Framework for calculating $\langle j_T \rangle$, $\langle k_T \rangle$ from two particle correlations in $d - Au$ and $p - p$

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Abstract

We derived a general set of formulae for calculating the RMS values for jet fragmentation momentum, j_T, and parton transverse momentum, k_T, in two particle correlation framework. Possible systematic errors and limitations in our approach are discussed.

Contents

1 Introduction

The high p_T particles are dominantly produced in hard-scattering of two partons; one from target and one from the beam. Each of outgoing partons then fragments into a jet of hadrons. Due to the momentum conservation, the two outgoing jets are roughly back-to-back. The hadron production rate can be described by the so called factorization theorem, which is a convolution of three important gradients: 1) the distribution of partons in the initial hadrons, $f_{h\to q}(x)$, 2) the distribution of hadrons resulting from the fragmentation of the outgoing partons, $D_q^h(z)$, and 3) the parton - parton elementary cross section which can be calculated from pQCD $d\hat{\sigma}/d\hat{t}$.

The spray of particles from a single jet typically collimate in space, each has a momentum transverse to the jet direction, commonly denoted as $\overrightarrow{j_T}$. On the other hand, particles from different jets in a single hard-scattering tend to be back-to-back in the plane transverse to the beam. They are not exactly back-to-back though, because of the finite $\overrightarrow{j_T}$ from the fragmentation process and the finite transverse momentum, $\overrightarrow{k_T}$, carried by each of the two incoming partons.

Unlike e^+e^- collision, in hadron-hadron or hadron-nucleus collisions, the isolation of a single jet is difficult due to the accompanying soft processes and initial and final state radiation of the scattering partons. Jets, especially for those with energy below 10 GeV/c, can not be cleanly identified on a event-by-event bases. Two particle correlation method provides an alternative way to measure jet properties on a statistical bases. In section.2, we derive a set of formulae, which relates the RMS values of $\overrightarrow{j_T}$ and $\overrightarrow{k_T}$, $\sqrt{\left\langle \overrightarrow{j_T}^2 \right\rangle}$ and $\sqrt{\left\langle \overrightarrow{k_T}^2 \right\rangle}$, to the measured azimuthal angular width from the two correlation function. In section.3, we compare our formulae with the formulae used in other analysis and discuss the limitations of our approach.

2 Derivation of j_T, k_T formulae

2.1 Definition of kinematic variables

Fig.1 shows a jet and one of it's particle in three dimension. The original jet has a total momentum of $\overrightarrow{p_{jet}}$ and its projection onto the azimuth plane is $\overrightarrow{p_{T,jet}}$. The particle belonging to the jet has momentum of $\overrightarrow{p}$ and its projection onto the azimuth plane is $\overrightarrow{p_T}$. The $\overrightarrow{j_T}$ is the component of the track momentum perpendicular to jet direction, which can be expressed as,

$$\overrightarrow{j_T} = \overrightarrow{p} - \overrightarrow{p_{jet}}\frac{\overrightarrow{p}\cdot\overrightarrow{p_{jet}}}{|\overrightarrow{p_{jet}}|^2} \tag{1}$$

It's component perpendicular to the plane defined by the beam and jet direction is j_{Ty} (i.e. $\overrightarrow{j_{Ty}}\cdot\overrightarrow{p_{jet}} = 0$ and $\overrightarrow{j_{Ty}}\cdot\overrightarrow{p_{T,jet}} = 0$). j_{Ty} defined this way is also equal to the component of p_T perpendicular to $p_{T,jet}$, i.e,

$$\overrightarrow{j_{Ty}} = \overrightarrow{p_T} - \overrightarrow{p_{T,jet}}\frac{\overrightarrow{p_T}\cdot\overrightarrow{p_{T,jet}}}{|\overrightarrow{p_{T,jet}}|^2} \tag{2}$$

Fig.2 depicts the typical picture of the correlation of two particles within the same jet. The jet itself and the two particles have been projected onto the azimuth plane. The jet direction is shown by the dashed line. The trigger particle and the associated particle have a transverse momentum of $\overrightarrow{p_{T,trig}}$ and $\overrightarrow{p_{T,asso}}$, respectively. If we denote the angle between jet-trigger, jet-associated and trigger-associated as, ϕ_{tq}, ϕ_{aq} and ϕ_{ta}, respectively, then the following relations are true,

$$\phi_{ta} = \phi_{tq} + \phi_{aq} \tag{3}$$

$$j_{Ty,trig} = p_{T,trig} sin(\phi_{tq}) \tag{4}$$

$$j_{Ty,asso} = p_{T,asso} sin(\phi_{aq}) \tag{5}$$

$$p_{out,N} = p_{T,asso} sin(\phi_{ta}) \tag{6}$$

If one define $\overrightarrow{p_{out2D,N}}$ as the component of associated particle momentum perpendicular to the trigger momentum,

$$\overrightarrow{p_{out2D,N}} = \overrightarrow{p_{asso}} - \overrightarrow{p_{trig}} \frac{\overrightarrow{p_{asso}} \cdot \overrightarrow{p_{trig}}}{|\overrightarrow{p_{trig}}|^2} \tag{7}$$

then $p_{out,N}$ is <u>approximately</u> the projection $\overrightarrow{p_{out2D,N}}$ onto the azimuth plane.

