1. Which of the following are true
   (a) {a,b} $\subset$ {b,a}; False. The two sets are equal, so one is not a proper subset of the other
   (b) {a,b} $\subseteq$ {b,a}; True. The two sets are equal, so they are subsets of each other
   (c) {c} $\in$ {a,b,c}; False. While c is an element of {a,b,c}, the set containing c is not.
   (d) {a,b} $\in$ $\wp${a,b,c,d}; True. $\wp x$ produces a set which contains all possible subsets of x. {a,b} $\subseteq$ {a,b,c,d}, therefore {a,b} $\in$ $\wp${a,b,c,d}
   (e) {{a}} $\in$ $\wp${a,b,c,d}; False. While {{a}} is a subset of the power set, it is not itself an element in the power set because it is not a subset of the original set {a,b,c,d}.
2. Calculate the following
   (a) $\wp\{1,2,3,4\}$
   {}, {1}, {2}, {1,2}, {3}, {1,3}, {2,3}, {1,2,3}, {4}, {1,4}, {2,4}, {1,2,4}, {3,4}, {1,3,4}, {2,3,4}, {1,2,3,4}
   (b) $\{a,b,c,d,e\} \cup \{d,e,f,g\}$
   $\{a,b,c,d,e,f,g\}$
   (c) {a,b,c,d,e} $\cap$ {d,e,f,g}
   {d,e}
   (d) $\{1,3,7\} \cap \{5,4,8\}$
   {}
   (e) $\cup\{\{1,2,3,4\},\{2,3,4,5\},\{3,4,5,6\}\}$
   {1,2,3,4,5,6}
   (f) $\cap\{\{1,2,3\},\{4,5\},\{6,7,8\}\}$
   {}
3. Which of the following is true?
   (a) $A \cap B \subset B$; False. A and B could both be the same set, in which case $A \cap B = B$ which would mean $A \cap B \not\subset B$
   (b) $A - B = B - A$; False. Consider A = {a} and B = {}. A - B = {a} and B - A = {}. {a} $\neq$ {}, therefore the above claim can be false.
4. Write 3 in set-theory notation
   $0 = \emptyset$, $1 = \{\emptyset\}$, $2 = \{\emptyset, \{\emptyset\}\}$, $3 = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

5. Short answer

   (a) Russel's paradox is a proof against naieve/unrestricted set comprehensions. This type of comprehension is of the form
   {x : x has property P }
   This type of comprehension leads to a paradox when P indicates sets which do not contain themselves. If we let C be the set {x : x does not contain itself }, a contradiction arrises. If C contains itself, then it does not meet the definition and cannot be a member of itself. It C does not contain itself, then it does meet the definition and must contain itself. Whether C contains itself or not, a contradiction is formed. This is the idea behind Russel's paradox.

   (b) Cantor's diagonalization proof serves to show there is no one-to-one mapping from the natural numbers to the real numbers. By showing this relationship does not exist, we can reason about the different types of infinity, namely *countable* and *uncountable*. Countable infinities are infinitely large sets which can be successfully enumerated in theory. Uncountable infinities are infinitely large sets which can never be fully enumerated. The proof goes something like this.

   Imagine you have written down an infinite number of binary strings in a list L. The list has indices from 0 to infinity. No matter how many strings you have written down, we can always find one that you have missed. To accomplish this, create a new string whose elements are [!L[i][i] for i in range(len(L))], or in other words, the ith element of the new string is the negation of the ith element of the ith element of the list. Basically, you create a new string which is different from every other string in at least one location. This proves that you cannot enumerate (count) some infinities, while we know you can count others. This proves that some infinities are larger than others.

   (c) The continuum hypothesis is the idea that there is no set whose size is between that of the natural numbers and the real numbers. Formally, this hypothesis may be stated as $\neg\exists y(|\mathbb{N}| < |y| < |\mathbb{R}|)$, meaning that there is no set y whose cardinality is greater than that of the natural numbers, but less than that of the real numbers. This idea essentially states that infinity sizes are non continuous, and that there are discrete steps between one size of infinity and the next. According to what I remember John saying in class, this hypothesis can be assumed as an axiom, or denied as an axiom, but I don't believe there is a way to prove either way.