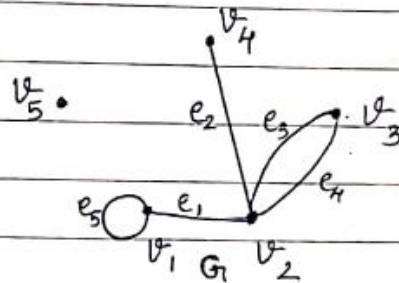


# Graph Theory.

- A graph is an ordered pair  $(V, E)$  where  $V$  = non-empty set of vertices.  
 $E$  = set of edges.
- A vertex is a point / node. An edge is a curve / line joining two vertices. A loop is an edge joining a vertex to itself. Parallel / Multiple edges means more than one edges joining the same pair of vertices.  
eg:  $G = (V, E)$   
where  $V = \{v_1, v_2, v_3, v_4, v_5\}$ ,  $E = \{e_1, e_2, e_3, e_4, e_5\}$



Here,  $e_1 = v_1 v_1$  (edge  $e_1$  joins vertices  $v_1, v_1$ )  
 $e_2 = v_2 v_4$   
 $e_3 = e_4 = v_2 v_3 \rightarrow$  multiple edges.  
 $e_5 = v_1 v_2 \rightarrow$  loop

- A graph without loops and multiple edges is called a simple graph. ~~A graph with loop~~  
A graph which is not simple is called multigraph.
- If edge  $e$  is formed by vertices  $v_1$  and  $v_2$  then  $v_1, v_2$  are called adjacent vertices.  
If vertex  $v$  is common between ~~two~~ edges  $e_1$  and  $e_2$  then  $e_1, e_2$  are called adjacent edges.  
Eg: In above example,  
 $v_1, v_2$  are adjacent vertices while  $v_1, v_4$  are not adjacent vertices.  
 $e_1, e_2$  are adjacent edges but  $e_2, e_3$  are not.

- Degree of a vertex  $v$  is the number of edges starting or ending at the vertex  $v$ , written as  $d(v)$ .

Eg: In graph  $G$  of page 1,

$$d(v_1) = 3 \quad (\text{as loop is counted 2 times})$$

$$d(v_2) = 4$$

$$d(v_3) = 2$$

$$d(v_4) = 1 \quad \therefore v_4 \text{ is called pendant vertex.}$$

$$d(v_5) = 0 \quad \therefore v_5 \text{ is called isolated vertex.}$$

- Handshaking Theorem:

The sum of degrees of all vertices in a graph is twice the number of edges in that graph

$$\text{i.e. } \sum d(v) = 2E$$

In graph  $G$  of page 1:

$$\sum d(v) = d(v_1) + d(v_2) + d(v_3) + d(v_4) + d(v_5)$$

$$= 3 + 4 + 2 + 1 + 0$$

$$= 10.$$

$$E = \text{no. of edges in } G = 5$$

$$\therefore \sum d(v) = 10 = 2 \times 5 = 2E.$$

- Types of graph:

### 1. Complete Graph

A simple graph in which every vertex is adjacent with every other vertex.

ie. A simple graph on  $n$  vertices in which degree of every vertex is  $n-1$ .

It is denoted by  $K_n$ .

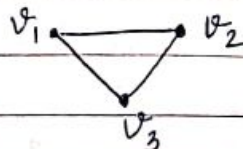
Eg:  $K_1$

$v_1$

$K_2$

$v_1 \text{ --- } v_2$

$K_3$



no. of edges

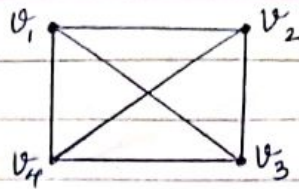
0

1

3

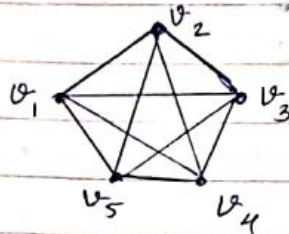


$K_4$



6

$K_5$

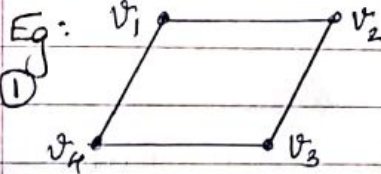


10

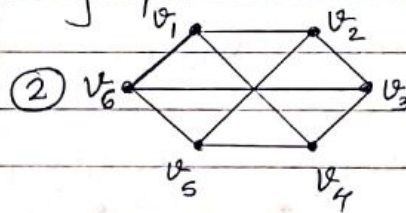
Note: The total no. of edges in  $K_n = \frac{n(n-1)}{2}$

