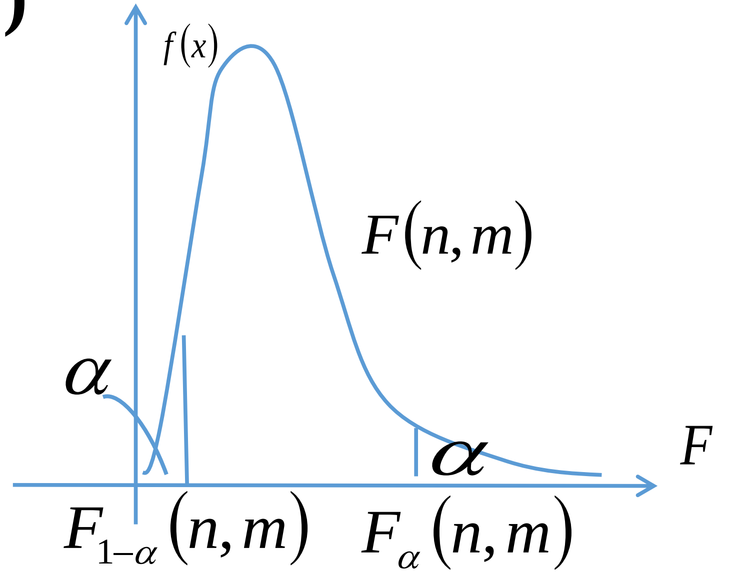
4.F分布

（1）定义：$U \sim \chi^2(n), V \sim \chi^2(m)$，$U$与$V$独立，则称 $F=\dfrac{U/n}{V/m}$

服从自由度为**(*n*，*m*)**的**F**分布，记为**F(*n*，*m*)**

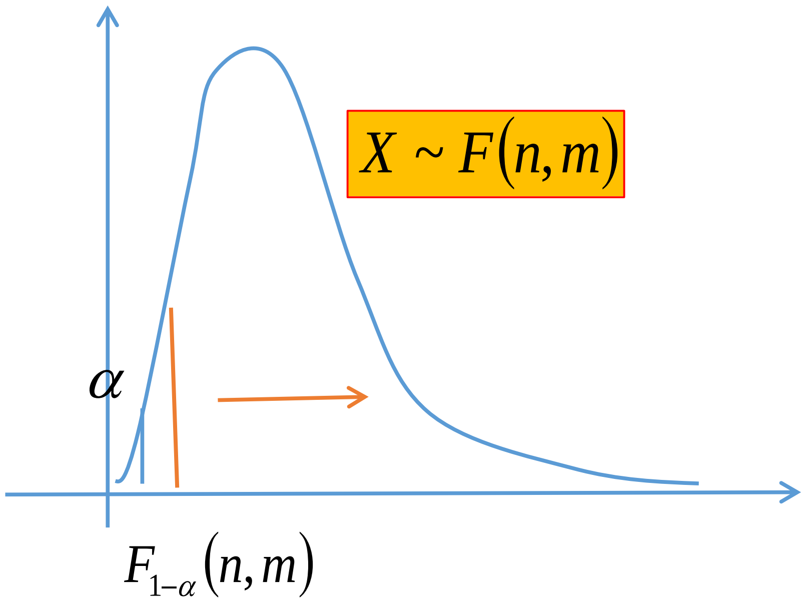
（2）性质：i）若 $X \sim F(n,m)$，则$\dfrac{1}{X} \sim F(m,n)$;