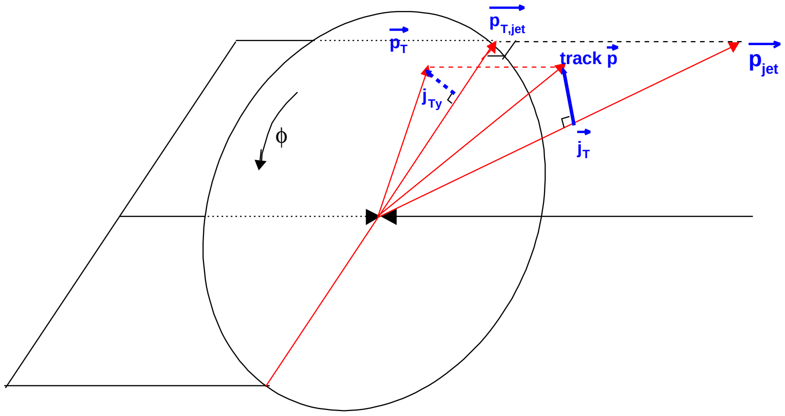

Figure 1: 3D view of a jet and one particle belong to this jet. The parallelogram represents the plane defined by the jet and beam direction. The circle represents the azimuth plane, which is perpendicular to the parallelogram.

When the trigger particle and associated particle are from the back-to-back jets in a single hard-scattering process, the typical configuration is plotted in Fig.3. If the two jets are exactly back-to-back and have identical transverse momentum, the situation would be very similar to that for the same jet correlation. In this case, the distribution of ϕ_{ta} should be identical, except that it would be centered around π instead of 0. However, because the colliding partons have finite initial transverse momentum $\overrightarrow{k_T}$, the dijet system is boosted to have a transverse momentum of $\overrightarrow{k_{T,1}} + \overrightarrow{k_{T,2}}$. The angle between the two jets, ϕ_{qq}, is no longer

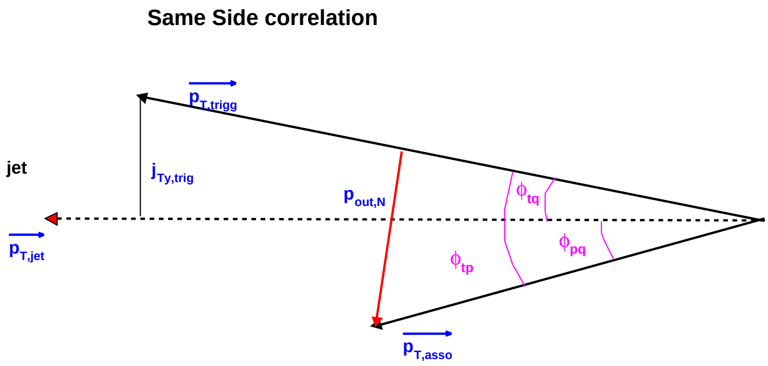

Figure 2: Schematic view of the kinematic variables of the two particles from the same jet. The momentum plotted have been projected onto azimuth plane.

0. Initial k_T also leads to momentum imbalance between the back-to-back jets. Since the typical k_T is on the order of 1-2 GeV/c, the imbalance effect can be sizable for low p_T jets. Including the k_T effect, the trigger jet transverse momentum would be $\overrightarrow{p_{T,jet}} + \overrightarrow{k_{T,1}}$, and the opposite side jet transverse momentum would be $-\overrightarrow{p_{T,jet}} + \overrightarrow{k_{T,2}}$. From Fig.3, we obtain the following relations,

$$\begin{aligned}
\phi_{ta} &= \phi_{tq} + \phi_{aq} + \phi_{qq} && (8)\\
j_{Ty,trig} &= p_{T,trig} sin(\phi_{tq}) && (9)\\
j_{Ty,asso} &= p_{T,asso} sin(\phi_{aq}) && (10)\\
p_{out,F} &= p_{T,asso} sin(\phi_{ta}) && (11)\\
sin(\phi_{qq}) &= \frac{\left(\overrightarrow{p_{T,jet}} + \overrightarrow{k_{T,1}}\right) \times \left(-\overrightarrow{p_{T,jet}} + \overrightarrow{k_{T,2}}\right) \cdot \hat{z}}{|\overrightarrow{p_{T,jet}} + \overrightarrow{k_{T,1}}||\overrightarrow{p_{T,jet}} - \overrightarrow{k_{T,2}}|} && (12)
\end{aligned}$$

where the $\hat{z}$ represents the unit vector along the beam direction z. This vector is perpendicular to $\overrightarrow{p_{T,jet}}$, $\overrightarrow{k_{T,1}}$ and $\overrightarrow{k_{T,2}}$.

Before we proceed with the derivation, let's define a list of quantities that are often used in the rest of the note.

$$\begin{aligned}
x_E &= -\vec{p}_{T,asso} \cdot \vec{p}_{T,trig} / p_{T,trig}^2 && (13)\\
x_h &= p_{T,asso} / p_{T,trig} && (14)\\
x_{j,trig} &= j_{T_y,trig} / p_{T,trig} = sin(\phi_{tq}) && (15)\\
x_{j,asso} &= j_{T_y,asso} / p_{T,asso} = sin(\phi_{aq}) && (16)\\
x_{k,trig} &= \sqrt{2} k_{T_y} z_{trig} / p_{T,trig} && (17)\\
z_{trig} &= \frac{p_{T,trig}}{p_{T,jet}}, z_{asso} = \frac{p_{T,asso}}{p_{T,jet}} && (18)
\end{aligned}$$

note that $x_E = x_h cos(\phi_{ta})$.

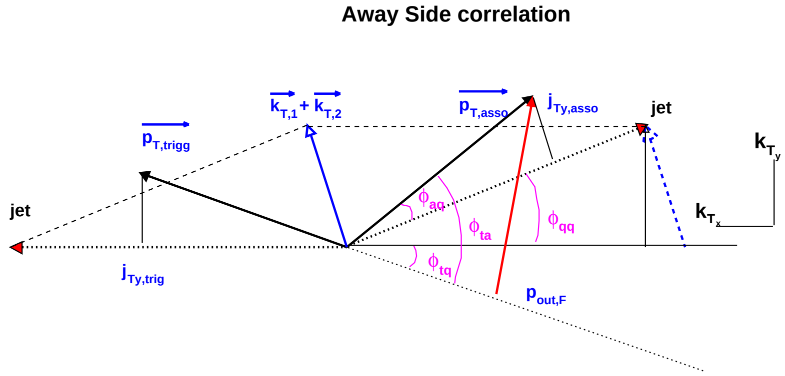

Figure 3: Schematic view of the kinematic variables of the two particles from two jets in one hard-scattering process. The momentum plotted have been projected onto azimuth plane.

