1. Short answer

   (a) The syntax of a logic is the structure of the symbols that make up the logic. It only defines the way the characters come together to form parts of a larger whole. For example, the syntax of the $\exists$ symbol is to follow it with a variable, then have a sentence which includes uses of that variable, such as $\exists xFx$. The semantics of a logic is the meaning behind a given sentence. The semantics of the sentence $\exists xFx$ is the meaning of the sentence, 'Something exists which is an F'. Both syntax and semantics play crucial roles in the expressiveness of a logic, and you cannot have one without the other

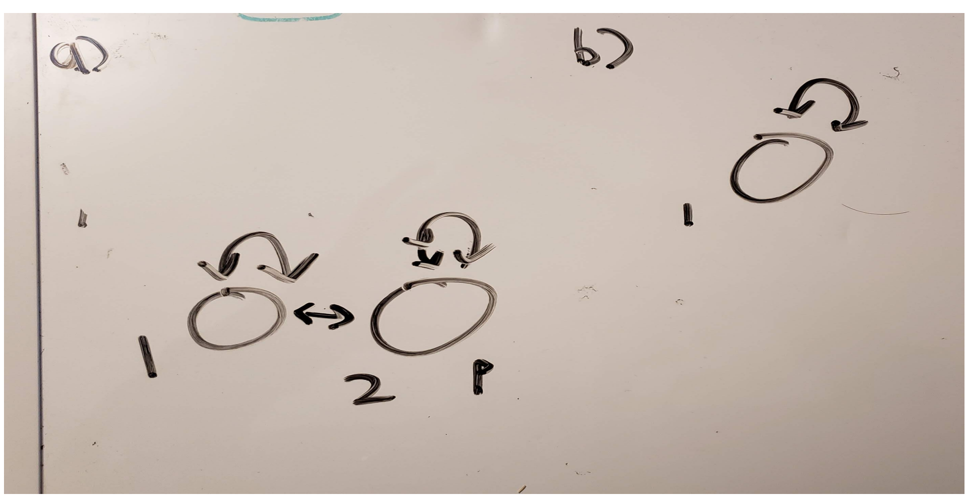
   (b) The difference between $\Diamond\exists xPx$ and $\exists x\Diamond Px$ is where the P might be happening, and in which domain the P may fall. In the sentence $\Diamond\exists xPx$, we are claiming that something in some world is a P. The P thing does not necessarily exist in our world, but it does exist in some world. To contrast this, the sentence $\exists x\Diamond Px$ says there is something in this world which could be a P. If we let P indicate somebody being a magic user, the second sentence ($\exists x\Diamond Px$) says that somebody in our world is possibly magic. The first sentence ($\Diamond\exists xPx$) says that it is possible that some person in some world (which may not be our own) is magic.

2. Symbolization

   (a) $\Box(M \rightarrow J)$ with M = mail has been checked and J = James is home

   (b) $\Diamond(V \wedge I)$ with V = Vase produced before 1000 B.C. and I = Hittites had iron weapons. The formula can be understood as *'It is possible that the vase was produced before 1000 B.C. and that Hittites had iron weapons'*

3. Countermodels



(a) To understand part a, we want to get $\Diamond\Diamond\Diamond P \wedge \neg P$ to form a counter example of $\neg\Diamond\Diamond\Diamond P \vee P$ World 1 does not have P, so we get $\neg P$. World 2 is accessible from world 1 and world 1 is accessible from world 1, which leads us to $\Diamond\Diamond\Diamond P$. We have shown $\Diamond\Diamond\Diamond P \wedge \neg P$, which forms a counterexample to $\neg\Diamond\Diamond\Diamond P \vee P$.

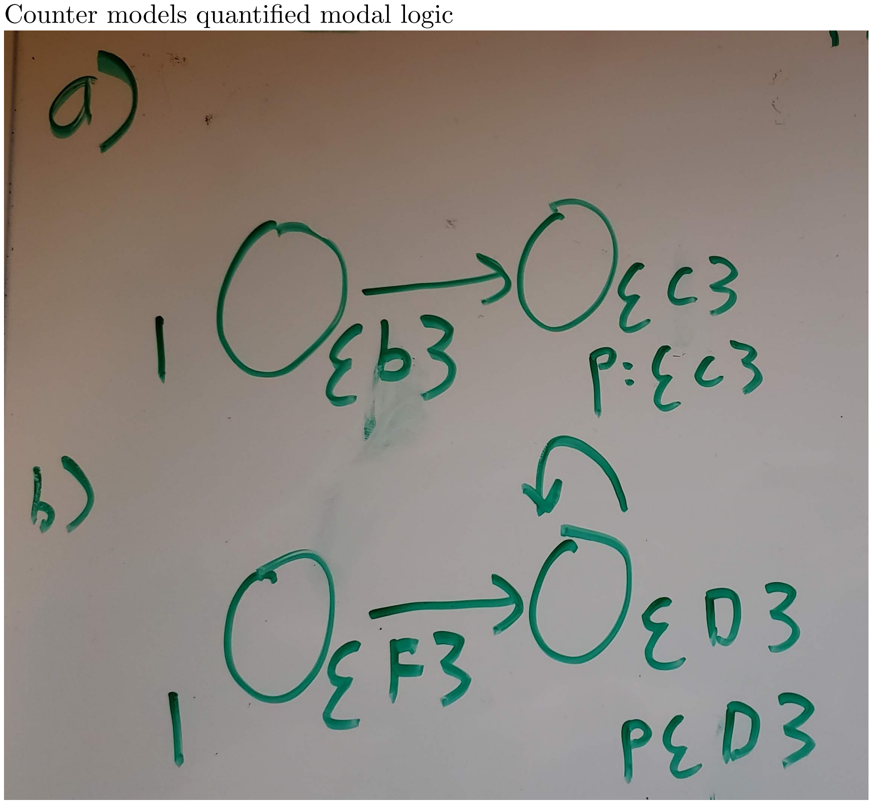
(b) To give a counter example for b, we want to show $P \rightarrow \Box Q$ and $\neg\Box Q$. World 1 is not a P world, so we can conclude $P \rightarrow \Box Q$ through explosion. World 1 is accessible from world 1, and it is not a Q world, so we can conclude $\neg\Box Q$. By showing $P \rightarrow \Box Q \wedge \neg\Box Q$, we have given a counterexample for $\neg(P \rightarrow \Box Q) \vee \Box Q$

4. Symbolizing quantified modal logic

   (a) $(\forall x \Diamond F) \rightarrow \Diamond\forall x Fx$

   (b) $[\Box(\exists x Fx)] \rightarrow \exists x \Box Fx$

5. Counter models quantified modal logic



Let '{a,b,c}' indicate a,b,c are in the domain of the adjacent world.
let 'P {a}' means 'a' is a P in the adjacent world