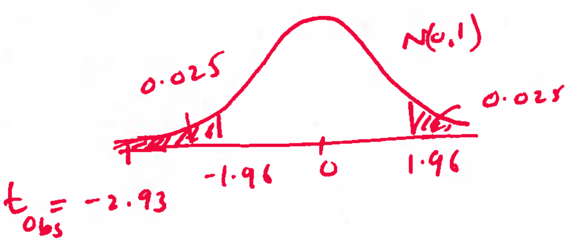
$\bar{x} = 98.3$, $s = 4.1$ $n = 50$, $\alpha = 0.05$ $df = n-1 = 49$.

test statistic

$$t_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{98.3 - 100}{4.1/\sqrt{50}}$$ 1

$t_{obs} = -2.93 \sim t_{49}$ 1

Since $n-1$ is large t_{49} distribution can be approached by standard normal distribution. 0.5



0.5

$t_{obs} = -2.93 < -1.96 = -z_{0.025}$

0.5 t_{obs} value is in the rejection region.

0.5 Therefore we reject H_0.

c) State the conclusions in a proper English sentence.

We conclude that the population mean reading under these conditions differs from 100 at the significance level $\alpha = 0.05$

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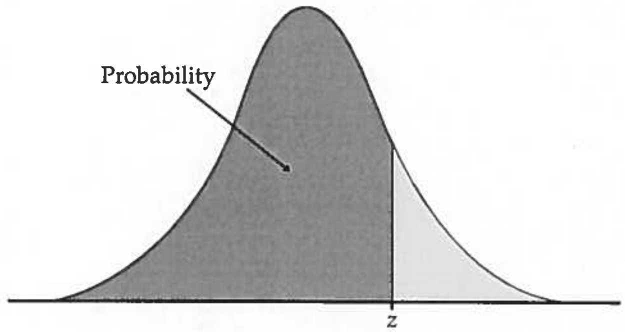

Table entry for z is the area under the standard normal curve to the left of z.

TABLE A

Standard normal probabilities (continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998