# Comparison of Image Patches Using Local Moment Invariants

Atilla Sit and Daisuke Kihara

Abstract—We propose a new set of moment invariants based on Krawtchouk polynomials for comparison of local patches in 2D images. Being computed from discrete functions, these moments do not carry the error due to discretization. Unlike many orthogonal moments which usually capture global features, Krawtchouk moments can be used to compute local descriptors from a region-of-interest in an image. This can be achieved by changing two parameters, and hence shifting the center of interest region horizontally and/or vertically. This property enables comparison of two arbitrary local regions. We show that Krawtchouk moments can be written as a linear combination of geometric moments, so easily converted to rotation, size, and position independent invariants. We also construct local Hubased invariants by using Hu invariants and utilizing them on images localized by the weight function given in the definition of Krawtchouk polynomials. We give the formulation of local Krawtchouk-based and Hu-based invariants, and evaluate their discriminative performance on local comparison of artificially generated test images.

Index Terms—Local image comparison, Krawtchouk polynomials, Krawtchouk invariants, Hu invariants, region of interest, discrete orthogonal functions, local descriptors, weighted Krawtchouk polynomials.

## I. INTRODUCTION

THE use of moment invariants in object recognition is widespread. Algebraic invariants was first used by Hu in recognition of 2D images [1]. Hu invariants then became popular in image processing due to their rotation, size, and position independence. Despite their recognition power, these moments are not orthogonal, thus reconstruction of images from these moments remains as a difficult task.

Moment invariants based on continuous orthogonal polynomials have also been used for image representation and comparison. A widely known one is the set of Zernike moments for their ability to retrieve images with minimal redundant information and having the property of rotational invariance, which is critical in object recognition. Being extracted from a continuous set of functions, these moments require discretization of basis functions in practice, which can cause inaccuracy

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within the computed moments, especially when the order of moments increases. These polynomials are defined only inside the unit disk, so calculation of Zernike moments requires coordinate transformation and proper approximation of the continuous moment integrals [2], [3], [4].

In order to correct these drawbacks, moment invariants based on discrete orthogonal functions have been proposed. One prominent example is Krawtchouk polynomials which were introduced by Mikhail Krawtchouk [5] and used for the first time in image analysis by Yap et al. [6]. They have been employed in image processing and pattern recognition fields including image reconstruction [6], [7], image watermarking [8], and face recognition [9], [10]. These polynomials are discrete, so no numerical approximation is included in deriving the moments of images that are constructed with discete pixels. Moreover, they are directly defined in the image coordinate space and hence their orthogonality property is well retained in the computed moments. Many popular moments, e.g. Zernike, Legendre, and Chebyshev moments, can represent only global features of images. However, Krawtchouk moments have the ability to extract not only global information, but also local descriptors from any region-of-interest in an image.

Local descriptors have become indispensible tools in image analysis and used in a number of applications such as object recognition [11], [12], [13], image retrieval [14], and shape matching [15]. They can be computed more efficiently than global descriptors, are robust to occlusion, generally less sensitive to viewpoint changes, and do not require segmentation [13], [16]. In general, a set of key points on an image is generated and then compared to those on another image through local descriptors computed for each keypoint. SIFT (Scale Invariant Feature Transform) [12] and other SIFT-based descriptors, such as PCA-SIFT [13] and GLOH [16], are some examples of local descriptors in the literature. SIFT method is a combination of a scale invariant region detector and a descriptor based on the gradient distribution in the detected regions. The descriptors are represented by a 3D histogram of gradient locations and orientations, so they usually require a large number of dimensions to build a reliable descriptor [16]. Moment invariants are still popular in object recognition due to their independence and easy computation for any order and degree.

In this paper, we propose a new set of local descriptors based on Krawtchouk polynomials, called *local Krawtchouk-based invariants*, which are not only rotation, position, and size independent, but also preserve their ability to extract features from any local interest region in an image. This can be achieved by changing two parameters  $p_x$  and  $p_y$ , given

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in the definition of 2D Krawtchouk polynomials. The local recognition ability of these polynomials has been realized in [7], but not well-studied so far. The locality property of these moments was discussed briefly, yet the invariants introduced in [7] still remained global.

We show that local Krawtchouk-based invariants of an image can be formed by a linear combination of geometric moment invariants of the image localized by the weight function associated with the 2D Krawtchouk polynomials. The weight function plays a critical role in the locality property of these descriptors. It contains two parameters  $p_x$  and  $p_y$ , which shift the center of local interest region horizontally and vertically, respectively. The weight functions, however, vary from one place to another. In order to compute a standardized weight, we first compute it from a fixed local region and then use it elsewhere by simply translating the graph of the weight to the region-of-interest. That way, we guarantee that the performance of our descriptors does not depend on the interest region. Note that this also ensures to have the invariance under translation. The recognition power of these invariants is finally compared with that of *local Hu-based invariants* which can be computed from images weighted by the weight function of Krawtchouk polynomials. Computing such moment invariants is very efficient in practice compared to computing descriptors using conventional approaches [16]. A recent work on local moment invariants can be found in Doretto and Yao [17], where they compute region moments, which are based on image moments and linearly related to image features. These region moments are computed over explicitly defined small rectangles, whereas our local invariants are extracted similarly to global ones, with region sizes depending on the weight function and degree of invariants.

This paper is organized as follows. In Section II, we briefly introduce Krawtchouk polynomials. After giving the necessary background on 2D weighted Krawtchouk polynomials in Section III, we present the formulation of our new local Krawtchouk-based and local Hu-based invariants in Section IV. Section V is devoted to comparison of the two sets of invariants for their local recognition performance on artificially generated datasets. In Section VI, we conclude with a summary of this work.