## 2. Regular Graph

A graph in which every vertex has same degree  $k$  is called a ' $k$ -regular graph'.



2-regular

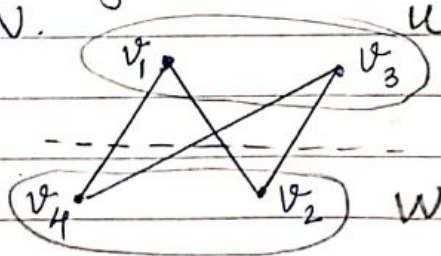


3-regular

## 3. Bipartite Graph

A graph  $G=(V, E)$  is called bipartite if the vertex set  $V$  is partitioned into two sets  $U$  and  $W$  such that every edge in  $E$  has one vertex in  $U$  and other in  $W$ .

Eg:

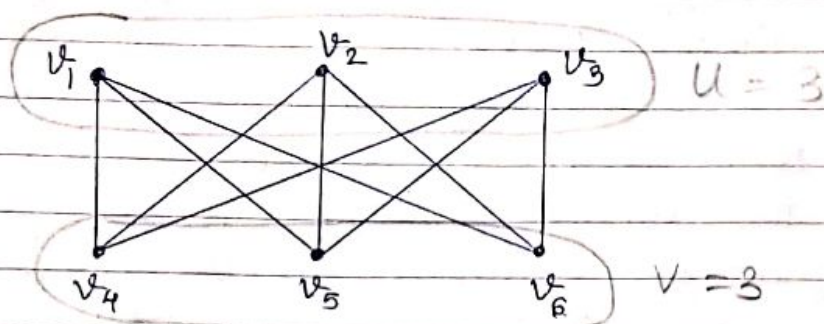
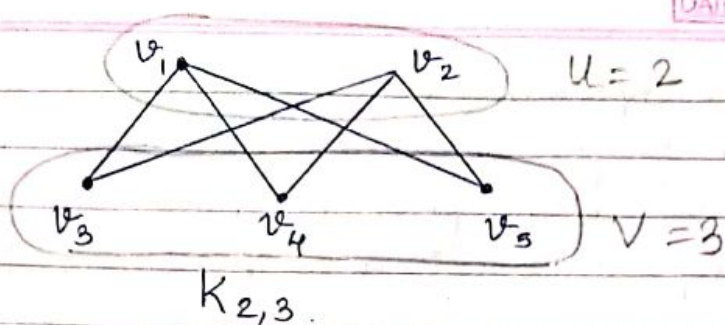


Redrawing the graph ① above.  
(2-regular)

## 4. Complete Bipartite Graph

A simple bipartite graph which is complete.  
It is denoted by  $K_{m,n}$ .

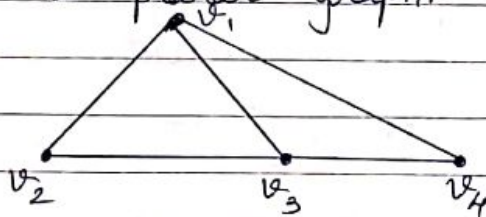
Eg :



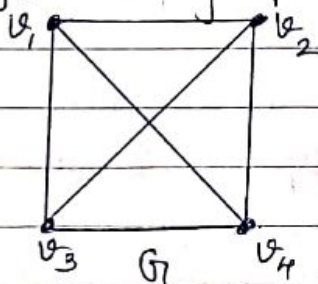
## 5. Planar Graph

A graph in which no two edges cross each other is called a planar graph.

