1 True and False

(a) False

(b) True

(c) False

(d) True

(e) False

2 Symbolization

(a) $M(a) \wedge \forall y([M(y) \wedge y \neq a] \rightarrow H(a,y))$

(b) $D(a) \wedge H(b,a) \wedge \forall x[(H(b,x) \wedge x \neq a) \rightarrow S(a,x)]$

(c) For all persons x which I've met who knew they were wrong, for every person y which I've met who didn't know they were wrong, x had a worse temper than y
In other words, Everyone who knows they are wrong has a worse temper than everyone who doesn't
$\forall x[(M(i,x) \wedge W(x)) \rightarrow \forall y([M(i,y) \wedge \neg W(y)] \rightarrow T(x,y))]$

(d) $\forall x(\neg L(x,x))$ or, alternatively $\neg \exists x L(x,x)$

(e) There exists a time such that there was no place that had somebody living in it
$\exists x[T(x) \wedge \neg \exists y(P(y) \wedge \exists z(R(z) \wedge L(z,y,z)))]$

(f) $\forall x \forall y \forall z[(F(x,y) \wedge F(y,z)) \rightarrow \neg F(x,z)]$

(g) $\neg \exists x[O(x) \wedge P(x) \wedge \exists y(O(y) \wedge W(y) \wedge R(x,y))]$

3 Proofs

(a) $Na \wedge \neg Nb \vdash a \neq b$

1	$Na \wedge \neg Nb$	PR
2	Na	Simp 1
3	$\neg Nb$	Simp 1
4	$\quad a = b$	ASS for IP
5	$\quad Nb$	LL 2,4
6	$\quad Nb \wedge \neg Nb$	Conj 3,5
7	$\quad \perp$	Contradiction 6
8	$a \neq b$	IP 2-7

(b) $\neg\exists x[Px \wedge \exists y(Py \wedge Lxy)] \vdash \forall x[Px \to \forall y(Py \to \neg Lxy)]$

1	$\neg\exists x[Px \wedge \exists y(Py \wedge Lxy)]$	PR
2	$\forall x\neg[Px \wedge \exists y(Py \wedge Lxy)]$	QN 1
3	$\neg[Pc \wedge \exists y(Py \wedge Lcy)]$	UI 2
4	$\neg Pc \vee \neg\exists y(Py \wedge Lcy)$	DeM 3
5	Pc	ASS for CP
6	$\neg\exists y(Py \wedge Lcy)$	DS 4,5
7	$\forall y\neg(Py \wedge Lcy)$	QN 6
8	$\neg(Pb \wedge Lcb)$	UI7
9	$\neg Pb \vee \neg Lcb$	DeM 8
10	Pb	ASS for CP
11	$\neg Lcb$	DS 9,10
12	$Pb \to \neg Lcb$	CP10-11
13	$\forall y(Py \to \neg Lcy)$	UG 12
14	$Pc \to \forall y(Py \to \neg Lcy)$	CP 5-13
15	$\forall x[Px \to \forall y(Py \to \neg Lxy)]$	UG 14

(c) $\exists x Ax \vee \exists x Bx \vdash \exists x(Ax \vee Bx)$

1	$\exists x Ax \vee \exists x Bx$	PR
2	$\exists x Ax$	ASS for CP
3	Ac	ASS for EI
4	$Ac \vee Bc$	Add 3
5	$\exists x(Ax \vee Bx)$	EG 4
6	$\exists x(Ax \vee Bx)$	EI3-5
7	$\exists x Ax \rightarrow \exists x(Ax \vee Bx)$	CP 2-6
8	$\exists x Bx$	ASS for CP
9	Bd	ASS for EI
10	$Bd \vee Ad$	Add 9
11	$Ad \vee Bd$	Commutation 10
12	$\exists x(Ax \vee Bx)$	EG 11
13	$\exists x(Ax \vee Bx)$	EI9-12
14	$\exists x Bx \rightarrow \exists x(Ax \vee Bx)$	CP8-13
15	$\exists x(Ax \vee Bx) \vee \exists x(Ax \vee Bx)$	CD1,7,14
16	$\exists x(Ax \vee Bx)$	Taut 15