#### II. KRAWTCHOUK POLYNOMIALS

First, we introduce Krawtchouk polynomials and their computation methods as previously discussed in [6], [7], [18]. The *n*th order Krawtchouk polynomials are defined as

$$K_n(x; p, N) = \sum_{k=0}^n a_{k,n,p,N} x^k = {}_2F_1(-n, -x; -N; \frac{1}{p})$$
 (1)

where  $x, n = 0, ..., N, N > 0, p \in (0,1)$  and the function  ${}_{2}F_{1}$  is the hypergeometric function which is defined as:

$$_{2}F_{1}(a,b;c;z) = \sum_{k=0}^{\infty} \frac{(a)_{k}(b)_{k}}{(c)_{k}} \frac{z^{k}}{k!}.$$
 (2)

The symbol  $(a)_k$  in (2) is the Pochhammer symbol given by

$$(a)_k = a(a+1)(a+2)\dots(a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)}.$$
 (3)

Note that the series in (2) terminates if either a or b is a nonpositive integer. Hence, the polynomial coefficients  $a_{k,n,p,N}$  in (1) can be obtained by simplifying the summation. The first three Krawtchouk polynomials are

$$K_0(x; p, N) = 1,$$

$$K_1(x; p, N) = 1 - \left(\frac{1}{Np}\right)x,$$

$$K_2(x; p, N) = 1 - \left(\frac{2}{Np} + \frac{1}{N(N-1)p^2}\right)x + \left(\frac{1}{N(N-1)p^2}\right)x^2.$$

The weighted Krawtchouk polynomials can be defined by

$$\bar{K}_n(x;p,N) = K_n(x;p,N) \sqrt{\frac{w(x;p,N)}{\rho(n;p,N)}}$$
(4)

where

$$w(x; p, N) = \binom{N}{x} p^x (1-p)^{N-x},$$
 (5)

$$\rho(n; p, N) = (-1)^n \left(\frac{1-p}{p}\right)^n \frac{n!}{(-N)_n}.$$
 (6)

The set of weighted Krawtchouk polynomials

$$\bar{S} = \{\bar{K}_n(x; p, N) : n = 0, \dots, N\}$$
 (7)

becomes a complete orthonormal set of basis functions on the discrete space  $\{0,\dots,N\}$  with the orthonormality condition

$$\sum_{x=0}^{N} \bar{K}_n(x; p, N) \bar{K}_{n'}(x; p, N) = \delta_{nn'}.$$
 (8)

To compute the weighted Krawtchouk polynomials, we use the three-term recurrence relation given in [7]. Such a recursive computation is shown to be more efficient than computing high order polynomials directly using (1) and (4). However, due to error propagation, computing polynomials recursively may still be numerically unstable for large N as noted by Zhang et al. [18]. To achieve numerical stability, we use an ascending three-term recurrence relation given in [18] to compute the first  $\lfloor \frac{N}{2} \rfloor + 1$  weighted Krawtchouk polynomials, namely  $\bar{K}_n(x; p, N)$  for  $n = 0, \dots, \lfloor \frac{N}{2} \rfloor$ . The remaining polynomials are computed by using a descending recurrence relation. In the end, the functions  $\bar{K}_n(x; p, N)$  can be computed for all n = 0, ..., N, but not for all possible values of x. This incomplete data can be easily complemented without further computation by using a symmetry relation that weighted Krawtchouk polynomials satisfy. For more details of this symmetry and bi-recursive algorithm, the readers are referred to [18] and [19].

# III. TWO-DIMENISONAL WEIGHTED KRAWTCHOUK MOMENTS

In this section, we give a brief formulation of 2D weighted Krawtchouk moments. Note that the functions  $\bar{K}_n$  defined by (4) are orthonormal in the one-dimensional discrete set  $\{0,\ldots,N\}$ , but they can be easily extended to 2D as mentioned in [6], [7].

Let

$$A = \{0, \dots, N\} \times \{0, \dots, M\}$$
 (9)

be a discrete field in the 2D space. We define the set of 2D weighted Krawtchouk polynomials on A as

$$\bar{S} = \{ \bar{K}_n(x; p_x, N) \cdot \bar{K}_m(y; p_y, M) : n = 0, \dots, N, m = 0, \dots, M \}.$$
 (10)

Note that  $\bar{S}$  is orthonormal on A with the orthonormality condition

$$\sum_{x=0}^{N} \sum_{y=0}^{M} \bar{K}_{n}(x; p_{x}, N) \, \bar{K}_{m}(y; p_{y}, M)$$

$$\cdot \, \bar{K}_{n'}(x; p_{x}, N) \, \bar{K}_{m'}(y; p_{y}, M) = \delta_{nn'} \delta_{mm'},$$
(11)

which follows immediately from the orthonormality of 1D functions given by (8).

Let f(x, y) be a 2D function defined on A. The weighted 2D Krawtchouk moments of order n + m of f are defined by

$$\bar{Q}_{nm} = \sum_{x=0}^{N} \sum_{y=0}^{M} f(x,y) \cdot \bar{K}_n(x; p_x, N) \, \bar{K}_m(y; p_y, M). \quad (12)$$

Note that by using (11) and solving (12) for f(x,y), the 2D function f(x,y) can be written in terms of the Krawtchouk moments, i.e.

$$f(x,y) = \sum_{n=0}^{N} \sum_{m=0}^{M} \bar{Q}_{nm} \cdot \bar{K}_{n}(x; p_{x}, N) \, \bar{K}_{m}(y; p_{y}, M).$$
 (13)

This means that the image f(x,y) can be reconstructed perfectly if all the moments  $\bar{Q}_{nm}$  are used for  $n=0,\ldots,N,$   $m=0,\ldots,M.$  An approximate reconstruction  $\hat{f}$  of f can be written as

$$\hat{f}(x,y) = \sum_{n=0}^{\hat{N}} \sum_{m=0}^{\hat{M}} \bar{Q}_{nm} \cdot \bar{K}_n(x; p_x, N) \, \bar{K}_m(y; p_y, M). \tag{14}$$

where  $0 \le \hat{N} \le N$ ,  $0 \le \hat{M} \le M$ . Note that equation (12) can be written as a matrix product

$$\mathbf{Q} = \mathbf{K}_{x}^{T} \mathbf{F} \mathbf{K}_{y}, \tag{15}$$

where

$$\mathbf{Q} = [Q_{nm}], \quad n = 0, \dots, N, \quad m = 0, \dots, M,$$

$$\mathbf{K}_x = [\bar{K}_n(x; p_x, N)], \quad x = 0, \dots, N, \quad n = 0, \dots, N,$$

$$\mathbf{F} = [f(x, y)], \quad x = 0, \dots, N, \quad y = 0, \dots, M,$$

$$\mathbf{K}_y = [\bar{K}_m(y; p_y, M)], \quad y = 0, \dots, M, \quad m = 0, \dots, M,$$

and  $\mathbf{K}_{x}^{T}$  denotes the transpose of  $\mathbf{K}_{x}$ . Since the matrices  $\mathbf{K}_{x}$  and  $\mathbf{K}_{y}$  are orthogonal, the image f can be reconstructed – from matrix representation of (13) – by