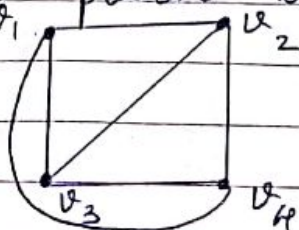
Eg :



Sometimes, the given graph is not planar, but upon drawing the same graph in a little different way, the graph becomes planar.



Here,  $G$  is not planar but if the same graph is drawn like



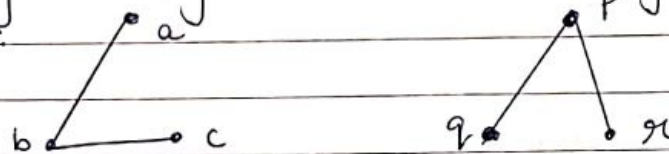
Now  $G$  is planar  $\therefore$  Given graph  $G$  is planar.



## Isomorphism of graphs:

Two graphs  $G = (V, E)$  and  $G' = (V', E')$  are said to be isomorphic if there is a one-to-one correspondence  $f: V \rightarrow V'$  such that for  $v_1, v_2 \in V$ ,  $v_1, v_2 \in V$ ,  $f(v_1) = v_1'$ ,  $f(v_2) = v_2'$  and  $v_1$  is adjacent to  $v_2$  then  $v_1'$  is adjacent to  $v_2'$ . i.e. [If we can find a bijective function from vertices to  $G$  to vertices of  $G'$  s.t. two vertices adjacent in  $G$  have their images adjacent in  $G'$  i.e. adjacency preserving.]

Eg:



Here  $a$  and  $b$  are adjacent

• If  $f(a) = q$ ,  $f(b) = p$

then  $p, q$  are adjacent

$\therefore$  proper mapping - can be considered.

• If  $f(a) = q$ ,  $f(b) = r$

then  $q$  and  $r$  are not adjacent

$\therefore$  improper mapping - cannot be considered.]

Note: If  $f(a) = b$  then  $\deg a = \deg b$ .

How to check whether given graphs are isomorphic or not?

Step 1: Check whether no. of vertices in  $G =$  no. of vertices in  $G'$ .  
If Yes then proceed to step 2, otherwise ~~conclude~~ conclude ' $G$  and  $G'$  are not isomorphic'.

Step 2: Check whether no. of edges in  $G =$  no. of edges in  $G'$ .  
If Yes then proceed to step 3, otherwise conclude ' $G$  and  $G'$  are not isomorphic'.

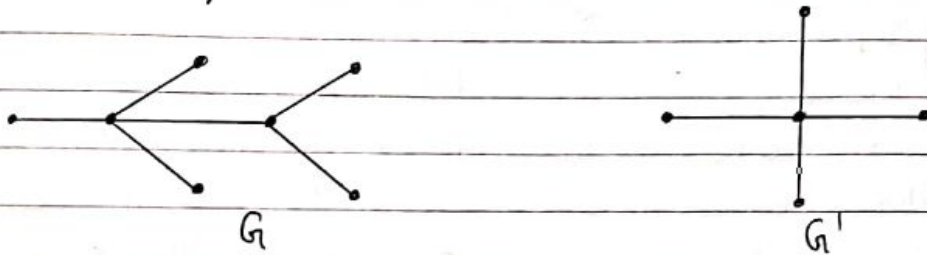
Step 3: Check whether degree of vertices in  $G$  and  $G'$  are same or not.

If Yes then, proceed to step 4, otherwise conclude ' $G$  and  $G'$  are not isomorphic'.

Step 4: Define a function  $f: G \rightarrow G'$  on vertices satisfying adjacency of vertices i.e. adjacency preserving map. If finding such  $f$  is possible then  $G$  and  $G'$  are 'isomorphic' otherwise they are not isomorphic.