2.2 The formulae for $p_{out,N}, p_{out,F}$

For each pair of trigger and associated particles, we measures the p_{out} value, $p_{out,N}$ and $p_{out,F}$ for same side correlation and away side correlation, respectively. If we assume that ϕ_{tq}, ϕ_{aq} and ϕ_{qq} are statistically independent, we can obtain the following relations from Eq. 8 for the away side correlation [1],

$$\begin{aligned} \left\langle sin^2\phi_{ta}\right\rangle &= \left\langle (sin\phi_{tq}cos\phi_{aq}cos\phi_{qq})^2\right\rangle + \left\langle (sin\phi_{aq}cos\phi_{tq}cos\phi_{qq})^2\right\rangle + \\ & \left\langle (sin\phi_{qq}cos\phi_{aq}cos\phi_{tq})^2\right\rangle + \left\langle (sin\phi_{tq}sin\phi_{aq}sin\phi_{qq})^2\right\rangle \end{aligned} \tag{19}$$

all cross terms drop out because their expectation values are 0. [2]

This equation leads to the following relations for p_{out},

$$\begin{aligned} \left\langle p^2_{out,F}\right\rangle &= \left\langle (p_{T,asso}sin\phi_{tq}cos\phi_{aq}cos\phi_{qq})^2\right\rangle + \left\langle (p_{T,asso}sin\phi_{aq}cos\phi_{tq}cos\phi_{qq})^2\right\rangle \\ &+ \left\langle (p_{T,asso}sin\phi_{qq}cos\phi_{aq}cos\phi_{tq})^2\right\rangle + \left\langle (p_{T,asso}sin\phi_{tq}sin\phi_{aq}sin\phi_{qq})^2\right\rangle \end{aligned} \tag{20}$$

$$\left\langle p^2_{out,N}\right\rangle = \left\langle (p_{T,asso}sin\phi_{tq}cos\phi_{aq})^2\right\rangle + \left\langle (p_{T,asso}sin\phi_{aq}cos\phi_{tq})^2\right\rangle \tag{21}$$

where we used the fact that $\phi_{qq} = 0$ for the same side jet.

Let's denote the k_T projection perpendicular and parallel to the jet direction by k_{Ty} and k_{Tx}, respectively. Further, if $p_{T,jet} \gg k_T$, Eq.12 can be simplified as

$$\begin{aligned} sin(\phi_{qq}) &= \frac{k_{Ty,1} + k_{Ty,2}}{p_{T,jet}} + \frac{k_{Tx,2}k_{Ty,2} - k_{Tx,1}k_{Ty,1}}{p^2_{T,jet}} + \mathcal{O}(\frac{1}{\mathrm{p}^3_{\mathrm{T,jet}}}) \\ &\approx \frac{k_{Ty,1} + k_{Ty,2}}{p_{T,jet}} \end{aligned} \tag{22}$$

[1]Note that in this note, we use $\langle x^2\rangle$ to represent the square of the RMS value for x and $\langle x\rangle$ to represent the mean of x.

[2]The cross terms are 0.5 $\langle cos2\phi_{qq}sin2\phi_{tq}sin2\phi_{aq}$ + the rotations of the indexes$\rangle$.

Thus it's RMS value can be approximated as,

$$\sqrt{\langle sin^2(\phi_{qq})\rangle} \approx \sqrt{2\left\langle \frac{k_{Ty}^2}{p_{T,jet}^2}\right\rangle} = \sqrt{2\left\langle \frac{k_{Ty}^2 z_{trig}^2}{p_{T,trig}^2}\right\rangle} = \sqrt{\langle x_{k,trig}^2\rangle} \tag{23}$$

Where we have assumed the statistical independence of $k_{Ty,1}$ and $k_{Ty,2}$ and have used Eq.18. Eq.23 is one of the important relations that were used in all j_T and k_T analysis[3, 4]. But we should bear in mind that this is an approximation and we shall discuss the limitations imposed by this approximation in Section3.

From Eq.19-Eq.23, we can derive the following general relations for far side p_{out},

$$\begin{aligned}
\langle p_{out,F}^2\rangle &= \langle (x_h j_{T_y,trig})^2(1-x_{j,asso}^2)(1-x_{k,trig}^2)\rangle \qquad (24)\\
&+\left\langle j_{T_y,asso}^2(1-x_{j,trig}^2)(1-x_{k,trig}^2)\right\rangle\\
&+2\left\langle (x_h z_{trig} k_{T_y})^2(1-x_{j,asso}^2)(1-x_{j,trig}^2)\right\rangle\\
&+2\left\langle (x_h z_{trig} k_{T_y})^2\left(\frac{j_{T_y,trig} j_{T_y,asso}}{x_h p_{T,trig}^2}\right)^2\right\rangle\\
&= \left\langle j_{T_y}^2\right\rangle \langle x_h^2(1-x_{j,asso}^2)(1-x_{k,trig}^2)\rangle \qquad (25)\\
&+\left\langle j_{T_y}^2\right\rangle \langle (1-x_{j,trig}^2)(1-x_{k,trig}^2)\rangle\\
&+2\left\langle k_{T_y}^2\right\rangle \langle (x_h z_{trig})^2(1-x_{j,asso}^2)(1-x_{j,trig}^2)\rangle\\
&+2\left\langle k_{T_y}^2\right\rangle \langle (z_{trig} x_{j,trig})^2\rangle
\end{aligned}$$

We have used Eq.23, and have factorized out $j_{T_y,trig}$,$j_{T_y,asso}$ and k_{T_y} because they are statistically independent from the other variables. We also uses the following relations,

$$\left\langle j_{T_y,trig}^2\right\rangle = \left\langle j_{T_y,asso}^2\right\rangle = \left\langle j_{T_y}^2\right\rangle \tag{26}$$

$$\langle (j_{T_y,asso} j_{T_y,trig})^2\rangle = \left\langle j_{T_y}^2\right\rangle^2 \tag{27}$$

These relations are true if we ignore the correlations between $j_{T_y,asso}$ and $j_{T_y,trig}$ [3].