$$\mathbf{F} = \mathbf{K}_x \mathbf{Q} \mathbf{K}_y^T. \tag{16}$$

All the computational work in this paper is implemented in MATLAB thanks to its characteristic of easy manipulation of matrices and fast matrix operations. Fig. 1 presents some reconstructions of the cameraman image using  $(p_x, p_y)$  parameters (0.5, 0.5), (0.45, 0.75), and (0.1, 0.1), and  $\hat{N}$ ,  $\hat{M}$  values of 5, 20, 50, and 299. As can be seen from the last

column of Fig. 1 (i.e.  $\hat{N}=\hat{M}=299$ ), the image can be recovered fully, if all the moments are used for reconstruction. If lower number of moments is used as in (14), then the reconstruction quality decreases as we observe from right to left in Fig. 1. However, using smaller numbers for  $\hat{N}$  and  $\hat{M}$ , the reconstructed images contain only local information which may be useful for local recognition of images and comparison. Note that the parameters  $p_x$  and  $p_y$  play a vital role here to determine the center of local region-of-interest. It is clear that the center of a local region corresponding to  $(p_x,p_y)$  is at  $(x_c,y_c)$  where  $x_c=Np_x$  and  $y_c=Mp_y$ . In Fig. 1, these centers are at (150,150), (135,225), and (30,30), respectively for  $(p_x,p_y)$  pairs of (0.5,0.5), (0.45,0.75), and (0.1,0.1).

Extension of Krawtchouk polynomials and their moments to three-dimensional space can also be performed in a similar fashion to what is described in this section. For details of such an extension to 3D moments and their potential applications, interested readers are referred to [20], [21], [22].

### IV. LOCAL MOMENT INVARIANTS

The 2D weighted Krawtchouk moments defined in (12) are not only orthogonal, but also capable of capturing local features from a region-of-interest in an image. This property has been realized in [7], but not extensively analyzed so far. Two studies in the literature [7], [23] pioneer the invariant extraction from Krawtchouk moments. In the former one, Yap *et al.* [7] have introduced a set of Krawtchouk moment invariants, which are restricted only to global image comparison. The latter focuses on obtaining rotationally invariant descriptors from radially defined Krawtchouk moments, but they do not possess the above-mentioned local feature, either.

Unlike the previous work, in this paper, we construct a new set of invariants based on Krawtchouk moments, which have the ability to capture features from a local interest region. We demonstrate that the weighted Krawtchouk moments can be expressed as a linear combination of the geometric moments, thus they can be easily converted to rotation, size, and position independent invariants, called *local Krawtchouk-based invariants*. We also formulate *local Hu-based invariants* by using the traditional seven Hu invariants [1] and utilizing them on images localized by the weight function given in the definition of Krawtchouk polynomials. To begin with, we first give a brief introduction on geometric moment invariants. From this point on in the paper, we will substitute the parameters N and M used in Sections II and III with N-1 and M-1 respectively, to match the  $N\times M$  pixel points of a 2D image.

# A. Geometric Invariants

We first give a brief background on geometric invariants, which are described in detail in [1], [7]. The geometric moments of an object with an  $N \times M$  density function f(x,y) is defined as

$$M_{nm} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} x^n y^m f(x, y).$$
 (17)

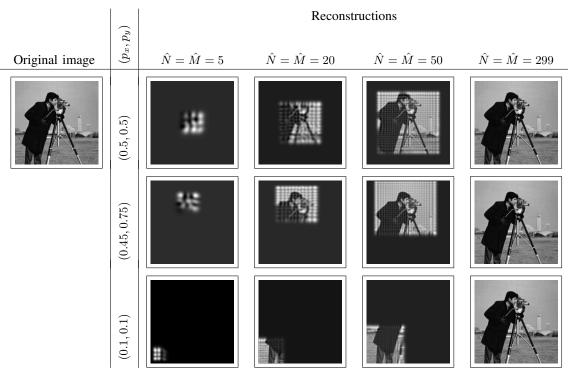


Fig. 1. Cameraman image and its reconstructions using the weighted Krawtchouk polynomials for  $\hat{N} = \hat{M} = 5, 20, 50$ , and 299, and  $(p_x, p_y)$  pairs of (0.5, 0.5), (0.45, 0.75), and (0.1, 0.1). The pixel size for the images is  $300 \times 300$ .

The translation invariant *central moments* are defined as

$$\mu_{nm} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} (x - \bar{x})^n (y - \bar{y})^m f(x, y), \qquad (18)$$

where  $\bar{x}=M_{10}/M_{00}$ , and  $\bar{y}=M_{01}/M_{00}$  are the centroids of the density. If  $\mu_{nm}$  are the central moments, then the moments defined by

$$\eta_{nm} = \frac{\mu_{nm}}{(M_{00})^{\frac{n+m}{2}+1}}$$

are scale invariant as well. The set of geometric moments of f, which is rotation, scale, and translation invariant [7] can be written as

$$\nu_{nm} = (M_{00})^{-\frac{n+m}{2}-1} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x,y)$$

$$\cdot [(x-\bar{x})\cos\theta + (y-\bar{y})\sin\theta]^{n}$$

$$\cdot [-(x-\bar{x})\sin\theta + (y-\bar{y})\cos\theta]^{m}$$
(19)

where

$$\tan 2\theta = \frac{2\mu_{11}}{\mu_{20} - \mu_{02}}. (20)$$

Note that the angle  $\theta$  is limited to the interval  $-\pi/4 \le \theta \le \pi/4$  if computed directly from (20). The exact angle  $\theta$  is obtained in [24] in the range of  $-\pi/2 \le \theta \le \pi/2$ .

### B. Local Krawtchouk-based Invariants

In this section, we introduce a new set of invariants, called *local Krawtchouk-based invariants*. We show that these invariants are not only rotation, size, and position independent, but also contain discriminative local features from any region-of-interest in an image.