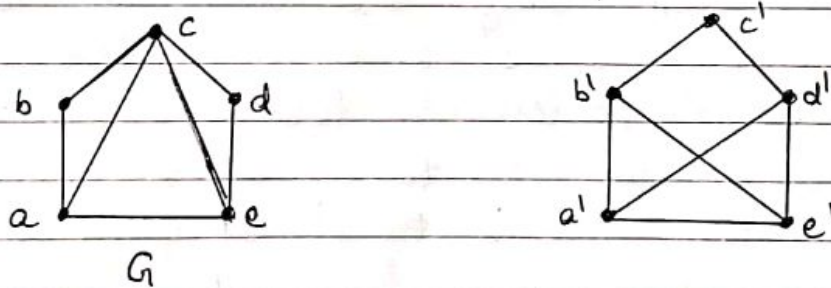
Eg: Check whether the following pair of graphs are isomorphic or not.

1.



Step 1: no. of vertices in  $G = 7$   
no. of vertices in  $G' = 5$   
 $\therefore G$  and  $G'$  are not isomorphic.

2.



Step 1: no. of vertices in  $G =$  no. of vertices in  $G' = 5$

Step 2: no. of edges in  $G =$  no. of edges in  $G' = 10$

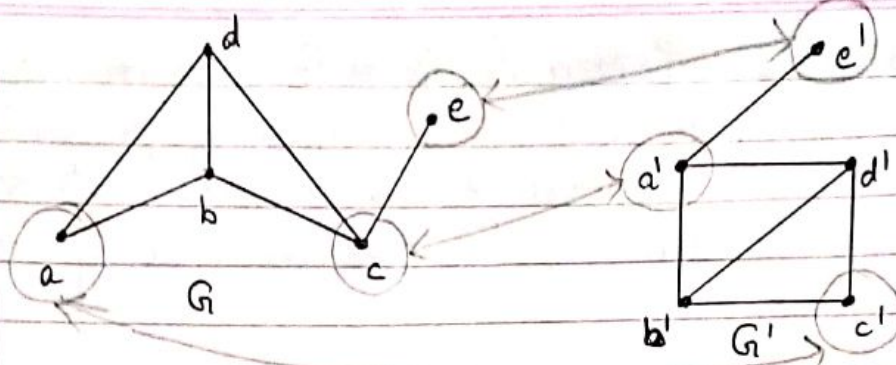
Step 3:

Degree	2	3	4
no. of vertices in $G$	2	2	1
no. of vertices in $G'$	1	4	—

Degree of vertices is not same  
 $\therefore G$  and  $G'$  are not isomorphic.



3.



Step 1: no. of vertices in  $G = \text{no. of vertices in } G' = 5$

Step 2: no. of edges in  $G = \text{no. of edges in } G' = 6$

Step 3:

Degree	1	2	3
No. of vertices in $G$	1	1	3
No. of vertices in $G'$	1	1	3

Step 4: Mapping vertex to vertex with same degree.

- From step 3, there is only one vertex of degree 1 in  $G$  and  $G'$   
 $\therefore$  mapping  $e$  to  $e'$  i.e.  $f(e) = e'$
- Similarly, only one vertex of degree 2  
 $\therefore$  mapping  $a$  to  $c'$  i.e.  $f(a) = c'$
- Now, only left to find  $f(b)$ ,  $f(c)$ ,  $f(d)$ .
- $e$  is adjacent to  $c$  in  $G$   
 $e'$  is adjacent to  $a'$  in  $G'$   
 $f(e) = e'$

$\therefore$  mapping  $c$  to  $a'$  i.e.  $f(c) = a'$

$a \rightarrow c'$

$b \rightarrow ?$

$c \rightarrow a'$

$d \rightarrow ?$

$e \rightarrow e'$

- Now, only left to find  $f(b)$ ,  $f(d)$ .  
 Here both are valid and free selection is possible.  
 $\therefore$  let  $f(b) = b'$ ,  $f(d) = d'$   
 OR  $f(b) = d'$ ,  $f(d) = b'$

Consider a map  $f: G \rightarrow G'$  defined by

$$f(a) = c'$$

$$f(b) = b'$$

$$f(c) = a'$$

$$f(d) = d'$$

$$f(e) = e'$$

OR.