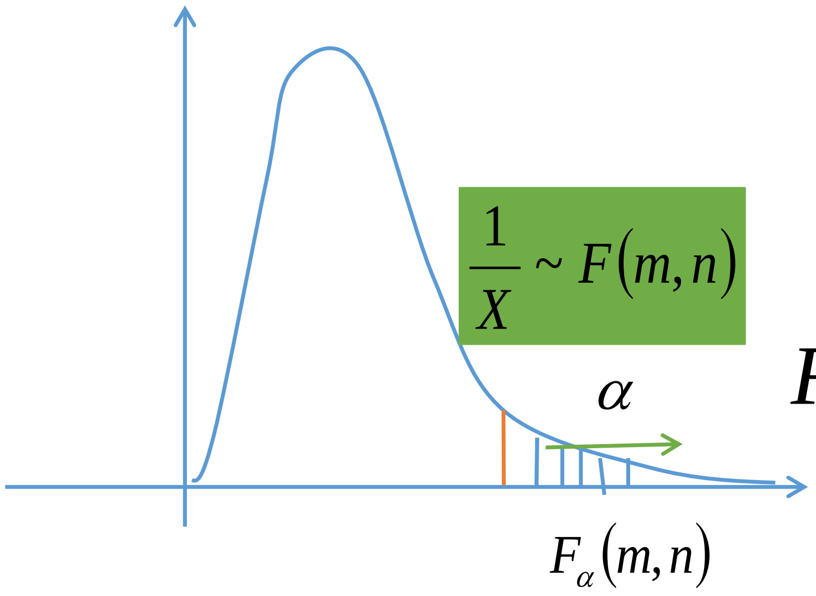
ii）$F_{1-\alpha}(n,m)=\dfrac{1}{F_{\alpha}(m,n)}$;



iii）若$t \sim t(n)$；则$t^2=\left(\dfrac{X}{\sqrt{Y/n}}\right)^2=\dfrac{X^2/1}{Y/n} \sim F(1,n)$

简单了解





$$F_{1-\alpha}(n,m) = \frac{1}{F_{\alpha}(m,n)}$$

$$P\left(X > F_{1-\alpha}(n,m)\right) = 1-\alpha$$

$$F_{\alpha}(m,n) = \frac{1}{F_{1-\alpha}(n,m)}$$

$$P\left(\frac{1}{X} < \frac{1}{F_{1-\alpha}(n,m)}\right) = 1 - P\left\{\frac{1}{X} \geq \frac{1}{F_{1-\alpha}(n,m)}\right\} = 1-\alpha$$

$$P\left\{\frac{1}{X} \geq \frac{1}{F_{1-\alpha}(n,m)}\right\} = \alpha,$$

又有 $P\left\{\frac{1}{X} \geq F_{\alpha}(m,n)\right\} = \alpha$

（3）分位点：（以α=0.05维离）

1）双侧分位点：$F_{0.975}(n,m)=1/F_{0.025}(m,n)$ 及 $F_{0.025}(n,m)$

非小概率事件区间：$\{1/F_{0.025}(m，n)，F_{0.025}(n，m)\}$

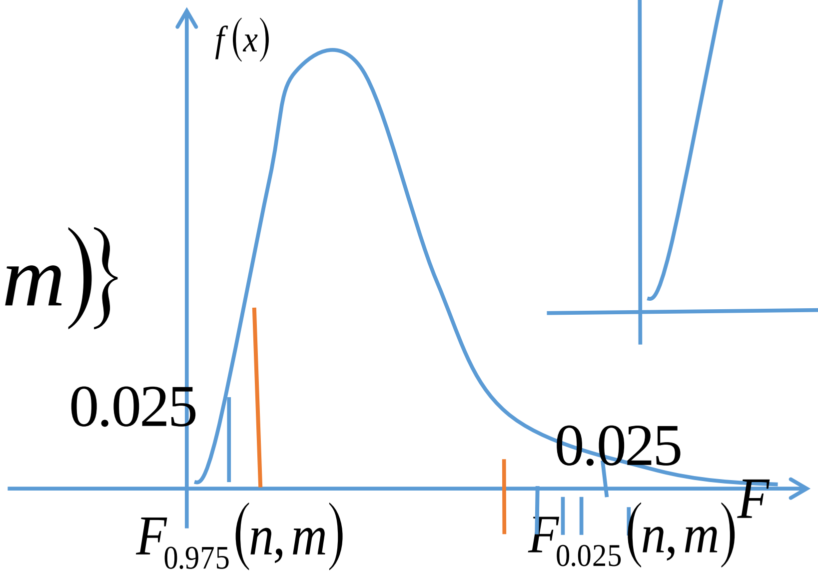
小概率事件区间：$\{0, 1/F_{0.025}(m，n)\} \cup \{F_{0.025}(n，m), +\infty\}$