(d) $\exists x Hx; \forall x \forall y[(Hx \wedge Hy) \rightarrow x = y] \vdash \exists x[Hx \wedge \forall y(Hy \rightarrow x = y)]$

1	$\exists x Hx$	PR
2	$\forall x \forall y[(Hx \wedge Hy) \rightarrow x = y]$	PR
3	Hc	ASS for UI
4	$\forall y[(Hc \wedge Hy) \rightarrow c = y]$	UI2
5	$(Hc \wedge Hg) \rightarrow c = g$	UI4
6	Hg	ASS for CP
7	$Hc \wedge Hg$	Conj 3,6
8	$c = g$	MP7,5
9	$Hg \rightarrow c = g$	CP 6-8
10	$\forall y(Hy \rightarrow c = y)$	UG 9
11	$Hc \wedge \forall y(Hy \rightarrow c = y)$	Conj 3,10
12	$\exists x[Hx \wedge \forall y(Hy \rightarrow x = y)]$	EG 11
13	$\exists x[Hx \wedge \forall y(Hy \rightarrow x = y)]$	EI3-12

(e) $\forall x(Ax \wedge Bx) \vdash \forall x Ax \wedge \forall x Bx$

1	$\forall x(Ax \wedge Bx)$	PR
2	$Ac \wedge Bc$	UI 1
3	Ac	Simp 2
4	$\forall x Ax$	UG 3
5	Bc	Simp 2
6	$\forall x Bx$	UG 5
7	$\forall x Ax \wedge \forall x Bx$	Conj 4,6

(f) $\forall x(Lx \iff \forall y My) \vdash \forall x Lx \vee \forall x \neg Lx$

1	$\forall x(Lx \iff \forall y My)$	PR
2	$p = p$	Butt
3	$Lc \iff \forall y My$	UI 1
4	$\forall y My \rightarrow Lc$	BE 3
5	$Lc \rightarrow \forall y My$	BE 3
6	$\neg Lc \vee \forall y My$	CE 5
7	$\quad \forall y My$	ASS for CP
8	$\quad Lc$	MP4,7
9	$\quad \forall x Lx$	UG 8
10	$\forall y My \rightarrow \forall x Lx$	CP7-9
11	$\quad \neg \forall y My$	ASS for CP
12	$\quad \neg Lc$	DS 6,11
13	$\quad \forall x \neg Lx$	UG 12
14	$\neg \forall y My \rightarrow \forall x \neg Lx$	CP 11-13
15	$\quad \forall y My$	ASS for CP
16	$\quad \forall y My$	REP 15
17	$\forall y My \rightarrow \forall y My$	CP
18	$\neg \forall y My \vee \forall y My$	CE 17
19	$\forall y My \vee \neg \forall y My$	Comm 18
20	$\forall x Lx \vee \forall x \neg Lx$	CD 18, 10,17

(g) $\exists x([(Ax \wedge Bxa) \wedge \forall y((Ay \wedge Bya) \to y = x)] \wedge Dxb); Ac \wedge Bca \vdash Dcb$

1	$\exists x([(Ax \wedge Bxa) \wedge \forall y((Ay \wedge Bya) \to y = x)] \wedge Dxb)$	PR
2	$Ac \wedge Bca$	PR
3	$\quad (Ag \wedge Bga) \wedge \forall y((Ay \wedge Bya) \to y = g) \wedge Dgb$	ASS for UI
4	$\quad Dgb$	Simp 3
5	$\quad (Ag \wedge Bga) \wedge \forall y((Ay \wedge Bya) \to y = g)$	Simp 3
6	$\quad \forall y((Ay \wedge Bya) \to y = g)$	Simp 5
7	$\quad (Ac \wedge Bca) \to c = g$	UI 6
8	$\quad c = g$	MP 2, 7
9	$\quad Dcb$	LL 4,8
10	Dcb	EI 1,3-9

4 Symbolize and prove

1. There are exactly two omniscient beings. Elohim is omniscient. We may infer that there is an omniscient being distinct from Elohim. (Ox: x is omniscient; e: Elohim)

   $\exists x \exists y[Ox \wedge Oy \wedge x \neq y \wedge \forall z(Oz \to (z = x \vee z = y))]; Oe \vdash \exists x(Ox \wedge x \neq e)$