$$f(a) = c'$$

$$f(b) = d'$$

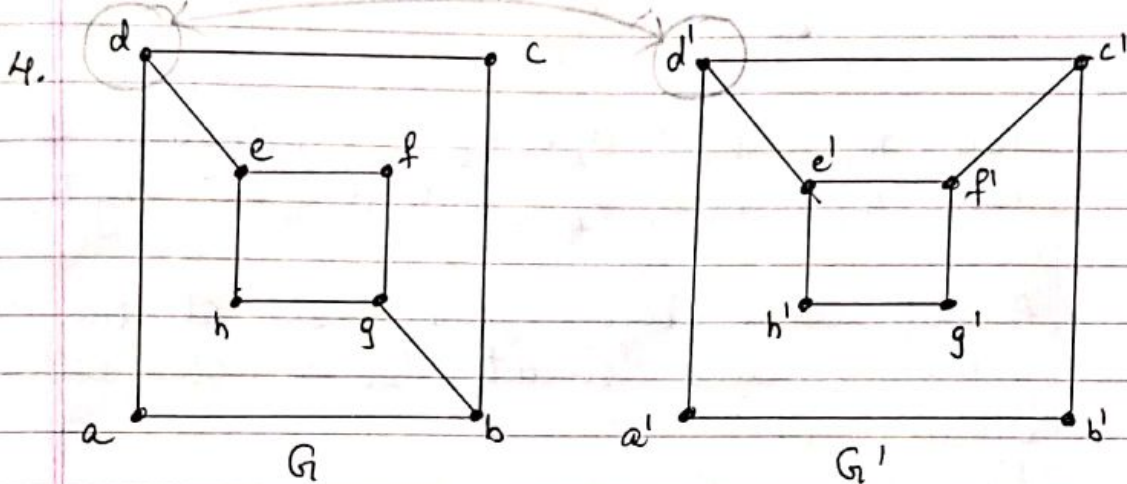
$$f(c) = a'$$

$$f(d) = b'$$

$$f(e) = e'$$

This is an adjacency preserving map.

$\therefore G$  is isomorphic to  $G'$ .



Step 1: no. of vertices in  $G$  = no. of vertices in  $G' = 8$

Step 2: no. of edges in  $G$  = no. of edges in  $G' = 10$

Step 3:

Degree

No. of vertices in  $G$  2 3

No. of vertices in  $G'$  4 4

Step 4:

Let  $f(d) = d'$  ~~not possible~~

Now,  $d$  is (connected) adjacent to  $a, e, c$  of which  $\deg a = \deg c = 2$ ,  $\deg e = 3$ .

And,  $d'$  is adjacent to  $a', e', c'$  of which  $\deg e' = \deg c' = 3$ ,  $\deg a' = 2$ .

$\therefore d$  cannot be mapped to  $d'$ .

Similarly,

$d$  cannot be mapped to  $e', f', c'$ .

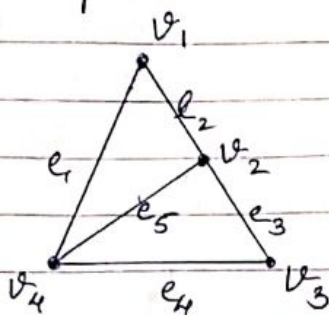
$\therefore$  Find  $f: G \rightarrow G'$  is not possible.

$\therefore G$  is not isomorphic to  $G'$ .



Path is an alternating sequence of vertices and edges of a graph in which neither vertex nor edge is repeated.

length of path = no. of edges in the path  
Eg:



Path  $\pi_1 = v_1, e_1, v_4, e_5, v_2$ , length = 2.

Path  $\pi_2 = v_1, e_1, v_4, e_4, v_3, e_3, v_2, e_2, v_1$ , length = 4.

A path that begins and ends at the same vertex is called 'circuit'. It is also called a 'closed path'.

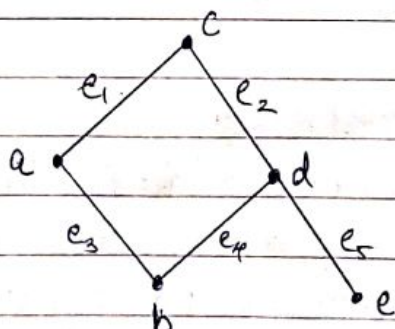
Eg: path  $\pi_2$  is a circuit.

path  $\pi_1$  is not a circuit.