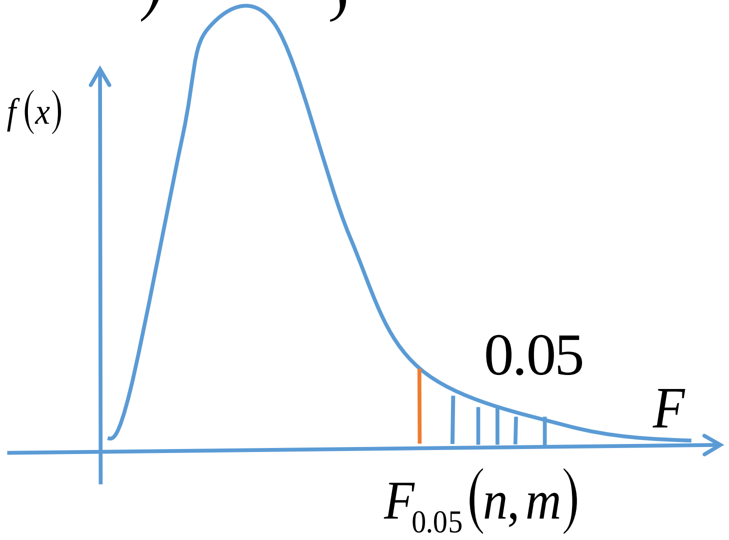
2）单侧分位点：

单侧上限分位点：$F_{0.05}(n,m)$

非小概率事件区间：$\{0，F_{0.05}(n,m)\}$

小概率事件区间：$\{F_{0.05}(n,m),+\infty\}$

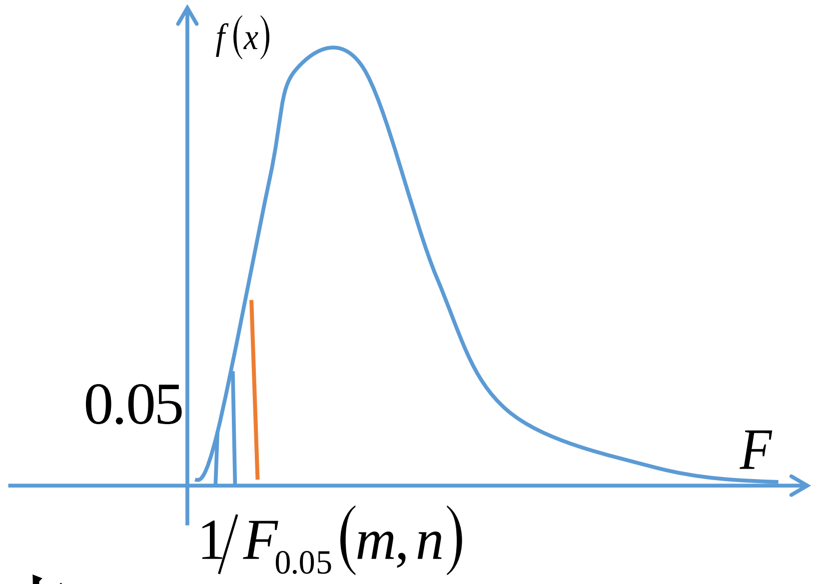




单侧上限分位点（过低异常）：$F_{0.95}(n,m)=1/F_{0.05}(m,n)$

非小概率事件区间：$\left(1/F_{0.05}(m,n),+\infty\right)$

小概率事件区间： $\left(0\ 1/F_{0.05}(m,n)\right)$



例1.（1）当α=0.05，求 $X\sim F(9,15)$双侧分位点。

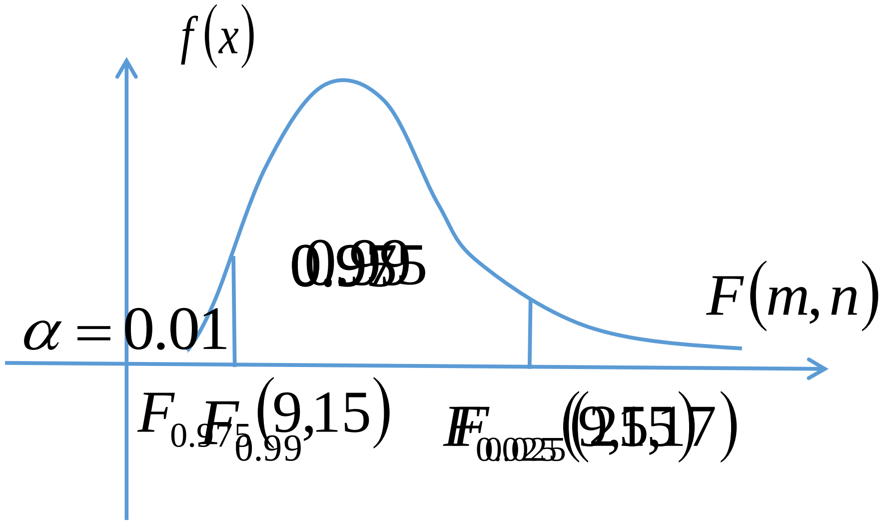
（2）当α=0.01，求 $X\sim F(13,11)$单侧下限分位点。

（3）当α=0.025，求 $X\sim F(25,17)$单侧上限分位点。

解：（1）当α=0.05， $F(9,15)$双

$$F_{0.025}(9,15)=3.12$$

$$F_{0.975}(9,15)=\frac{1}{F_{0.025}(15,9)}=\frac{1}{3.77}$$



（2）当α=0.01， $F(13,11)$下

$$F_{0.99}(13,11)=\frac{1}{F_{0.01}(11,13)}=\frac{2}{4.1+3.96}$$

（3）当α=0.025，$F(25,17)$上 $F_{0.025}(25,17)=2.56$

例1.总体$X \sim N(0,1)$, $X_1, X_2 \cdots X_n$是简单随机样本，下列统计量各服从什么分布？

