1 Short Answer

(a) An axiomatic system is complete if and only if for all sentences S in the languages, there is a valid proof for either S or its negation (¬S). In other words, there is no sentence which can neither be proven nor disproven.

(b) Hilbert set out to create an axiom system capable of representing mathematics which is both complete and consistent. Godel proved that this axiom system was impossible to create with his two incompleteness theorems, especially the first.

(c) A decidable property is a property which can be determined through application of a (finite runtime) algorithm. The algorithm can spit out 'yes' not 'no', but it may not run for infinite time. A complete axiomatic system is a system which is able to prove all sentences as either true or false.

(d) Godel's second incompleteness theorem states that no consistent axiom system capable of representing arithmetic can prove itself to be consistent. Prior to this theorem, many researchers (namely Hilbert) tried to prove an axiom system which was complete and sound. Godel's first theorem proved that you cannot have both completeness and soundness. Godel's second theorem told them it was impossible to even prove consistency (within the system). This revelation certainly dashed the hopes of Hilbert and those working with him, and showed people the limits of our formal systems.

2 Robinson

1. $x = y \to x' = y'$

1	x = y	ASS FOR CP
2	x' = x'	BUTT
3	x' = y'	LL 1,2
4	$x = y \to x' = y'$	CP1-3

2. $1 + 0 = 0 + 1$

1	$1 + 0 = 1$	Q4
2	$0 + 0' = (0 + 0)'$	Q5
3	$0 + 0 = 0$	Q4
4	$0 + 0' = 0'$	LL 2,3
5	$0 + 1 = 1$	DEF 4
6	$1 + 0 = 0 + 1$	LL 1,5

3. $3 + 1 = 2 \cdot 2$

1	$2 \cdot 2 = 4$	T22
2	$1 + 2' = (1 + 2)'$	Q5
3	$1 + 2 = 3$	T12
4	$1 + 2' = 3'$	LL 2,3
5	$1 + 3 = 4$	DEF 4
6	$1 + 3 = 3 + 1$	T16
7	$3 + 1 = 4$	LL 5,6
8	$3 + 1 = 2 \cdot 2$	LL 1,7

3 Peano

1. x' + y = (x + y)'
   Base Case, y=0:

1	$x' + 0 = x'$	P4
2	$x + 0 = x$	P4
3	$x' + 0 = (x + 0)'$	LL 1,2

Inductive Step $\Phi y \rightarrow \Phi y'$:

1	$\quad x' + y = (x + y)'$	ASS FOR CP
2	$\quad x' + y' = (x' + y)'$	P5
3	$\quad x' + y' = (x + y)''$	LL 1,2
4	$\quad x + y' = (x + y)'$	P5
5	$\quad x' + y' = (x + y')'$	LL 3,4
6	$x' + y = (x + y)' \rightarrow x' + y' = (x + y')'$	CP 1-5

The relationship holds for y = 0, and if it holds for y, then it holds for y'. Therefore, this relationship holds for all y by P3. Dr. Haderlie said we didn't need to explicitly show the P3 steps because they are always the same, so I am leaving this here.

2. $0 \cdot x = 0$

   Base Case x=0:

1	$0 \cdot 0 = 0$	P6

   Inductive Step $\Phi x \to \Phi x'$:

1	$0 \cdot x = 0$	ASS FOR CP
2	$0 \cdot x' = (0 \cdot x) + 0$	P7
3	$0 \cdot x' = 0 + 0$	LL 1,2
4	$0 + 0 = 0$	P4
5	$0 \cdot x' = 0$	LL 3,4
6	$0 \cdot x = 0 \to 0 \cdot x' = 0$	CP 1-5

   For x = 0, $0 \cdot x = 0$. If we assume $0 \cdot x = 0$, then we can derive $0 \cdot x' = 0$. Therefore, for all x, $0 \cdot x = 0$

3. 0 is even ($Ex :: \exists y(y \cdot 2 = x)$)

1	$0 \cdot 2 = 0$	T33
2	$\exists y(y \cdot 2 = 0)$	EG 1