Similarly for near side p_{out},

$$\begin{aligned}
\langle p_{out,N}^2\rangle &= \left\langle j_{T_y}^2\right\rangle \langle x_h^2(1-x_{j,asso}^2)\rangle \qquad (28)\\
&+\left\langle j_{T_y}^2\right\rangle \langle (1-x_{j,trig}^2)\rangle\\
&= \left\langle j_{T_y}^2\right\rangle \langle 1+x_h^2-2x_{j,trig}^2\rangle
\end{aligned}$$

[3] There might be some correlation due to the trigger bias, the generally relation is $\left\langle j_{T_y}^4\right\rangle > \langle (j_{T_y,asso} j_{T_y,trig})^2\rangle \gtrapprox \left\langle j_{T_y,asso}^2\right\rangle \left\langle j_{T_y,trig}^2\right\rangle = \left\langle j_{T_y}^2\right\rangle^2$

By combining above equations, we have

$$
\begin{aligned}
\left\langle p^2_{out,F}\right\rangle &- \left\langle p^2_{out,N}\right\rangle \left\langle (1-x^2_{k,trig})\right\rangle \qquad (29)\\
&= \left\langle j^2_{T_y}\right\rangle \left[\left\langle x^2_h(1-x^2_{j,asso})\right\rangle \left\langle x^2_{k,trig}\right\rangle - \left\langle x^2_h(1-x^2_{j,asso})x^2_{k,trig}\right\rangle\right]\\
&\quad + \left\langle j^2_{T_y}\right\rangle \left[\left\langle x^2_{j,trig}x^2_{k,trig}\right\rangle - \left\langle x^2_{j,trig}\right\rangle \left\langle x^2_{k,trig}\right\rangle\right]\\
&\quad +2\left\langle k^2_{T_y}\right\rangle \left\langle (x_h z_{trig})^2(1-x^2_{j,asso})(1-x^2_{j,trig})\right\rangle\\
&\quad +2\left\langle k^2_{T_y}\right\rangle \left\langle (z_{trig}x_{j,trig})^2\right\rangle\\
&= \left\langle j^2_{T_y}\right\rangle \left[\left\langle (x^2_h-2x^2_{j,trig})\right\rangle \left\langle x^2_{k,trig}\right\rangle - \left\langle (x^2_h-2x^2_{j,trig})x^2_{k,trig}\right\rangle\right]\\
&\quad +2\left\langle k^2_{T_y}\right\rangle \left\langle (x_h z_{trig})^2(1-x^2_{j,asso})(1-x^2_{j,trig})\right\rangle\\
&\quad +2\left\langle k^2_{T_y}\right\rangle \left\langle (z_{trig}x_{j,trig})^2\right\rangle
\end{aligned}
$$

We have used the relation that $\left\langle x^2_h x^2_{j,asso}\right\rangle = \left\langle x^2_{j,trig}\right\rangle$. Apart from the assumption of statistical independence of the angles and the approximation Eq.23, the relation given by Eq.25, 28 and 29 are exact. These formulae allow us to extract the RMS values for j_T and k_T from the measured the p_{out} distributions. All formulae used in previous correlation analysis are more or less approximations to these three formulae. [4]

Using the knowledge of the typical p_T range of the $p_{T,trig}$ and $p_{T,asso}$ in different analysis, Eq.26-Eq.29 can be further simplified. The typical value of j_{T_y} is about 300-400 MeV/c, typical $\sqrt{2}k_{T_y}z_{trig}$ are about 1-1.5 GeV/c [1]. For $\pi^{\pm} - h$ correlation, trigger p_T typically is larger than 5 GeV/c, and associated particle p_T is above 1 GeV/c. So $x_{j,trig} \lesssim 0.08$, $x_{j,asso} \lesssim 0.4$, $x_{k,trig} \lesssim 0.3$ and $x_h \gtrsim 0.2$. Thus we can expand Eq. 26 and Eq. 28 in Tayler series, and ignore the high order terms,

$$
\begin{aligned}
\left\langle p_{out,F}\right\rangle^2 &= \left\langle j^2_{T_y}\right\rangle \left\langle 1+x^2_h-x^2_{k,trig}-x^2_h x^2_{k,trig}-2x^2_{j,trig}+2x^2_{j,trig}x^2_{k,trig}\right\rangle \qquad (30)\\
&\quad +2\left\langle k^2_{T_y}\right\rangle \left\langle (z_{trig})^2(x^2_h - x^2_h x^2_{j,trig}+x^4_{j,trig})\right\rangle\\
&= \left\langle j^2_{T_y}\right\rangle \left\langle (1+x^2_h)(1-x^2_{k,trig})+\mathcal{O}\left(0.007\right)\right\rangle\\
&\quad +2\left\langle k^2_{T_y}\right\rangle \left\langle (z_{trig}x_h)^2+\mathcal{O}\left(0.007\right)\right\rangle
\end{aligned}
$$

Eq.29 becomes

$$
\begin{aligned}
\left\langle p^2_{out,F}\right\rangle - \left\langle p^2_{out,N}\right\rangle \left\langle (1-x^2_{k,trig})\right\rangle &= \left\langle j^2_{T_y}\right\rangle \left[\left\langle x^2_h\right\rangle \left\langle x^2_{k,trig}\right\rangle - \left\langle x^2_h x^2_{k,trig}\right\rangle + \mathcal{O}\left(0.001\right)\right] \qquad (31)\\
&+ 2\left\langle k^2_{T_y}\right\rangle \left\langle z_{trig}x_h\right)^2+\mathcal{O}\left(0.007\right)\right\rangle\\
&= 2\left\langle k^2_{T_y}\right\rangle \left\langle z^2_{trig}[x^2_h(1+\mathcal{O}\left(0.02\right))]\right\rangle
\end{aligned}
$$

[4] In addition, they also approximate p_{out} with angles.