(1) $X_1^2 + X_2^2$ (2) $\dfrac{X_1 - X_2}{\sqrt{X_3^2 + X_4^2}}$ (3) $\dfrac{X_2}{|X_6|}$ (4) $\dfrac{\left(\frac{n}{3}-1\right)\sum_{i=1}^{3} X_i^2}{\sum_{i=4}^{n} X_i^2}$

解 (1) $X_1^2 + X_2^2 \sim \chi^2(2)$

(2) $X_1 - X_2 \sim N(0, 2)$, $\dfrac{X_1 - X_2}{\sqrt{2}} \sim N(0, 1)$, $X_3^2 + X_4^2 \sim \chi^2(2)$

$X_1, X_2 \cdots X_n$相互独立，则 $\dfrac{(X_1 - X_2)/\sqrt{2}}{\sqrt{X_3^2 + X_4^2 / 2}} = \dfrac{X_1 - X_2}{\sqrt{X_3^2 + X_4^2}} \sim t(2)$

(3) $\dfrac{X_2}{|X_6|}$, $X_2 \sim N(0,1)$, $X_6^2 \sim \chi^2(1)$, $\dfrac{X_2}{\sqrt{X_6^2/1}} = \dfrac{X_2}{|X_6|} \sim t(1)$

(4) $\dfrac{\left(\frac{n}{3}-1\right)\sum_{i=1}^{3} X_i^2}{\sum_{i=4}^{n} X_i^2}$, $\sum_{i=1}^{3} X_i^2 \sim \chi^2(3)$, $\sum_{i=4}^{n} X_i^2 \sim \chi^2(n-3)$,

$$\frac{\sum_{i=1}^{3} X_i^2 \Big/ 3}{\sum_{i=4}^{n} X_i^2 \Big/ n-3} = \frac{\left(\frac{n}{3}-1\right)\sum_{i=1}^{3} X_i^2}{\sum_{i=4}^{n} X_i^2} \sim F(3,n-3)。$$