

Algorithms for numerical solution of initial value problems

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Question 1

$$y_{n+3} + (2b - 3)(y_{n+2} - y_{n+1}) - y_n = hb(f_{n+2} + f_{n+1}), b \in \mathbb{R}$$

Method stability determined by roots of first characteristic polynomial of magnitude ≤ 1 with values of b .

$$y_{n+3} + (2b - 3)y_{n+2} + (3 - 2b)y_{n+1} - y_n$$

$$\text{Giving alphas: } \alpha_3 = 1 \quad \alpha_2 = 2b - 3 \quad \alpha_1 = 3 - 2b \quad \alpha_0 = -1$$

Which gives us the first characteristic polynomial:

$$\rho(\xi) = \xi^3 + (2b - 3)\xi^2 + (3 - 2b)\xi - 1$$

$$\text{Factorizing gives: } (\xi - 1)(\xi^2 + 2b\xi - 2\xi + 1)$$

Root at $\xi = 1$. Quadratic formula gives the roots:

$$\left[\xi = -\sqrt{b^2 - 2b} - b + 1, \xi = \sqrt{b^2 - 2b} - b + 1, \xi = 1 \right]$$

The interval of values for the real constant b for which the method is stable is $0 < b < 2$

Question 2

A linear multistep method is of order p if:

$$C_0 = C_1 = \dots = C_p = 0, C_{p+1} \neq 0$$

Constants to check for equality to 0:

$$C_0 = \alpha_0 + \alpha_1 + \alpha_2 + \alpha_3$$

$$C_1 = \alpha_1 + 2\alpha_2 + 3\alpha_3 - (\beta_0 + \beta_1 + \beta_2 + \beta_3)$$

$$C_j = \frac{1}{j!}(\alpha_1 + 2^j\alpha_2 + 3^j\alpha_3) - \frac{1}{(j-1)!}(\beta_1 + 2^{j-1}\beta_2 + 3^{j-1}\beta_3)$$

Question 2

Using:

$$\sum_{i=0}^k \alpha_i y_{n+i} = h \sum_{i=0}^k \beta_i f_{n+i}, n = 0, 1, \dots$$

Gives alphas:

$$\alpha_3 = 1$$

$$\alpha_2 = 2b - 3$$

$$\alpha_1 = -(2b - 3)$$

$$\alpha_0 = -1$$

And betas:

$$\beta_3 = 0$$

$$\beta_2 = b$$

$$\beta_1 = b$$

$$\beta_0 = 0$$

Question 2

Calculate the constants

$$C_0 = -1 - (2b - 3) + (2b - 3) + 1 = 0$$

$$C_1 = -(2b - 3) + 2(2b - 3) + 3 - 2b = 0$$

$$C_2 = \frac{1}{2}(-(2b - 3) + 4(2b - 3) + 9) - 3b = \frac{1}{2}(6b - 9 + 9) - 3b = 0$$

$$\begin{aligned} C_3 &= \frac{1}{6}[-(2b - 3) + 8(2b - 3) + 27] - \frac{1}{2}(b + 4b) \\ &= \frac{1}{6}[7(2b - 3) + 27] - \frac{1}{2}5b = \frac{14b}{6} + 1 - \frac{5b}{2} \\ &= 1 - \frac{b}{6} \neq 0, \text{ for } b \in (0, 2) \end{aligned}$$

So,

$$p = 2$$

Question 3

For t_1 :

$$t_1 = t_0 + h_E = h_E$$

$$y_{1,1} = h_E(\mu - \beta(h_E)y_{1,0}y_{3,0}) + y_{1,0}$$

$$y_{2,1} = h_E\left(\beta(h_E)y_{1,0}y_{3,0} - \frac{1}{\lambda}y_{2,0}\right) + y_{2,0}$$

$$y_{3,1} = h_E\left(\frac{1}{\lambda}y_{2,0} - \frac{1}{\eta}y_{3,0}\right) + y_{3,0}$$

If we had numbers, we should now solve these three equations to get numerical answer for $y_{1,1}$ $y_{2,1}$ and $y_{3,1}$

For t_2 :

$$t_2 = 2h_E$$

$$y_{1,2} = h_E(\mu - \beta(h_E)y_{1,0}y_{3,0}) + y_{1,0} + (\mu - \beta(2h_E)y_{1,0}y_{3,0})h_E$$

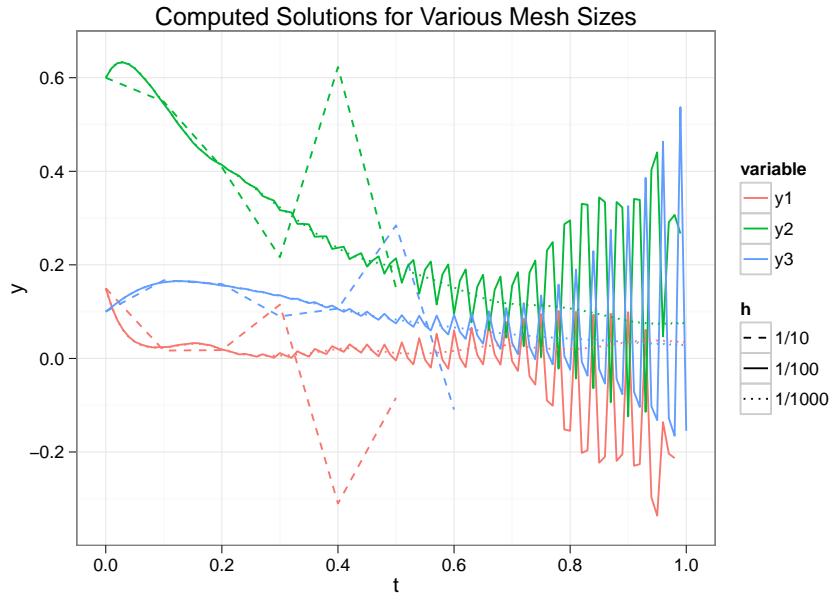
$$y_{2,2} =$$

$$h_E\left(\beta(h_E)y_{1,0}y_{3,0} - \frac{1}{\lambda}y_{2,0}\right) + y_{2,0} + h_E\left(\beta(2h_E)y_{1,0}y_{3,0} - \frac{1}{\lambda}y_{2,0}\right)$$

$$y_{3,2} = h_E\left(\frac{1}{\lambda}y_{2,0} - \frac{1}{\eta}y_{3,0}\right) + y_{3,0} + h_E\left(\frac{1}{\lambda}y_{2,0} - \frac{1}{\eta}y_{3,0}\right)$$

Again we should solve these equations simultaneously to $y_{1,2}$ $y_{2,2}$ and $y_{3,2}$.

Question 4



Question 4

