FOURIER SERIES

Maths III Assignment

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I would also like to thank my parents for their help and support.

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MATLAB

MATLAB (an abbreviation of "matrix laboratory") is a proprietary multi-paradigm programming language and numeric computing environment developed by *MathWorks*. *MATLAB* allows matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages.

MATLAB (software)

L-shaped membrane logo[18]

Developer(s)	MathWorks
Initial release	1984; 37 years ago
Stable release	R2020b / September 17, 2020; 5 months ago
Written in	C/C++, MATLAB
Operating system	Windows, macOS, and Linux[19]
Platform	IA-32, x86-64
Type	Numerical computing
License	Proprietary commercial software
Website	mathworks.com

Fig 1: An overview of *MATLAB*

Fourier Series

Fourier series is a periodic function composed of harmonically related sinusoids, combined by a weighted summation. With appropriate weights, one cycle (or period) of the summation can be made to approximate an arbitrary function in that interval.

A function f(x), that can be integrated over length L, which would be period of the Fourier Series

Eg: $x \in [0,1]$ and L=1
$x \in [-\pi, \pi]$ and L=2 π

The analysis process determines the weights, indexed by integer n, which is also the number of cycles of the nth harmonic in the analysis interval. Therefore, the length of a cycle, in the units of x is L/n. The nth harmonics are $\sin(2\pi x \frac{n}{L})$ and $\cos(2\pi x \frac{n}{L})$ and their amplitudes (weights) are found by integration over the interval of length L.

Fourier coefficients:

$a_n = \frac{2}{L}\int_L \; f(x).\cos(2\pi x \frac{n}{L})\, dx$, $b_n = \frac{2}{L}\int_L \; f(x).\sin(2\pi x \frac{n}{L})\, dx$

$a0 = \frac{2}{L}\int_0^{2\pi} f(x)dx$

$f_N(x) = \frac{a_0}{2} + \sum_{n=1}^{N}(a_n \cos(2\pi x \frac{n}{L}) + b_n \sin(2\pi x \frac{n}{L}))$

Dirichlet conditions

Dirichlet conditions are sufficient conditions for a real valued, periodic function f to be equal to the sum of its Fourier series at each point where f is continuous. Moreover, the behaviour of the Fourier series at points of discontinuity is determined as well (it is the midpoint of the values of the discontinuity). These conditions are named after Peter Gustav Lejeune Dirichlet.

The conditions are:

1. f must be absolutely integrable over a period.

2. f must be of bounded variation in any given bounded interval.

3. f must have a finite number of discontinuities in any given bounded interval, and the discontinuities cannot be infinite.

Applications:

The Fourier series finds its applications in many fields such as electrical engineering, vibration analysis, acoustics, optics, signal processing, image processing, quantum mechanics, econometrics, thin-walled shell theory, etc.

It turns out that any kind of a wave can be written as a sum of sines and cosines. So for example, if a voice is recorded then

voice=$\sin(x) + \frac{1}{10}\sin(2x) + \frac{1}{100}\sin(3x) + \cdots$

and shows how when one adds sines and/or cosines the graph of cosines and sines becomes closer and closer to the original graph one is trying to approximate.

Fourier series can help approximate any kind of wave.

Second, when fourier series converge, they converge very fast. So one of many many applications is compression. Everyone's favorite MP3 format uses this for audio compression. You take a sound, expand its fourier series. It'll most likely be an infinite series BUT it converges so fast that taking the first few terms is enough to reproduce the original sound.

Coded Procedure

Ex 1: *f(x) = x * sinx*

```
syms x;
T = pi;
FS = 0;
f = x*sin(x);
a0 = 1/T*int(f,x,0,2*pi);
for n = 1:30
    an_sym = 1/T*int(f*cos(n*x),x,0,2*pi);
    an = double(an_sym);
    bn_sym = 1/T*int(f*sin(n*x),x,0,2*pi);
    bn = double(bn_sym);
    FS = FS + a0/2 + (an*cos(n*x) + bn*sin(n*x));
end
fplot(FS, [0 30]);
```

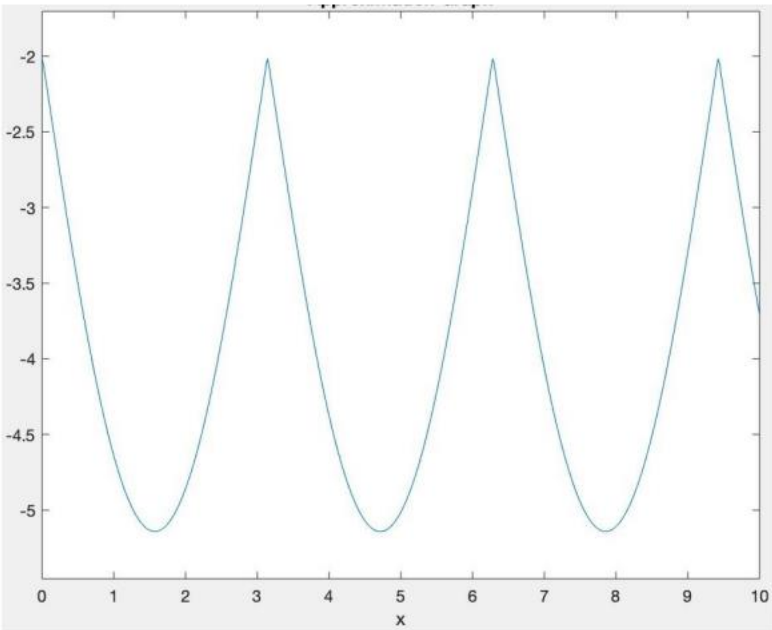

Fig 2: Graph of x*sin(x)