Let f(x,y) be a 2D image function defined on  $A=\{0,\ldots,N-1\}\times\{0,\ldots,M-1\}$ , and  $w(x,y;p_x,p_y)$  be the 2D weight function corresponding to  $(p_x,p_y)$  by

$$w(x, y; p_x, p_y) = \sqrt{w(x; p_x, N - 1)w(y; p_y, M - 1)}.$$
 (21)

The weighted Krawtchouk moments of f(x,y) at the  $(p_x,p_y)$  position can be written in term of geometric moments of the weighted function

$$\tilde{f}(x,y) = w(x,y; p_x, p_y) f(x,y) \tag{22}$$

as follows:

$$\bar{Q}_{nm} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} \bar{K}_n(x; p_x) \, \bar{K}_m(y; p_y) \, f(x, y) 
= \left[ \rho(n; p_x, N - 1) \rho(m; p_y, M - 1) \right]^{-\frac{1}{2}} 
\cdot \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} K_n(x; p_x) \, K_m(y; p_y) \, \tilde{f}(x, y) 
= \left[ \rho(n; p_x, N - 1) \rho(m; p_y, M - 1) \right]^{-\frac{1}{2}} 
\cdot \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} \tilde{f}(x, y) \left( \sum_{i=0}^{n} a_{i,n,p_x,N-1} x^i \right) 
\cdot \left( \sum_{j=0}^{m} a_{j,m,p_y,M-1} y^j \right) 
= \left[ \rho(n; p_x, N - 1) \rho(m; p_y, M - 1) \right]^{-\frac{1}{2}} 
\cdot \sum_{i=0}^{n} \sum_{j=0}^{m} a_{i,n,p_x,N-1} a_{j,m,p_y,M-1} \tilde{M}_{ij}, \tag{23}$$

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where

$$\tilde{M}_{ij} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} x^i y^j \tilde{f}(x, y). \tag{24}$$

Note that the geometric moments  $\tilde{M}_{ij}$  and hence the weighted Krawtchouk moments  $\bar{Q}_{nm}$  are not invariant under translation, rotation, and scaling. Similarly to (19), we can define geometric moments of  $\tilde{f}$ , which are invariant under these transformations by

$$\tilde{\nu}_{ij} = (\tilde{M}_{00})^{-\frac{i+j}{2}-1} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} \tilde{f}(x,y) \\ \cdot [(x-\tilde{x})\cos\theta + (y-\tilde{y})\sin\theta]^{i} \\ \cdot [-(x-\tilde{x})\sin\theta + (y-\tilde{y})\cos\theta]^{j}$$
 (25)

where  $\tilde{x} = \tilde{M}_{10}/\tilde{M}_{00}$ ,  $\tilde{y} = \tilde{M}_{01}/\tilde{M}_{00}$ . The angle  $\theta$  can be computed from equations (18) and (20), with the function f replaced by  $\tilde{f}$ , and the centroids  $\bar{x}$  and  $\bar{y}$  replaced by  $\tilde{x}$  and  $\tilde{y}$ , respectively.

Fig. 2 shows density plots of 2D weight functions  $w(x,y;p_x,p_y)$  in (21) for three different  $(p_x,p_y)$  pairs. Note that the coverage of each weight differs as the parameters  $(p_x,p_y)$  changes. This difference is also visible in Fig. 1 as the local coverages in each case are distinct for fixed values of  $\hat{N}$  and  $\hat{M}$ . The coverage is the largest for  $(p_x,p_y)=(0.5,0.5)$  and smaller for other  $(p_x,p_y)$  pairs.

Note that different coverages for fixed  $\hat{N}$  and  $\hat{M}$  result in the loss of translation and scale invariance. One way to overcome this problem is to determine a suitable  $(p_x,p_y)$  pair, say  $(p_x^*,p_y^*)$ , and use the corresponding weight  $w(x,y;p_x^*,p_y^*)$  for every local region-of-interest by shifting the graph of w to that location, in other words, use the weight  $w(x-Np_x^*+Np_x,y-Mp_y^*+Mp_y;p_x^*,p_y^*)$  instead of  $w(x,y;p_x,p_y)$ .

In order to shift the centroid of the weighted image  $\tilde{f}(x,y)$  to  $(Np_x^*, Mp_y^*)$ ,  $\tilde{\nu}_{ij}$  in (25) is modified to

$$\tilde{\lambda}_{ij} = (\tilde{M}_{00})^{-1} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} \tilde{f}(x,y)$$

$$\cdot \{ [ (x - \tilde{x}) \cos \theta + (y - \tilde{y}) \sin \theta ] / (\tilde{M}_{00})^{1/2} + Np_x^* \}^i$$

$$\cdot \{ [ -(x - \tilde{x}) \sin \theta + (y - \tilde{y}) \cos \theta ] / (\tilde{M}_{00})^{1/2} + Mp_y^* \}^j.$$
(26)

Let 
$$A(x,y;\theta)=(x-\tilde{x})\cos\theta+(y-\tilde{y})\sin\theta$$
 and  $B(x,y;\theta)=-(x-\tilde{x})\sin\theta+(y-\tilde{y})\cos\theta$ . Then,  $\tilde{\lambda}_{ij}$  can

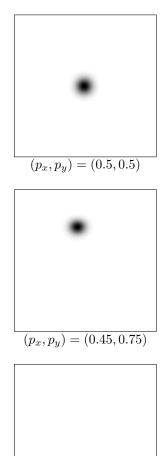


Fig. 2. Density plots of 2D weight functions for different choices of  $p_x$  and  $p_y$ . The image size is  $300 \times 300$  in each case.

 $(p_x, p_y) = (0.1, 0.1)$ 

be written using binomial expansion as:

$$\tilde{\lambda}_{ij} = (\tilde{M}_{00})^{-1} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} \tilde{f}(x,y)$$

$$\cdot \left\{ \sum_{k=0}^{i} \binom{i}{k} \left( A(x,y;\theta) / (\tilde{M}_{00})^{1/2} \right)^{k} (Np_{x}^{*})^{i-k} \right\}$$

$$\cdot \left\{ \sum_{l=0}^{j} \binom{j}{l} \left( B(x,y;\theta) / (\tilde{M}_{00})^{1/2} \right)^{l} (Mp_{y}^{*})^{j-l} \right\}$$

$$= \sum_{k=0}^{i} \sum_{l=0}^{j} \binom{i}{k} \binom{j}{l} (Np_{x}^{*})^{i-k} (Mp_{y}^{*})^{j-l}$$

$$\cdot (\tilde{M}_{00})^{-\frac{k+l}{2}-1} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} \tilde{f}(x,y) (A(x,y;\theta))^{k} (B(x,y;\theta))^{l}$$

$$= \sum_{l=0}^{i} \sum_{k=0}^{j} \binom{i}{k} \binom{j}{l} (Np_{x}^{*})^{i-k} (Mp_{y}^{*})^{j-l} \tilde{\nu}_{kl}.$$

Thus,  $\tilde{\lambda}_{ij}$  is a linear combination of invariants  $\tilde{\nu}_{kl}$  in (25) for  $k=0,\ldots,i$  and  $l=0,\ldots,j$ . Therefore, these new geometric moments are rotation, translation, and scale

invariant, and yet centered at the point  $(Np_x^*, Mp_y^*)$ . If we set  $(p_x, p_y) = (p_x^*, p_y^*)$  in (23) and replace  $\tilde{M}_{ij}$  by their invariant counterparts  $\tilde{\lambda}_{ij}$  from (26), we get a new set of moments which are invariant under rotation, translation, and scaling, i.e.

$$\tilde{Q}_{nm} = [\rho(n; p_x^*, N-1)\rho(m; p_y^*, M-1)]^{-\frac{1}{2}} \cdot \sum_{i=0}^{n} \sum_{j=0}^{m} a_{i,n,p_x^*,N-1} a_{j,m,p_y^*,M-1} \tilde{\lambda}_{ij}.$$
(27)

We call this new set of moments as *local Krawtchouk-based* invariants.