1	$\exists x \exists y[Ox \wedge Oy \wedge x \neq y \wedge \forall z(Oz \rightarrow (z = x \vee z = y))]$	PR
2	Oe	PR
3	$\exists y[Oc \wedge Oy \wedge c \neq y \wedge \forall z(Oz \rightarrow (z = c \vee z = y))]$	ASS for EI
4	$Oc \wedge Od \wedge c \neq d \wedge \forall z(Oz \rightarrow (z = c \vee z = d))$	ASS for EI
5	Oc	Simp 4
6	Od	Simp 4
7	$c \neq d$	Simp 4
8	$\forall z(Oz \rightarrow (z = c \vee z = d))$	Simp 4
9	$Oe \rightarrow (e = c \vee e = d)$	UI 8
10	$e = c \vee e = d$	MP 2, 9
11	$d = c$	ASS for IP
12	$d \neq d$	LL 11,7
13	$\bot$	Contradiction 12
14	$d \neq c$	IP11-13
15	$e = c$	ASS for CP
16	$d \neq e$	LL 14,15
17	$Od \wedge d \neq e$	Conj 6,16
18	$\exists x(Ox \wedge x \neq e)$	EG 17
19	$e = c \rightarrow \exists x(Ox \wedge x \neq e)$	CP 15-18
20	$e = d$	ASS for CP
21	$c \neq e$	LL 7,20
22	$Oc \wedge c \neq e$	Conj 5,21
23	$\exists x(Ox \wedge x \neq e)$	EG 22
24	$e = d \rightarrow \exists x(Ox \wedge x \neq e)$	CP 20-23
25	$\exists x(Ox \wedge x \neq e) \vee \exists x(Ox \wedge x \neq e)$	CD10,19,24
26	$\exists x(Ox \wedge x \neq e)$	Tautology 25
27	$\exists x(Ox \wedge x \neq e)$	EI 4-26
28	$\exists x(Ox \wedge x \neq e)$	EI 3-27

2. Every act is caused by a desire. Every desire is caused by a brain process. For all x, y, and z, if x is caused by y and y is caused by z, then x is caused by z. Thus, every act is caused by a brain process. (Ax: x

is an act; Dx: x is a desire, Dxy: x is caused by y; Bx: x is a Brain process)

$\forall x(Ax \to \exists y(Dy \wedge Cxy)); \forall x(Dx \to \exists y By \wedge Cxy); \forall x \forall y \forall z[(Cxy \wedge Cyz) \to Cxz] \vdash \forall x[Ax \to \exists y(By \wedge Cxy)]$

1	$\forall x(Ax \to \exists y(Dy \wedge Cxy))$	PR
2	$\forall x(Dx \to \exists y By \wedge Cxy)$	PR
3	$\forall x \forall y \forall z[(Cxy \wedge Cyz) \to Cxz]$	PR
4	$Ac \to \exists y(Dy \wedge Ccy))$	UI 1
5	Ac	ASS for CP
6	$\exists y(Dy \wedge Ccy))$	MP 4,5
7	$Dd \wedge Ccd$	ASS for EI
8	Dd	Simp 7
9	Ccd	Simp 7
10	$Dd \to \exists y By \wedge Cdy$	UI 2
11	$\exists y By \wedge Cdy$	MP 8,10
12	$Be \wedge Cde$	ASS for EI
13	$\forall y \forall z[(Ccy \wedge Cyz) \to Ccz]$	UI 3
14	$\forall z[(Ccd \wedge Cdz) \to Ccz]$	UI 13
15	$(Ccd \wedge Cde) \to Cce$	UI 14
16	Be	Simp 12
17	Cde	Simp 12
18	$Ccd \wedge Cde$	Conj 9,17
19	Cce	MP 15,18
20	$Be \wedge Cce$	Conj 16,19
21	$\exists y(By \wedge Ccy)$	EG 20
22	$\exists y(By \wedge Ccy)$	EI 12-21
23	$\exists y(By \wedge Ccy)$	EI 7-22
24	$Ac \to \exists y(By \wedge Ccy)$	CP 5-23
25	$\forall x[Ax \to \exists y(By \wedge Cxy)]$	UG 24