Connected graph

A graph is called connected if there is a path between every pair of vertices in that graph.

Eg:



From a-b is a direct path  $a, e_3, b$

a-c ——— " ———  $a, e_1, c$

a-d  $\Rightarrow \pi_1: a, e_1, c, e_2, d$   
 $\pi_2: a, e_3, b, e_4, d$

$$a - c \Rightarrow \pi: a e_1 c e_2 d e_5 e$$

$$\pi_2: a e_3 b e_4 d e_5 e$$

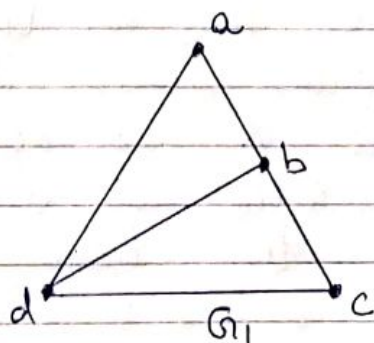
Similarly, we can find a path ~~from~~ between every other pair.

NOTE: Here,

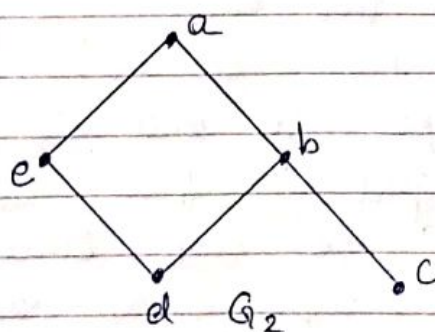
- $\pi: a e_1 c e_2 d$  can also be written as 'acd' as there is only one edge  $e_1$  between 'a' and 'c'.
- Writing 'acd' as a path is easier than writing  $a e_1 c e_2 d$ .
- Length of path  $\pi = 2 = \text{no. of edges}$ .

### Eulerian Graph.

- A path or circuit which includes every 'edge' of the graph is said to be an Eulerian path / circuit.
- If a graph has an Eulerian circuit, it is called Eulerian graph.



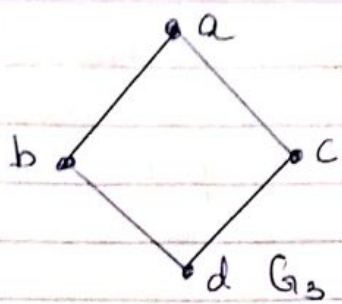
- $a-b-c-d-a$  is a circuit but it leaves edge  $b-d$  so it is not Eulerian circuit.
- $d-a-b-d-c-b$  is an Eulerian path.



- There is no possible Eulerian circuit so the given graph  $G_1$  is not Eulerian.

- $c-b-a-e-d-b$  is Eulerian path.
- There is no possible Eulerian circuit, so graph  $G_2$  is not Eulerian.





$a-b-d-c-a$  is an Eulerian circuit  
 $\therefore$  The graph  $G_3$  is Eulerian.

How to check whether the given graph is Eulerian or not?

- Theorem:  
 A connected graph is Eulerian iff and only if degree of every vertex is even.

Eg: In above example, there were odd degree vertices in  $G_1$  and  $G_2$  but in  $G_3$  all vertices had even degree  
 So, only  $G_3$  is Eulerian.

- Theorem:  
 A connected graph  $G$  has an Eulerian path but not an Eulerian circuit if there are exactly two vertices of odd degree  
 i.e. If a graph has exactly two vertices of odd degree -  $u$  and  $v$  then there is an Eulerian path between  $u$  and  $v$ .

NOTE: As there are two odd degree ~~vertices~~ vertices, the graph will not have Eulerian circuit.

Eg: In above example,  $G_1$  and  $G_2$  had 2 vertices of odd degree so they had Eulerian path but were not Eulerian.  $G_3$  had all vertices of ~~even~~ even degree so it had Eulerian circuit. Hence  $G_3$  has / does not have Eulerian path.