#### C. Local Hu-based Invariants

Note that we can also represent the local weighted function  $\tilde{f}$  by using Hu invariants. These invariants are first defined in [1] as a set of global descriptors which are rotation, scale, and translation invariant, but they can easily be adapted to local invariants, using the weight of Krawtchouk polynomials, in the following sense. Let

$$\tilde{\mu}_{nm} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} (x - \tilde{x})^n (y - \tilde{y})^m \tilde{f}(x, y)$$
 (28)

be the central moments of  $\tilde{f}$ , where  $\tilde{x}=\tilde{M}_{10}/\tilde{M}_{00}$ ,  $\tilde{y}=\tilde{M}_{01}/\tilde{M}_{00}$  are the centroids of  $\tilde{f}$ , and  $\tilde{M}_{nm}$  are the geometric moments of  $\tilde{f}$  defined in (24).

If we let

$$\tilde{\eta}_{nm} = \frac{\tilde{\mu}_{nm}}{(\tilde{M}_{00})^{\frac{n+m}{2}+1}},$$

then we can define Hu invariants of  $\tilde{f}$ , or the *local Hu-based* invariants of f as follows:

$$\tilde{I}_{1} = \tilde{\eta}_{20} + \tilde{\eta}_{02}, 
\tilde{I}_{2} = (\tilde{\eta}_{20} - \tilde{\eta}_{02})^{2} + 4\tilde{\eta}_{11}^{2}, 
\tilde{I}_{3} = (\tilde{\eta}_{30} - 3\tilde{\eta}_{12})^{2} + (3\tilde{\eta}_{21} - \tilde{\eta}_{03})^{2}, 
\tilde{I}_{4} = (\tilde{\eta}_{30} + \tilde{\eta}_{12})^{2} + (\tilde{\eta}_{21} + \tilde{\eta}_{03})^{2}, 
\tilde{I}_{5} = (\tilde{\eta}_{30} - 3\tilde{\eta}_{12})(\tilde{\eta}_{30} + \tilde{\eta}_{12}) 
\cdot [(\tilde{\eta}_{30} + \tilde{\eta}_{12})^{2} - 3(\tilde{\eta}_{21} + \tilde{\eta}_{03})^{2}] 
+ (3\tilde{\eta}_{21} - \tilde{\eta}_{03})(\tilde{\eta}_{21} + \tilde{\eta}_{03}) 
\cdot [3(\tilde{\eta}_{30} + \tilde{\eta}_{12})^{2} - (\tilde{\eta}_{21} + \tilde{\eta}_{03})^{2}], 
\tilde{I}_{6} = (\tilde{\eta}_{20} - \tilde{\eta}_{02})[(\tilde{\eta}_{30} + \tilde{\eta}_{12})^{2} - (\tilde{\eta}_{21} + \tilde{\eta}_{03})^{2}] 
+ 4\tilde{\eta}_{11}(\tilde{\eta}_{30} + \tilde{\eta}_{12})(\tilde{\eta}_{21} + \tilde{\eta}_{03}), 
\tilde{I}_{7} = (3\tilde{\eta}_{21} - \tilde{\eta}_{03})(\tilde{\eta}_{30} + \tilde{\eta}_{12}) 
\cdot [(\tilde{\eta}_{30} + \tilde{\eta}_{12})^{2} - 3(\tilde{\eta}_{21} + \tilde{\eta}_{03})^{2}] 
- (\tilde{\eta}_{30} - 3\tilde{\eta}_{12})(\tilde{\eta}_{21} + \tilde{\eta}_{03}) 
\cdot [3(\tilde{\eta}_{30} + \tilde{\eta}_{12})^{2} - (\tilde{\eta}_{21} + \tilde{\eta}_{03})^{2}].$$
(29)

Here,  $\tilde{I}_7$  is the skew invariant which can distinguish mirror images.

#### V. RESULTS AND DISCUSSION

In this section, we present experimental results and evaluate the discriminative power of the two proposed methods, local Krawtchouk-based and Hu-based moment invariants. Note that an analogous comparison was performed in [7]. For Krawtchouk-based invariants, we use the feature vector (similar to the one used in [7])

$$V = [\tilde{Q}_{20}, \tilde{Q}_{02}, \tilde{Q}_{12}, \tilde{Q}_{21}, \tilde{Q}_{30}, \tilde{Q}_{03}], \tag{30}$$

and the vector

$$I = [\tilde{I}_1, \tilde{I}_2, \tilde{I}_3, \tilde{I}_4, \tilde{I}_5, \tilde{I}_6, \tilde{I}_7]$$
(31)

for Hu-based invariants. We use the Euclidean distance for local image comparison, namely

$$d(\mathbf{z}^{q}, \mathbf{z}^{t}) = \sum_{i=1}^{T} (z_{i}^{q} - z_{i}^{t})^{2}$$
(32)

where  $\mathbf{z}^q$  and  $\mathbf{z}^t$  are the feature vectors for the query and target images, respectively, and T is the dimension of the feature vector.

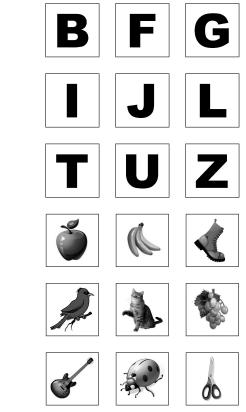


Fig. 3. Nine small binary images of uppercase letters (first three rows) used to generate the first dataset, and nine small gray-scale clip art images (last three rows) used to generate the second dataset.