The last relation come from the fact that assume $j_{T_y} \lesssim 0.5 k_{T_y} z$ [5], so $\left\langle j_{T_y}^2 \right\rangle [\langle x_h^2 \rangle \langle x_{k,trig}^2 \rangle - \langle x_h^2 x_{k,trig}^2 \rangle] \lesssim 0.2 k_{T_y}^2 z_{trig}^2 x_h^2 x_{k,trig}^2 \lesssim 0.02 k_{T_y}^2 z_{trig}^2 x_h^2$.

Alternatively, if we only expands the j_T terms, we can have a more accurate relation from Eq.29,

$$\begin{aligned} \langle p_{out,F}^2 \rangle &- \langle p_{out,N}^2 \rangle \langle 1 - x_{k,trig}^2 \rangle \\ &= 2 \left\langle k_{T_y}^2 \right\rangle \langle z_{trig}^2 [x_h^2 - x_{j,trig}^2 (x_h^2 - x_{j,trig}^2)] \rangle \\ &+ \mathcal{O}(0.005) \end{aligned} \tag{32}$$

The terms that were ignored in above derivation is at most 2 percent for $\pi^{\pm} - h$ correlation. But actually, these formulae can also be used in other correlation analysis with different ranges of trigger and associated particle p_T. In generally, if $p_{T,trig} > 3$ GeV/c and $p_{T,asso} > 1$ GeV/c, Eq.31 can be used to calculate k_T to $< 5\%$ level.

2.3 The formulae for j_T and k_T

In many cases, people measure the distribution of the angle between trigger particle and associated particle, ϕ_{ta}, instead of p_{out}[6]. These two quantities are closely related via Eq.6 and Eq.11, hence the relation between their RMS values are,

$$\begin{aligned} \langle p_{out}^2 \rangle &= \langle p_{T,asso}^2 sin^2 \phi_{ta} \rangle \\ &= \langle p_{T,asso}^2 \rangle \langle sin^2 \phi_{ta} \rangle \\ &= \langle p_{T,asso}^2 \rangle [sin \langle \phi_{ta}^2 \rangle - \frac{\langle \phi_{ta}^4 \rangle}{3} + \frac{19 \langle \phi_{ta}^6 \rangle}{90} + \mathcal{O}(0.01)] \\ &= \langle p_{T,asso}^2 \rangle [sin \sigma^2 - \sigma^4 + \frac{19\sigma^6}{6} + \mathcal{O}(0.01, |\phi| < 1)] \end{aligned} \tag{33}$$

Where we have used the Taylor expansion of $sin^2(\phi)$, and replace the expectation values for difference terms with the RMS value of the angular distribution, σ, assuming that ϕ_{ta} follows Gauss distribution [7]

The difference between $sin^2\phi$ and $sin\phi^2$ are plotted in Fig. 4. This difference is $< 8\%$ at $|\phi_{ta}| < 0.5$, and is less than 20% at $|\phi_{ta}| < 1.4$ [8]. In our analysis, $\sigma_{\phi_{ta}}$ is typically less than 0.3 rad for same side correlation, and is less than 0.6 rad for the away side correlation. Thus, the difference for same side is negligible; while the difference on the away side could be a few percent.

[5] $\langle z \rangle = 0.7$ according to Jan Rak [1]

[6] The so called azimuth correlation function.

[7] In general, if x follows Gauss distribution.

$$\langle |x|^n \rangle = \left\{ \begin{array}{l} (2m-1)!!\sigma^{2m} (n = 2m) \\ (2m)!!\sqrt{\frac{2}{\pi}}\sigma^{2m+1} (n = 2m+1) \end{array} \right\} = (n-1)!!\sqrt{\frac{\pi}{2}}^n \langle |x| \rangle^n \tag{34}$$

[8] More accurate correction is $-\frac{\langle \phi_{ta}^4 \rangle}{3} + \frac{19\langle \phi_{ta}^6 \rangle}{90} - \frac{\langle \phi_{ta}^{10} \rangle}{120} \approx -\sigma^4 + \frac{19\sigma^6}{6} - 8\sigma^{10}$. The difference is less than 0.005 in [-1.6,1.6] range.

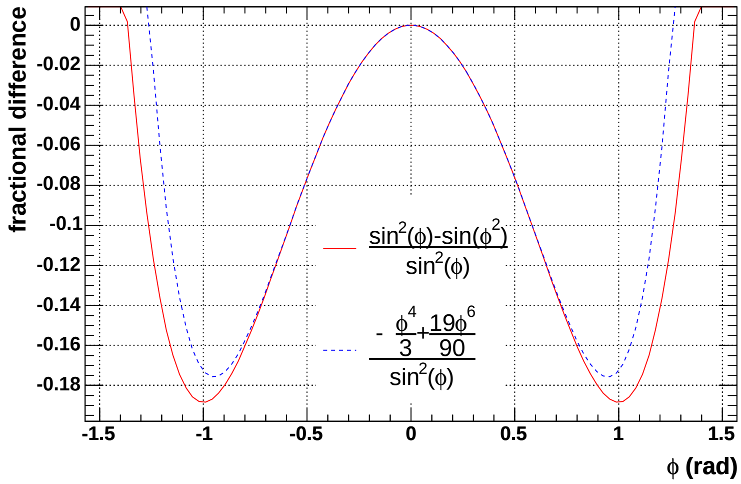

Figure 4: The fractional difference between $sin^2\phi$ and $sin\phi^2$ and it's six order polynomial approximation.