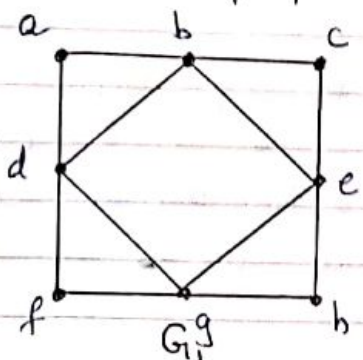
[Q] Can a circuit be treated a path?



Check whether the following graphs have Eulerian path and/or circuit.

[Whenever we say that the graph has Eulerian path/circuit, we need to specify that particular path/circuit. Just mentioning it has path/circuit is not ~~a~~ proper].

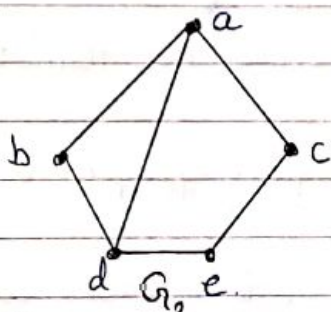
(1)



In graph  $G_1$ ,  
All vertices have even degree.  
 $\therefore$  The graph is Eulerian.  
 $\pi_1: a-b-c-e-h-g-f-d-g-e-b-$   
 $d-a$   
is an Eulerian circuit.

Every circuit is a path.  
 $\therefore \pi_1$  is an Eulerian path.

(2)

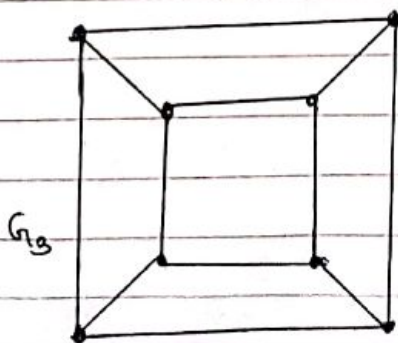


In graph  $G_2$ ,  
All vertices do not have even degree.  
 $\therefore$  The graph is not Eulerian.

But there are exactly two vertices  $a, d$  of odd degree so an Eulerian path between  $a$  and  $d$  exists.  $\pi_2: a-b-d-a-c-e-d$ .

NOTE: There is no Eulerian path between any other pair of vertices.

(3)



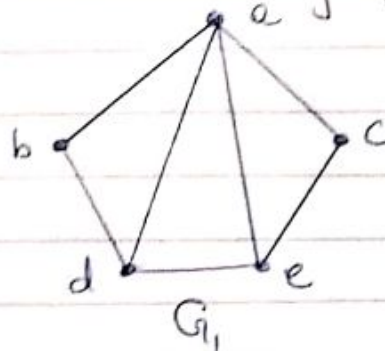
In Graph  $G_3$ ,  
All vertices have odd degree.  
 $\therefore$  The graph does not have an Eulerian path ~~neither~~ nor Eulerian circuit.



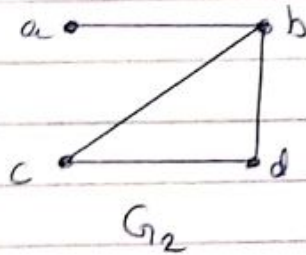
## Hamiltonian Graph.

- A path or circuit which includes every 'vertex' of the graph is said to be an Hamiltonian path or circuit.
- If a graph has an Hamiltonian circuit, it is called Hamiltonian graph.

Eg.



In graph  $G_1$ ,  
 $\pi_1 = a - b - d - e - c - a$  is  
 a Hamiltonian circuit.  
 Hence,  $G_1$  is Hamiltonian.



In Graph  $G_2$ ,  
 $\pi_2 = a - b - d - c$  is a ~~Hamiltonian~~ <sup>Hamiltonian</sup> path.  
 • There is no Hamiltonian circuit.

How to check whether the given graph is Hamiltonian or not?