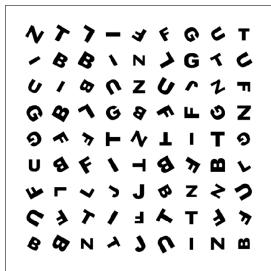
Two datasets are used for performance evaluation. The first set consists of nine capital letters  $\{B, F, G, I, J, L, T, U, Z\}$  as shown in Fig. 3, with different scales and orientations. Each binary image is of size  $64 \times 64$ . These small images are rotated by the angles

$$\phi = 0^{\circ}, 30^{\circ}, 60^{\circ}, 90^{\circ}, 120^{\circ}, 150^{\circ}, 180^{\circ}, 210^{\circ}, 240^{\circ}, 270^{\circ}, 300^{\circ}, 330^{\circ},$$
(33)

and scaled by the factors

$$s = 0.8, 0.9, 1.0,$$
 (34)

to obtain a set of  $9\times12\times3=324$  target images. These images are randomly placed in all  $(p_x,p_y)$  positions to construct larger pictures of size  $640\times640$ , where  $p_x,p_y=0.1,\ldots,0.9$ . In our experiment, four such large pictures are constructed, each with  $9\times9=81$  subregions to ensure that all of the 324 target subimages are used. See Fig. 4 for a sample picture.



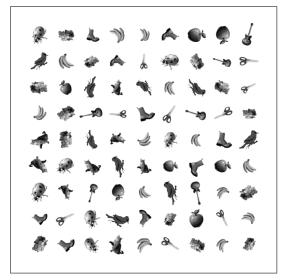


Fig. 4. An example picture from the first test (top) consisting of binary target subimages of nine capital letters, and an example picture from the second test (bottom) consisting of nine gray-scale target subimages.

To construct the second dataset, nine clip art icons are downloaded from Microsoft Office Online. They are then transformed to  $64\times64$  gray-scale images as shown in Fig. 3. Similarly to the letters set, we rotate these images by the angles  $\phi$  in (33) and scaled them by the factors in (34) to expand the set to  $9\times12\times3=324$  target images. These images are then randomly positioned in a bigger picture of size  $640\times640$  as explained above. In this experiment, again 4 such pictures are generated and one of these is shown in Fig. 4.

In order to assess the performance of the two invariant sets, a large query set is formed by using the 324 subimages and placing them in the aforementioned 81 different positions. Note that for each subimage, 12 (angles)  $\times$  3 (scales)  $\times$  81 (positions) = 2,916 different inputs can be obtained so that we have a total number of 9 (images)  $\times$  2,916 = 26,244 queries. Krawtchouk-based and Hu-based invariants of all queries are computed, and then compared to those of the subregions in target sets. The Euclidean distance in (32) is used for quantifying the similarity between a query and a subimage. For each query, subimages from the target set are scanned and the one with the minimum distance to the query is recorded. If this subimage is the same as the query image, then the query is labeled as "correctly classified." We can then define the recognition accuracy R as

$$R = \frac{\text{Number of "correctly classified" images}}{\text{Total number of query inputs}}.$$
 (35)

The denominator of R in (35) is equal to 2,916 per image, and 26,244 if all queries are used in the test. These queries are compared against the salt-and-pepper noise degraded versions of the target images constructed above, with noise densities 0%, 1%, 2%, 3% and 4%, where 0% means noise-free targets. In each case, the feature vectors V and I given in (30) and (31) are computed. The recognition accuracy of local Krawtchoukbased invariants (K) is then compared to that of local Hubased invariants (H), and the results for the letter and clip art image datasets are summarized in Table I and II, respectively. Throughout all these experiments, to compute the weighted function  $\tilde{f}$  in (22), the translated weight function  $w(x-Np_x^*+Np_x,y-Mp_y^*+Mp_y;p_x^*,p_y^*)$  with  $p_x^*=p_y^*=0.1$  is used, as this  $(p_x^*,p_y^*)$  pair yields an appropriate patch size to cover a single target subimage each time.

It is evident from Table I that local Krawtchouk-based invariants show better performance in recognition than local Hu-based invariants for all cases presented. The recognition accuracy of Krawtchouk-based invariants is 100% for all the noisy and noise-free cases, except for the letter J with 4% noise. In this case, only 215 (out of 2,916) J queries are not correctly classified, so the accuracy becomes 92.6%. Among these 215 queries which fail to have the letter J at the first place, the correct match still ranks second for 57, third for 32, and fourth for 70 cases. For 36 of these queries, the correct subimage ranks eighth, whereas for the remaining 20, it ranks thirteenth. The performance of the local Hu-based invariants is still noteworthy with accuracy of 100% in 7 of the 9 letters for noise-free and 1% noisy cases, and 6 out of 9 letters for other noisy cases. The inaccuracies happen in two cases, J and U, even with noise-free targets. The performances for these letters are 94.1% and 96.4%, respectively. In these cases, for more than 98% of the queries, the correct targets have ranked within top 5% of all target subimages.

In Table II presented are the accuracy results for gray-scale clip art images. Local Krawtchouk-based invariants perform better in noise-free condition with 100% for all images. However, their discriminative power decreases as the amount of noise incerases. For noise up to 2%, Krawtchouk-based invariants have better accuracy but for 3% or more noise,

TABLE I
RECOGNITION ACCURACIES (%) FROM THE FIRST TEST ON BINARY IMAGES OF
CAPITAL LETTERS

	Noise-free		1% noise		2% noise		3% noise		4% noise	
	K	Н	K	Н	K	Н	K	Н	K	Н
В	100	100	100	100	100	100	100	100	100	100
F	100	100	100	100	100	100	100	100	100	100
G	100	100	100	100	100	100	100	100	100	100
I	100	100	100	100	100	100	100	100	100	100
J	100	94.1	100	89.6	100	84.3	100	85.6	92.6	87.5
L	100	100	100	100	100	99.5	100	97.7	100	97.1
T	100	100	100	100	100	100	100	100	100	100
U	100	96.4	100	97.5	100	98.8	100	96.4	100	95.3
Z	100	100	100	100	100	100	100	100	100	100
Average	100	98.9	100	98.6	100	98.1	100	97.7	99.2	97.8

