

1 Symbolize

Definitions:

$C(x)$ x is a cat

$D(x)$ x is a dog

$S(x, y)$ x has been seen by y

$B(x, y)$ x has bitten y

$T(x, y)$ x is taller than y

a)

No dog bites any cat that sees it

There does not exist a dog which bites a cat who sees the dog

There does not exist a dog who has both been seen by and bitten a given cat

There does not exist a dog such that there exists a cat where the following hold:

1) the cat has seen the dog

2) the cat has been bitten by the dog

$$\neg \exists x [D(x) \wedge \exists y (C(y) \wedge S(x, y) \wedge B(x, y))]$$

Another possible interpretation is:

If x is a Dog and y is a Cat and dog has been seen by cat, then dog has not bitten cat

$$\forall x [D(x) \rightarrow \forall y (C(y) \wedge S(x, y) \rightarrow \neg B(x, y))]$$

b)

The tallest dog bites at least one cat

There exists a dog which no dog is taller than and there exists a cat that the tallest dog has bitten

$$\exists x [D(x) \wedge \neg \exists y (D(y) \wedge T(y, x)) \wedge \exists z (C(z) \wedge B(x, z))]$$

c)

There are at least two cats that bit each other

There exists a cat x and a cat y such that x is not y, x has bitten y, and y has bitten x

$$\exists x [C(x) \wedge \exists y (C(y) \wedge x \neq y \wedge B(x, y) \wedge B(y, x))]$$

2 Prove

1	$\neg \exists y(\exists x F(x) \rightarrow F(y))$	ASS for IP
2	$\forall y \neg(\exists x F(x) \rightarrow F(y))$	QN 1
3	$\neg(\exists x F(x) \rightarrow F(c))$	UI 2
4	$\exists x F(x) \wedge \neg F(c)$	CN 3
5	$\exists x F(x)$	Simp 4
6	$\neg F(c)$	Simp 4
7	$F(c)$	ASS for EI
8	$\neg F(c)$	Rep 6
9	$F(c) \wedge \neg F(c)$	Conj 7,8
10	\perp	Contradiction 9
11	\perp	UI 5,7-10
12	$\exists y(\exists x F(x) \rightarrow F(y))$	IP 1-11

3 Symbolize and Prove

a)

Identity is transitive

$\forall xyz[(x = y \wedge y = z) \rightarrow x = z]$

1	$\neg \forall x \forall y \forall z [(x = y \wedge y = z) \rightarrow x = z]$	ASS for IP
2	$\exists x \neg \forall y \forall z [(x = y \wedge y = z) \rightarrow x = z]$	QN 1
3	$\neg \forall y \forall z [(c = y \wedge y = z) \rightarrow c = z]$	ASS for EI
4	$\exists y \neg \forall z [(c = y \wedge y = z) \rightarrow c = z]$	QN 3
5	$\neg \forall z [(c = b \wedge b = z) \rightarrow c = z]$	ASS for EI
6	$\exists z \neg [(c = b \wedge b = z) \rightarrow c = z]$	QN 5
7	$\neg [(c = b \wedge b = f) \rightarrow c = f]$	ASS for EI
8	$(c = b \wedge b = f) \wedge c \neq f$	CN 7
9	$c \neq f$	Simp 8
10	$(c = b \wedge b = f)$	Simp 8
11	$c = b$	Simp 10
12	$b = f$	Simp 10
13	$c = f$	LL 11,12
14	$c = f \wedge c \neq f$	Conj 12,9
15	\perp	Contradiction 14
16	\perp	EI 7-15
17	\perp	EI 5-16
18	\perp	EI 3-17
19	$\forall x \forall y \forall z [(x = y \wedge y = z) \rightarrow x = z]$	IP 1-18

b)

If $1 + 1 = 2$ and $3 - 1 = 2$ then $1 + 1 = 3 - 1$

let x represent $1+1$

let y represent 2

let z represent $3 - 1$

Show $(x = y \wedge y = z) \rightarrow x = z$

1	$x = y \wedge y = z$	ASS for CP
2	$x = y$	SIMP 1
3	$y = z$	SIMP 1
4	$x = z$	LL 2,3
5	$(x = y \wedge y = z) \rightarrow x = z$	CP 1-4

If we had access to functions like $\text{Plus}(x,y)$ which return $x + y$, we could prove this more clearly. We only have access to T/F predicates, so I think this proof is the best I can do