Note that the additional correction in Eq.33 is different from the seagull correction [5]. Seagull correction reflect the fact that j_T and k_T can't be bigger than $p_{T,asso}$ and $p_{T,jet}$, respectively, while our correction here is the correction on the small angle approximations. The six order polynomial correction describe the difference up to 1% level for $|\phi| < 1$.

We are ready to write down the final formulae for calculating the RMS value of j_T, k_T. Since j_T, k_T are two dimensional vectors, the RMS values for 2D is simply factor of $\sqrt{2}$ larger than that for 1D. The results we present here is for 1D RMS value, $V_{y,1D} = \sqrt{\left\langle V_y^2 \right\rangle}$. We do not use the mean of absolute value as most other people do.

The formulae for j_T, k_T using directly measured p_{out} RMS values are (SET1),

$$j_{T_y,1D} = \frac{\left\langle p_{out,N,1D} \right\rangle}{\sqrt{\left\langle 1 + x_h^2 - 2x_{j,trig}^2 \right\rangle}} \tag{35}$$

$$(k_{T_y} z_{trig})_{1D} = \sqrt{\frac{\left\langle p_{out,F,1D} \right\rangle^2 - \left\langle p_{out,N,1D} \right\rangle^2 \left\langle 1 - x_{k,trig}^2 \right\rangle}{2\left\langle x_h^2 - x_{j,trig}^2 (x_h^2 - x_{j,trig}^2) \right\rangle}} \tag{36}$$

The formulae for j_T, k_T using directly measured correlation width are (SET2),

$$j_{T_y,1D} = \sqrt{\frac{\left\langle p_{T,asso}^2 \right\rangle (sin\sigma_N^2 - \sigma_N^4)}{\left\langle 1 + x_h^2 - 2x_{j,trig}^2 \right\rangle}} \tag{37}$$

$$(k_{T_y} z_{trig})_{1D} = \sqrt{p_{T,asso}^2} \sqrt{\frac{(sin\sigma_F^2 - \sigma_F^4 + 16/9\sigma_F^6 - 8\sigma_F^{10}) - (sin\sigma_N^2 - \sigma_N^4) \left\langle 1 - x_{k,trig}^2 \right\rangle}{2\left\langle x_h^2 - x_{j,trig}^2 (x_h^2 - x_{j,trig}^2) \right\rangle}} \tag{38}$$

Eq.36 and Eq.38 can only give the formula $k_{Ty}z$ instead of the k_{Ty} itself. z has to be evaluated separately from the inclusive hadron spectra. Its value is found to be ≈ 0.7 for the case of $\pi^{\pm} - h$ correlation and slightly increase for larger trigger p_T. Further information on determination of $\langle z \rangle$ can be found in [8].

2.4 Formulae for fixed p_T correlation

The formula for fixed p_T correlation can be obtained rather trivially from Eq.35-Eq.38. The j_T formula is,

$$j_{T_y,1D} \quad = \quad \frac{\langle p_{out,N,1D} \rangle}{\sqrt{\langle 1 + x_h^2 - 2x_j^2 \rangle}} \tag{39}$$

The k_T formula is,

$$(k_{T_y} z)_{1D} \quad = \quad \sqrt{\frac{\langle p_{out,F,1D} \rangle^2 - \langle p_{out,N,1D} \rangle^2 \langle 1 - x_k^2 \rangle}{2 \langle x_h^2 - x_j^2 (x_h^2 - x_j^2) \rangle}} \tag{40}$$

Since $\langle p_{T,asso} \rangle = \langle p_{T,trig} \rangle$, $x_j = j_T / p_T$ where p_T can be either trigger or partners. $x_h = p_{T,asso}/p_{T,trig}$, since trigger and associated particles have the same p_T range, so the mean of x_h, $\langle x_h \rangle = 1$. However we have to remember that $\langle x_h^2 \rangle = \langle x_h \rangle^2 + \sigma^2 = 1 + \sigma^2 > 1$!!, where σ is the standard deviation of x_h. Clearly, the difference between $\langle x_h^2 \rangle$ and $\langle x_h \rangle^2$ are sensitive to the width of the p_T bin.

3 Discussion

3.1 Comparing with other formulae

There are many versions of j_T k_T formulae used by Mike, Nathan, Jan, and Wolf[4, 7, 8, 9]. These formulae are different levels of approximations to our previously derived formulae. To check the difference of those formulae from ours, we pick the following set as example.

$$j_{T_y,1D} = \frac{\langle p_{T,asso} \rangle \langle p_{T,trig} \rangle \sigma_N}{\sqrt{\langle p_{T,asso} \rangle^2 + \langle p_{T,asso} \rangle^2}} = \frac{\langle p_T \rangle \sigma_N}{\sqrt{1 + x_h^2}} \tag{41}$$

$$(k_{T_y} z_{trig})_{1D} \quad = \quad \frac{\sqrt{\frac{\pi}{2}} \langle p_T \rangle}{\sqrt{2} x_h} \sqrt{sin^2 \sqrt{\frac{2}{\pi}} \sigma_F - (1 + x_h^2) sin^2 \frac{\sigma_N}{\sqrt{\pi}}} \tag{42}$$

Comparing Eq.41,42 and Eq.35,36(or Eq.37,38), the most significant differences are,

1. Eq.42 missed the factor $\langle 1 - x_{k,trig}^2 \rangle$ which exists in our formulae, this term is not negligible, especially when trigger p_T is low. For a 3 GeV/c trigger, $x_{k,trig} \approx 0.35$ assuming $k_{T,y} = 1$ GeV/c and $z_{trig} = 0.7$.