K: Local Krawtchouk-based invariants, H: Local Hu-based invariants

TABLE II
RECOGNITION ACCURACIES (%) FROM THE SECOND TEST ON GRAY-SCALE CLIP ART
IMAGES

	Noise-free		1% noise		2% noise		3% noise		4% noise	
	K	Н	K	Н	K	Н	K	Н	K	Н
apple	100	100	100	100	100	100	100	100	100	99.3
bananas	100	100	100	100	100	100	100	100	100	100
boot	100	100	100	100	100	100	100	100	98.1	100
cardinal	100	100	100	96.5	100	91.5	100	87.9	100	78
cat	100	100	100	100	100	97.4	98.9	99.5	69.8	98.9
grapes	100	82.6	100	78.6	88	89.8	59.9	78.5	44.9	86.5
guitar	100	100	100	99.3	100	100	100	100	100	100
ladybug	100	96.8	100	100	100	100	97.4	96.3	51.8	91.9
scissors	100	100	98.9	100	91.4	100	83.9	100	66.6	100
Average	100	97.7	99.9	97.2	97.7	97.6	93.3	95.8	81.2	95

K: Local Krawtchouk-based invariants, H: Local Hu-based invariants

Hu-based invariants become more robust and their average accuracy is still above 95%, whereas that of Krawtchouk-based invariants drops down to 81.2%. Although local Krawtchouk-based invariants performed better in less noisy or noise-free conditions, local Hu-based invariants show that they are more reliable than local Krawtchouk-based invariants in more noisy conditions.

Some example matches from the first and second test can be found in Fig. 5 and 6, respectively. The first column of both figures are query subimages and the next ones are top five matches among the target subimages with 4% noise. In the first and last three rows of each figure shown are the results corresponding to local Krawtchouk-based (K) and local Hubased (H) invariants, respectively. For each set of invariants, we present three examples; one with all top 5 five results match the query; one with the top one matching the query but some of the rest do not; and one example of an incorrect classification with some matches still within top 5. The confusions happen when the query and target have roughly similar shapes or common parts, such as the couples L-I, J-L, J-U, grapes-ladybug, guitar-scissors, and cat-cardinal.

In Table III, we further analyze confusions happening in

TABLE III DISTRIBUTION OF INCORRECTLY CLASSIFIED QUERIES FROM THE FIRST DATASET  $^{\rm I}$ 

	Queries								
		J	I		U				
	K H		K	Н	K	Н			
F	_	_	_	_	_	16			
I	183	_	_	_	_	_			
J	_	_	_	52	_	122			
L	32	_	_	_	_	_			
Т	_	_		_	_	_			
U	_	364	_	32	_	_			
Total	215	364	_	84	_	138			

Binary images of letters with the target subimages under 4% noise.

the classification of letter queries under 4% noisy condition. For Krawtchouk-based invariants, the confusions happen only for the pairs J-I and J-L. Using Hu-based invariants, we see confusions for the pair J-U when either letter in the pair is a query. For gray scale clip art images, the distribution

	Queries													
	apple		boot		cardinal		cat		grapes		ladybug		scissors	
	K	Н	K	Н	K	Н	K	Н	K	Н	K	Н	K	Н
apple	_	_	_	_	_	_	_	_	_	32	_	_	_	_
cardinal	_	_	_	_	_	_	881	32	_	_	1,406	_	_	_
cat	_	_	56	_	_	642	_	_	265	_	_	_	_	_
grapes	_	20	_	_	_	_	_	_	_	_	_	236	_	_
guitar	_	_	_	_	_	_	_	_	_	_	_	_	973	_
ladybug	_	_	_	_	_	_	_	_	1,341	362	_	_	_	_
Total	_	20	56	_	_	642	881	32	1,606	394	1,406	236	973	_

TABLE IV DISTRIBUTION OF INCORRECTLY CLASSIFIED QUERIES FROM THE SECOND DATASET  $^{\rm I}$ 

<sup>&</sup>lt;sup>1</sup> Clip art images with the target subimages under 4% noise.

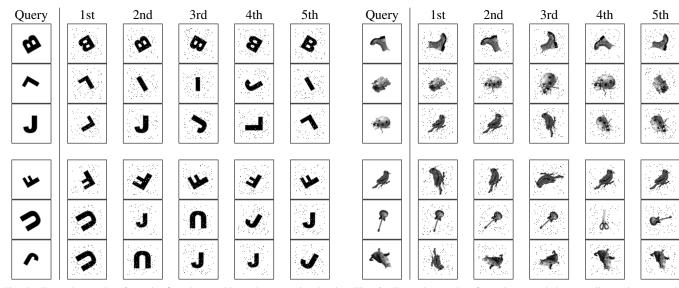


Fig. 5. Example matches from the first dataset (binary letters) using local Krawtchouk-based invariants (first three rows), and using local Hu-based invariants (last three rows). For each query subimage, top 5 matches from the dataset with 4% noise are shown.

Fig. 6. Example matches from the second dataset (clip art images) using local Krawtchouk-based invariants (first three rows), and using local Hu-based invariants (last three rows). For each query subimage, top 5 matches from the dataset with 4% noise are shown.

of incorrect classifications are provided in Table IV. The most significant confusions are observed for grapes-ladybug, ladybug-cardinal, and scissors-guitar pairs using Krawtchouk-based invariants, when the former ones in the pairs are queries. The cat-cardinal, grapes-apple, and grapes-ladybug pairs are the ones confused by Hu-based invariants when either of the subimages is inputted as a query.

TABLE V COMPUTATIONAL TIMES (SECONDS)

	Binary letters	Clip art images
Preparation	0.6445	0.7900
Descriptor (K)	0.3023	0.3159
Descriptor (H)	0.0026	0.0026
Search (K)	3.291e-4	3.304e-4
Search (H)	3.320e-4	3.315e-4

Table V shows the performance results from computing local Krawtchouk-based and local Hu-based invariants. The programs were run in Matlab 2009b version 7.9 on a standard

TABLE VI AVERAGE RECOGNITION ACCURACIES (%) FROM THE TWO DATASETS USING DIFFERENT NUMBER OF INVARIANTS

	N	Noise-fre	e	4% noise			
	$V_{1:2}$	$V_{1:4}$	$V_{1:6}$	$V_{1:2}$	$V_{1:4}$	$V_{1:6}$	
Binary letters	99.2	100	100	86.3	98.5	99.2	
Clip art images	96.9	100	100	63.1	79.1	81.2	
	$I_{1:2}$	$I_{1:6}$	$I_{1:7}$	$I_{1:2}$	$I_{1:6}$	$I_{1:7}$	
Binary letters	98.8	95.4	98.9	84	91.7	97.8	
Clip art images	97	98.9	97.7	76.9	93.8	95	

laptop with i3 CPU of 2.53 GHz and 4.00 GB memory. The preparation step includes the computation of the translated weight, the weighted image  $\tilde{f}$ , and the values  $\tilde{M}_{00}$ ,  $\tilde{M}_{01}$ ,  $\tilde{M}_{10}$ ,  $\tilde{x}$ ,  $\tilde{y}$ ,  $\tilde{\mu}_{11}$ ,  $\tilde{\mu}_{20}$ , and  $\tilde{\mu}_{02}$ , which are commonly needed for computing both invariants. The first three rows of Table V show the average CPU times spent on the corresponding tasks, where the average is taken among 324 target subimages. Including the preparation step, local Krawtchouk-based invariants of a typical subimage are computed within a second, whereas the

computation of local Hu-based invariants is 1.5 times faster. The former takes longer mainly due to the computation of geometric moments in (26), and the centroid shift in (27). In practice, given a typical query, the times required to scan 100 target subimages are also shown in Table V. Assuming the descriptors for target subimages were precomputed and stored, the search step can be performed in instant time.