Ex 2: $f(x) = \begin{cases} x, & -\pi \le x \le 0 \\ -x, & 0 < x \le \pi \end{cases}$

```
q1.m*

syms x;
T = pi;
FS = 0;
f1 = x;
f2 = -x;
a0 = 1/T*int(f,x,-pi,pi);
for n = 1:30
    an_sym = 1/T*int(f1*cos(n*x),x,-pi,0) + 1/T*int(f2*cos(n*x),x,0,pi);
    an = double(an_sym);
    bn_sym = 1/T*int(f1*sin(n*x),x,-pi,pi) + 1/T*int(f2*cos(n*x),x,0,pi);
    bn = double(bn_sym);
    FS = FS + a0/2 + (an*cos(n*x) + bn*sin(n*x));
end
fplot(FS, [0 30]);
```

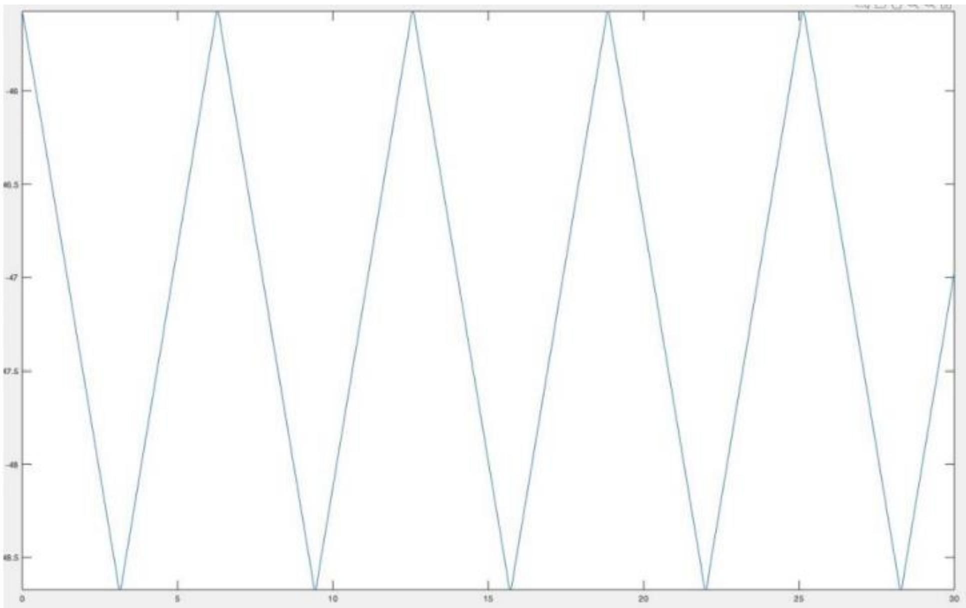

Fig 3: Graph of f(x)

Ex 3: $f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ 1, & 0 < x \le \pi \end{cases}$

```
q1.m
syms x;
T = pi;
FS = 0;
a0 = 1/T*int(0,x,-pi,0) + 1/T*int(1,x,0,pi);
for n = 1:30
    an_sym = 1/T*int(1*cos(n*x),x,0,pi);
    an = double(an_sym);
    bn_sym = 1/T*int(1*sin(n*x),x,0,pi);
    bn = double(bn_sym);
    FS = FS + a0/2 + (an*cos(n*x) + bn*sin(n*x));
end
fplot(FS, [0 30]);
```

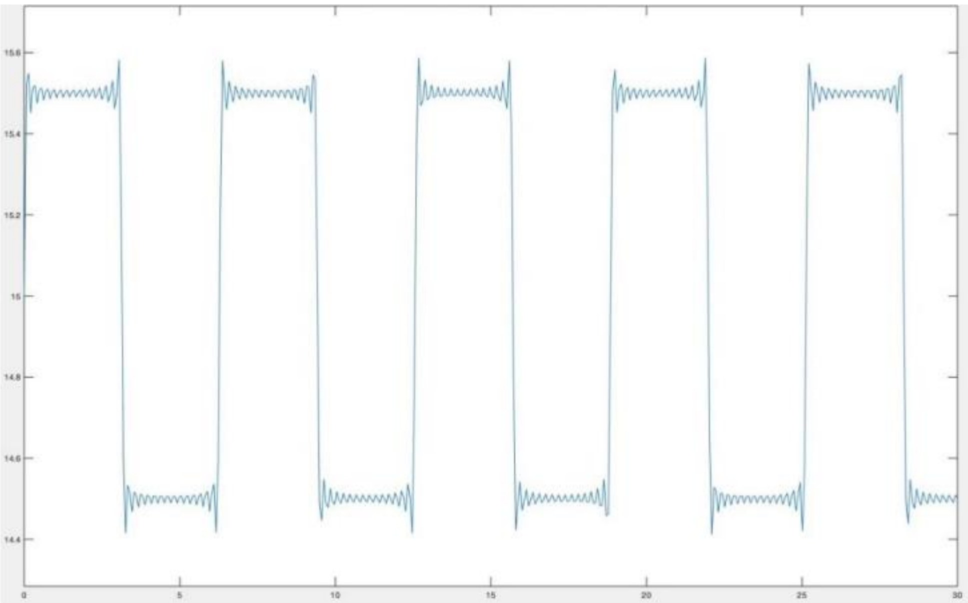

Fig 4: Graph of f(x)

Bibliography

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