2. In general $\left\langle \sqrt{x^2} \right\rangle \neq \langle x \rangle$, in most analysis, $\langle x \rangle$ was used instead of $\left\langle \sqrt{x^2} \right\rangle$. Examples is the $\langle p_T \rangle$, $\langle x_h \rangle$. This can create up to 10% difference in the final results if the p_T bin width for trigger and associated particles are large.

3. The difference between $sin^2(\sigma)$ and $sin(\sigma^2)$ or between $sin(c \times \sigma)$ and $c \times sin(\sigma)$. The difference between $sin^2(\sigma)$ and $sin(\sigma^2)$ is already shown in Fig.4. The difference between $sin(c\times\sigma)$ and $c\times sin(\sigma)$ can be seen from Fig.5. The fact that a multiplicative factor exist in $sin(c \times \sigma)$, where $c = \sqrt{1/\pi}$ or $\sqrt{1/\pi}$ is a artifact resulting from evaluating the mean of the absolute value instead of evaluating directly the RMS value. Clearly, as long as the we stay with reasonable small angle $|\phi| < 0.5$, the difference is less than 10%. For large width, we have to apply additional correction for Eq.42.

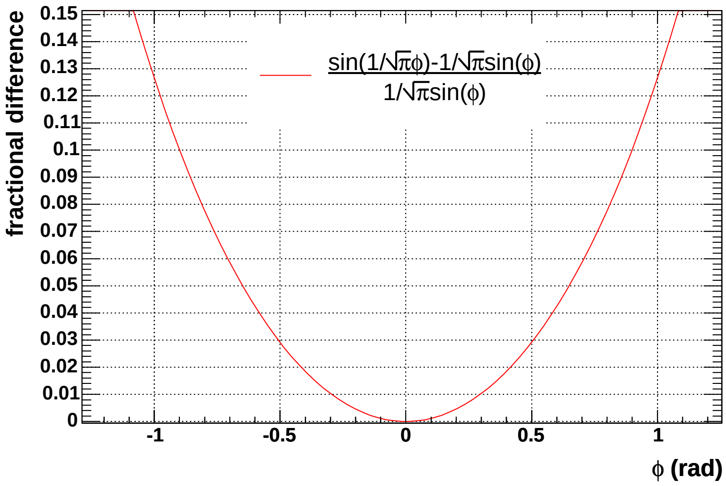

Figure 5: The fractional difference between $sin\frac{1}{\sqrt{\pi}}\phi$ and $\frac{1}{\sqrt{\pi}}sin\phi$.

3.2 Limitations

In deriving the j_T, k_T formulae, i.e. Eq.35 to Eq.38, we have tried to be as general as possible. But in any case, we still made a number of assumptions, who's systematics errors need to be evaluated. There are mainly four important assumptions or simplifications,

1. We have used approximation Eq.23 for the azimuth angle between two partons, ϕ_{qq}. Obviously, the higher order term $\mathcal{O}(\frac{1}{\mathrm{p}_{\mathrm{T,jet}}^2})$ become important when k_T becomes comparable to the jet momentum $p_{T,jet}$. The error on the approximation can be estimated using a simple MC simulation. In this simulation, we generate back-to-back jets with

fixed initial p_T, each jet then obtains a $\vec{k_T}$, it's x and y component k_{Tx} and k_{Ty} were generated randomly according to a Gauss distribution with a mean of 0 and width of 1 GeV/c. We then calculate the angle between the two jets and fill a histogram. Fig.6 shows the distribution of $sin(\phi_{qq})$ calculated according to Eq.12 and it's approximation Eq.23, assuming that the initial jet momentum is $p_{T,jet} = 2$ GeV/c. Clearly, since $k_{Ty} = 1$ GeV/c is almost 50% of the initial jet energy, the approximation is not valid anymore. In fact, it totally missed the shape of the distribution, and has a much larger RMS values. The true distribution stop at -1 and 1, while the estimated distribution extends beyond 1. The sharp increases close to 1 and -1 for the 'exact distribution' come from the fact that the k_T can be larger than the jet momentum, which means that there is a singularity point in Eq.12 because the denominator can become zero. Fig.7 and Fig.8 shows the results for higher initial jet p_T of 3 GeV/c and

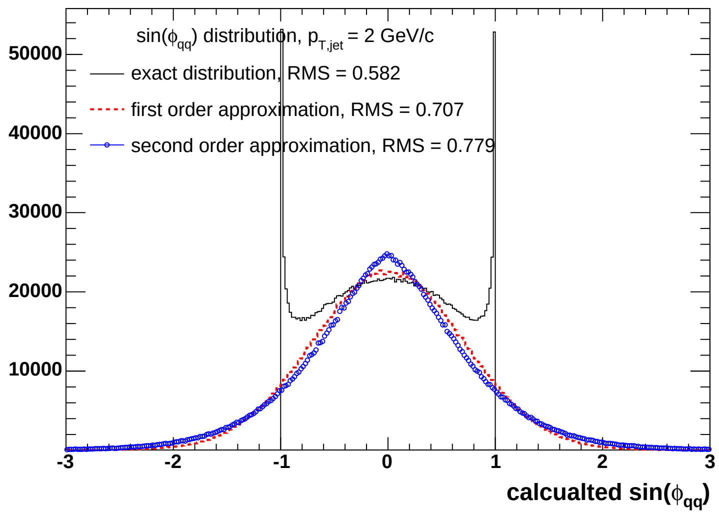

Figure 6: The calculated sin of the angle between the jets with initial p_T of 2 GeV/c, $sin(\phi_{qq})$ using exact form Eq.12(solid line), first order approximation Eq.23(dashed line) and second order approximation Eq.22(open circles). The RMS value of the three distributions are also shown.

5 GeV/c, respectively. Clearly, as the jet p_T becomes much larger than the typical k_T, the difference between exact solution and approximation become smaller, as expected.