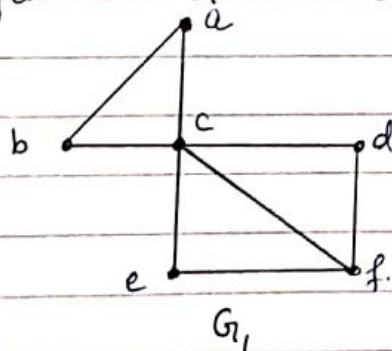
- Checking for Hamiltonian is not as easy as checking for Eulerian.
- ~~Finding~~ Every connected graph
- Finding Hamiltonian path/circuit is not difficult.
- When concluding that the graph is not Hamiltonian, below stated theorems should be used.

Theorem

1. If  $G$  is connected graph with  $n$  vertices and if sum of degrees of each pair of vertices is greater than or equal to  $n-1$  then there is Hamiltonian circuit.
2. In a simple connected graph with  $n$  vertices, if degree of each vertex is greater than or equal to  $n/2$  then there is Hamiltonian Circuit.

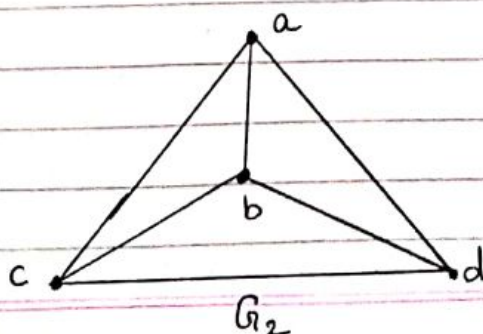
Check whether the following graphs have Hamiltonian path and/or circuit.

①



In Graph  $G_1$ ,  $n = 6$ .  $\therefore n/2 = 3$   
 $\deg(a) = 2$   
 $\therefore \deg(a) \neq n/2$   
 $\therefore$  Hamiltonian circuit does not exist.  
 But Hamiltonian path exist -  
 $\pi_1: a-b-c-d-f-e$ .

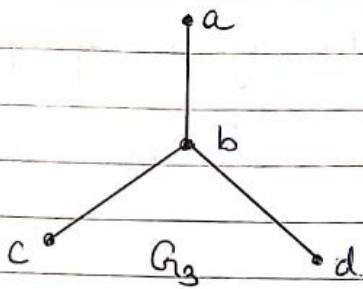
②



In graph  $G_2$ ,  $n = 4$ ,  $\therefore n/2 = 2$   
~~deg(a)~~ degree of all vertices = 3  
 which is greater than  $n/2$   
 $\therefore$  Hamiltonian circuit exist  
 $\pi_2: a-c-b-d-a$ .



(3)



In graph  $G_3$ ,  $n=4$ ,  $n/2=2$

$\deg(a) \neq n/2$

$\therefore$  There is no Hamiltonian circuit.

Also, clearly there is no Hamiltonian path.

## 7 Post Experiment Exercise

### B. Questions

1 Explain arrays in shell with an example

Ans. Shell supports array variable. This can hold multiple values at the same time. Array provides a method of grouping a set of variables. Instead of creating a new name for each variable that is required, you can use a single array variable that stores all the other variables.

ex	name[0]="Lional"	echo "Second names \${name[@]}"
	name[1]="Richard"	echo "All names: \${name[@]}"
	name[3]="Stephen"	
	name[4]="Nikola"	

O/P Second name is Richard

All names: Lional Richard Stephen Nikola.

2. Write a shell program to count no of files in a directory.

Ans. #!/bin/bash

START=\$HOME

#change directory to commandline if passed, otherwise use home.

[ \$# -eq 1 ] && START=\$1 || :

if [ ! -d \$START ]

then

echo "\$START not a directory!"

exit 1

fi

variable

DIRS=\$(find "\$START" -type d)

#loop through each dir to get all sub dir files

for d in \$DIRS

do

["\$d" != "." -a "\$d" != ".."] && echo "\$d directory has

\$(ls -l \$d | wc -l) files"

done.

FOR EDUCATIONAL USE



## Conclusion

In this experiment we performed shell scripting in Unix terminal. We searched for an element within a list, computed GCD & LCM of 2 numbers and checked whether a file is a directory or not.

Shellscripts are powerful tools that let us execute program directly from the OS.

References : 1. Yashwanth Kanetkar, UNIX Shell Programming.  
2. Sumitabha Das, UNIX Concepts & Applications.  
-end-