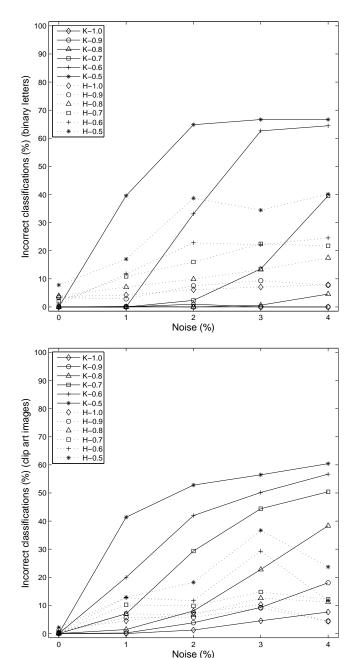


Fig. 7. Effect of noise and scaling on distribution of incorrectly classified queries from the first dataset (binary letters, top) and the second dataset (clip art images, bottom).

In Table VI, we evaluate the effect of varying the number of invariants on average recognition accuracies from the two datasets. For this experiment, we employed  $V_{1:2} = [\tilde{Q}_{20}, \tilde{Q}_{02}], \ V_{1:4} = [\tilde{Q}_{20}, \tilde{Q}_{02}, \tilde{Q}_{12}, \tilde{Q}_{21}], \ \text{and} \ V_{1:6} = V \ \text{of}$  the Krawtchouk-based feature vector V in (30), and  $I_{1:2} = V$ 

 $[\tilde{I}_1, \tilde{I}_2], I_{1:6} = [\tilde{I}_1, \tilde{I}_2, \tilde{I}_3, \tilde{I}_4, \tilde{I}_5, \tilde{I}_6],$  and  $I_{1:7} = I$  of the Hubased feature vector I in (31). In general, the more the number of invariants, the higher the recognition accuracy. In noise-free case, the number of invariants does not have a significant effect on accuracies, and it ranges only between 96.9% and 100% for Krawtchouk-based, and between 95.4% and 98.9% for Hu-based invariants. Under 4% noise, the effect of varying the number of invariants is apparent as the differences between accuracies are now higher than noise free case. In any case, compared to binary letters, clip art image dataset is more sensitive to both noise and change in the number of invariants.

Fig. 7 presents the effects of noise and scaling on the distribution of incorrectly classified queries. The plots at the top and bottom summarize the results for the first letter dataset and the second clip-art image dataset, respectively. To understand the effect of scaling better, we have added three more scales into the scale set, which is now equal to  $s = \{0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$ . Note that the number of all possible queries and the number of target subimages have doubled after this change. The effect of noise in both plots is evident. As expected, the number of incorrect matches generally increases as the noise rate goes up.

In the binary letters case (top plot), local Krawtchouk-based invariants are almost unaffected by the noise for the scales s = 1.0, 0.9, 0.8. For s = 0.7 and 0.6, they are tolerant up to 2% and 1% noise, respectively, whereas for s = 0.5, they are intolerant to noise anymore. Under noise-free condition, Hubased invariants are slightly affected by scaling. For s = 0.5, they are more tolerant than Krawtchouk-based invariants for all the noisy cases. For s = 0.6, they perform better for the noise of 2% or higher, and for s = 0.7, they are more tolerant for 4% noise only. In the clip art images case (bottom plot), scaling has almost no effect on both invariants under noisefree conditions. For the scales s=0.5 and 0.6, Hu-based invariants perform better for all the noisy cases. For the scale s = 0.7, Krawtchouk-based invariants have a slightly better performance than Hu-based invariants under 1% noise, but the latter are much better for higher noise rates. In general, Hubased invariants have tendency to be more tolerant to scaling under high rate noisy cases.

### VI. CONCLUSION

In this paper, we have developed two new sets of local descriptors, *local Krawtchouk-based* and *local Hu-based invariants*, for identification and comparison of local regions in images. The former set is based on 2D Krawtchouk moments. While obtaining these rotation, size, and position independent invariants, we preserve the ability of Krawtchouk moments to extract local features of an image from any region-of-interest. This locality property is due to the weight function given in the definition of Krawtchouk polynomials. The weight contains two parameters  $p_x$  and  $p_y$ , shifting the center of local interest region horizontally and/or vertically. We have noticed that, for each  $(p_x, p_y)$  pair, the coverage of the weight function is different, which prevents Krawtchouk moments from being translation invariant. To overcome this problem, we have fixed a  $(p_x, p_y)$  pair, computed the weight at that location and used it

for other local regions by translating the graph of the weight function as needed. To construct local Hu-based invariants, we have used the global Hu invariants and utilized them on images localized by the above-mentioned standardized weight function. Since we use a fixed  $(p_x,p_y)$  pair in determining the weight, the performance of our descriptors does not depend on the local region of interest. Also, we have utilized only six Krawtchouk-based and seven Hu-based invariants for local comparison. The number of our invariants is very small compared to conventional local descriptors, which makes the computation more efficient.

We have tested the discriminative powers of the two moment invariants on two datasets, one constructed from binary images of capital letters, and the other from gray-scale clip art images. As shown in the test results, local Krawtchouk-based invariants have better recognition power than local Hu-based invariants, when the test is performed on binary letter images. For gray-scale images, Krawtchouk-based invariants are still more powerful under noise-free and less noisy circumstances. When the noise is larger, local Hu-based invariants have better recognition accuracy. As a future direction, it will be interesting to investigate how to compare local images of different sizes with Krawtchouk-based invariants. As shown in Fig. 1, Krawtchouk moments have ability to control patch sizes by changing the order of moments.

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