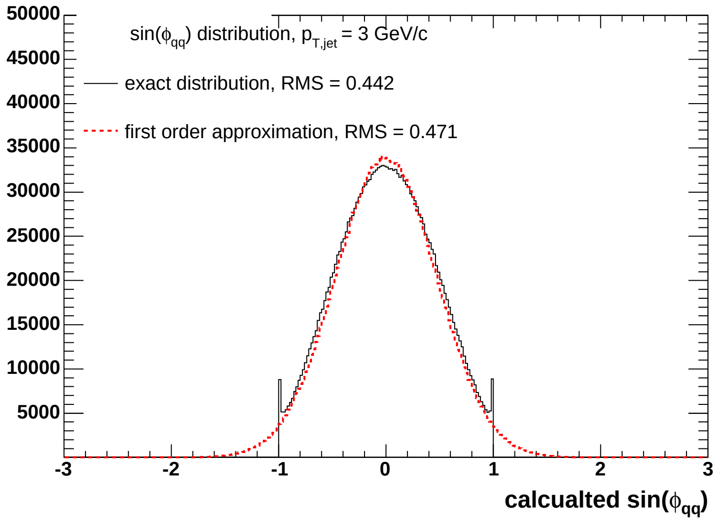

Figure 7: The calculated sin of the angle between the jets with initial p_T of 3 GeV/c, $sin(\phi_{qq})$ using exact form Eq.12(solid line) and first order approximation Eq.23(dashed line). The RMS value of the three distributions are also shown.

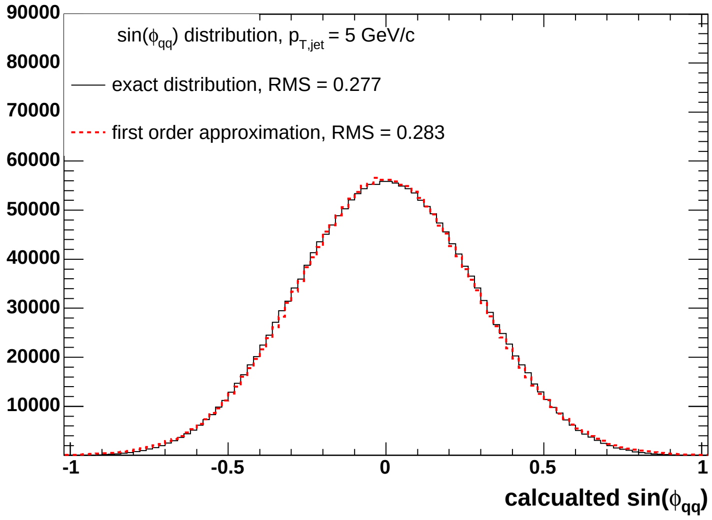

Figure 8: The calculated sin of the angle between the jets with initial p_T of 5 GeV/c, $sin(\phi_{qq})$ using exact form Eq.12(solid line) and first order approximation Eq.23(dashed line). The RMS value of the three distributions are also shown.

2. The assumption of the statistical independence of angles between trigger-associated(ϕ_{ta}), trigger-jet(ϕ_{tq}), associated-jet(ϕ_{aq}) and jet-jet(ϕ_{qq}). We believe this assumption is true.

3. The assumption that p_{out} or the angle ϕ_{ta} follows Gauss statistics. This is only true when $|p_{out}| \ll p_{T,asso}$ or $|\phi_{qq}| \ll 1$. Otherwise, one have to take into account 1) the Seagull effect, and 2) the contamination in the same side jet peak from the tail of the away side jet. The magnitude of these two effects sets the lower thresholds for both the $p_{T,trig}$ and $p_{T,asso}$. We believe that going beyond these p_T thresholds would introduce large systematic errors and would make the results unstable. Some of the study in this direction has been done by Jan Rak[8]. Based on his study and our own study with the real data, we believe both $p_{T,trig}$ and $p_{T,asso}$ should be above 2 GeV/c for stable j_T and the $p_{T,trig}$ should be above 3 GeV/c for extracting reliable k_T. Further systematic study is clearly desired.

4. When using Eq.24, Eq.28 and Eq.29, we have assumed the statistical independence of several terms in the brackets in order to simplify the formula. One counterexample is $\langle z_{trig}x_h \rangle$, in general it does not equal to $\langle z_{trig} \rangle \langle x_h \rangle$ because the two terms are strongly correlated, in fact $z_{trig}x_h = p_{asso}/p_{quark} = z_{asso}$, which does not depends trigger momentum! The effects of all such correlations among the variables are very sensitive to the size of the momentum bin used for the trigger and associated particles. We believe, the error we make for the p_T ranges used in the $\pi^{\pm} - h$ correlation is less than 10%. The effect for other correlation analysis need to be checked with detailed Monte-carlo study.

Finally, we want to make the point that the physics content carried with RMS values of j_T and k_T is limited. Ideally, we would like to extract the full j_T, k_T spectra instead of just the RMS value. Mathematically, it is conceivable that two distributions can have the same RMS values but totally different shapes. Due to the parton bremsstrahlung, both j_T and k_T distributions tends to have power law type of tails which very likely to be missed by current j_T and k_T analysis(because we assume Gaus statistics). We recommend that the measurement of the full p_{out} distributions would overcome some of the shortcomings.

References

[1] PPG29, /phenix/WWW/p/draft/rak/ppg029.

[2] Analysis note 183.

[3] Addendum to Analysis note 183.

[4] Analysis note 221, M. J. Tannenbaum, Two formulas for jet fragmentation.

[5] Analysis note 251.

[6] Analysis note 257.

[7] /phenix/WWW/p/draft/ncgrau/Run3/pi0h_AN/anaNote.pdf.

[8] /phenix/WWW/p/draft/rak/ppg029/note.ps.

[9] Analysis note 283.